Physics 8 — Wednesday, November 1, 2017

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- Homework study/help sessions (optional): Bill will be in DRL 2C6 Wednesdays from 4–6pm (today). Grace will be in DRL 4C2 on Thursdays from 6:30–8:30pm.
- This week, you read O/K ch2&3 in Perusall (statics; selected determinate systems). Next week, you’ll read O/K ch4&5 in Perusall (load tracing; strength of materials).
- For the Onouye/Kane chapters, you’ll do both the Perusall annotations and the usual reading questions. I grade the reading questions, as usual, and Perusall’s AI algorithm assigns you a score for the annotations, which I will count as part of your “in class” work. I am still pondering how best to use Perusall’s numerical scores. My main goal is that you do the reading in a way that maximizes your learning.
- I was tempted to ask you to send in another photo of today’s in-class work, but instead I’ll just record who is here today.
Use a cable to hold bottom of “arch” together so that we can use scale to measure tension. **Weight** \((mg)\) of each side is 20 N. We’ll exploit mirror symmetry and analyze just one side of arch.

Right side shows EFBD for right-hand board.
How many unknown variables is it possible to determine using the equations for static equilibrium in a plane?

(A) one  
(B) two  
(C) three  
(D) four  
(E) five
Static equilibrium lets us write down three equations for a given object: \( \sum F_x = 0 \), \( \sum F_y = 0 \), \( \sum M_z = 0 \). Let’s first sum up the “moments” (a.k.a. torques) about the top hinge.

Which statement correctly expresses \( \sum M_z = 0 \) (a.k.a. \( \sum \tau = 0 \))?

(Let the mass and length of each wooden board be \( L \) and \( m \).)

Don’t answer quite yet!

(A) \(-mg \left( \frac{L}{2} \right) \cos \theta + F^N L \cos \theta - TL \sin \theta = 0\)

(B) \(-mg \left( \frac{L}{2} \right) \cos \theta + F^N L \cos \theta + TL \sin \theta = 0\)

(C) \(-mg \left( \frac{L}{2} \right) \sin \theta + F^N L \sin \theta - TL \cos \theta = 0\)

(D) \(-mg \left( \frac{L}{2} \right) \sin \theta + F^N L \sin \theta + TL \cos \theta = 0\)

(E) \(+mg \left( \frac{L}{2} \right) \cos \theta + F^N L \cos \theta - TL \sin \theta = 0\)

(F) \(+mg \left( \frac{L}{2} \right) \cos \theta + F^N L \cos \theta + TL \sin \theta = 0\)

(G) \(+mg \left( \frac{L}{2} \right) \sin \theta + F^N L \sin \theta - TL \cos \theta = 0\)

(H) \(+mg \left( \frac{L}{2} \right) \sin \theta + F^N L \sin \theta + TL \cos \theta = 0\)
Let’s start with torque (about top hinge) due to tension $T$.

- Usual convention: clockwise = negative, ccw = positive.
- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the “lever arm.”
- Magnitude of torque is $|\tau| = (r_\perp)(F)$. 

$$\tau = (r_\perp)(F)$$
Alternative method: use \((r)(F_\perp)\) instead of \((r_\perp)(F)\).

- Draw vector \(\vec{r}\) from pivot to point where force is applied.
- Draw force vector \(\vec{F}\), with its line-of-action passing through the point where the force is applied.
- Decompose \(\vec{F}\) to find component \(F_\perp\) perpendicular to \(\vec{r}\).
- Magnitude of torque is \(|\tau| = (r)(F_\perp)|.\)
Now you try it for the normal force $\vec{F}^N$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the “lever arm.”
- Magnitude of torque is $|\tau| = r_\perp F$.

Which component of $\vec{r}$ is perpendicular to the normal force $\vec{F}^N$?

(A) horizontal component
(B) vertical component
Now you try it for the normal force $\vec{F}^N$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the “lever arm.”
- Magnitude of torque is $|\tau| = r_\perp F$.

How long is the horizontal component of $\vec{r}$ (i.e. the $\vec{r}$ component which is perpendicular to $\vec{F}$) ?

(A) $L \cos \theta$
(B) $L \sin \theta$
(C) $L \tan \theta$
OK, now back to the original question: Let’s sum up the “moments” (a.k.a. torques) about the top hinge.

Which statement correctly expresses $\sum M_z = 0$ (a.k.a. $\sum \tau = 0$)? (Let the mass and length of each wooden board be $L$ and $m$.)

(A) $-mg \left( \frac{L}{2} \right) \cos \theta + F_N L \cos \theta - TL \sin \theta = 0$
(B) $-mg \left( \frac{L}{2} \right) \cos \theta + F_N L \cos \theta + TL \sin \theta = 0$
(C) $-mg \left( \frac{L}{2} \right) \sin \theta + F_N L \sin \theta - TL \cos \theta = 0$
(D) $-mg \left( \frac{L}{2} \right) \sin \theta + F_N L \sin \theta + TL \cos \theta = 0$
(E) $+mg \left( \frac{L}{2} \right) \cos \theta + F_N L \cos \theta - TL \sin \theta = 0$
(F) $+mg \left( \frac{L}{2} \right) \cos \theta + F_N L \cos \theta + TL \sin \theta = 0$
(G) $+mg \left( \frac{L}{2} \right) \sin \theta + F_N L \sin \theta - TL \cos \theta = 0$
(H) $+mg \left( \frac{L}{2} \right) \sin \theta + F_N L \sin \theta + TL \cos \theta = 0$

Next: what about $\sum F_x = 0$ and $\sum F_y = 0$?
Next: what about \( \sum F_x = 0 \) and \( \sum F_y = 0 \) ?
We said $mg = 20 \text{ N}$, so we expect the string tension to be

$T = \frac{10 \text{ N}}{\tan \theta}$

How would this change if we suspended a weight $Mg$ from the hinge? (By symmetry, each side of arch carries half of this $Mg$.)
Another equilibrium problem!

The top end of a ladder of inertia $m$ rests against a smooth (i.e. slippery) wall, and the bottom end rests on the ground. The coefficient of static friction between the ground and the ladder is $\mu_s$. What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Let’s start by drawing an EFBD for the ladder.
Why must we say the wall is slippery?

Is the slippery wall more like a pin or a roller support?

What plays the role here that string tension played in the previous problem?

Does the combination of two forces at the bottom act more like a pin or a roller support?

Which forces would an engineer call “reaction” forces?
Which choice of pivot axis will give us the simplest equation for $\sum M_z = 0$? (We’ll get an equation involving only two forces if we choose this axis.)

(A) Use bottom of ladder as pivot axis.
(B) Use center of ladder as pivot axis.
(C) Use top of ladder as pivot axis.
How would I write \( \sum M_z = 0 \) about the bottom end of the ladder? (Take length of ladder to be \( L \).)

(A) \( F_W L \cos \theta + mgL \sin \theta = 0 \)

(B) \( F_W L \cos \theta + mg \frac{L}{2} \sin \theta = 0 \)

(C) \( F_W L \cos \theta - mgL \sin \theta = 0 \)

(D) \( F_W L \cos \theta - mg \frac{L}{2} \sin \theta = 0 \)

(E) \( F_W L \sin \theta + mgL \cos \theta = 0 \)

(F) \( F_W L \sin \theta + mg \frac{L}{2} \cos \theta = 0 \)

(G) \( F_W L \sin \theta - mgL \cos \theta = 0 \)

(H) \( F_W L \sin \theta - mg \frac{L}{2} \cos \theta = 0 \)

What do we learn from \( \sum F_x = 0 \) and \( \sum F_y = 0 \)?
What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

Let’s answer the original question:

What is the minimum angle between the ground and the ladder such that the ladder does not slip?

We stopped after this.
Suppose we add to this picture a woman of mass \( M \) who has climbed up a distance \( d \) along the length of the ladder. Now how do we write the moment equation \( \sum M_z = 0 \) ?

(A) \[ F_W L \sin \theta - mg \frac{L}{2} \cos \theta + Mg \frac{d}{2} \cos \theta = 0 \]

(B) \[ F_W L \sin \theta - mg \frac{L}{2} \cos \theta + Mg \frac{d}{2} \sin \theta = 0 \]

(C) \[ F_W L \sin \theta - mg \frac{L}{2} \cos \theta + Mgd \cos \theta = 0 \]

(D) \[ F_W L \sin \theta - mg \frac{L}{2} \cos \theta + Mgd \sin \theta = 0 \]

(E) \[ F_W L \sin \theta - mg \frac{L}{2} \cos \theta - Mg \frac{d}{2} \cos \theta = 0 \]

(F) \[ F_W L \sin \theta - mg \frac{L}{2} \cos \theta - Mg \frac{d}{2} \sin \theta = 0 \]

(G) \[ F_W L \sin \theta - mg \frac{L}{2} \cos \theta - Mgd \cos \theta = 0 \]

(H) \[ F_W L \sin \theta - mg \frac{L}{2} \cos \theta - Mgd \sin \theta = 0 \]

What do we learn from \( \sum F_x = 0 \) and \( \sum F_y = 0 \) ?
What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

For a given $\theta$, how far up can she climb before the ladder slips?
Here’s a trickier equilibrium problem:

4*. You want to hang a 22 kg sign (shown at right) that advertises your new business. To do this, you attach a 7.0 kg beam of length 1.0 m to a wall at its base by a pivot $P$. You then attach a thin cable to the beam and to the wall in such a way that the cable and beam are perpendicular to each other. The beam makes an angle of 37° with the vertical. You hang the sign from the end of the beam to which the cable is attached. (a) What must be the minimum tensile strength of the cable (the amount of tension it can sustain) if it is not to snap? (b) Determine the horizontal and vertical components of the force the pivot exerts on the beam.

What forces act on the beam?

What 3 equations can we write for the beam?
A tightly stretched “high wire” has length $L = 50 \text{ m}$. It sags by $d = 1.0 \text{ m}$ when a tightrope walker of mass $M = 51 \text{ kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

What happens to the tension as we make the sag smaller and smaller?
Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.

How would you find the tensions $T_1$ and $T_2$?

Once you know $T_1$ and $T_2$, what are the horizontal and vertical forces exerted by the two supports on the cable?
How many equations does the “method of joints” allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4  (B) 8  (C) 12  (D) 15
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