

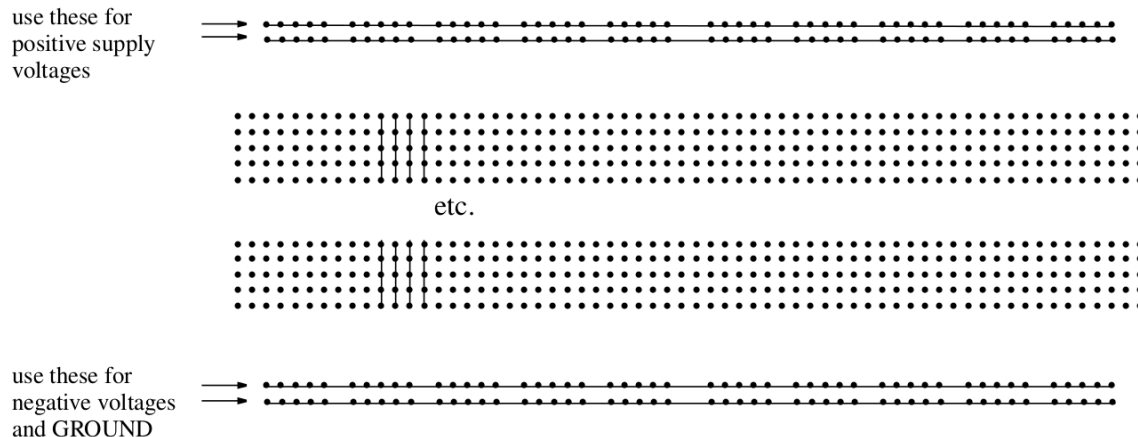
Physics 364, Fall 2012, Lab #2
(*RC circuits, filters, diode rectifier, AM radio receiver*)
start Friday, September 14 — finish Wednesday, September 19.

Course materials and schedule are at positron.hep.upenn.edu/p364

Once everyone has arrived, Jose will give a short tutorial on adjusting your scope probes to work with the oscilloscope. Try to imagine how happy Joseph Fourier would be to see you turning a tiny screwdriver to tweak a capacitor, until your eye confirms that the components of a square wave appear in the perfect $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$ proportions! **Now that you know how to use your scope probe, you want to use it, in 10× mode, for your measurements going forward.**

Incidentally, you want to make sure that your scope inputs are set for DC coupling (not AC), even though this lab is all about time-varying waveforms. When you use “AC coupled” mode, the scope is internally putting a capacitor in series with the input. This can be useful if you want to zoom in on a time-varying signal of very small amplitude that is riding on top of a large DC offset, but it is not what you want for general use. (Can you see why the capacitor *blocks* DC signals and lets only the time-varying component get through?)

One more thing! I reproduce below a diagram illustrating the connections on each section of your breadboard, just in case it’s not yet second nature to you. I should have included this with Lab 1 — sorry about that.



Part 1: voltage divider revisited

Using a pair of 2.2 k Ω resistors, draw and then build a voltage divider that will take a 1 kHz sine wave at 10 V_{pp} as input and will produce a 5 V_{pp} sine wave as output. Use the oscilloscope to observe $V_{in}(t)$ and $V_{out}(t)$. Make a rough sketch of the input and output waveforms, pointing out the relative amplitude and phase of V_{in} and V_{out} . (If your function generator and oscilloscope disagree by about a factor of two on the amplitude of V_{in} , check that the function generator’s output is configured to drive a

“high impedance” load rather than a $50\ \Omega$ load.) This first part is a bit dull, so don’t spend too much time on it. One thing you should do while you’re here is to continue getting to know the scope. Move the trigger threshold up and down and see how it affects the horizontal position of the traces on the screen. Why does this happen? Also, try using the vertical and horizontal cursors on the scope to check that the period and the amplitudes of the sine waves are what you expect.

Part 2: RC low-pass filter

(a) Now replace one of the two resistors with a capacitor to form a *low-pass* filter having $f_{3\text{dB}} \approx 1.6\ \text{kHz}$. What capacitor value did you choose? (The parts drawers should have a cap that is within 10% of the desired value.) Which resistor did you replace? To check that your choice (of which resistor to replace) makes sense, look at the voltage-divider equation

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{Z_2}{Z_1 + Z_2}$$

and remember that a capacitor looks like a short-circuit ($Z \rightarrow 0$) at high frequency and looks like an open-circuit ($Z \rightarrow \infty$) at low frequency. There’s no calculation to do here — just mentally check that the low- and high-frequency limits make sense.

(b) Now measure $|V_{\text{out}}/V_{\text{in}}|$ for $f = 0\ \text{Hz}$ (i.e. for DC), and in factor-of-two steps in frequency from $f = 200\ \text{Hz}$ up through $f \approx 100\ \text{kHz}$. At what frequency does $|V_{\text{out}}/V_{\text{in}}| = 1/\sqrt{2}$? Do you see that beyond $f_{3\text{dB}}$, the response falls (asymptotically) by 6dB/octave? In other words, for each doubling in frequency, the amplitude should decrease by a factor of two. If you have time, make a log-log graph on a spreadsheet (or your favorite data-plotting program) of $|V_{\text{out}}/V_{\text{in}}|$ vs. f and see if you find the expected shape: pretty flat for $f \ll f_{3\text{dB}}$ and falling with a slope of -1 (a factor of 10 in amplitude for each factor of 10 in frequency) for $f \gg f_{3\text{dB}}$; if you’re short on time, just sketch roughly what you would expect the graph to look like.

(c) Measure the phase difference between $V_{\text{out}}(t)$ and $V_{\text{in}}(t)$ for three frequencies: for $f \ll f_{3\text{dB}}$, for $f = f_{3\text{dB}}$, and for $f \gg f_{3\text{dB}}$. Where is the phase shift 45° ? Where you find a large phase shift, which waveform is ahead of the other one — in other words, which direction does the phase shift go? How did you measure the relative phase? (There are several ways to do it. Ask for help if you don’t know how to do it. The classic method is to look at the time difference between the two waveforms’ positive-slope zero crossings, then multiply that Δt by $\frac{360^\circ}{\text{period}}$.)

(d) [If you haven’t reached this section by 10:15 on Friday, just skip it and move on to Part 3. I’d like everyone to be at least started on Part 3 by the end of Friday’s class.] Finally, what is the worst-case *Thevenin impedance* (more commonly called *output impedance*) of the low-pass filter? (You can work this out in your head.) Remember that a better (stronger, stiffer) voltage source has a smaller R_{th} (which we can generalize to Z_{th} when capacitors and inductors are included). So the worst case

(for $|\mathbf{Z}_{\text{thv}}|$ of a voltage source) would be the largest possible value. Hint: this worst case happens in the $f \rightarrow 0$ limit, because $|\mathbf{Z}_{\text{cap}}| \propto 1/f$. With your answer in mind, do you think that V_{out} of your circuit will change appreciably if it is required to drive a 220 k Ω load connected to its output? (The whole point of a filter is to send a signal (with modified frequency content) to some downstream circuit fragment. What we're asking here is how picky our filter will be about what sort of load it is willing to drive, before its output begins to droop — and other bad things happen, too.) Go ahead and connect a 220 k Ω load resistor; then draw the new schematic diagram in your notebook; then check whether V_{out} has changed appreciably for $f = \frac{1}{2\pi RC}$ and for $f \ll \frac{1}{2\pi RC}$. You should find that the filter has no trouble driving a 220 k Ω load. Now (finally!) if you replace the 220 k Ω load resistor with a 2.2 k Ω resistor instead, you should find that two unwanted things happen: first, V_{out} is considerably smaller, even at very low frequencies; second (this is more insidious), the “corner frequency” $f_{3\text{dB}}$ is shifted when too small a load resistance is connected. (For a filter whose largest $|\frac{V_{\text{out}}}{V_{\text{in}}}|$ is different from 1.0, you need to define $f_{3\text{dB}}$ as the frequency at which $|\frac{V_{\text{out}}}{V_{\text{in}}}|$ is a factor $\frac{1}{\sqrt{2}}$ smaller than its maximum value. This will also be the frequency at which the phase shift is 45°.) What is the new $f_{3\text{dB}}$ for your overloaded RC filter?

Keep your low-pass filter intact for the next part.

Part 3: RC “integrator” (such as it is)

(a) Now remove the load resistors from the RC low-pass filter from Part 2, so that it is back to the state it started in in Part 2a. Supply a 10 V_{pp} **square wave** at 100 Hz as $V_{\text{in}}(t)$. You should see the familiar $1 - e^{-t/RC}$ shape as the capacitor charges up (though it will go from -5 V up to $+5$ V instead of starting at 0 V), and the familiar $e^{-t/RC}$ shape as it discharges. Measure how long V_{out} takes to climb 63% of the way up from its minimum, and how long it takes to fall back down to just 37% of the way up from the minimum. Do you find the expected time constant?

(b) At the bottom of page 2 of my Lab 2 reading notes, we saw that $\frac{d}{dt}V_{\text{out}} = \frac{1}{RC}(V_{\text{in}} - V_{\text{out}})$, so in the limiting case $|V_{\text{out}}| \ll |V_{\text{in}}|$, we get

$$V_{\text{out}} \approx \frac{1}{RC} \int V_{\text{in}} dt.$$

Since you already know that this circuit is a low-pass filter with $f_{3\text{dB}} \approx 1.5$ kHz, you should be able to arrange to have $|V_{\text{out}}| \ll |V_{\text{in}}|$ just by adjusting the square frequency to be considerably larger than 1.5 kHz. Try gradually increasing the frequency until V_{out} looks like a more and more plausible integral of V_{in} . Once the integration looks OK, try changing the shape from square to triangle, and see whether the circuit seems to be integrating. You already tried “integrating” a sine wave in Part 2: you got back a sine wave with its phase shifted 90°, which is just (minus) a cosine wave.

[Try to pace yourself to reach this point by the end of Friday, if you can.]

Part 4: RC high-pass filter / “differentiator”

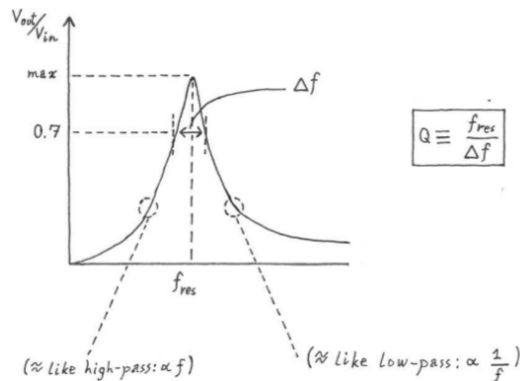
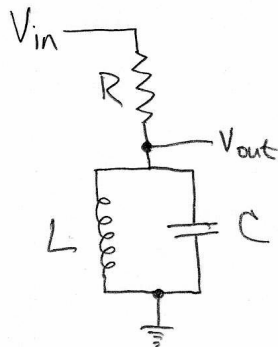
(a) Now interchange the resistor and capacitor from Part 3 so that you have a *high-pass* filter. Before measuring it, think about what you expect a log-log graph of $|\frac{V_{out}}{V_{in}}|$ vs. f to look like. I think you already know where f_{3dB} will be. Now go ahead and measure $|\frac{V_{out}}{V_{in}}|$ for sinusoidal $V_{in}(t)$ at $f = 12$ kHz, 6 kHz, 3 kHz, 1.5 kHz, 800 Hz, 400 Hz, 200 Hz, and 100 Hz. Make V_{in} quite large (e.g. 10 V_{pp}) so that you can still see V_{out} even where the filter’s response is very small. For the lowest frequencies, does V_{out} fall a factor of two for each octave in frequency? (Don’t bother to graph the data this time.) How about the phase shift? $V_{in}(t)$ and $V_{out}(t)$ should be in phase with one another for the frequencies at which $|V_{out}| \approx |V_{in}|$ (i.e. the highest frequencies), and they should be 90° apart at $f \ll f_{3dB}$. Where they are $\approx 90^\circ$ apart, which one is ahead of the other? At what frequency do you expect $V_{in}(t)$ and $V_{out}(t)$ to be 45° apart?

(b) Now try a square-wave $V_{in}(t)$ at a frequency $f \ll f_{3dB}$, e.g. around 150 Hz, so that $\frac{d}{dt}V_{out} \ll \frac{d}{dt}V_{in}$. In this limit (notes page 3),

$$V_{out} \approx RC \frac{dV_{in}}{dt}.$$

Does the circuit appear to be differentiating V_{in} ? What if you give it a triangle-wave input? How about a sine?

Part 5: RLC bandpass filter



On page 7 of this week’s notes, we saw the *band-pass* filter shown above (left). It again looks like a generalized voltage divider, where the impedance on top is $\mathbf{Z}_1 = R$ and the impedance on the bottom is

$$\mathbf{Z}_2 = \mathbf{Z}_L \parallel \mathbf{Z}_C = j\omega L \parallel \frac{1}{j\omega C} = \frac{j\omega L}{1 - \omega^2 LC}.$$

The full expression for

$$\frac{V_{out}}{V_{in}} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{\mathbf{Z}_2}{R + \mathbf{Z}_2}$$

is messy, but it's easy to look at a few limiting cases. You can see that $\mathbf{Z}_2 \rightarrow 0$ for both $f \rightarrow 0$ and $f \rightarrow \infty$, so V_{out} should vanish at both of these limits. And you can see that $|\mathbf{Z}_2| \rightarrow \infty$ at $\omega = \frac{1}{\sqrt{LC}}$, so V_{out} should be maximized at the resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

Ideally $\frac{|V_{\text{out}}|}{|V_{\text{in}}|} = 1$ at f_0 , but real-world inductors have some finite series resistance, which makes the peak wider and less tall. With ideal components, the bandwidth (illustrated above, right) is

$$\Delta f \approx \frac{1}{2\pi RC}$$

(I got this wrong in the notes, by the way).

(a) First build the bandpass filter shown above, using $R = 20 \text{ k}\Omega$, $L = 47 \text{ }\mu\text{H}$, $C \approx 600 \text{ pF}$. You'll need to improvise a bit to get close to 600 pF: remember that capacitors add in parallel. Now imagine your (resonant) filter to be the Liberty Bell (before it cracked) and whack it with a big 10 V_{pp} square wave at about 1 kHz. At what frequency does it ring? Is it reasonably close to your calculated f_0 ? How quickly does it decay to $1/e \approx 37\%$ of its initial amplitude? Is the decay time reasonably close to RC ? (Probably τ will be somewhat shorter than RC because of the inductor's internal series resistance.)

(b) Now measure your filter's frequency response using a sine wave input. Start out at $f \approx f_0$, and use the knob on the function generator to move the frequency up and down in steps of about 10 kHz. You should be able to see where V_{out} is maximal, and how far you have to move f to get V_{out} to fall to $\frac{1}{\sqrt{2}} \approx 70\%$ of its maximum value. Does it peak near the expected f_0 ? Is Δf about right (e.g. $\Delta f = 1/(2\pi\tau)$, where τ is the decay time found in part a)? (When I tried it, I got OK but not great agreement for Δf .)

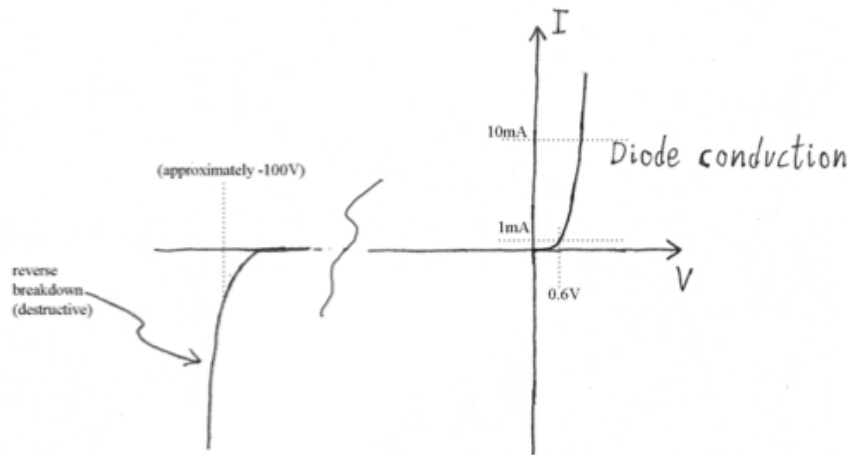
(c) [*Skip this section unless you reach this point by 10:30 on Monday.*] Try replacing the 20 k Ω resistor first with 200 k Ω and then with 2 k Ω , and see if the effect on Δf (or on $Q = f_0/\Delta f$) makes sense. Next, try adding one or more additional capacitors in parallel with C and see how it affects f_0 . In Part 7, we're going to tune this circuit to find a nearby AM radio station. The AM band goes from 540 kHz to 1610 kHz, in 10 kHz steps. Reducing f_0 by about 10% is most easily done by increasing C by about 20% (because of the square-root), etc. Remember that capacitors add in parallel.

Before you leave this part, adjust your capacitor choices as needed so that the measured maximum of V_{out} occurs at 950 kHz (± 5 kHz). Also, adjust R if necessary so that the measured Δf is in the range $10 \text{ kHz} < \Delta f < 100 \text{ kHz}$, so that we can pick out one strong AM station later. *Keep this circuit together for use in Part 7!*

[Try to pace yourself to reach this point by the end of Monday.]

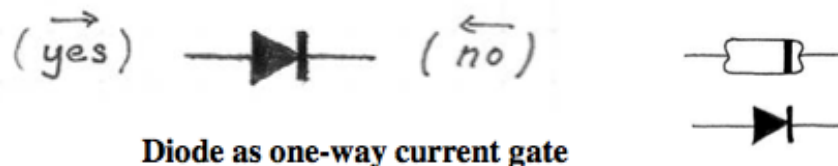
Part 6: Diode rectifier

We're once again talking about diodes without really formally introducing them. Eggleston's Chapter 3 is all about the physics of diodes, but we won't read that until Week 5, when we start to discuss transistors. So for now you'll continue to see diodes informally, as if you were learning a new language by immersion. The figure below¹ shows the diode I - V curve that you measured last week: for $V > 0$, I increases exponentially with V ; for $V < 0$, $I \approx 0$ until (zap!) the diode breaks down.



: Diode I -vs- V curves: reverse current is in nano-amp range

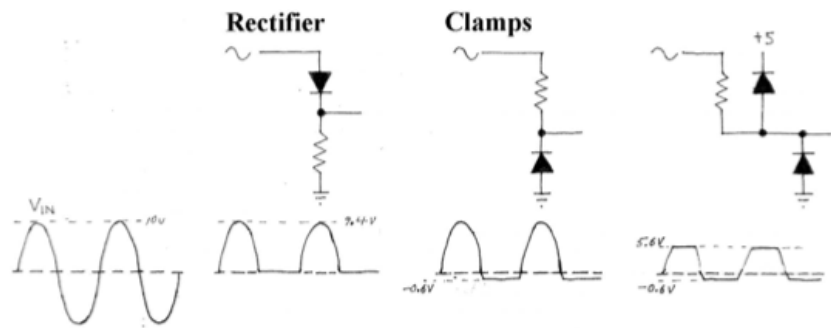
A simple-minded way to look at a diode is as a device that allows current to flow only in one direction, as illustrated below. The current is free to move in the direction of the arrow, but not in the opposite direction. By the way, real diodes are marked with a black band on the end to which the arrow points.



Diode as one-way current gate

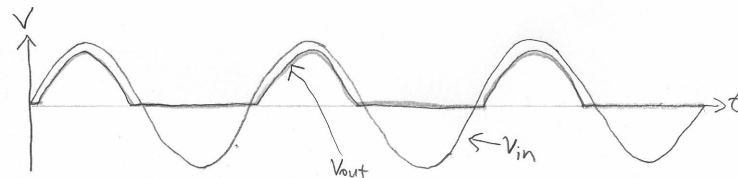
The figure below shows a few common applications of diodes. *Rectifying* means converting a signal in which current flows alternately back-and-forth into a signal in which current flows in only one direction. (This is one step in converting AC into DC.) *Clamping* means limiting the range of a voltage signal, e.g. to protect a circuit from being damaged if you touch it after rubbing your shoes on the carpet. You briefly saw an example of clamping at the end of Lab 1. Here we'll try building a rectifier, because soon we'll need to rectify our AM radio signal.

¹Shamelessly borrowed from the Harvard course.



3 dividers made with diodes: one rectifier, two clamps

(a) Let's again modify the voltage divider from Part 1, but this time replacing R_1 with a diode, such as 1N4148 or 1N5817. Pick up one of each: the 1N4148 is in the normal parts drawers, and the 1N5817 was specially ordered for us. (We'll see in a moment that the 1N5817 has a smaller forward voltage drop — about 0.25 V, compared with about 0.7 V for the 1N4148.) Try the 1N4148 first. Make sure the diode points downward, as drawn in the above-left schematic, so that we keep the positive rather than the negative part of V_{in} . Drive V_{in} with a sine wave of frequency around 1 kHz, and try varying the amplitude to compare the shape of $V_{in}(t)$ and $V_{out}(t)$. If you get the scope to show V_{in} and V_{out} on the same scale, you should see two key features. First, the negative part should be missing from V_{out} . Second, V_{out} should peak about 0.7 V lower than V_{in} . This is not such a big deal for large V_{in} , but it makes it difficult to rectify small signals: if the amplitude of V_{in} were only around 1 V, you wouldn't have much of a signal left after the rectifier. For V_{in} around 10 V_{pp}, you should see something like the figure below.

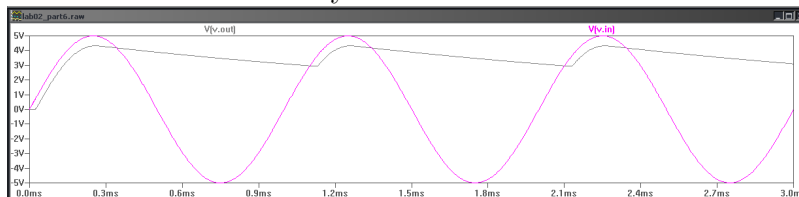


(b) Now replace the 1N4148 with the 1N5817. You should now see a much smaller amplitude difference between V_{in} and V_{out} , because the 1N5817 (a “Schottky diode”) has a voltage drop about 4× smaller than that of the 1N4148. This feature will be very helpful for rectifying small signals, e.g. if the amplitude of V_{in} is less than 1 V. Schottky diodes are also used in some power supplies², because less power $P = IV$ is wasted in the diode.

(c) Next, put a 1 μ F capacitor to ground after the rectifier (i.e. in parallel with the resistor), and draw the new schematic diagram. Notice that V_{out} starts out at the most recent peak value of V_{in} (minus a diode drop), and then falls off with time constant RC . The result should look something like the graph below, which I made using the LTspice circuit simulator. This is a highly simplified version of the AC-to-

²Especially in switched-mode power supplies, e.g. LT3957.

DC conversion that is done in a DC power supply. (There would be a transformer upstream of this circuit, and a transistor-based voltage regulator downstream.³) Stare at your schematic diagram for a moment. The way to think about this circuit is to imagine the diode squirting charge onto the top of the capacitor when $V_{\text{in}} > V_{\text{cap}}$ (plus a diode drop). Meanwhile, the charge on the capacitor bleeds out through the resistor with time constant RC . Do you see it?



(d) Try driving V_{in} with an amplitude-modulated sine wave, and see that the rectifier (with sensibly chosen RC) picks out the *envelope* of the modulated sine wave. The idea of amplitude modulation is that we want to use a small range of frequencies around a *carrier frequency* f_c (e.g. $f_c \approx 1$ MHz) to transmit some information (e.g. the intensity of your voice vs. time) that takes the form $A(t)$. Then instead of transmitting a straightforward sine wave of the form $V(t) = V_0 \cos(2\pi f_c t)$, you instead transmit a modulated sine wave of the form

$$V(t) = (1 - A(t))V_0 \cos(2\pi f_c t).$$

The maximum magnitude of $A(t)$ is called the modulation depth. We want to get the function generator to output $V(t)$ with $f_c = 950$ kHz, amplitude $V_0 \approx 10$ V, and a 1 kHz modulation frequency with 25% modulation depth: in other words, $A(t) = (0.25) \cos(2\pi (1 \text{ kHz}) t)$.

Now replace the $1 \mu\text{F}$ capacitor with a value chosen so that RC is much larger than the period of the 950 kHz carrier, but much smaller than that of the 1 kHz modulation.⁴ If you look at $V_{\text{in}}(t)$ on the scope, you should see the 950 kHz sine wave modulated by a 1 kHz sine wave. If you observe $V_{\text{out}}(t)$ on the scope, you should see only the envelope function. If this is not what you see, or if you do not understand what you are seeing, pause to ask Bill, Jose, or Zoey for help!

How small can you make the amplitude of V_{in} and still see the envelope clearly when you observe V_{out} ? I think that with the 1N5817 diode, you should be able to get down to somewhere around $1 V_{\text{pp}}$.

(Keep this rectifier circuit intact for Part 7!)

³Also, there is a trick using four diodes in a “full-wave bridge” configuration that lets you use both halves of the sine wave!

⁴I chose 1 kHz to be in the range of audio frequencies. To set the scale for audio frequencies: the lowest and highest keys on a piano are 27.5 Hz (an A note) and 4186 Hz (a C note); the telephone network is sensitive to 300–3300 Hz; and hi-fidelity music transmission needs good response from 20 Hz–20 kHz.

Part 7: AM radio receiver

Now let's piece together several of the circuit fragments from this lab to make a simple AM radio receiver. For starters, keep the rectifier and AM signal generator from part 6 intact. Put the bandpass filter from part 5 between the signal generator and the rectifier. Draw the schematic diagram for what you now have assembled.

Move the carrier frequency up and down a bit and look at the response to make sure that the peak response is around 950 kHz. (It turns out that the strongest AM radio station in the Philadelphia area is WPEN, which is an ESPN sports station broadcasting at 950 kHz.)

Probe with the scope before and after the rectifier to check that you see the 950 kHz carrier frequency coming out of the bandpass filter, but see just the envelope coming out of the rectifier. If you see a 1 kHz signal at the end of the chain, try connecting an amplified speaker (or an unamplified earbud). You should hear a tone pretty close to the B two octaves above middle C. If this all works, you're ready to try replacing the signal generator with Bill's ad-hoc radio antenna.

Ask Bill, Jose, or Zoey when you're ready for the antenna and the speaker!

Note that I deliberately do not provide a schematic diagram for this part, so that you have to think a bit for yourself about how the pieces go together. Note also that I cheated a bit and amplified the signal coming out of the antenna (using an opamp circuit that you will build in Lab 3), because my antenna's output on its own was not large enough to rectify easily — i.e. its amplitude, after the bandpass filter, was smaller than a diode drop.

Try to spend a moment sketching out what an amplitude-modulated waveform looks like. Also spend a moment thinking through what each section of the schematic is doing. First you pick out 950 kHz from the other nearby radio stations. Then you trim away the negative part of $V_{in}(t)$ with the rectifier. Then the RC combination at the downstream end of the rectifier separates the envelope from the carrier, exploiting the fact that audio frequencies are so much smaller than radio frequencies.

If you are not yet out of time and still have the antenna cable at your lab bench, try putting your scope into FFT mode (there is a Fast Fourier Transform math function on the scope), and look at V_{in} (from the antenna cable) for the peaks of the various nearby AM radio stations. You will probably need help for this from Bill, Jose, or Zoey.

Fully optional Part 8: full-wave bridge rectifier

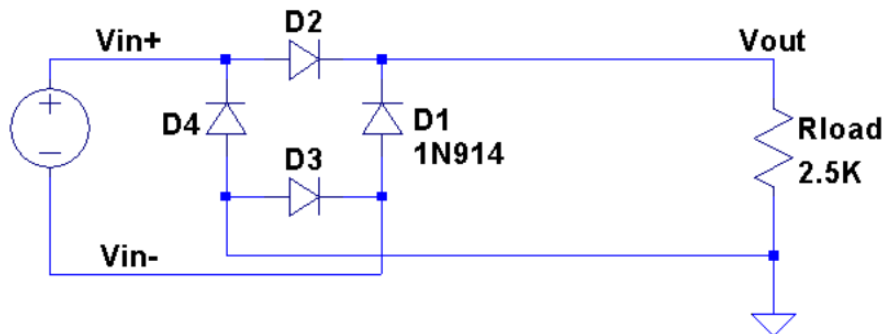
Do this only if you finish everything else early. I don't expect that more than 2 or 3

people will get this far — don't worry about it.

The electricity that comes out of most wall outlets in North America takes the form of a sinusoidal voltage source, 110 volts rms, 60 Hz. Imagine that you are trying to turn this AC line voltage into 5 volts DC to power your mp3 player. In real life, we would first use a transformer to get the AC line voltage into a more convenient (and safer) range, say 10 volts in amplitude. Instead, you will start by programming your function generator to output a 60 Hz sine wave, 10 volts in amplitude.

Now we need to turn this into a voltage whose average value exceeds +5V. (We'll probably learn later in the course how to turn an unregulated 6 or 7 volt power supply into a regulated 5 volt supply. A few of you actually saw such a device inside Mystery Box #3 in Lab 1.)

We could start with the circuit in part 6, but it has a significant drawback: it uses only half of each 60 Hz cycle. Instead, build the full-wave bridge rectifier drawn below. (You will probably have a much easier time sorting out the orientation of the diodes if you lay out your circuit so that it looks like the schematic diagram.) Measure and sketch V_{in} and V_{out} .



You may have trouble if the scope probe's ground clip or the negative lead from the function generator causes unwanted grounding, as the circuit will not work if V_{in-} is shorted to the bottom of R_{load} . If this happens in your lab station, ask Bill or Jose for help inserting some DC-blocking capacitors.

Now add a capacitor (the filter capacitor) in parallel with the load. Try a few capacitor values — starting out by calculating a value such that the RC time constant for discharging the capacitor is several tens of milliseconds, and then going up and down by a factor of 2 or more. See how the amplitude of the 120 Hz droop is affected. How do your measurements compare with the expected exponential decay (which you can approximate using $e^{-t/RC} \approx 1 - t/(RC)$ to speed up your calculations).