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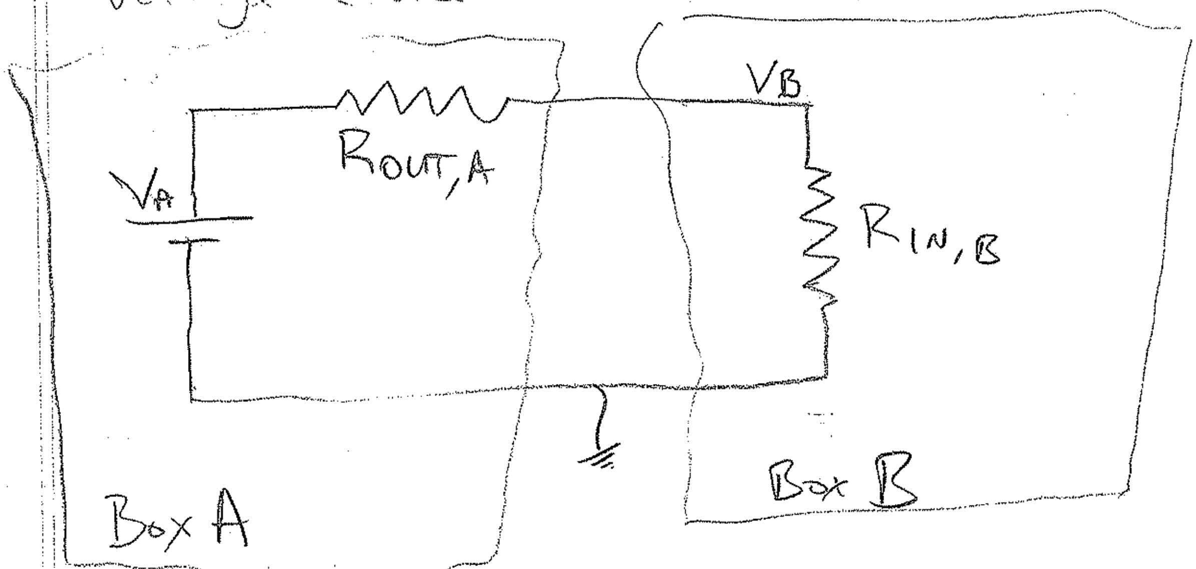
[First work through quiz question #1 on board.]

Common source of confusion last week: meaning & usefulness of input/output impedance (or resistance).

Suppose Black Box A with output impedance $R_{OUT,A}$ tries to drive signal V_A to Black Box B which has input impedance $R_{IN,B}$. What signal does B see?

Model it as — you guessed it — a voltage divider.



A sends $V_A \equiv V_{\text{thevenin}} \equiv V_{OC} \equiv$ A's output with no load connected

B sees $V_B = V_A \cdot \dfrac{R_{IN,B}}{R_{OUT,A} + R_{IN,B}}$

If $R_{IN,B} \gg R_{OUT,A}$, then $V_B \approx V_A$.

i.e. $\lim\limits_{\frac{R_{INB}}{R_{OUTA}} \to \infty} V_B / V_A = 1$

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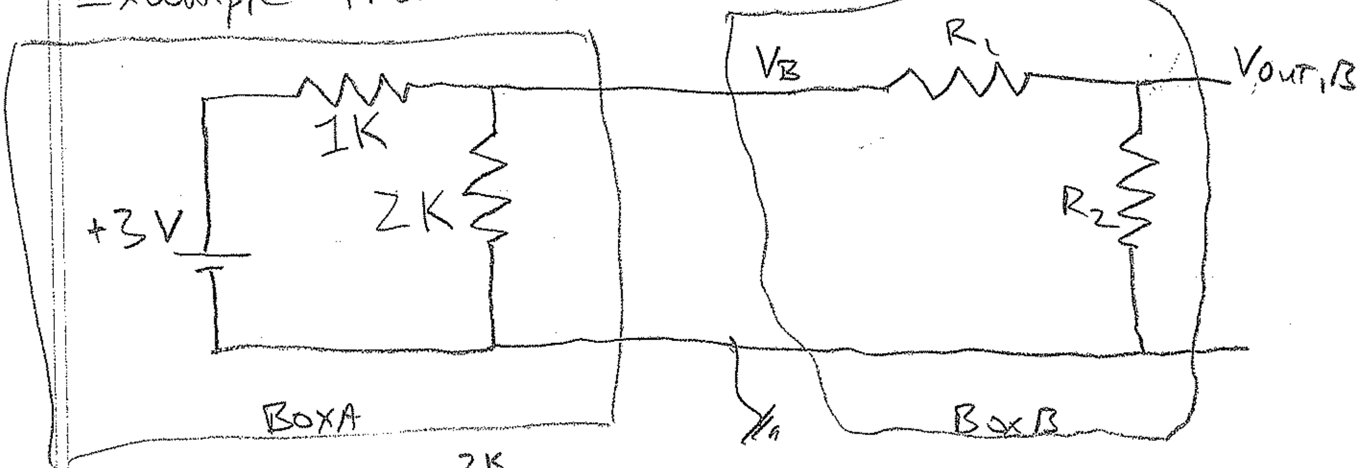
Example from Lab 1:



$V_{THEV,A} = 3V \cdot \frac{2K}{3K} = 2V$

$R_{OUT,A} \equiv R_{THEV} \equiv R_{EQ} = 1K // 2K \simeq 667\Omega$

$R_{IN,B} = R_1 + R_2$

what B sees is $V_B = V_{TH,A} \cdot \frac{R_{IN}}{667\Omega + R_{IN}} = V_{TH,A} \cdot \frac{(R_1+R_2)}{667\Omega + (R_1+R_2)}$

$V_B = 2V \cdot \frac{(R_1+R_2)}{667\Omega + (R_1+R_2)}$

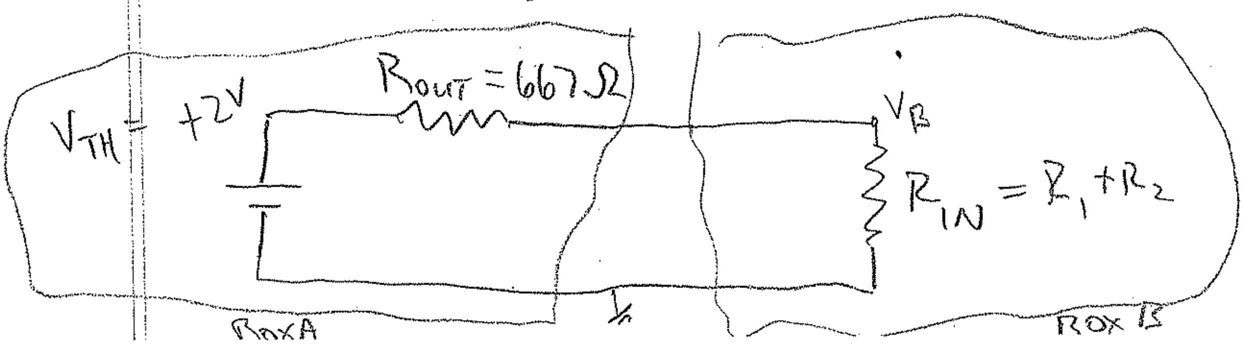
And $V_{OUT,B} = V_B \cdot \frac{R_2}{R_1+R_2}$ as usual.

If $R_1 = 100K$, $R_2 = 200K$, then

$V_B = 2V \cdot \frac{300K}{300.667K} \simeq 1.996\ V$, $V_{out_B} \simeq 1.33\ V$

If $R_1 = 1K$, $R_2 = 2K$, then

$V_B = 2V \cdot \frac{3K}{3.667K} \simeq 1.64\ V$, $V_{out_B} \simeq 1.09\ V$



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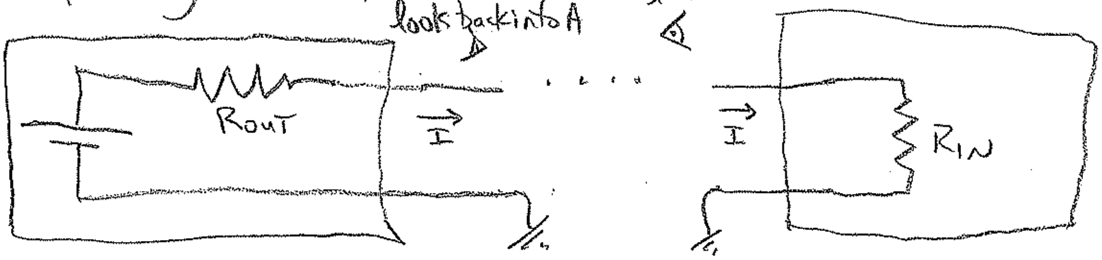
Computing R_{IN}, R_{OUT}.



$$\text{impedance} = \frac{\Delta V}{\Delta I}$$

how much do you have to change I in order to see a given ΔV?

if you change V, how much does I change?

If A tries to raise V, how much more current must it supply to do so?

If B draws more current from A, how much will V sag?

looking back at A, $-\dfrac{dV_{OUT}(A)}{dI_{OUT}(A)} = R_{OUT}^{(A)}$

looking into B, $\dfrac{dV_{IN}(B)}{dI_{IN}(B)} = R_{IN}^{(B)}$

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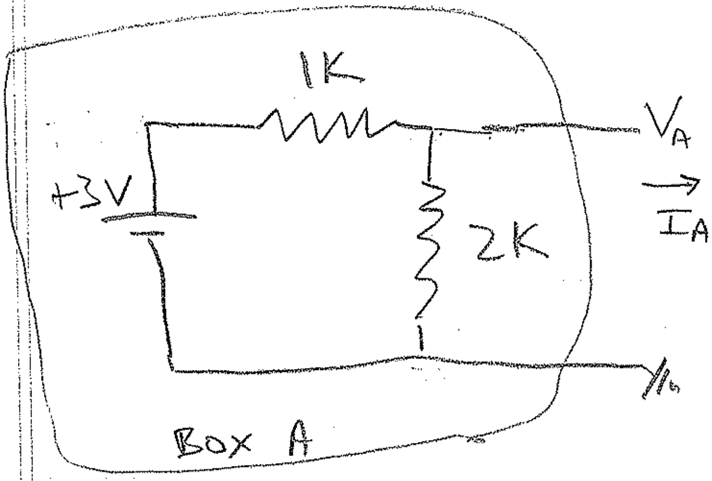
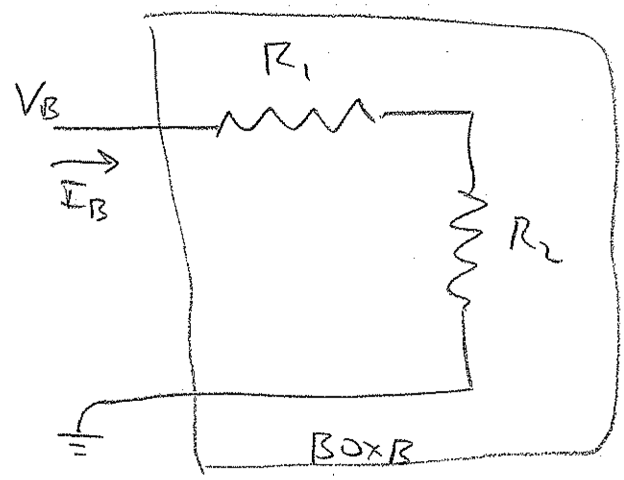


$$-\frac{dV_A}{dI_A} = 667\Omega$$
$$= 1K // 2K$$



$$\frac{dV_B}{dI_B} = R_1 + R_2$$

Useful trick for computing R_{OUT}, R_{IN}:

replace all voltage sources with short circuit
replace all current sources with open circuit

evaluate resistance between the two terminals.

(DISCLAIMERS — see text book
only works for ideal, independent sources)

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Generalize Ohm's law to include phase shifts.

$$V = IR \rightsquigarrow V = IZ$$

Z = impedance, complex quantity

Relationship still <u>linear</u>. Recall two facts about linear systems:

- A linear system driven by $A\cos(2\pi f t)$ responds as $B\cos(2\pi f t + \phi)$, i.e. same frequency, but in general different amplitude & phase
- differentiation is a linear operation

Impedance quantifies response (including phase shift) of a linear circuit as a function of frequency

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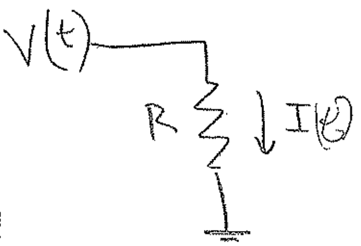
$$V(t) = V_0 \cos(\omega t) = \mathrm{Re}\left\{ V_0 e^{j\omega t} \right\}, \quad j \equiv \sqrt{-1}$$

for a resistor, $Z = R$



If $V(t) = V_0 e^{j\omega t}$
then $I(t) = (V_0/R)\, e^{j\omega t}$

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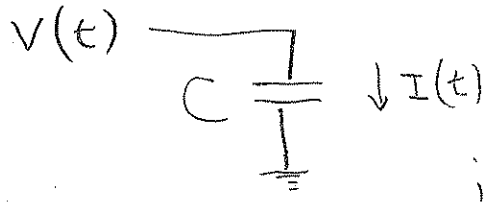
consider a capacitor



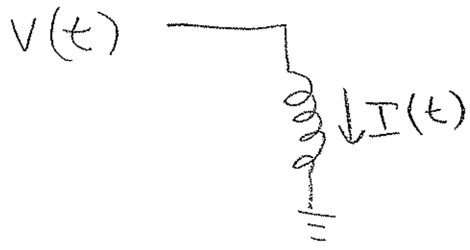
$$Q = CV$$
$$I = dQ/dt = C\frac{dV}{dt}$$

if $V = V_0 e^{j\omega t}$

then $I = j\omega C\, V_0 e^{j\omega t} = j\omega C V$

$$V = IZ \Rightarrow \boxed{Z = \frac{1}{j\omega C}}$$

consider an inductor



$$V = L\, dI/dt$$
$$V_0 e^{j\omega t} = L \frac{d}{dt} I_0 e^{j\omega t}$$
$$V_0 e^{j\omega t} = j\omega L\, I_0 e^{j\omega t}$$
$$V = j\omega L\, I$$
$$V = IZ \Rightarrow \boxed{Z = j\omega L}$$

Ohm's Law generalized to include R, L, C driven sinusoidally (using complex exponential shorthand):

$Z = R$ resistor

$Z = \frac{1}{j\omega C}$ capacitor $\left(|Z| \propto \frac{1}{f}\right)$ D.C. open, high-frequency sh

$Z = j\omega L$ inductor $\left(|Z| \propto f\right)$ D.C. short, high-frequency op

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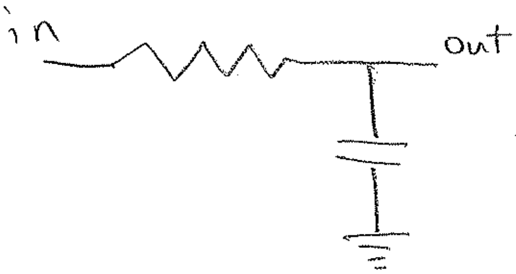
New and fascinating sorts of voltage dividers:



$$\frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

$$\lim_{\omega \to 0} = 1$$

$$\lim_{\omega \to \infty} = 0 \quad \text{(with } -90^\circ \text{ phase shift)}$$

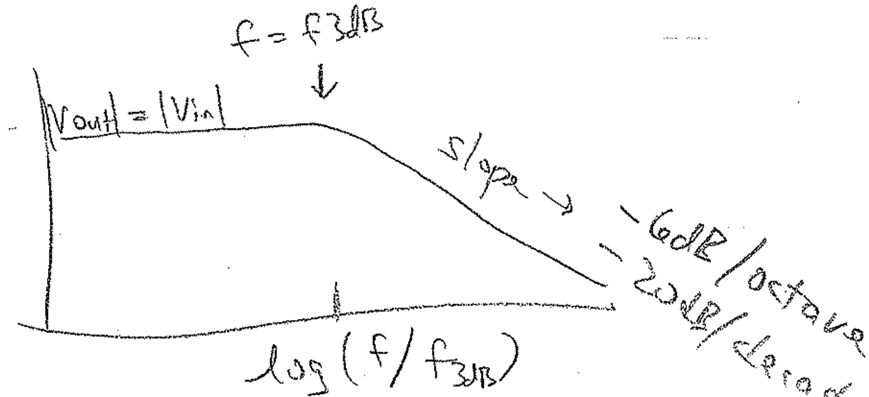
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+(2\pi f RC)^2}}$$

"Low pass filter"

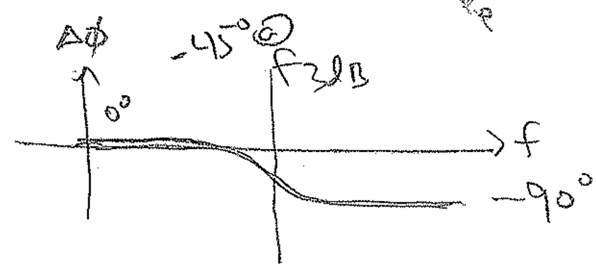
at $f = \frac{1}{2\pi RC}$, $\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}}$ ("f_{3dB}")

at $f \gg \frac{1}{2\pi RC}$, $\xrightarrow[f\to\infty]{\text{limit}} \frac{1}{2\pi f RC} = \frac{f_{3dB}}{f}$

$\log \left|\frac{V_{out}}{V_{in}}\right|$



"Bode plot"



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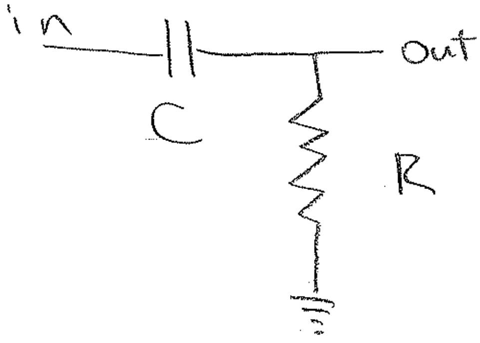


$$\frac{V_{out}}{V_{in}} = \frac{R}{(1/j\omega C) + R} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\lim_{\omega \to 0} = 0 \quad \text{(with } +90^\circ \text{ phase shift)}$$

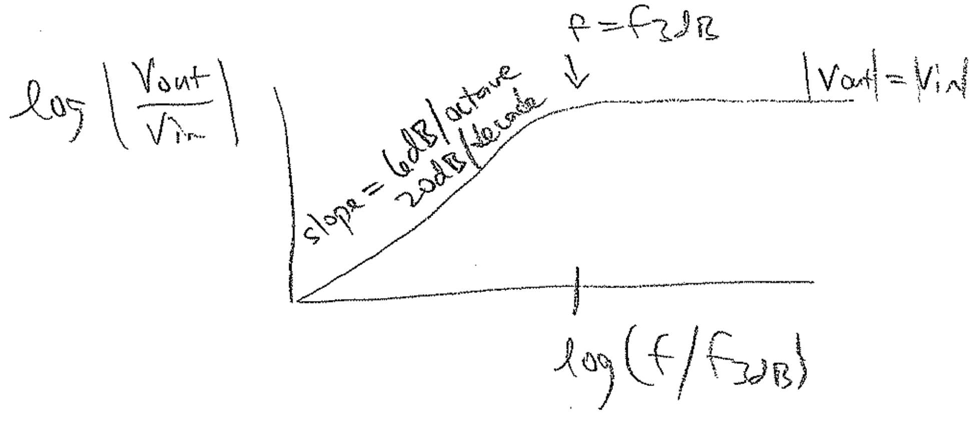
$$\lim_{\omega \to \infty} = 1$$

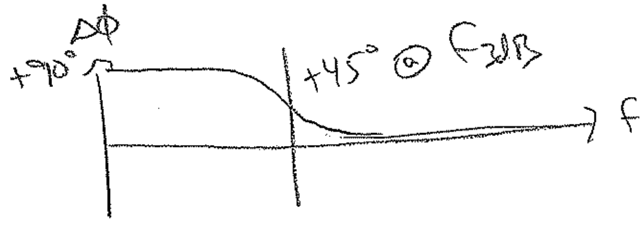
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} = \frac{2\pi f RC}{\sqrt{1 + (2\pi f RC)^2}}$$

"High pass filter"

at $f = \frac{1}{2\pi RC}$, $\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}}$

at $f \ll \frac{1}{2\pi RC}$, $\lim_{f \to 0} \rightsquigarrow 2\pi f RC = \frac{f}{f_{3dB}}$





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[Now work through quiz problem #2 on board]

[Now go through Lab 2 LTspice examples]

Next week (Lab #3, HW #3)
will be op amp circuits
read Bugg chapter 7
and HH 4.00 through 4.10 (pp 175-188)