

Physics 9, fall 2020, reading due before 2020-11-04

(Whenever you start reading this document, you might want to double-check that there is not a more recent version online. I put the document's update date on the bottom edge of each page. An accompanying asynchronous lecture video (mostly demonstrations this time) is at [or \(panopto\)](http://panopto). This file is http://positron.hep.upenn.edu/p9/files/reading_20201104.pdf

The aim of this PDF is to summarize some key results from Giancoli's chapters 13,14,15 (heat and thermal physics).

Heat and thermal physics.

Heat is energy that is transferred into or out of a system due to a temperature difference between that system and its environment. (Giancoli prefers to refer to two objects rather than to a system and its environment: heat is energy that is transferred from one object to another because of a difference in temperature.) We use the symbol capital Q for heat; its units are joules, the units of energy. (Caution: earlier, we used the symbol Q to denote the rate (cubic meters per second) of fluid flow. And later we will reuse the symbol Q to denote electric charge (measured in coulombs). But in the context of thermal physics, the symbol Q means heat, measured in energy units: joules, calories, BTU, etc.

Remember from mechanics that $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$. As in chemistry, one calorie is 4.18 J. In the US, one dietary Calorie is 4180 J, though elsewhere the energy content of food is often labeled in kilocalories (kcal) to avoid the confusion between a calorie and a Calorie. The historical definition of a calorie is the energy required to raise the temperature of one gram of water by one degree Celsius. In the history of US customary units, a BTU is the energy required to raise the temperature of one pound of water by one degree Fahrenheit: $1 \text{ BTU} = 1055 \text{ J}$. Since US HVAC systems are often rated in BTU/hour, it may help to know that $1 \text{ kW} = 3400 \text{ BTU/hour}$.

Remember also from mechanics that one watt equals one joule per second: $1 \text{ W} = 1 \text{ J/s}$. A person typically consumes about 2000 food Calories per day. So the average power dissipated by one person is

$$\frac{2000 \text{ food Calories}}{\text{day}} \times \frac{4180 \text{ J}}{\text{food Calorie}} \times \frac{1 \text{ day}}{24 \times 60 \times 60 \text{ s}} = 97 \text{ W} \approx 100 \text{ W} .$$

The approximate value 100 watts per person is worth remembering. Nearly all of that 100 W/person is eventually dissipated into thermal energy. So if you start with an empty auditorium and then add 100 people, your HVAC system needs to remove 10 kW of additional thermal power from the auditorium. I notice this in classrooms:

shortly after several dozen students arrive, the air-conditioning turns on. You may have also noticed this at (pre-pandemic) apartment parties: once the living-room gets crowded, someone opens a window, even in winter.

Former US energy secretary Steven Chu was fond of pointing out that since the USA has an overall power usage of roughly 3×10^{12} W (including industry, etc.) and a population of roughly 300 M people, we average about 10 kW per person in the US, or roughly 100 “energy helpers” per person. In other words, total US energy use per unit time (including industry) is about $100 \times$ a typical person’s 100 W metabolic rate.

Thermal energy is the incoherent (disordered) internal energy of an object due to the random motion of its atoms. So heat is the transfer of thermal energy from an object at higher temperature to an object at lower temperature. Thermal energy can be transferred into or out of a system by three mechanisms: conduction, convection, and radiation.

Thermal **radiation** is the emission of electromagnetic waves (infrared, visible, ultraviolet, etc.) by an object as a result of the thermal motion of that object’s atoms. A familiar example is the light emitted by the Sun. Light from the Sun travels to Earth mostly through empty space: unlike conduction and convection, radiation does not require a medium (matter) in which to propagate. The rate at which an object emits thermal radiation is (heat per unit time)

$$\frac{dQ}{dt} = eA\sigma T^4 \quad \text{with } \sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}$$

where A is the object’s surface area, T is the object’s temperature (on the kelvin scale!), and σ is the Stefan-Boltzmann constant. The object’s emissivity, e , with $0 \leq e \leq 1$, is a dimensionless measure of how “black” it is, i.e. how efficiently it emits or absorbs thermal radiation; think of how quickly a silver or white object absorbs the Sun’s rays vs. a black object. Because of the T^4 factor, if you double the (kelvin-scale) temperature of an object you multiply $\times 16$ the rate at which the object radiates thermal energy.

The Sun is 150 million kilometers ($R_{es} = 1.5 \times 10^{11}$ m) from Earth, has a surface temperature $T_{\text{sun}} = 5780$ K, and is 696 thousand kilometers ($R_{\text{sun}} = 6.96 \times 10^8$ m) in radius. Taking $e \approx 1$, the total power output of the Sun, P_{sun} , is

$$\frac{dQ}{dt} = eA\sigma T^4 = \sigma (4\pi R_{\text{sun}}^2) T_{\text{sun}}^4 = 3.85 \times 10^{26} \text{ W}$$

The process that generates that enormous quantity of power is nuclear fusion.

Remember, from our study of acoustics, that intensity is power per unit area. Other things being equal, the Sun’s radiation is more intense when you are very close to

the Sun than when you are very far away from the Sun. Just as we saw for sound waves propagating in open air, the intensity (measured in W/m^2) of the Sun's rays at a distance r from the Sun is $\frac{\text{power}}{4\pi r^2}$. The intensity of the Sun's radiation reaching Earth's upper atmosphere is the so-called **solar constant**:

$$\text{intensity} = \frac{P_{\text{sun}}}{4\pi R_{\text{es}}^2} = \frac{3.85 \times 10^{26} \text{ W}}{(4\pi)(1.5 \times 10^{11} \text{ m})^2} = 1360 \text{ W}/\text{m}^2 .$$

It turns out that not all of this $1360 \text{ W}/\text{m}^2$ is absorbed by Earth. If it were, Earth would look dark when observed from space. About 30% of incoming sunlight is reflected by clouds, water, land, etc. With a clear sky, the solar intensity reaching Earth's surface with the Sun directly overhead is about $1025 \text{ W}/\text{m}^2$ (see "Solar constant" in Wikipedia).



If Earth absorbs 70% of the $1360 \text{ W}/\text{m}^2$ of solar intensity incident at the top of Earth's atmosphere (I ought to better justify the factor 0.70), then the total solar power absorbed by Earth is

$$\frac{dQ_{\text{absorbed}}}{dt} = 0.70 \times 1360 \text{ W}/\text{m}^2 \times 4\pi R_{\text{earth}}^2 = 1.2 \times 10^{17} \text{ W} ,$$

which is an enormous amount of power: it is about $6000\times$ as large as the 20 terawatts ($20 \times 10^{12} \text{ W}$) of power currently consumed by all of human industry, transportation, heating, etc. (about $\frac{1}{6}$ of which is used by the USA). Because Earth's temperature is (very nearly) constant over time, Earth must re-radiate very nearly all of that power back out into space.

$$\frac{dQ_{\text{emitted}}}{dt} = e\sigma AT^4 = \sigma (4\pi R_{\text{earth}}^2)(T_{\text{earth}})^4 = 1.2 \times 10^{17} \text{ W} ,$$

Using emissivity $e = 1$ and solving for T_{earth} yields $T_{\text{earth}} \approx 255 \text{ K}$, which is -18°C (0°F). Brrr! That's what you would get if there were no atmosphere to insulate Earth, like a blanket (or a greenhouse). Averaged over seasons and area, $T \approx 288 \text{ K}$ ($+15^\circ\text{C}$, 60°F) is the actual temperature.

The idea of steady-state temperature is worth pausing to understand. When you warm an object with the Sun's rays, the object absorbs heat, thus increasing the object's internal thermal energy, thus increasing the object's temperature. As the object's temperature increases, the rate at which the object loses thermal energy to its environment (whether by radiation, conduction, or convection) increases. At some point, the rate at which thermal energy is absorbed equals the rate at which thermal energy is lost: we call the temperature at which that happens the steady-state temperature. If you're curious about the distinction between equilibrium temperature and steady-state temperature, you can check out this 15-minute MIT video: <https://youtu.be/NLSKAbefDTA> .

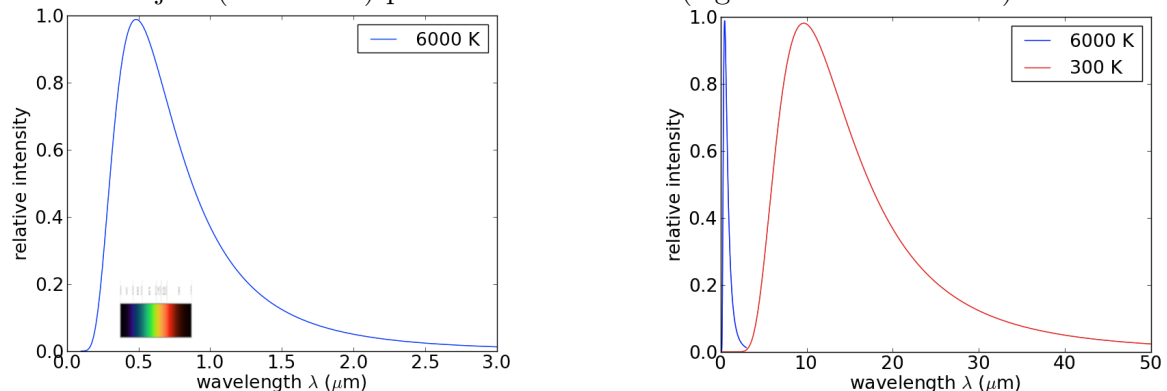
For example, if I leave my black smartphone out in the mid-day sunlight, the phone’s temperature initially rises. As T increases, the rate dQ_{out}/dt of heat transferred out of the phone increases, because radiation $\propto T^4$, while conduction $\propto T - T_{\text{ambient}}$, and convection also increases as $T - T_{\text{ambient}}$ increases. Eventually, T stabilizes at the temperature at which $dQ_{\text{out}}/dt = dQ_{\text{in}}/dt$.

Next, imagine that after my smartphone temperature stabilizes in the mid-day sunlight, I then cover it up with an upside-down transparent glass bowl. Now the temperature further increases, because the Sun’s light can still come in about about the same rate, but it is harder now for the heat to escape: dQ_{out}/dt is lower for a given T because of the insulating layer of glass. Eventually, T stabilizes at the new, higher point where $dQ_{\text{out}}/dt = dQ_{\text{in}}/dt$.

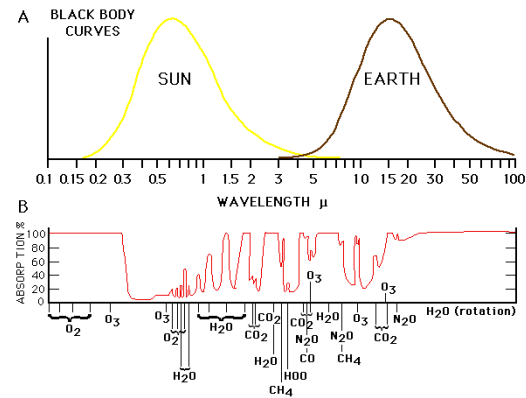
What happens if we surround Earth with an insulating layer that, for a given T_{earth} , cuts dQ_{out}/dt in half. You know that water vapor is one such insulator: it is cooler in the morning after a clear night, and warmer after a cloudy night. The weather report sometimes mentions “radiational cooling” after a clear night. Water vapor acts something like the glass roof of a greenhouse.

It turns out that the more jiggly molecules in Earth’s atmosphere (e.g. H_2O vapor, CO_2 , CH_4 , which have many “degrees of freedom”) are transparent to incoming visible light but absorb and re-emit outgoing infrared radiation. Dense clouds can, of course, block both incoming and outgoing radiation. Here is a three-minute video, made by Prof. Nelson here at Penn Physics, illustrating that even a thin layer of CO_2 gas will absorb and re-emit infrared radiation: <https://youtu.be/0eI9zxZoipA> .

The wavelength distribution (the “spectrum”) for a 6000 K object (like the Sun) and for a 300 K object (like Earth) are compared below. Hotter objects emit shorter-wavelength thermal radiation. The radiation for a 6000 K object (like the Sun) peaks in the visible range of wavelengths (e.g. about half a micron), while the radiation for a 300 K object (like Earth) peaks in the infrared (e.g. about ten microns).



A gas that is relatively transparent for visible light but is relatively opaque for infrared light will allow sunlight to warm Earth's surface but will tend to impede the infrared light whose escape cools Earth's surface: the gas would serve as a sort of blanket, like the glass of a greenhouse. Such gases are thus known as greenhouse gases (GHG). The figure on the right shows the absorption vs. wavelength of Earth's atmosphere.



With a blanket of greenhouse gases, the incoming solar power absorbed by Earth is still

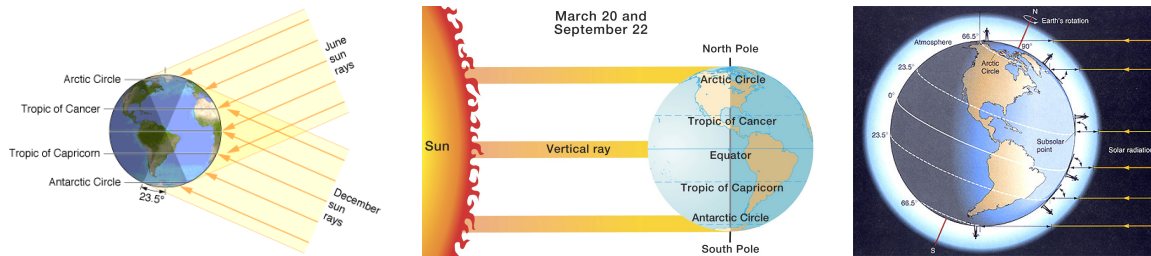
$$\frac{dQ_{\text{absorbed}}}{dt} = 0.70 \times 1360 \text{ W/m}^2 \times 4\pi R_{\text{earth}}^2 = 1.2 \times 10^{17} \text{ W} ,$$

but the thermal energy emitted by Earth (for a given T_{earth}) is reduced. Thus, to keep the incoming and outgoing heat in balance, T_{earth} will increase. As a very rough illustrative example, suppose that Earth's atmosphere absorbs all of the infrared radiation emitted by Earth and re-scatters it half upward and half downward. Then the power radiated by Earth's surface must double, so that the net power leaving the top of Earth's atmosphere equals the power coming in from the Sun. Inserting that very crude factor of two, which would thus increase T_{earth} by a factor $2^{(1/4)}$.

$$\frac{dQ_{\text{emitted}}}{dt} = e\sigma AT_{\text{earth}}^4 = 2 \times 1.2 \times 10^{17} \text{ W} \quad \Rightarrow \quad T_{\text{earth}} = 2^{(1/4)} \times 255 \text{ K} = 303 \text{ K}$$

That's closer to the right answer (which is $15^\circ\text{C} = 288 \text{ K}$), but too hot. If instead of trapping 50%, the atmosphere instead traps 39% of Q_{out} , we get 288 K at Earth's surface. A more effective insulating blanket yields a warmer surface. That's the basic idea, but in practice a large number of details need to be accounted for carefully: the living Earth is much more complicated than the physics-problem idealization of a black spherical rock surrounded by a layer of greenhouse gases.

One more thing about the Sun's rays: Note that the solar radiation absorbed by a given surface is proportional to $\cos \theta$, where θ is the angle between the Sun's rays and the surface normal. In your architectural studio work, you may find this angle-dependence in various solar-modeling software. (Several years ago, it was common to use Autodesk Ecotect.) The $\cos \theta$ factor explains the seasonal effect of latitude on temperature.



Example to ponder: The solar power incident on one square meter of flat, level ground near Philadelphia, at noon on March 21, is about $1000 \text{ W} \times \cos(40^\circ) \approx 770 \text{ W}$. Note that 70% of $1350 \text{ W/m}^2 \approx 1000 \text{ W}$, i.e. the solar intensity reaching Earth's surface is about 1000 W/m^2 . Also note that Philadelphia's latitude is about 40° north of the equator.

Another example to ponder: The solar power incident on one square meter of a vertical, south-facing window near Philadelphia, at noon on March 21, is about $1000 \text{ W} \times \sin(40^\circ) \approx 640 \text{ W}$. See if you can follow how the cosine in the previous example became a sine in this example.

Thermal **conduction** is the transfer of internal energy by microscopic collisions of particles and movement of electrons within a body. (Wikipedia.) The rate (watts) of heat conduction through a block of material whose two ends are held at temperatures T_1 and T_2 is proportional to cross-sectional area A (section perpendicular to heat flow), proportional to the temperature difference, and inversely proportional to the thickness ℓ (parallel to heat flow), with constant of proportionality k , the **thermal conductivity**, in $\frac{\text{W}}{\text{m}^\circ\text{C}}$.

$$\frac{dQ}{dt} = \frac{kA}{\ell}(T_1 - T_2) = \frac{A}{R}(T_1 - T_2)$$

The "R value" (often given in US customary units, unfortunately) lets you quickly compare 15 cm of fiberglass vs. 30 cm of fiberglass vs. 10 cm of styro-foam insulation, etc.

$$\frac{\ell}{k} = \frac{\text{thickness}}{\text{thermal conductivity}}$$

More area (in cross-section), a larger thermal conductivity, or a bigger temperature difference yields faster heat conduction. A thicker insulating layer or a larger 'R-value

TABLE 14-4
Thermal Conductivities

Substance	Thermal Conductivity, k	
	kcal ($\text{s} \cdot \text{m} \cdot \text{C}^\circ$)	J ($\text{s} \cdot \text{m} \cdot \text{C}^\circ$)
Silver	10×10^{-2}	420
Copper	9.2×10^{-2}	380
Aluminum	5.0×10^{-2}	200
Steel	1.1×10^{-2}	40
Ice	5×10^{-4}	2
Glass	2.0×10^{-4}	0.84
Brick	2.0×10^{-4}	0.84
Concrete	2.0×10^{-4}	0.84
Water	1.4×10^{-4}	0.56
Human tissue	0.5×10^{-4}	0.2
Wood	0.3×10^{-4}	0.1
Fiberglass	0.12×10^{-4}	0.048
Cork	0.1×10^{-4}	0.042
Wool	0.1×10^{-4}	0.040
Goose down	0.06×10^{-4}	0.025
Polyurethane	0.06×10^{-4}	0.024
Air	0.055×10^{-4}	0.023

yields slower heat conduction. Giancoli's Table 14-4 lists thermal conductivities.

Example: I keep the inside of my house at 20°C . If the thermal power dQ/dt conducted through the walls of my house is 10 kilowatts when the outdoor temperature is 10°C , what will be the thermal power conducted through the walls of my house when the outdoor temperature is -10°C ? (Hint: this is why the gas company often tells you the number of "degree days" for the winter months. Your heating bill should scale like the inside-outside temperature difference, integrated over time.) Answer: 30 kW. Tripling the inside-outside temperature difference triples the heat conduction.

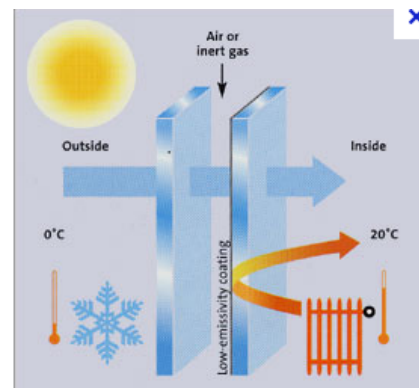
Example: concrete has a thermal conductivity k that is about $8.4\times$ that of wood. How thick a layer of concrete would I need to use in order to provide thermal insulation equivalent to that of a 2 cm layer of wood? Answer: 16.8 cm. This comparison shows the usefulness of the R-value — 2 cm of wood provides the same R-value as 16.8 cm of concrete.

Example: if 10 kilowatts of power (heat per unit time) is conducted through the $10\text{ m} \times 10\text{ m}$ roof of my house on a given winter day, how much power would be conducted through a similar roof (same kind of insulation, same thickness, etc.) that is $20\text{ m} \times 20\text{ m}$ on the same winter day? Answer: 40 kW. Quadrupling the area quadruples the heat conduction.

If you buy fiberglass insulation at a home-improvement store, the R-value is written on the paper backing. Remember $R = \ell/k$. A U.S. customary R value of 1 is (in metric units) $0.176 \frac{\text{m}^2\text{C}}{\text{W}}$. Note that R-values add (along the axis parallel to heat flow): doubling the thickness means doubling the R-value, which means halving the heat conducted per unit time.

TABLE 14-5 R-values

Material	Thickness	R-value ($\text{ft}^2 \cdot \text{h} \cdot \text{F}^{\circ}/\text{Btu}$)
Glass	$\frac{1}{8}$ inch	0.91
Brick	$3\frac{1}{2}$ inches	0.6–1
Plywood	$\frac{1}{2}$ inch	0.6
Fiberglass insulation	4 inches	12



Example of R-values adding: a double-glazed window. Compute the R-value (ℓ/k) for 6 mm of glass. Then for 10 mm of air. Then for a sandwich made of 3 mm of glass,

10 mm of air, and another 3 mm of glass. Use $k_{\text{glass}} = 0.84 \frac{\text{W}}{\text{m}^\circ\text{C}}$ and $k_{\text{air}} = 0.026 \frac{\text{W}}{\text{m}^\circ\text{C}}$. Answer: $0.0071 \frac{\text{m}^2 \cdot ^\circ\text{C}}{\text{W}}$ for 6 mm glass; $0.385 \frac{\text{m}^2 \cdot ^\circ\text{C}}{\text{W}}$ for 10 mm air; $0.392 \frac{\text{m}^2 \cdot ^\circ\text{C}}{\text{W}}$ for 3 mm of glass, 10 mm of air, then 3 mm of glass.

So in theory, two 3 mm panes of glass separated by 10 mm of air will *conduct* about 1/50 as much heat per unit time as a single 6 mm pane of glass. (But we ignored convection and radiation. And in real life, even a single layer of glass will build up an insulating layer of cool air next to it, increasing its effectiveness.)

Nearly every material expands as its temperature increases. (One fascinating exception is liquid water between 0°C and $+4^\circ\text{C}$.) Architectural structures are designed with thermal expansion in mind. For example, highway bridges have “expansion joints” to allow for the change in length of segments of the bridge deck as the temperature varies. Thermal expansion is characterized by a **coefficient of linear expansion**, denoted by the Greek letter alpha (α). If the temperature an object changes from T_1 to T_2 , then the length of that object changes from L_0 to $L = L_0(1 + (T_2 - T_1)\alpha)$. Giancoli’s Table 13-1 lists α values for common materials.

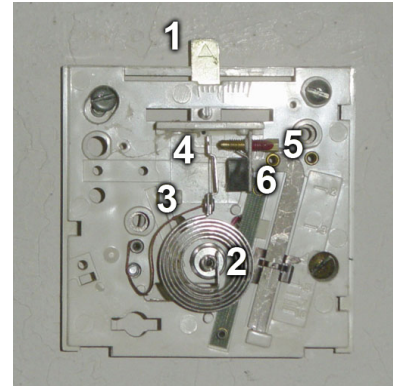
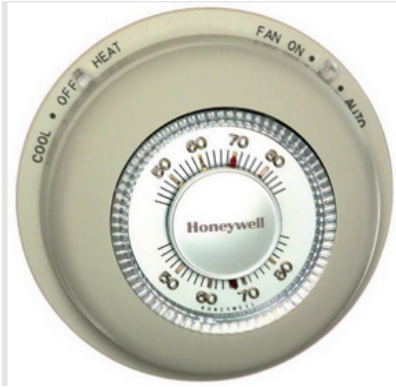


TABLE 13-1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α ($^\circ\text{C}^{-1}$)
<i>Solids</i>	
Aluminum	25×10^{-6}
Brass	19×10^{-6}
Copper	17×10^{-6}
Gold	14×10^{-6}
Iron or steel	12×10^{-6}
Lead	29×10^{-6}
Glass (Pyrex [®])	3×10^{-6}
Glass (ordinary)	9×10^{-6}
Quartz	0.4×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$
Marble	$1.4\text{--}3.5 \times 10^{-6}$

Example: a bridge deck is made of concrete slabs that are 10.0 meters long at 20°C . How wide should the expansion cracks between the slabs be (at 20°C) to prevent buckling if the range of temperatures is -20°C to $+40^\circ\text{C}$? Use $\alpha = 12 \times 10^{-6}/^\circ\text{C}$ for the coefficient of linear expansion of concrete. Answer: 2.4 mm.

Example: a bridge deck is made of concrete slabs that are 10.0 meters long at 20°C . If the expansion cracks between slabs are 2.4 mm wide at 20°C , how wide will they be at -20°C ? Answer: 7.2 mm. (Tricky: the slab shrinks, making the crack wider.)



Do you remember Richard Muller's story about Pyrex vs. ordinary glass? Have you ever seen an old thermostat that looks like this? How do you suppose this works? Related question: suppose I take a long strip of steel and a long strip of aluminum, and I solder them together, like a submarine sandwich, with the steel on top. What happens when I heat it up? (See demo video.)

It turns out that a simple coefficient of linear (or volume) expansion works pretty well for most materials. But these things are usually tabulated as a function of temperature.

$$\Delta L = \alpha L_0 \Delta T \quad \rightarrow \quad \frac{dL}{dT} = \alpha L_0$$

$$\Delta V = \beta V_0 \quad \rightarrow \quad \frac{dV}{dT} = \beta V_0$$

If $V = L^3$ and material is isotropic, then

$$\frac{dV}{dT} = \frac{d}{dT}(L^3) = 3L_0^2(\alpha L_0) = 3\alpha L_0^3 = (3\alpha)V_0$$

so $\beta \approx 3\alpha$ for most materials. Giancoli's Table 13-1 bears this out.

Thermal stress: Remember from Physics 8 that

$$\frac{\Delta L}{L_0} = \frac{1}{E} \frac{F}{A}$$

where E is Young's modulus. If I heat something up so that it tries to expand thermally, then

$$\frac{\Delta L}{L_0} = \alpha \Delta T.$$

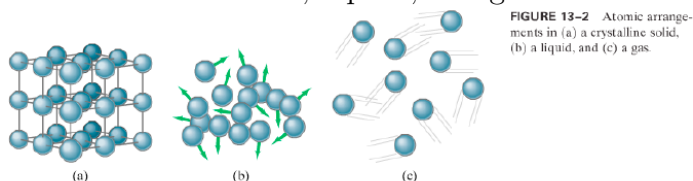
But suppose instead I hold it in place rigidly, so that it is unable to expand. Then the material is effectively being squished by a factor $\alpha \Delta T$. So holding the material at its original length when it heats up induces a stress (force per unit area):

$$\frac{F}{A} = E \alpha \Delta T$$

which the material may or may not be able to tolerate, depending on its compressive strength.

Example: A bridge deck is made of concrete slabs that are 10.0 meters long at 20°C. They are laid end-to-end with no expansion cracks!! So the length of each slab cannot increase. (Oops!) If the temperature rises to 30°C, how large is the compressive stress that develops in the concrete in order to maintain its original length? Use $\alpha = 12 \times 10^{-6}/^\circ\text{C}$ for the coefficient of linear expansion for concrete. Use $E = 20 \times 10^9 \text{ N/m}^2$ for the Young's modulus of concrete. Answer: I get stress $= 2.4 \times 10^6 \text{ N/m}^2 = \alpha E \Delta T$. This is about $\times 8$ below the compressive strength of concrete, so the slabs won't crush, but they may buckle, as a slender column would.

Giancoli illustrates the nature of solids, liquids, and gases like this:



When you read the periodic table of the elements, you will see masses of atoms (sometimes called “atomic weight”) expressed as a multiple of the **atomic mass unit** (u), which is defined as

$$1 \text{ u} = 1 \frac{\text{gram}}{\text{mole}} = 1.66 \times 10^{-27} \text{ kg}$$

This is very close (within 1%) to the mass of a proton: $m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg}$.

A dozen eggs is 12 eggs. A mole of protons is $N_A = 6.022 \times 10^{23}$ protons. A ^{12}C nucleus contains 6 protons + 6 neutrons. So a carbon atom has a mass of 12 u, or 12 gram/mol. A mole of protons has a mass (within 1%) of 1.0 gram. A mole of ^{12}C atoms has a mass (by definition of mole) of exactly 12 grams.

Example: An oxygen nucleus contains 8 protons + 8 neutrons = 16 nucleons. A hydrogen nucleus contains just 1 proton (and no neutrons). What is the mass of a mole of water (H_2O) molecules? Answer: 0.018 kg.

You might have learned the **ideal gas law** in high-school chemistry: $PV = nRT$, with $R = 8.315 \frac{\text{J}}{\text{mol}\cdot\text{K}}$ if you express pressure in N/m^2 and volume in m^3 , or else $R = 0.0821 \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$ if you express pressure in atmospheres and volume in liters. P is pressure, V is volume, n is number of moles of gas, T is temperature (in kelvin), and R is a constant of nature.

Note that a temperature difference of 1 K is the same as a temperature difference of

1°C. To get temperature in kelvin, take the temperature in Celsius and add 273.15.

Also note that a temperature difference of 1°C is $1.8\times$ as large as a temperature difference of 1°F.

Example: What is the final pressure P_f (in terms of the initial pressure P_i) of a sealed bottle of air after you raise its temperature from 27°C to 57°C? Answer: $P_f = 1.1P_i$.

Example: A cylinder initially contains one liter of air at atmospheric pressure. I then compress the gas *isothermally* (i.e. at constant temperature) to half its initial volume. What is the final pressure of the gas (in atmospheres)? Answer: $P_f = 2.00$ atm.

Example: What is the mass of one mole of nitrogen (N_2) molecules? (Each nitrogen **atom** contains 7 protons and 7 neutrons.) Answer: 28 grams. What is the volume of one mole of N_2 molecules, at room temperature (22°C)? Answer: $V = \frac{nRT}{P} = (.0821)(295) = 24.22$ L. What is the density of N_2 gas at $T = 22^\circ\text{C}$? Answer: $m/V = 1.16$ g/L = 1.16 kg/m³.

Example: Now do the same thing for oxygen (O_2) molecules. Each oxygen **atom** contains 8 protons and 8 neutrons. Answer: one mole has a mass of 32 grams. The volume is the same as above. So the density is 1.32 kg/m³.

Example: Dry air is 78% N_2 , 21% O_2 , 1% Ar, 0.04% CO_2 , ... For simplicity, let's call it 80% N_2 and 20% O_2 . What is the density of dry air at room temperature? Answer (using the above two results): $(0.8)(1.16) + (0.2)(1.32) = 1.19$ kg/m³.

The familiar ideal gas law from chemistry is $PV = nRT$ with $R = 8.315 \frac{\text{J}}{\text{mol}\cdot\text{K}}$, where n is expressed in moles. The analogous ideal gas law in terms of number of molecules (N) rather than number of moles (n) is $PV = Nk_B T$, where k_B is **Boltzmann's constant**: $k_B = 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$. Notice that when you divide $8.315 \frac{\text{J}}{\text{mol}\cdot\text{K}}$ by $6.022 \times 10^{23}/\text{mol}$ (Avogadro's number), you get Boltzmann's constant! The familiar ideal gas constant R is given by $R = k_B N_A$.

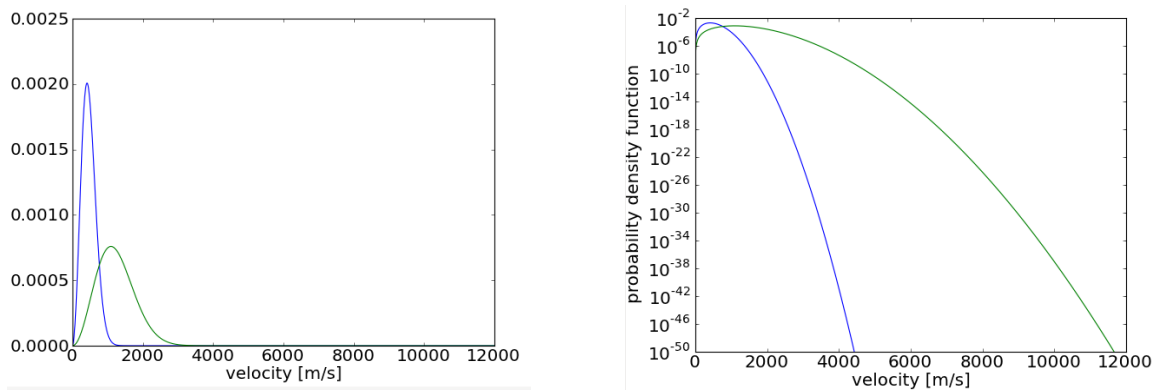
For a molecule of ideal gas at temperature T , the average thermal energy per *degree of freedom* is $\frac{1}{2}k_B T$. A single atom of monotomic gas can move from place to place in three dimensions, but can't rotate or vibrate, so it has 3 degrees of freedom. Thus, $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_B T$, where v_{rms} means the "root mean squared" speed. The average energy of an atom or molecule is directly proportional to temperature. For an ideal gas, $v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$. The lighter molecules tend to move faster!

For helium gas (4g/mol) at room temperature, $v_{\text{rms}} \approx 1360$ m/s. For nitrogen gas

(N₂, 28 g/mol) at 298 K, $v_{\text{rms}} \approx 520$ m/s. For gasoline vapor (C₈H₁₈, 114 g/mol) at 298 K, $v_{\text{rms}} \approx 260$ m/s. Richard Muller pointed out that this is why the helium escaped from Earth's atmosphere: some small fraction of the helium atoms move fast enough (11200 m/s) to escape Earth's gravity.

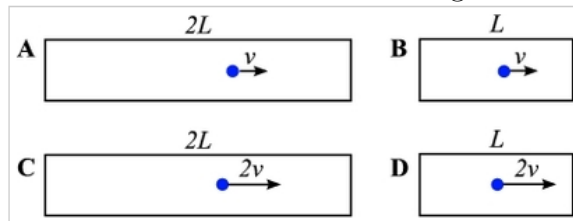
One of the two curves shown below (red or green) is the velocity distribution (at $T = 288$ K) for N₂ (nitrogen) and one is for He (helium). Which is which?

If you make the vertical axis logarithmic, you can see that indeed some tiny fraction of the helium atoms move faster than the 11 km/s “escape velocity” of Earth's gravity.



Example: Nitrogen (N₂) makes up about 78% of the air we breathe, while oxygen (O₂) accounts for approximately 21%. On average, are the nitrogen or the oxygen molecules moving faster? (The mass of an N₂ molecule is 28 u, while the mass of an O₂ molecule is 32 u.) Answer: The rms speed of the N₂ molecules is $\frac{32}{28}$ as large as the rms speed of the O₂ molecules, so the N₂ molecules move slightly faster, on average.

Example: A gas atom of mass m moves in a straight line with speed v or $2v$ in a box with length L or $2L$, as shown below. The atom collides elastically with the right wall, travels to the left wall, collides elastically, returns to the right wall and repeats this process. In which case does the atom exert the largest average force on the walls?



Answer: remember that force is the time rate of change of momentum. The average force is proportional to the change in momentum imparted to an atom when it collides with the wall, and is inversely proportional to the time between successive collisions.

So the average force is largest for the case of the fast-moving atom in a small box (D).

The above example helps us to understand the origins of the ideal gas law. Bigger thermal energy \rightarrow higher rms speed \rightarrow larger force per unit area (pressure). Smaller volume \rightarrow particles collide more often with walls \rightarrow larger force per unit area (pressure). More particles in container \rightarrow collisions with walls occur more often \rightarrow larger force per unit area (pressure). So if you are super ambitious, you can even derive $P = \frac{Nk_B T}{V}$ from $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_B T$ if you consider a single gas molecule moving back and forth in a straight line from end to end of a box.

As you increase the temperature of a substance, its molecules move faster. So its thermal energy increases. This thermal energy is just an “incoherent” form of kinetic energy: you’re adding up a whole bunch of $\frac{1}{2}mv^2$, but the velocities are all pointing in different directions, so the substance as a whole is not moving even though the individual molecules are moving. The internal thermal energy of a monatomic gas (like helium or argon) is $\frac{3}{2}k_B T$ per molecule, or $\frac{3}{2}RT$ per mole.

You sometimes need to know how much energy is needed to heat an object (like a brick wall, or an old metal baseboard radiator, or a swimming pool) by some number of degrees. Tables usually list **energy per unit mass per degree C**. This is known as **specific heat capacity** (or more commonly just “specific heat”). The quantity of heat needed to raise a quantity of mass m from temperature T_i to temperature T_f is $Q = mc(T_f - T_i)$, where c is the specific heat, whose SI units are $\frac{\text{J}}{\text{kg}\cdot^\circ\text{C}}$. Giancoli’s Table 14-1 lists specific heats.

Example: how many joules of heat are needed to raise 10 kg of water from 20°C to 30°C? Answer: $4.19 \times 10^5 \text{ J}$ or 419 kJ. How many watts of power are needed to do this in 1000 s (about 17 minutes)? Answer: 419 watts.

TABLE 14-1 Specific Heats
(at 1 atm constant pressure and 20°C unless otherwise stated)

Substance	Specific Heat, c		Substance	Specific Heat, c	
	kcal/kg·C°	J/kg·C°		kcal/kg·C°	J/kg·C°
Aluminum	0.22	900	Alcohol (ethyl)	0.58	2400
Copper	0.093	390	Mercury	0.033	140
Glass	0.20	840	Water		
Iron or steel	0.11	450	Ice (−5°C)	0.50	2100
Lead	0.031	130	Liquid (15°C)	1.00	4186
Marble	0.21	860	Steam (110°C)	0.48	2010
Silver	0.056	230	Human body (average)	0.83	3470
Wood	0.4	1700	Protein	0.4	1700

Imagine heating water in your tea kettle. Using specific heat, as we did above, tells us the energy needed to go from room temperature to just below the boiling point.

At the boiling point, the internal energy of the gas phase is higher than that of the liquid phase: one needs to overcome the attractive forces that keep molecules close together in a liquid. The energy per unit mass needed to go (at the boiling point) from the liquid phase to the gas phase is called the **latent heat of vaporization**.

Similarly, when you melt an ice cube, the internal energy of the liquid phase is higher than that of the solid phase at the melting point. The energy per unit mass needed to go (at the melting point) from the solid phase to the liquid phase is called the **heat of fusion**.

The point to remember is that even at a fixed temperature, you need to add some energy to turn a solid into a liquid or to turn a liquid into a gas. This added energy (per unit mass) is called the **latent heat**. Latent heat is extremely useful. For instance, your body takes advantage of the latent heat (heat of vaporization) of water when you sweat.

Example: Suppose that by exercising you double your metabolic rate, from about 100 watts to about 200 watts. How many milliliters of water (1 mL = 1 cc $\approx \frac{1}{30}$ ounce) do you need to sweat per minute, in order for your body to remove these extra 100 W by letting the sweat evaporate from your skin? The latent heat of vaporization for water is 2260 kJ/kg = 2260 J/g. The density of water is 1 g/mL. See if you can work out an answer (in milliliters per minute). My answer is:

$$\frac{100 \text{ J/s} \times 60 \text{ s/min}}{1.0 \text{ g/mL} \times 2260 \text{ J/g}} = 2.65 \text{ mL/minute}$$

which is about an ounce every 11 minutes, or a glass of water every hour or two while exercising. That seems plausible.

In an architectural context, one can cool a building by a process analogous to sweating, called *evaporative cooling*, also known as a “swamp cooler,” though this works best in arid climates. On the Penn campus (near 40th and Walnut), an evaporative cooling system produces chilled water, which then circulates around campus and is run through heat-exchangers for air-conditioning. I made this video from the top of a Penn parking garage: [link](#).

We’ve said that “dry air” is 78% N₂, 21% O₂, 1% Ar, 0.04% CO₂. The important constituent of ordinary air that we are missing is, of course, water vapor. The fraction of water vapor in the air varies from region to region (climate) and from day to day (weather). The maximum possible fraction of water vapor in the air (before you get rain, snow, fog, etc.) varies with temperature. To discuss the quantity of one gas (e.g. water vapor) that is dissolved in another gas (e.g. air), it helps to introduce the concept of **partial pressure**, which is the pressure due to that constituent alone.

Example: air at 1 atm is 21% O₂. What is the partial pressure of oxygen (in atmospheres)? Answer: 0.21 atm, which is about 21300 N/m².

At a given temperature, the maximum possible partial pressure of water vapor is called the “(saturated) vapor pressure of water.” When the partial pressure of water vapor equals the saturated vapor pressure, a pool of standing water is in equilibrium with water vapor: the rate of evaporation (liquid to gas) equals rate of condensation (gas to liquid). If the partial pressure of water vapor exceeds the saturated vapor pressure (e.g. if you take saturated air and cool it), precipitation occurs. Below this partial pressure, standing water will evaporate. When the saturated vapor pressure equals the external atmospheric pressure, boiling occurs.

In humid air, the rate at which sweat can evaporate from your skin (hence cooling your body, via latent heat) is lower than in dry air. So a hot, humid day feels more uncomfortable than a hot, dry day. HVAC climate-control systems monitor and regulate **relative humidity**:

$$\text{R.H.} = \frac{\text{partial pressure of water vapor}}{\text{saturated water vapor pressure}} \times 100\%$$

R.H. ~ 40%-50% is most comfortable.

Example: if $T = 30^\circ\text{C}$ and R.H.=50%, what is the partial pressure of water vapor (in N/m²)? In that case, what fraction of molecules in the air are H₂O molecules? Answer: 2120 N/m². So 2.1% of molecules in the air would be H₂O molecules.

TABLE 13–3 Saturated Vapor Pressure of Water

Temperature (°C)	Saturated Vapor Pressure	
	torr (= mm-Hg)	Pa (= N/m ²)
–50	0.030	4.0
–10	1.95	2.60 × 10 ²
0	4.58	6.11 × 10 ²
5	6.54	8.72 × 10 ²
10	9.21	1.23 × 10 ³
15	12.8	1.71 × 10 ³
20	17.5	2.33 × 10 ³
25	23.8	3.17 × 10 ³
30	31.8	4.24 × 10 ³
40	55.3	7.37 × 10 ³
50	92.5	1.23 × 10 ⁴
60	149	1.99 × 10 ⁴
70 [†]	234	3.12 × 10 ⁴
80	355	4.73 × 10 ⁴
90	526	7.01 × 10 ⁴
100 [‡]	760	1.01 × 10 ⁵
120	1489	1.99 × 10 ⁵
150	3570	4.76 × 10 ⁵

[†] Boiling point on summit of Mt. Everest.

[‡] Boiling point at sea level.

We learned in Physics 8 to associate *irreversible* processes (for which a movie played backwards looks impossible) with the dissipation of (coherent) mechanical energy into (incoherent) thermal energy. So you’re not surprised that after a few bounces, the initial mechanical energy of a ball dissipates into thermal energy; but a movie of this process played backwards would look like an impossible situation. Similarly, you’re accustomed to seeing heat flow spontaneously from a warm object to a cooler object. That’s why thermal insulation is needed to slow this process down. So you’re not surprised when your glass of ice-water reaches room temperature after sitting out for an hour; but again a movie of this process played backwards would show something

that looks impossible. Both of these situations involve an **increase in entropy**; and hence are **irreversible**. Let's ponder the meaning of entropy and its connection to probability.

You might have learned in a math class at some point that if I toss N coins, the probability of seeing m of those N coins land heads-up is given by the *binomial distribution*,

$$P(m, N) = \frac{N!}{m!(N-m)!} \left(\frac{1}{2}\right)^N$$

For $N = 2$, the probabilities for $m = 0, 1, 2$ are

$$\frac{1}{4}, \frac{2}{4}, \frac{1}{4}$$

For $N = 4$, the probabilities for $m = 0, 1, 2, 3, 4$ are

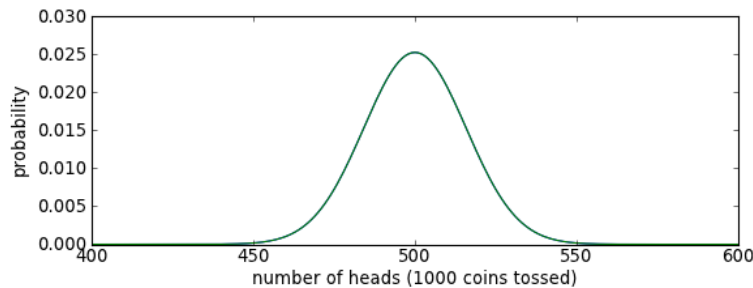
$$\frac{1}{16}, \frac{4}{16}, \frac{6}{16}, \frac{4}{16}, \frac{1}{16}$$

For $N = 8$, the probabilities for $m = 0, 1, 2, 3, 4, 5, 6, 7, 8$ are

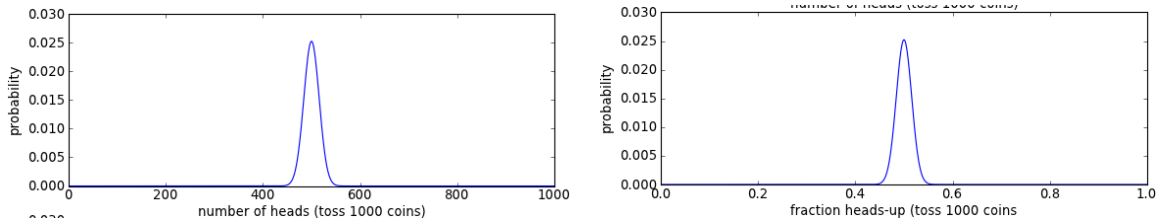
$$\frac{1}{256}, \frac{8}{256}, \frac{28}{256}, \frac{56}{256}, \frac{70}{256}, \frac{56}{256}, \frac{28}{256}, \frac{8}{256}, \frac{1}{256}$$

If I toss $N = 1000$ coins, the probability of finding m heads-up is

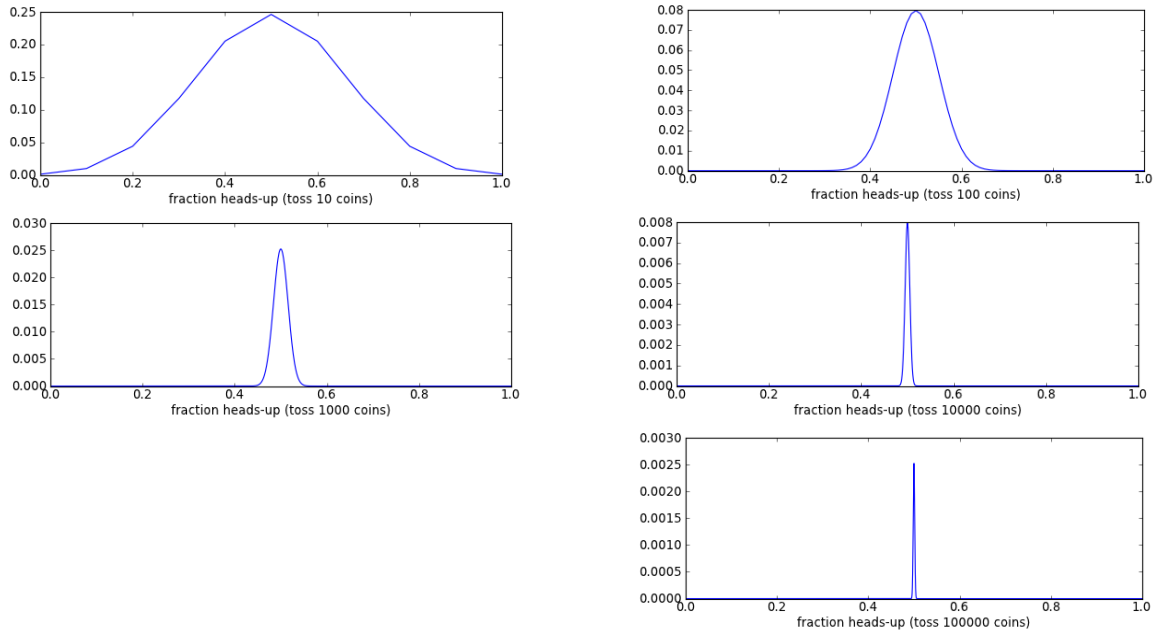
heads	probability	...	heads	probability
0	0.9×10^{-301}		1000	0.9×10^{-301}
1	0.9×10^{-298}		999	0.9×10^{-298}
450	0.02%		550	0.02%
480	1.1%		520	1.1%
499	2.5%		501	2.5%
500	2.5%			



Let's take that same graph (for tossing 1000 coins) and change the x -axis so that it tells us **what fraction** of the coins lands heads-up.



Now let's see how the right-hand graph changes as we vary the number of coins that we toss.



No matter how many coins you toss, the most probable outcome is that half of the coins will land heads-up. But for a small number of coins, it's not so unlikely to see a fraction that is quite different from 50%. As the number of coins increases, it becomes less and less probable that the fraction of heads is very different from 50%.

When you toss N coins, the number of values of m that have some reasonable probability of occurring is proportional to \sqrt{N} . So the number of heads that you expect to see goes roughly like

$$m \pm \delta m = \frac{N}{2} \times \left(1 \pm \frac{1}{\sqrt{N}} \right)$$

You expect to be within about 10% of 50/50 for 100 coins, to be within about 1% of 5000/5000 for 10000 coins, to be within about 0.1% of 500000/500000 for a million coins, etc. If I toss $N = 100$ coins, I am not surprised to find any value $40 \leq m \leq 60$, but I would be very surprised to find $m = 30$.

If I toss 10^{20} coins, the fraction of heads I expect to find is, on average, 0.5. How far

from 0.5 would be surprising?

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{10^{20}}} = 10^{-10}$$

So I would be surprised to find a fraction of heads that is less than about 0.4999999999 or larger than about 0.5000000001

Example: If I poll 10000 architects and find that 50% agree and 50% disagree with the assertion that the spire atop the new One World Trade Center legitimately makes 1 WTC taller than Chicago's Willis (Sears) Tower, what is the "margin of error" of my survey? Answer: roughly $\pm 1\%$, because $1/\sqrt{10000} = 0.01$.

(Note for statistics buffs: I think if you calculate more carefully, the rms fluctuation is more like $\pm \frac{1}{2}\%$, so my rule-of-thumb $\pm 1\%$ roughly corresponds to the 95% confidence interval in this case.)

Just to reiterate: Suppose I repeatedly toss 1000 coins such that **on average** the number of heads-up coins is 500. I don't always get exactly 500 heads. The range of heads-up coins that would be "not surprising" (or not suspicious) is approximately

$$(500 - \sqrt{500}) \leq N_{\text{heads}} \leq (500 + \sqrt{500})$$

So anything in the 475–525 range would be unsurprising. Even 460 or 540 heads would not be too surprising. Finding 400 heads (or 600 heads) would be extremely surprising. In general, when you count things that are randomly sampled, and you expect on average to count N of them, the "unsurprising" range of fluctuations from sample to sample is roughly $\pm\sqrt{N}$.

Example: Suppose I survey 20000 people and find that 9970 of them prefer chocolate ice cream, while 10030 of them prefer vanilla ice cream. If I repeat the survey (with different people, randomly chosen) again and again each week, roughly how much do I expect that 9970 number to vary, just because of the finite size of the number of people surveyed? I don't expect to get exactly 9970 votes for chocolate each week. Instead, I expect the number to vary from survey to survey by about how much? Answer: varies by about ± 100 .

Imagine that you have 10^{23} gas molecules in a box. Then you mark off half of the volume with a ruler and ask "What fraction of the molecules are in the left half of the box?" This is like tossing 10^{23} coins and asking what fraction f is heads-up. You expect f to be almost exactly 0.5. If you do the math, you expect something like

$$0.4999999999997 < f < 0.5000000000003$$

So for all practical purposes, exactly half of the molecules are in each half of the volume. That's called "equipartition of space."

The number of heads m (or equivalently the fraction of heads m/N) could be called the "macro state," or the observable state of the system. If I tossed 10^{20} coins that had one side painted black and one side painted white, I could measure the "macro state" without looking at every coin. Similarly, the observable state ("macro state") of 10^{23} gas molecules is given by P , V , and N . (The "basic state," a.k.a. "micro state," is described by writing down *every particle's* position and velocity.) The number of particles is so huge that the chance that all of the molecules are going to huddle up on one side of the room is beyond infinitesimal. The most probable macrostate is the one with the largest number of basic states. Once a closed system has reached thermal equilibrium (e.g. once it has a well-defined temperature), it will stay in the most probable state or states.

The logarithm of [the number, Ω , of different basic states that contribute to a given macrostate] is called the **entropy** of that macrostate:

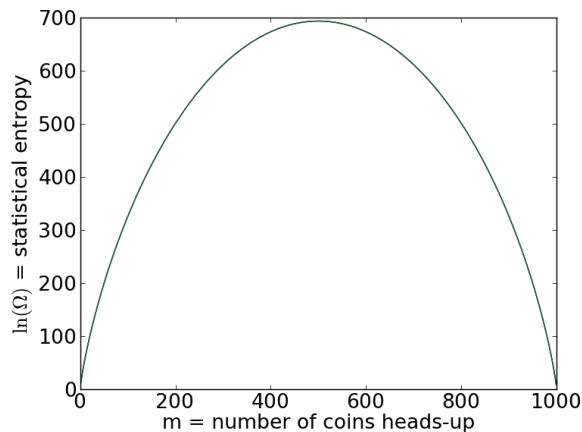
$$S = \ln(\Omega)$$

for the coin toss, the macrostate corresponds to knowing N and m , and Ω corresponds to $\frac{N!}{m!(N-m)!}$. An interesting nerd fact ("Stirling's formula"): as $N \rightarrow \infty$,

$$\ln(N!) \rightarrow N (\ln(N) - 1) \approx N \ln(N)$$

So the "statistical entropy," S , for m coins heads-up is

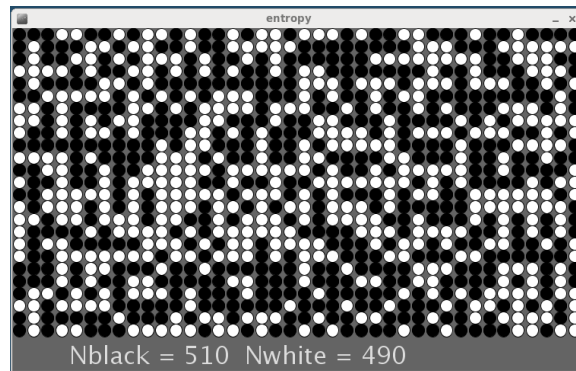
$$S = \ln(\Omega) = \ln\left(\frac{N!}{m!(N-m)!}\right) \approx N \ln N - m \ln m - (N-m) \ln(N-m)$$



Suppose I start out with a jar of 1000 coins that are carefully arranged to be entirely heads-up, and I shake the jar for a long time. What will I find? Shaking will tend

toward more probable (higher entropy) states. After shaking, I will find that the entropy has increased to the largest possible value (plus or minus very small fluctuations). Once you reach “equilibrium,” i.e. the $m_{\text{heads}} \approx 500$ state, you will never spontaneously go back to the $m_{\text{heads}} \approx 0$ state. It’s just too improbable. Once you shake for long enough to get close to 500, you’ll stay pretty close to 500 (roughly within $500 \pm \sqrt{500}$ or so).

Similarly, the **second law of thermodynamics** states that the entropy of a closed system will never decrease with time. (It can stay the same or can increase.) The 2nd law is just a statement that a closed system evolves toward the most probable macrostate. (I should post a video of the animation that I coded that is shown in this screenshot.)



For a system that is not closed (i.e. it can exchange energy with its environment), the entropy law states that the combined entropy of the system + its environment will never decrease with time. (It can stay the same or can increase.) This is equivalent to treating “system + environment” as a (much larger) closed system.

Increases in entropy are associated with irreversible processes, like the dissipation of mechanical energy into heat when I drop a tennis ball and let it bounce until it comes to rest. Reversible processes (i.e. processes for which a movie played either forward or backward looks like physics that is possible) correspond to zero change in entropy.

For a system that is in equilibrium with a thermal reservoir at temperature T , you can relate the system’s change in entropy to the “energy transferred thermally” (a.k.a. heat) into the system:

$$\Delta S = \frac{Q}{k_B T}$$

or in other words,

$$Q = k_B T \Delta S$$

A net heat (Q) flow into a system increases the system’s entropy. (Giancoli and Mazur (the Physics 8 book) have different conventions about the factor of k_B . Mazur uses

“statistical entropy,” which no units; Giancoli (and everyone else, as far as I know) uses “thermodynamic entropy,” which is in J/K (joules per kelvin).)

For an ideal gas, we learned $PV = Nk_B T$. The thermal energy of the gas (if monatomic like He, Ar, etc.) is $E_{\text{thermal}} = \frac{3}{2}Nk_B T = \frac{3}{2}PV$. (Complication that we’ll ignore: the $\frac{3}{2}$ becomes $\frac{d}{2}$ for the non-monatomic case.)

If I compress the gas without letting any heat escape, I will increase its thermal energy. You know from experience (e.g. bicycle pump) that work (W) is required to compress a gas: as I push down on a piston, I exert a force that opposes the gas pressure. So we can increase the energy of a gas by doing mechanical work (W) on the gas, which decreases the gas’s volume:

$$W_{(\text{ON gas})} = - \int P \, dV$$

$W > 0$ (work done ON the gas) increases the gas’s energy and decreases the gas’s volume. The transfer of energy into a system via coherent mechanical movement (via an external force) is called **work**. You also know from experience that you can increase the temperature (and therefore the energy) of the gas by **heating** it, i.e. by putting it in contact with an object of higher temperature. The transfer of incoherent thermal energy into a system (usually because of a temperature difference) is called **heat** (Q). FYI: Don’t confuse “heat” (Q) with “thermal energy” (E_{thermal} , a.k.a. U). Because the word “heat” is used ambiguously, Mazur’s book refers to Q as “energy transferred thermally.”

So we have two ways to add energy to a gas: W and Q .

$$\Delta E_{\text{thermal}} = W + Q$$

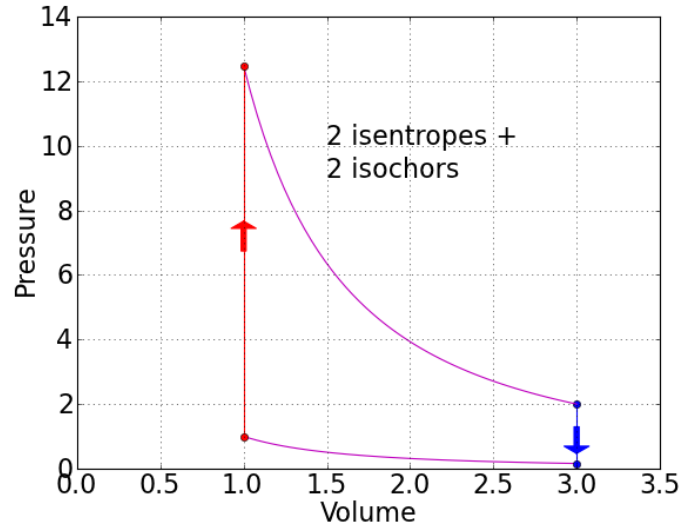
W is due to (coherent) mechanical interactions, like a piston moving up and down or an electric motor pumping on the gas. Q is due to (incoherent) thermal interactions, like the jiggling molecules of an adjacent heat bath (thermal reservoir) causing the molecules of the gas to reach the temperature of the heat bath.

If $Q = 0$ the entropy of the gas does not change. If $Q > 0$ the entropy of the gas increases. If $Q < 0$ the entropy of the gas decreases. For heat transfer Q into the gas at constant temperature T ,

$$\Delta S_{\text{gas}} = \frac{Q}{k_B T}$$

$Q < 0$ is only possible if there is some compensating change in entropy somewhere else, such that the overall $\Delta S \geq 0$.

Heating ($Q > 0$) or cooling ($Q < 0$) the gas while $W = 0$ looks like $\Delta V = 0$ (isochore, “constant volume”). Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) while $Q = 0$ looks like $P_f V_f^{(5/3)} = P_i V_i^{(5/3)}$ (isentropes, “constant entropy”). Giancoli says “adiabatic” where Mazur says “isentropic.”



In the **unlikely event** that you want to know where the $\frac{5}{3}$ in $PV^{(5/3)}$ comes from, it’s because (for a monatomic gas, $d = 3$)

$$0 = \Delta S = N \ln \left(\frac{V_f}{V_i} \right) + \frac{3}{2} N \ln \left(\frac{T_f}{T_i} \right)$$

then using the ideal gas law, $PV \propto T$,

$$0 = \Delta S = N \ln \left(\frac{V_f}{V_i} \right) + \frac{3}{2} N \ln \left(\frac{P_f V_f}{P_i V_i} \right)$$

$$0 = \Delta S = \frac{5}{2} N \ln \left(\frac{V_f}{V_i} \right) + \frac{3}{2} N \ln \left(\frac{P_f}{P_i} \right)$$

$$1 = \left(\frac{V_f}{V_i} \right)^5 \times \left(\frac{P_f}{P_i} \right)^3$$

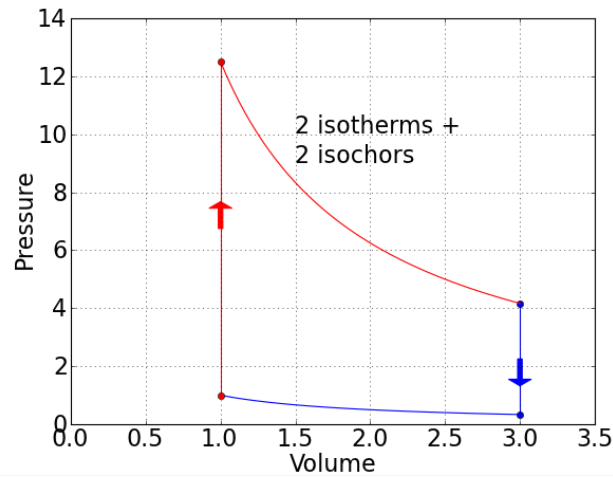
so finally (for an isentropic process on a monatomic gas)

$$P_i V_i^{(5/3)} = P_f V_f^{(5/3)}$$

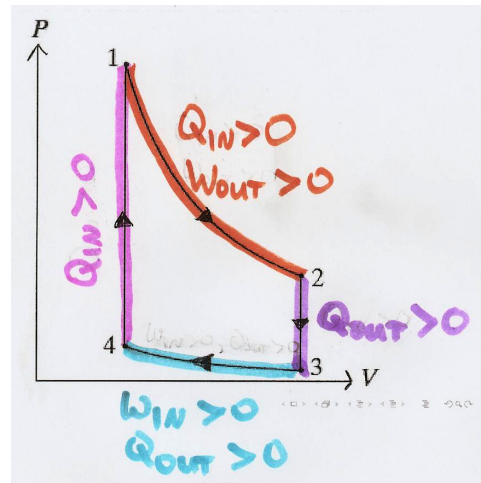
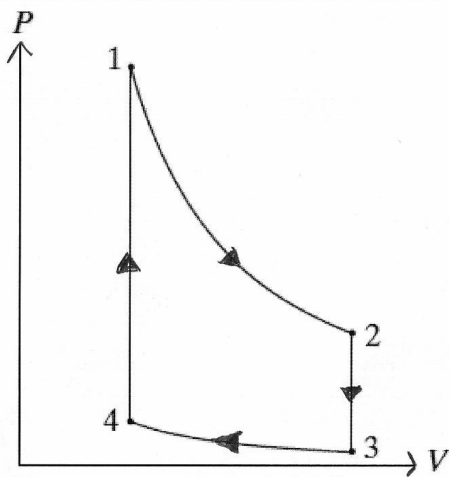
The starting point was Mazur eq 19.61. But you don’t need to know this!

Heating ($Q > 0$) or cooling ($Q < 0$) the gas while $W = 0$ looks like $\Delta V = 0$ (isochore). Doing work on the gas ($W > 0$) or letting the gas do work ($W < 0$) at

constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). (For an engine, $W > 0$ happens at low T .)



Example: The cycle shown below consists of two isotherms ($\Delta T = 0$) and two isochores ($\Delta V = 0$). The cycle is $4 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow \dots$. On which parts of the cycle is positive work done ON the gas ($W_{in} > 0$)? Answer: $3 \rightarrow 4$ only.



Example: For the same cycle shown above, on which parts of the cycle is positive work done BY the gas ($W_{out} > 0$)? Answer: $1 \rightarrow 2$ only.

Example: For the same cycle shown above, over one complete cycle, is W_{out} larger than, smaller than, or equal to W_{in} ? Answer: $W_{out} > W_{in}$ (which is what we want for a **heat engine**).

Example: For the same cycle shown above, on which parts of the cycle is the internal

energy of the gas constant? Answer: $1 \rightarrow 2$ and $3 \rightarrow 4$ (the two isotherms).

Rules for “steady devices” (engine, heat pump, etc.)

- Over one cycle $\Delta E_{\text{thermal}} = W + Q = 0$ (the energy of the gas returns to the value at which it started):

$$W_{\text{input}} + Q_{\text{input}} = W_{\text{output}} + Q_{\text{output}}$$

- The total entropy cannot decrease. Since the gas returns to its initial state at the end of each cycle ($\Delta S_{\text{cycle}} = 0$), this implies that the entropy of the environment cannot decrease:

$$\Delta S_{\text{environment}} = \frac{Q_{\text{output}}}{k_B T_{\text{output}}} - \frac{Q_{\text{input}}}{k_B T_{\text{input}}} \geq 0$$

- Notice that moving heat from the device out to the environment increases S_{env} , while moving heat from the environment in to the device decreases S_{env} .

$$\frac{Q_{\text{output}}}{T_{\text{output}}} \geq \frac{Q_{\text{input}}}{T_{\text{input}}}$$

- If you’re trying to do useful work with the heat (Q_{input}), some of the heat (Q_{output}) is always thrown away as a by-product.

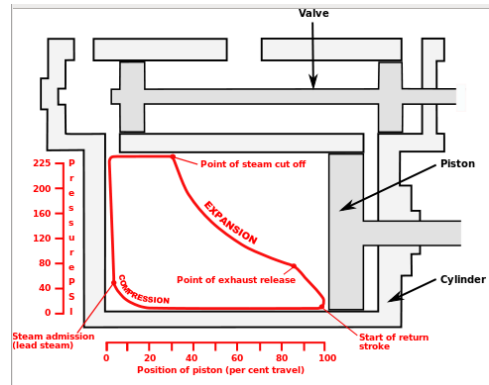
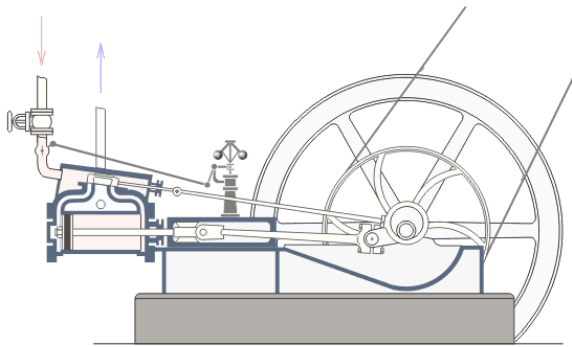
Efficiency of a heat engine.

What you “pay” to run the engine is Q_{input} , which comes from e.g. burning fuel. What you want from the engine is mechanical work: $W_{\text{output}} - W_{\text{input}} = -W$. (Warning: Giancoli uses the opposite sign convention for W . For now I am using Mazur’s sign convention.) The **efficiency** of a heat engine is defined as

$$\eta = \frac{W_{\text{output}} - W_{\text{input}}}{Q_{\text{input}}}$$

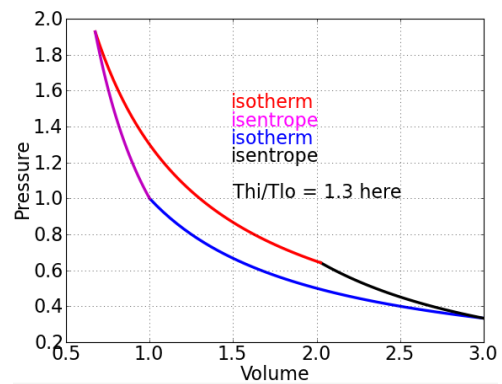
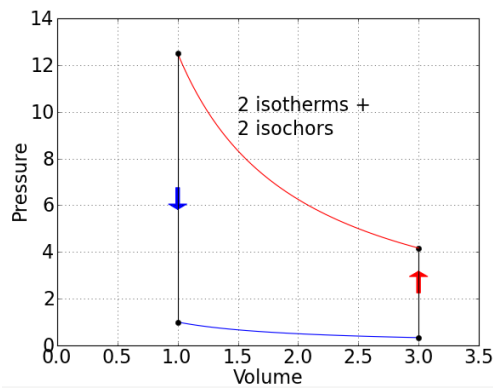
An engine takes in Q_{input} at some high temperature T_{input} and exhausts waste heat Q_{output} at some low temperature T_{output} . Engines need $T_{\text{input}} > T_{\text{output}}$ and work best when $T_{\text{input}} \gg T_{\text{output}}$. The entropy law says that engines can never do better than the theoretical $\Delta S_{\text{environment}} = 0$ case (the case of a **reversible** engine):

$$\eta \leq 1 - \frac{T_{\text{output}}}{T_{\text{input}}} \quad \boxed{\text{efficiency} \leq 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}}$$



https://en.wikipedia.org/wiki/Steam_engine#/media/File:Steam_engine_in_action.gif

For a **refrigerator** or a **heat pump**, the PV diagram goes counterclockwise. The following (left figure below) is a dumb example of a fridge, but it is one that is easy to illustrate using coffee cans. **Doing work on the gas** ($W > 0$) or **letting the gas do work** ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). (For a fridge, $W > 0$ happens at high T .)



This example (right figure above) is a **reversible** ($\Delta S_{\text{env}} = 0$) fridge cycle. **Doing work on the gas** ($W > 0$) or **letting the gas do work** ($W < 0$) at constant temperature T looks like $P_f V_f = P_i V_i$ (isotherm). This cycle, which has two isotherms and two isentropes, is called the **Carnot cycle**.

If you instead run this same (Carnot) cycle clockwise, you get the Carnot heat engine, which is of theoretical interest (because it has optimal efficiency) but not of practical interest (because the net work done per cycle is very small — as you can see from the fact that the enclosed area between the curves is so small).

If I compress a gas (or let it expand) while the gas is sealed in a thermally insulated vessel (like a thermos bottle with a piston), so that the gas can't exchange heat with

the environment, that process is called **isentropic** a.k.a. **adiabatic**.

If I compress a gas (or let it expand) while the gas is in thermal contact with in a large container of water (a “heat bath” a.k.a. “thermal reservoir”), so that the gas maintains the same constant temperature as the reservoir, that process is called **isothermal**.

We saw that the figure of merit for a heat engine was the **efficiency** with which the engine turned input heat into useful mechanical work. By contrast, for a refrigerator or a heat pump, the figure of merit is the **coefficient of performance (COP)**.

For a refrigerator (or an air conditioner), what you “pay” to run the fridge is $W = W_{\text{input}} - W_{\text{output}}$, which comes from the *compressor’s* electric motor. What you want from the fridge is the cooling: Q_{input} . The **coefficient of performance** of a fridge is defined as

$$\text{COP}_{\text{cooling}} = \frac{Q_{\text{input}}}{W}$$

A fridge performs best when the kitchen (T_{output}) is not too much warmer than the desired fridge temperature (T_{input}). Unlike an engine, a fridge prefers a small ΔT . The entropy law dictates that fridges can never do better than the theoretical $\Delta S_{\text{env}} = 0$ case (ideal “reversible” fridge):

$$\text{COP}_{\text{cooling}} \leq \frac{T_{\text{input}}}{T_{\text{output}} - T_{\text{input}}} \quad \text{COP}_{\text{cooling}} \leq \frac{T_{\text{cold}}}{T_{\text{hot}} - T_{\text{cold}}}$$

It is difficult to overstate the impact of refrigeration on the quality of human life, for food preservation, vaccine delivery, and indoor climate control.

For a heat pump, what you “pay” to run the heat pump is W , which comes from an electric motor — same as with a fridge. What you want from heat pump is the **heating**: Q_{output} The **coefficient of performance** of a heat pump is

$$\text{COP}_{\text{heating}} = \frac{Q_{\text{output}}}{W}$$

A heat pump performs best when the house (T_{output}) is not too much warmer than the heat-exchange-coil (T_{input}). A heat pump prefers a small ΔT . (So bury coil underground.) The entropy law dictates that heat pumps can never do better than the theoretical $\Delta S_{\text{env}} = 0$ case (ideal “reversible” pump):

$$\text{COP}_{\text{heating}} \leq \frac{T_{\text{output}}}{T_{\text{output}} - T_{\text{input}}} \quad \text{COP}_{\text{heating}} \leq \frac{T_{\text{hot}}}{T_{\text{hot}} - T_{\text{cold}}}$$

Confusingly, T_{output} (to which heat is output) is the indoor temperature, and T_{input} (from which heat is input) is the outdoor temperature.

Example: A heat pump has a COP of 3.0 and is rated to do work at 1500 W. How much heat can it add to a room per second? Answer: 4500 W.

Example: Suppose that a heat pump operates with $T_{\text{hot}} = 35^\circ\text{C}$ and $T_{\text{cold}} = 0^\circ\text{C}$. What is the theoretical maximum COP that one could ever hope to achieve for these T_H and T_C values? Answer:

$$\begin{aligned} COP_{\text{heating}} &= \frac{Q_{\text{hot}}}{W} = \frac{Q_{\text{hot}}}{Q_{\text{hot}} - Q_{\text{cold}}} \leq \frac{T_{\text{hot}}}{T_{\text{hot}} - T_{\text{cold}}} \\ COP_{\text{heating}} &\leq \frac{308\text{K}}{308\text{K} - 273\text{K}} \leq 8.8 \end{aligned}$$

but in practice under these conditions the best heat-pump systems achieve $COP_{\text{heating}} \approx 4.5$ in laboratory tests and more like 3.5 in real-life application. It is still a useful guideline that COP_{heating} gets worse as the indoor/outdoor temperature difference increases. That is why a *ground source heat pump* usually achieves a much better COP than an *air source heat pump*.

Example: A freezer has a COP of 3.8 and uses 200 W of electrical power. How long would it take to freeze an ice-cube tray that contains 600 g of water at 0°C ? The latent heat of fusion for water is $L = 333\text{ kJ/kg}$. Answer:

$$t = \frac{mL}{COP \times 200\text{ W}} = 260\text{ s} \approx 4.3\text{ minutes}$$

This seems unrealistically fast for an ordinary fridge, but dedicated ice-cube makers use metal rods to maximize the thermal contact between the fridge and the water.

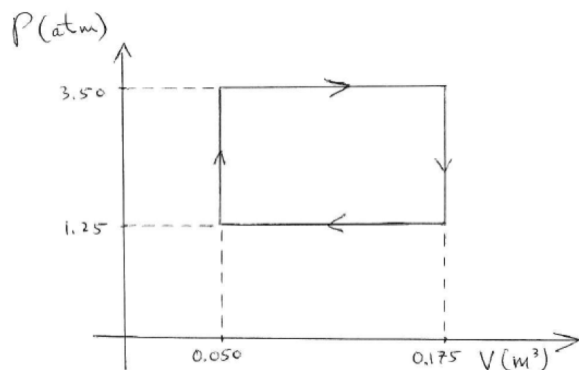
Note that

$$COP_{\text{cooling}} = \frac{Q_{\text{cold}}}{W} = \frac{Q_{\text{cold}}}{Q_{\text{hot}} - Q_{\text{cold}}} \leq \frac{T_{\text{cold}}}{T_{\text{hot}} - T_{\text{cold}}}$$

So in particular the “ideal COP_{cooling} ” has T_{cold} in the numerator, while the “ideal COP_{heating} ” has T_{hot} in the numerator.

One more confusing detail. In “US customary units,” one uses the (Seasonal) Energy Efficiency Ratio (SEER, or EER), where heat is measured in BTU, while work is measured in watt-hours. Since a British Thermal Unit is 1055 J, a COP of 1.0 corresponds to an SEER of 3.4. So if a very good COP value these days is around 4.0, then a very good SEER is around 13.6.

Example: The PV diagram for a certain (somewhat contrived, to keep the math simple) heat engine cycle is shown below. What is the efficiency ($\eta = (W_{\text{out}} - W_{\text{in}})/Q_{\text{in}}$) of this engine, if it exhausts $Q_{\text{out}} = 43.5$ kJ of thermal energy per cycle? (Remember $1 \text{ atm} = 101325 \text{ N/m}^2$.)



Answer: $W_{\text{out}} = 44.3$ kJ, $W_{\text{in}} = 15.8$ kJ, $Q_{\text{in}} = W_{\text{out}} + Q_{\text{out}} - W_{\text{in}} = 72.0$ kJ.
 $\eta = 0.396$.

Example: In one cycle, a steady device transfers 1200×10^3 J of energy from a thermal reservoir at 600 K to a thermal reservoir at 300 K. (This is a pretty useless device: since $Q_{\text{out}} = Q_{\text{in}}$, we must have $W = 0$.) Find the change in entropy (after one complete cycle), (a) For the device, and (b) for the environment. (Mazur writes $\Delta S = Q/(k_B T)$. Giancoli writes $\Delta S = Q/T$. Let's use Giancoli's convention here, since it makes the math easier.) Answer: The change in entropy for the device itself must be zero after one complete cycle of a steady device. For the environment, $\Delta S_{\text{env}} = \frac{Q_{\text{output}}}{T_{\text{output}}} - \frac{Q_{\text{input}}}{T_{\text{input}}} = \frac{1200 \text{ kJ}}{300 \text{ K}} - \frac{1200 \text{ kJ}}{600 \text{ K}} = 2000 \text{ J/K}$

Extended example:

If I plug a 2000 watt electric space heater into the electric outlet and turn it on, how many watts of electrical power are consumed by the space heater? Answer: 2000 W.

How many watts of heat are delivered to the room? Answer: 2000 W.

If I plug a 2000 watt heat pump into the electrical supply and turn it on, how many watts of electrical power are consumed by the heat pump? Answer: 2000 W.

If the heat pump has $COP_{\text{heating}} = 4.0$, how many watts of heat are delivered to the room? Answer: 8000 W.

Does COP_{heating} tend to get bigger or smaller as the temperature difference between "the hot place" and "the cold place" gets larger? Answer: smaller.

If I plug a 2000 watt air conditioner into the electrical supply and turn it on, how many watts of electrical power are consumed by the air conditioner? Answer: 2000 W.

If the air conditioner has a $COP_{\text{cooling}} = 4.0$, how many watts of heat are removed from the room? Answer: 8000 W.

Does COP_{cooling} tend to get bigger or smaller as the temperature difference between “the hot place” and “the cold place” gets larger? Answer: smaller.

By contrast, does the efficiency of a heat engine (steam engine, combustion engine, jet engine, etc.) tend to get bigger or smaller as the temperature difference between “the hot place” and “the cold place” gets larger? Answer: bigger.

What is the range of physically possible values for the efficiency of a heat engine? Answer: $0 \leq \eta \leq 1$.

Can the COP of a heat pump be larger than one? Answer: Yes.

Can the efficiency of an engine be larger than one? Answer: No.

Copied from older ‘equations’ file.

Atomic mass unit: $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$. Proton mass: $m_p = 1.6726 \times 10^{-27} \text{ kg}$. Neutron mass: $m_n = 1.6749 \times 10^{-27} \text{ kg}$. As you saw if you did the extra-credit reading on Special Relativity (Einstein’s $E = mc^2$, etc.), the mass of a nucleus is slightly smaller than the sum of the masses of its protons and neutrons, because of the negative binding energy that holds the nucleus together. So you can argue that mass is really just one more form of energy! The atomic mass unit u is defined to be $\frac{1}{12}$ of the mass of a ^{12}C nucleus, which is a bound state of 6 protons and 6 neutrons.

Avogadro’s number: $N_A = 6.022 \times 10^{23}$. Just as 12 of something is called a dozen, 6.022×10^{23} of something is called a mole. The mass of a mole of protons is 1.007 grams, i.e. almost exactly a gram. A mole of atomic mass units is $N_A \times 1 \text{ u} = 1.0000 \text{ g} = 1.0000 \times 10^{-3} \text{ kg}$.

A Fahrenheit degree is $\frac{5}{9}$ of a Celsius degree, and 0°C is 32°F . According to the Wikipedia, the Fahrenheit scale is considered obsolete everywhere except the United States, the Cayman Islands, and Belize.

The Kelvin scale measures absolute temperature. A change of one Kelvin is the same as a change of 1°C , but with an offset such that $0^\circ\text{C} = 273.15 \text{ K}$.

Thermal expansion: $\Delta L = \alpha L_0 \Delta T$, $\Delta V = \beta V_0 \Delta T$. Typically $\beta = 3\alpha$. (Here α is the linear coefficient of thermal expansion, and β is the volume coefficient of thermal expansion.)

Thermal stress (if ends are not allowed to move when object is heated or cooled):
 $F/A = E\alpha\Delta T$

Ideal gas law (works where density is low enough that the gas molecules interact primarily with the walls of the container, and not so much with one another):

$$PV = nRT$$

where T is in Kelvin and n is in moles. If you measure P in Pa (same as N/m^2) and V in m^3 , then $R = 8.315 \frac{J}{mol \cdot K}$. If you measure P in atm and V in liters, then $R = 0.0821 \frac{L \cdot atm}{mol \cdot K}$. A mol of ideal gas at STP (1 atm, $0^\circ C$) has a volume of 22.4 L.

The volume per mole of ideal gas at temperature T at 1 atm is

$$\frac{V}{n} = \frac{RT}{P} = \frac{(0.0821 \frac{L \cdot atm}{mol \cdot K})T}{1 \text{ atm}} = (22.4 \text{ L}) \left(\frac{T}{273 \text{ K}} \right) = (0.0224 \text{ m}^3) \left(\frac{T}{273 \text{ K}} \right)$$

If you measure N in molecules (not moles), P in N/m^2 , V in m^3 , and T in Kelvin, then $PV = Nk_B T$, where $k_B = 1.38 \times 10^{-23} \text{ J/K}$ is **Boltzmann's** constant. The root-mean-squared speed of a gas molecule, v_{rms} is given by $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_B T$, with T in Kelvin. So the average K.E. of a gas molecule is proportional to absolute temperature.

Because of the random motions of molecules, a concentrated blob of ink, spray of perfume, etc., will **diffuse** from a region of high concentration to a region of low concentration. The number per unit time of molecules diffusing through area A is $\frac{dN}{dt} = AD \frac{dC}{dx}$, where C is the concentration of molecules per unit volume, and D is called the diffusion constant (unit is m^2/s).

Increasing the temperature of a given mass of a given substance requires heat $Q = mc\Delta T$, where c is the **specific heat capacity** of the substance. (Mind the sign: you get heat back out if you decrease the temperature.)

Melting or evaporating a mass m of a substance requires heat $Q = mL$, where L is the **latent heat** (of fusion for melting, of vaporization for boiling). (Mind the sign: you get heat back out for condensation or for freezing.)

Remember that **energy is conserved** (always, now that we know how to account for thermal energy). Work W represents the transfer of mechanical energy into ($W > 0$)

or out of ($W < 0$) a system. Heat Q represents the transfer of thermal energy into ($Q > 0$) or out of ($Q < 0$) a system. If we call the internal energy (including thermal energy) of the system U , then

$$\Delta U = W_{\text{in}} + Q_{\text{in}} - W_{\text{out}} - Q_{\text{out}}$$

is just the statement that energy is conserved.

Heat (symbol Q , standard (S.I.) unit = joules) is the transfer of thermal energy from a warmer object to a cooler object. Heat can be transferred via conduction, convection, and radiation.

Conduction is the incoherent movement (similar to diffusion) of thermal energy through a substance, from high T to low T . The heat conducted per unit time is

$$\frac{dQ}{dt} = \frac{k A \Delta T}{\ell} = \frac{A \Delta T}{R}$$

where k is thermal conductivity, A is cross-sectional area (perpendicular to direction of heat flow), ℓ is the thickness (parallel to direction of heat flow), and ΔT is the temperature difference across thickness ℓ . We can also define **R-value**, $\mathbf{R} = \ell/k$, and then use the second form written above. Be careful: if an R-value is given in imperial units ($\text{foot}^2 \cdot \text{hour} \cdot ^\circ\text{F}/\text{Btu}$), you must multiply it by 0.176 to get S.I. units ($\text{m}^2 \cdot \text{K}/\text{W}$).

Convection means e.g. I heat some water in a furnace, then a pump mechanically moves the hot water to a radiator: it is the transfer of thermal energy via the coherent movement of molecules. Convection also occurs if I heat some air, which then becomes less dense and rises (because of buoyancy, which is caused by gravity), moving the thermal energy upward. “Heat rises” because increasing temperature usually makes things less dense, hence more buoyant.

Radiation is the transfer of heat via electromagnetic waves (visible light, infrared, ultraviolet, etc.), which can propagate through empty space. For a body of emissivity e ($0 \leq e \leq 1$, $0 = \text{shiny}$, $1 = \text{black}$) at temperature T (kelvin), with surface area A , the heat radiated per unit time is

$$\frac{dQ}{dt} = e\sigma AT^4$$

where $\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$ is the Stefan-Boltzmann constant.

Useful tables: expansion coefficients (Giancoli Table 13-1, page 388); saturated vapor pressure of water (Giancoli Table 13-4, page 406); specific heat capacities (Giancoli

Table 14-1, page 421); latent heats (Giancoli Table 14-3, page 425); thermal conductivities (Giancoli Table 14-4, page 429).

Useful tables for elasticity, etc.: elastic modulus (Giancoli Table 9-1, page 254); ultimate strength (Giancoli Table 9-2, page 258).

Work done BY a gas is $W_{\text{out}} = \int P \, dV$. Work done ON a gas is $W_{\text{in}} = -\int P \, dV$. Mazur's convention is $W = -\int P \, dV$, i.e. if you don't specify "in" or "out" then W means $W_{\text{in}} - W_{\text{out}}$.

The state (P, V, T, S, energy) of a steady device is the same at the end of each cycle. Since the energy is unchanged after going around one complete cycle, all of the changes in energy must add up to zero, so then

$$W_{\text{in}} + Q_{\text{in}} = W_{\text{out}} + Q_{\text{out}}$$

If a system transfers thermal energy Q_{out} to its environment at constant temperature T_{out} , the change in the entropy of the environment is $\Delta S_{\text{env}} = Q_{\text{out}}/(k_B T_{\text{out}})$. (This is using Mazur's definition, $S = \ln \Omega$, for entropy, which is usually called the "statistical entropy." Most other books instead use $S = k_B \ln \Omega$, which is called the "thermodynamic entropy." Books that use $S = k_B \ln \Omega$ will instead write $\Delta S_{\text{env}} = Q_{\text{out}}/T_{\text{out}}$.)

If a system absorbs thermal energy Q_{in} from its environment at constant temperature T_{in} , the change in the entropy of the environment is $\Delta S_{\text{env}} = -Q_{\text{in}}/(k_B T_{\text{in}})$.

If thermal energy is transferred at non-constant temperature, you can use calculus to figure out $\Delta S = \int \frac{1}{T} \, dQ$. If thermal energy Q flows from system A at temperature T_A to system B at temperature T_B (and without any mechanical work done on or by either system), then $\Delta S_A = -Q/(k_B T_A)$, and $\Delta S_B = +Q/(k_B T_B)$.

If N molecules of ideal gas go from an equilibrium state with temperature T_i and volume V_i to a new equilibrium state with temperature T_f and volume V_f , the change in entropy of the gas is (where C_V is heat capacity per particle at constant volume)

$$\Delta S_{\text{gas}} = S_f - S_i = \frac{NC_V}{k_B} \ln(T_f/T_i) + N \ln(V_f/V_i)$$

The efficiency of a heat engine is

$$\eta = \frac{W_{\text{out}} - W_{\text{in}}}{Q_{\text{in}}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} \leq \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}}$$

In the special case of an ideal (or Carnot, or “reversible”) heat engine, the “ \leq ” becomes “ $=$ ” (which you can prove by using $\Delta S_{\text{env}} = \frac{Q_{\text{out}}}{k_B T_{\text{out}}} - \frac{Q_{\text{in}}}{k_B T_{\text{in}}} = 0$).

A heat pump moves thermal energy from a “low” temperature T_L to a “high” temperature T_H . The coefficient of performance (COP) for heating is (note that “out” means the output of the heat pump, not the outdoors)

$$\text{COP}_{\text{heating}} = \frac{Q_{\text{out}}}{W_{\text{in}} - W_{\text{out}}} = \frac{Q_{\text{out}}}{Q_{\text{out}} - Q_{\text{in}}} \leq \frac{T_{\text{out}}}{T_{\text{out}} - T_{\text{in}}} = \frac{T_H}{T_H - T_L}$$

where the “ \leq ” is “ $=$ ” for an ideal heat pump. For COP for cooling is

$$\text{COP}_{\text{cooling}} = \frac{Q_{\text{in}}}{W_{\text{in}} - W_{\text{out}}} = \frac{Q_{\text{in}}}{Q_{\text{out}} - Q_{\text{in}}} \leq \frac{T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}} = \frac{T_L}{T_H - T_L}$$

where once again “ \leq ” becomes “ $=$ ” if $\Delta S_{\text{env}} = 0$ for a complete cycle.