Your Name: ____________________

Teammates: ____________________

Physics 8, Fall 2023, Worksheet #27.

`http://positron.hep.upenn.edu/p8/files/ws27.pdf`

Upload PDF (smartphone scan or tablet edit) to Canvas shortly after class on Wed, Dec 6, 2023.

Problems marked with (*) must include your own drawing or graph representing the problem and at least one complete sentence describing your reasoning.

Discuss each problem with your teammates, then write up your own solution. Be sure to compare final results with your teammates, as a way to catch mistakes. It can also be very interesting when you and a teammate use different methods to arrive at a result. Do not hesitate to ask for help from other students or from the instructors — but don't just copy down other people's results!

Today's first two problems are intended to illustrate for you how the physics of beams is used in practice. Both problems are adapted from Onouye/Kane chapter 8, and are similar in spirit to beam questions that you will consider if you take Prof Farley's structures course in your senior year.

1. (Counts double.) The formula for the maximum vertical deflection (i.e. the deflection at mid-span) of a simply supported beam under uniform load w (vertical force per unit length) is $\Delta_{\rm max} = (5wL^4)/(384EI)$, where L is the length of the beam, E is Young's modulus, and I is the second moment of area (a.k.a. "area moment of inertia"). Typically the maximum allowable deflection of a beam of length L is $\frac{L}{360}$, to prevent plaster ceilings from cracking under excessive deflection, etc. (a) Using the $L/360$ rule, what is the maximum allowed vertical deflection, $\Delta_{\rm max}$, of a beam of length $L = 4.5$ m? (b) If the beam is designed to carry a load of 100 kg/m, what is w in N/m? (I chose these numbers to correspond to 50 pounds per square foot at a spacing between beams ("joists") of 16 inches.) (c) If the beam is made from southern pine timber having Young's modulus $E = 1.1 \times 10^{10}$ N/m^2, what minimum value of I (second moment of area) is required? (d) If the beam has a rectangular cross-section of width $b = 0.038$ m (1.5 inches), what minimum vertical depth h is required to obtain this value of I? Remember $I = bh^3/12$ for a rectangular cross-section. (e) Would a 2×10 wooden beam (cross section 1.5 in $\times$ 9.5 in) suffice for the required depth you calculated? (Note: in real life, deflection is one of several criteria that a beam must satisfy; other criteria include maximum bending stress, maximum shearing stress, and lateral stability, as discussed in O/K chapter 8.)

2. (Counts double.) In problem 1, we went step-by-step through a beam design whose only criterion was allowable deflection (which is a "stiffness" criterion). Now let's try a beam-design problem in which we evaluate allowable bending stress (which is a "strength" criterion), as in O/K §8.2. Imagine a set of floor joists, of length $L = 4.5$ m, spaced at 0.40 m intervals, designed to support a uniform load of 2400 N/m^2. (I chose these numbers to correspond roughly to 50 pounds per square foot load, 16 inch joist spacing, 15-foot span.) (a) What is the load, in N/m, carried by each floor joist? (Multiply load per unit area by joist spacing to get load per unit length of each joist.) (b) Considering each floor joist to be a simply-supported beam, draw the usual load, shear (V), and bending moment (M) diagrams for one floor joist. (c) From your bending-moment diagram, what is the maximum bending moment that the beam (joist) must resist? For the given loading and support conditions, this maximum should occur at mid-span. The answer should be in newton-meters. (d) Our floor joists will be made of southern pine timber having allowable bending stress $F_b = 10700$ kN/m^2 (that's 1.07×10^7 N/m^2, which is about 1550 psi in US customary units). Given that $S_{\text{required}} = M_{\text{max}}/F_b$, what is the required section modulus S for this floor-joist design? Your answer should be in m^3, but a meter is quite large compared to the transverse dimensions of a floor joist, so you will get a number that is a small fraction of a cubic meter. (e) For a rectangular beam, the second moment of area is $I = bh^3/12$, and the distance from neutral axis to extreme fibers is $c = h/2$. So the section modulus for a rectangular beam is $S = I/c = bh^2/6$. If $b = 0.038$ m (that's 1.5 inches, the width of "2×6," '2×8," "2×10," "2×12" etc. dimensional lumber that you would buy at Home Depot), what minimum value of h is required, to get the necessary minimum section modulus? (f) Convert your answer for part (e) to inches. Would you need a "2×6" ($h = 5.5$ in), a "2×8" ($h = 7.5$ in), a "2×10" ($h = 9.5$ in), or a 2×12 ($h = 11.5$ in) for each floor joist? (g) Remember that for identical conditions in problem 1, the "$L/360$" deflection rule required us to use "2×10" floor joists. Which design criterion (allowable bending stress vs. allowable deflection) turned out to be more stringent in this case? (Quote from my copy of the ARCH 435 notes, quoting Prof. Farley's lecture: "Sizing of a beam will almost always be dependent on the deflection equation; rarely shear or bending.")

3. You measure the oscillation frequency f_{whole} of a vertical block-spring system to be 2.0 Hz. You then cut the spring in half (leaving a spring with half the original length), hang the same block from one of the halves, and measure the frequency f_{half}. What is f_{half} ? (To answer this question, it helps to know that when two springs of spring constant k_1 and k_2 are connected head-to-tail (one in series with the other), the combined spring constant is $k = k_1k_2/(k_1 + k_2)$. So putting two identical springs of spring constant k_1 in series would result in $k = k_1/2$. Therefore, cutting a spring of spring constant k_1 in half (half the length) leaves a spring with spring constant $k = 2k_1$.)

4. A block of mass $m_1 = 2.5$ kg hangs on a vertical spring and oscillates with frequency $f = 1.0$ Hz. With an additional block of mass m_2 added to the spring, the frequency is half as large: 0.5 Hz. What is m_2? (Be careful to answer "What is m_2," not "What is $m_1 + m_2$.")

XC5*. Block B in the figure below is free to slide (with negligible friction) on the horizontal surface. With block C placed on top of B, the system undergoes simple harmonic motion with an amplitude of 0.10 m. Block B has a speed of 0.24 m/s at a displacement of 0.06 m from equilibrium. (a) Find the period of the motion. (b) What minimum value for the coefficient of static friction μ_s between B and C is needed if C is never to slip?

You may be wondering about the connection with Physics for Architects here. To my mind, this problem scenario (probably with different numerical values, and stated in a less tricky way) is one way you could model the phenomenon of objects sliding off of a bookshelf during an earthquake. Object B would represent the bookshelf, oscillating back and forth, and object C would represent the whatever object is resting on the bookshelf. You can see that a key quantity is the peak acceleration of object B.