Physics 8, Fall 2017, Final Exam. **Name:** ______________________________

This two-hour, closed-book exam has 20% weight in your course grade. You can use one sheet of your own handwritten notes and a calculator. **Turn in your sheet of notes with your exam.**

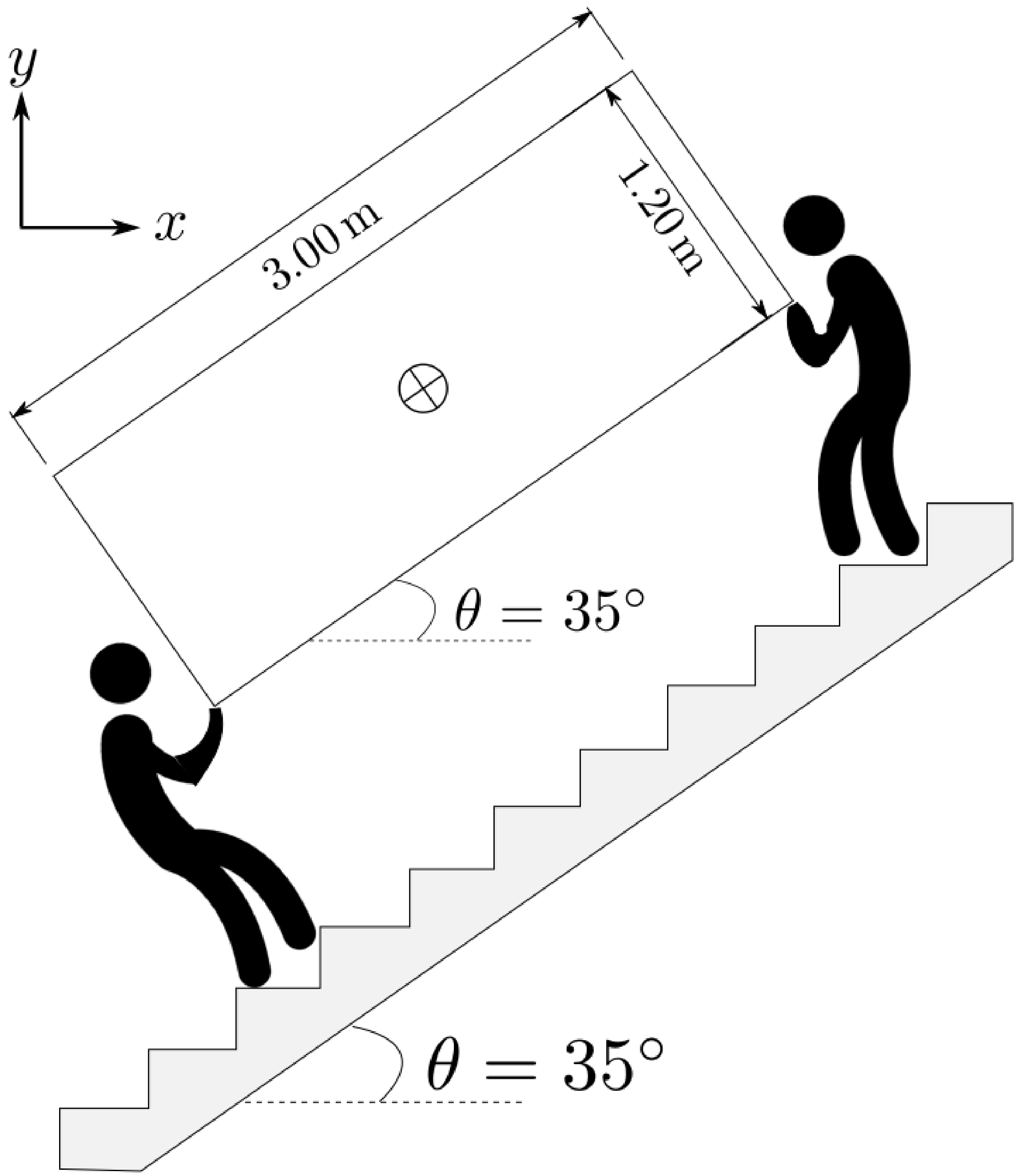
Please show your work on these sheets. Use the spare sheet at the back of the exam if needed. I also have a pad of graph paper up front if you need a sheet or two. The last page of the exam contains a list of equations that you might find helpful. **Please approximate** $\boxed{g = 10\ \mathrm{m/s^2}}$ **to simplify numerical values.**

Work alone. Keep in mind that here at Penn, every member of the University community is responsible for upholding the highest standards of honesty at all times: offering or accepting help with this exam would be a serious violation of Penn's Code of Academic Integrity.

My signature below certifies that I am familiar with Penn's Code of Academic Integrity and that I agree to comply with the Code during this exam.

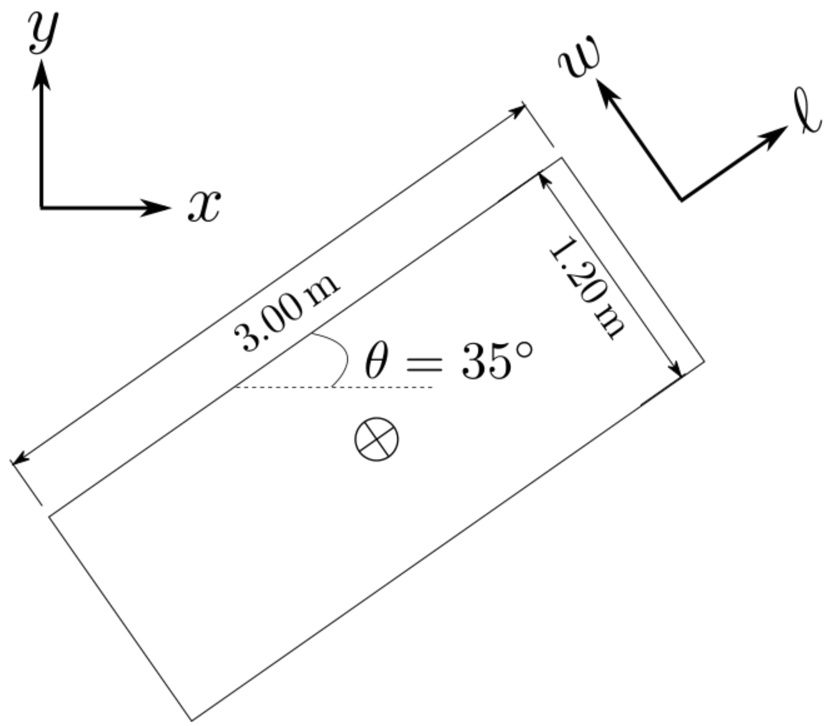
Signature: ______________________________

1. (20%) You and your friend are paused for a moment while you are in the middle of carrying a 150 kg box up a flight of stairs. The box is 3.00 m long and 1.20 m wide (high), and the contents of the box are somehow arranged so that the center of gravity is at the center of the box, as indicated. The stairs make a 35.0° angle to the floor. The box is carried at a 35.0° angle, so that the box's long side is parallel to the staircase. Assume that each person applies, with her hands, a **purely vertical force** to the corresponding corner of the box. In other words, the person below applies a force only in the $+y$ direction at the corner that she touches, and the person above applies a force only in the $+y$ direction at the corner that she touches, with the y axis as indicated. In case it helps: $\cos(35°) \approx 0.8192$, $\sin(35°) \approx 0.5736$. Use $g \approx 10\,\mathrm{m/s^2}$.



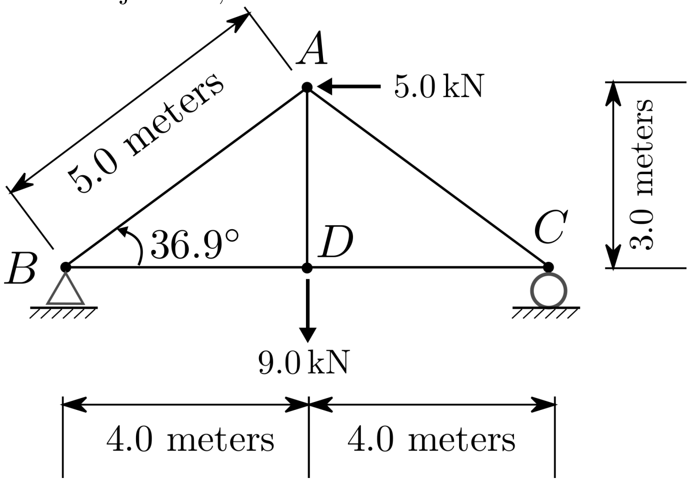
(Problem continues on next page.)

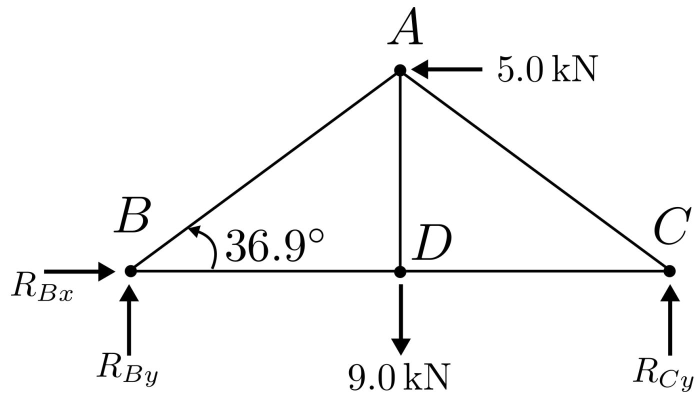
(a) Turn the figure below into an Extended Free Body Diagram by drawing onto the diagram, with correct lines of action, the force F_A exerted on the box by the person above on the staircase, the force F_B exerted on the box by the person below on the staircase, and the force F^G exerted on the box by Earth's gravity. All of these forces should point along the $\pm y$ axis. Then **decompose** F^G into components along the w and ℓ axes, which are parallel to the width and length of the box. Be sure that the components of your decomposed F^G have the correct lines of action.



(b) Determine the magnitudes (give numbers, in newtons) of the forces F^G, F_A, and F_B. You should find that it is easier to be person A than to be person B.

2. (30%) The truss shown below is simply supported at joints B and C, carries a 5.0 kN horizontal load at joint A, and carries a 9.0 kN vertical load at joint D.



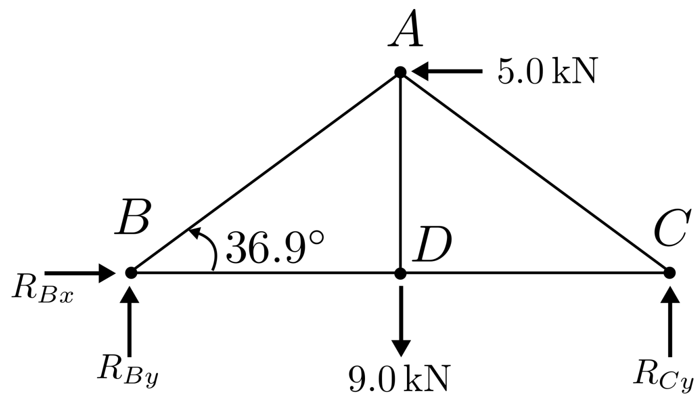


(a) I've drawn, above-right, an Extended Free Body Diagram for the truss as a whole. Using joint B as a pivot, write the moment equation about joint B to solve for the vertical "reaction" force R_{Cy} exerted on the truss by the roller support at joint C. As a check against careless mistakes, you should expect your answer to be one of the following: ±2.625 kN, ±3.500 kN, ±4.375 kN, ±5.000 kN, ±6.375 kN, ±9.000 kN, ±10.625 kN.

(b) Now use $\sum F_x$ and $\sum F_y$ for the truss as a whole to find the two reaction forces, R_{Bx} and R_{By} exerted on the truss by the hinge support at joint B. As a check against careless mistakes, you should expect each answer to be one of the following: ±2.625 kN, ±3.500 kN, ±4.375 kN, ±5.000 kN, ±6.375 kN, ±9.000 kN, ±10.625 kN.

(Problem continues on next page.)

(c) Using the method of joints, write $\sum F_y$ for joint D to solve for the bar tension $\boxed{T_{AD}}$. Indicate whether bar AD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: $\pm 2.625\,\mathrm{kN}$, $\pm 3.500\,\mathrm{kN}$, $\pm 4.375\,\mathrm{kN}$, $\pm 5.000\,\mathrm{kN}$, $\pm 6.375\,\mathrm{kN}$, $\pm 9.000\,\mathrm{kN}$, $\pm 10.625\,\mathrm{kN}$.



(d) Using the method of joints, write $\sum F_y$ for joint C to solve for the bar tension $\boxed{T_{AC}}$. Indicate whether bar AC is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: $\pm 2.625\,\mathrm{kN}$, $\pm 3.500\,\mathrm{kN}$, $\pm 4.375\,\mathrm{kN}$, $\pm 5.000\,\mathrm{kN}$, $\pm 6.375\,\mathrm{kN}$, $\pm 9.000\,\mathrm{kN}$, $\pm 10.625\,\mathrm{kN}$.

(e) Using the method of joints, write $\sum F_y$ for joint B to solve for the bar tension $\boxed{T_{AB}}$. Indicate whether bar AB is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: $\pm 2.625\,\mathrm{kN}$, $\pm 3.500\,\mathrm{kN}$, $\pm 4.375\,\mathrm{kN}$, $\pm 5.000\,\mathrm{kN}$, $\pm 6.375\,\mathrm{kN}$, $\pm 9.000\,\mathrm{kN}$, $\pm 10.625\,\mathrm{kN}$.

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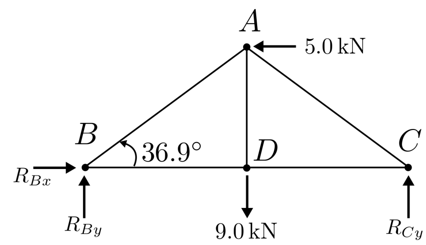
(f) Using the method of joints, write $\sum F_y$ for joint A to check that your results for T_{AB}, T_{AD}, and T_{AC} are consistent.



(g) Using the method of joints, write $\sum F_x$ for joint C to solve for the bar tension $\boxed{T_{CD}}$. Indicate whether bar CD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ±2.625 kN, ±3.500 kN, ±4.375 kN, ±5.000 kN, ±6.375 kN, ±9.000 kN, ±10.625 kN.

(h) Using the method of joints, write $\sum F_x$ for joint D to solve for the bar tension $\boxed{T_{BD}}$. Indicate whether bar BD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ±2.625 kN, ±3.500 kN, ±4.375 kN, ±5.000 kN, ±6.375 kN, ±9.000 kN, ±10.625 kN.

(i) Using the method of joints, write $\sum F_x$ for joint B to check that your results for T_{AB}, T_{BD}, and R_{Bx} are consistent.

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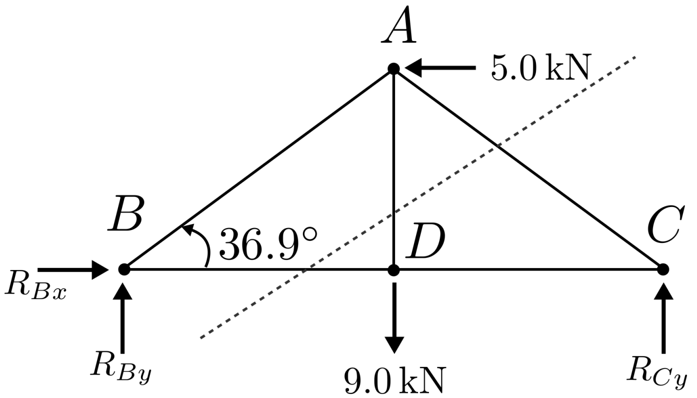
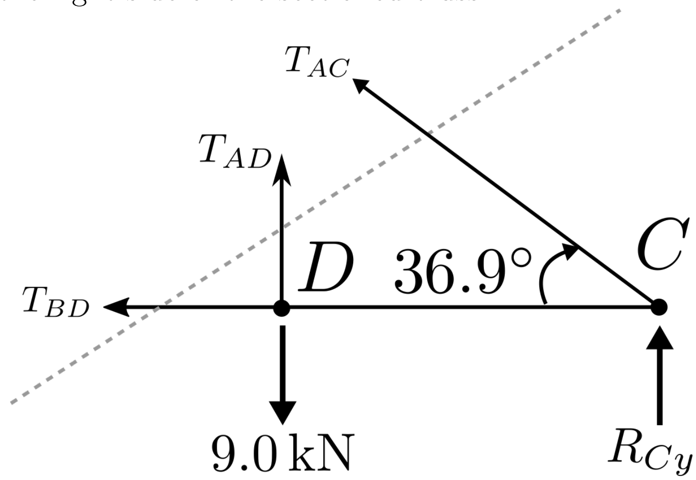
Now let's use the method of sections to analyze the right-hand side of the section shown in the figure below. The right figure shows the forces acting on the right side of the sectioned truss.

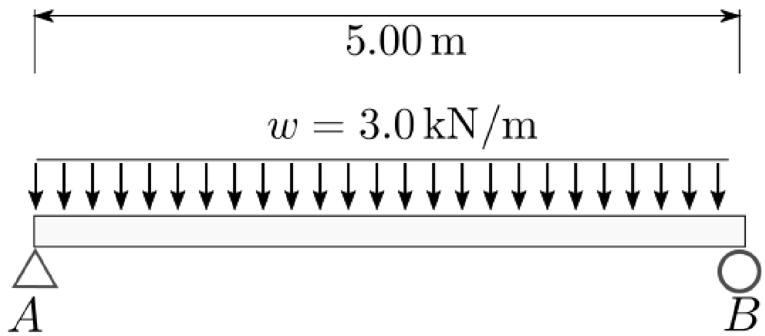




(j) Using moments about joint D, write an equation that lets you check the consistency of your results for T_{AC} and R_{Cy}. (Alas, this equation may feel a bit redunant to you.)

(k) Using moments about joint A (which is invisible here, but you know where it is), write an equation that lets you check the consistency of your results for T_{BD} and R_{Cy}.

3. (20%) (a) A simply-supported beam of length $L = 5.00\,\mathrm{m}$ carries a uniform distributed load $w = 3.00\,\mathrm{kN/m}$ along its entire length. Solve for the three "reaction" forces exerted by the supports: the upward force $\boxed{R_{Ay}}$ exerted by the support at A, the horizontal force $\boxed{R_{Ax}}$ exerted by the support at A, and the upward force $\boxed{R_{By}}$ exerted by the support at B.



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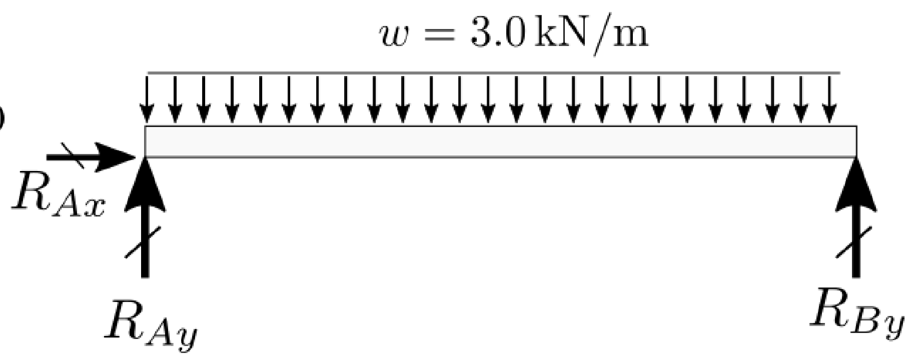

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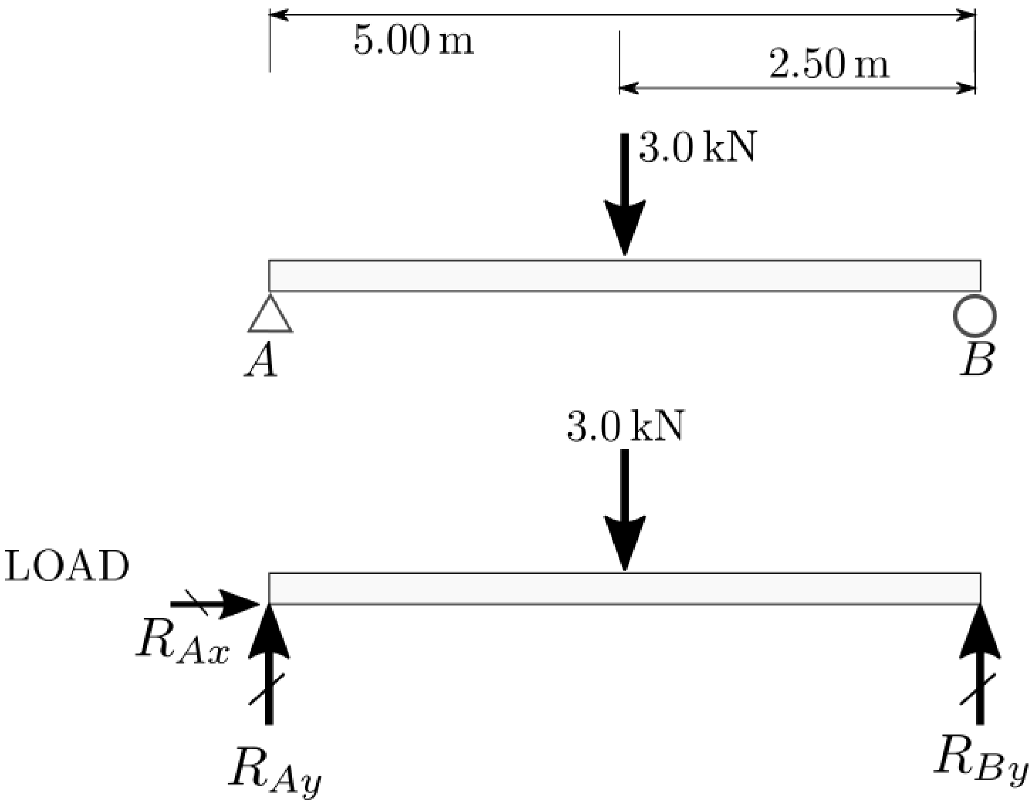
(b) A simply-supported beam of length $L = 5.00\,\text{m}$ carries a single concentrated load 3.0 kN at mid-span. Solve for the three "reaction" forces exerted by the supports: the upward force $\boxed{R_{Ay}}$ exerted by the support at A, the horizontal force $\boxed{R_{Ax}}$ exerted by the support at A, and the upward force $\boxed{R_{By}}$ exerted by the support at B.



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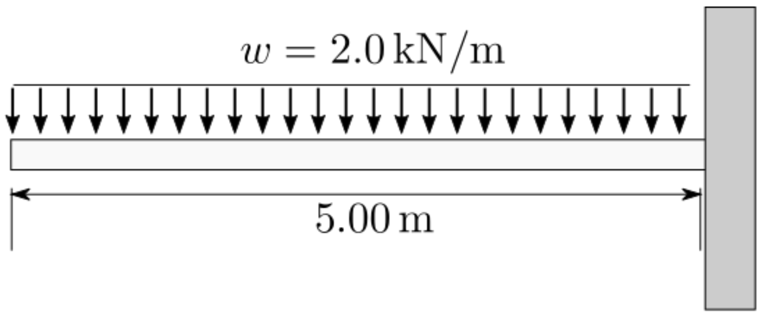
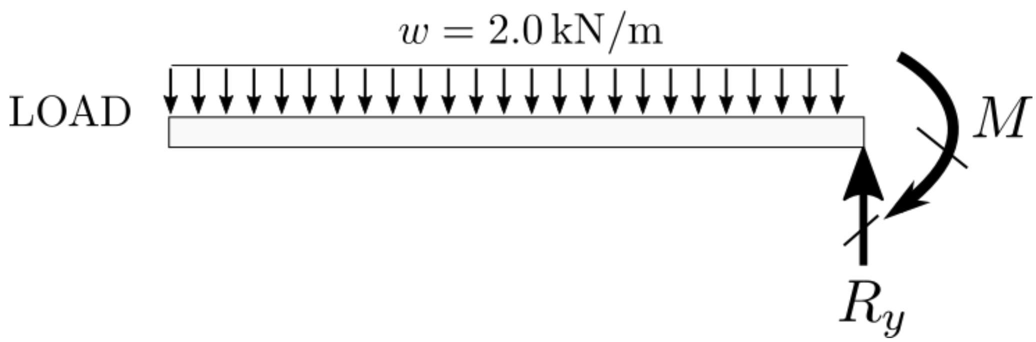
(c) A cantilever beam of length $L = 5.00\,\mathrm{m}$ carries a uniform distributed load $w = 2.00\,\mathrm{kN/m}$ along its entire length. Solve for the upward "reaction" force $\boxed{R_y}$ exerted by the wall on the right end of the beam. Also solve for the moment (torque) $\boxed{M}$ exerted by the wall on the beam: to do this, use the right end of the beam (at the wall) as a pivot, and let M be whatever torque is needed to make all torques (moments) add up to zero for equilibrium.





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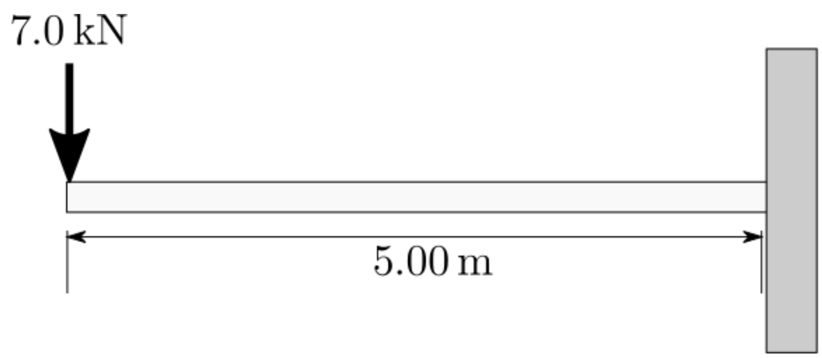
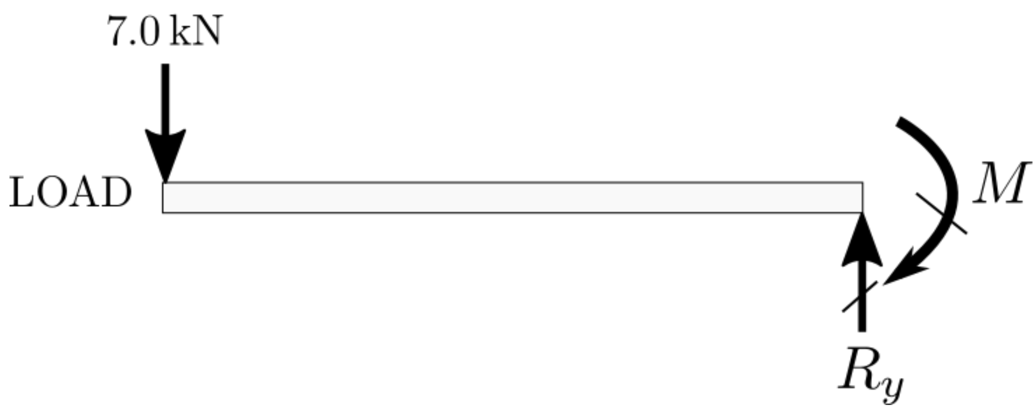
(d) A cantilever beam of length $L = 5.00\,\mathrm{m}$ carries a single concentrated load of $w = 7.0\,\mathrm{kN}$ at its left end. Solve for the upward "reaction" force $\boxed{R_y}$ exerted by the wall on the right end of the beam. Also solve for the moment (torque) $\boxed{M}$ exerted by the wall on the beam: to do this, use the right end of the beam (at the wall) as a pivot, and let M be whatever torque is needed to make all torques (moments) add up to zero for equilibrium.





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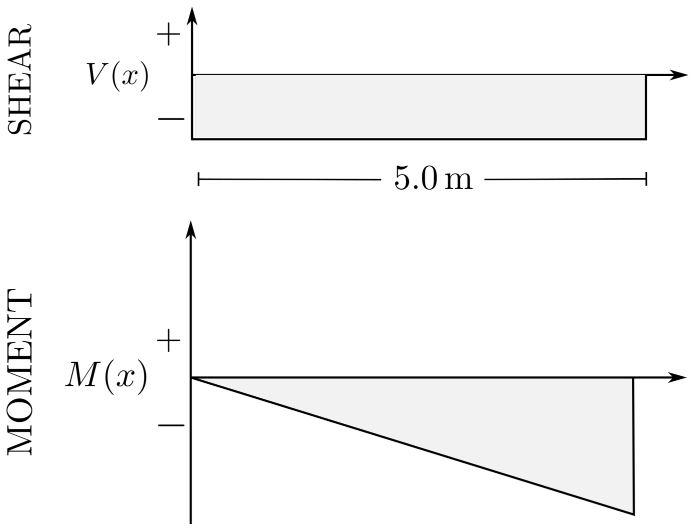
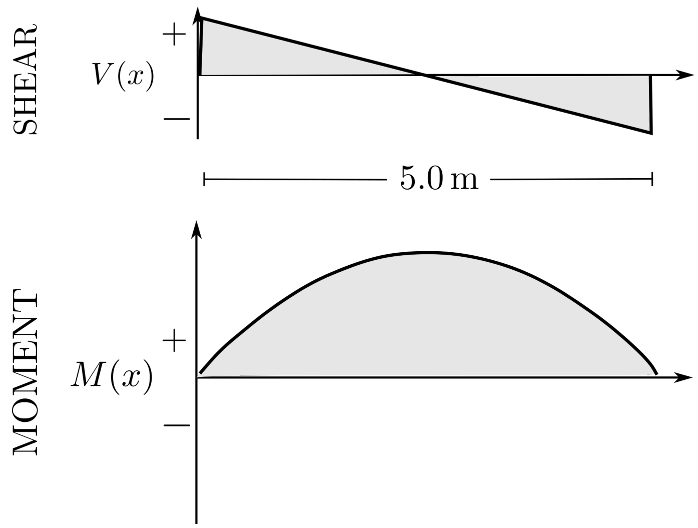
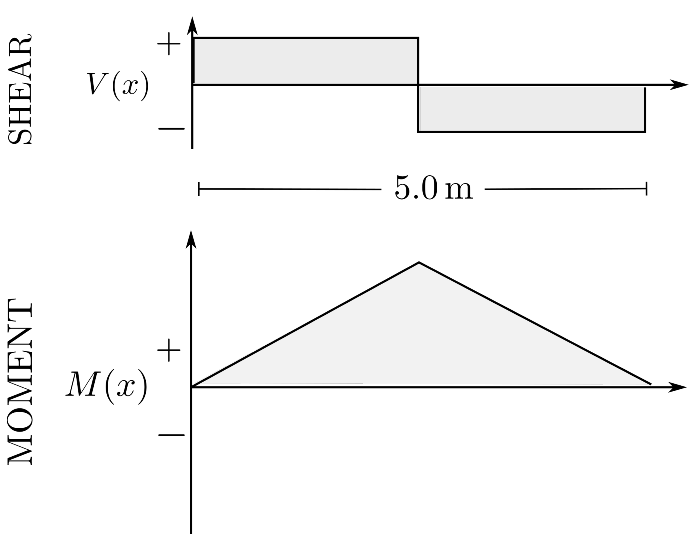
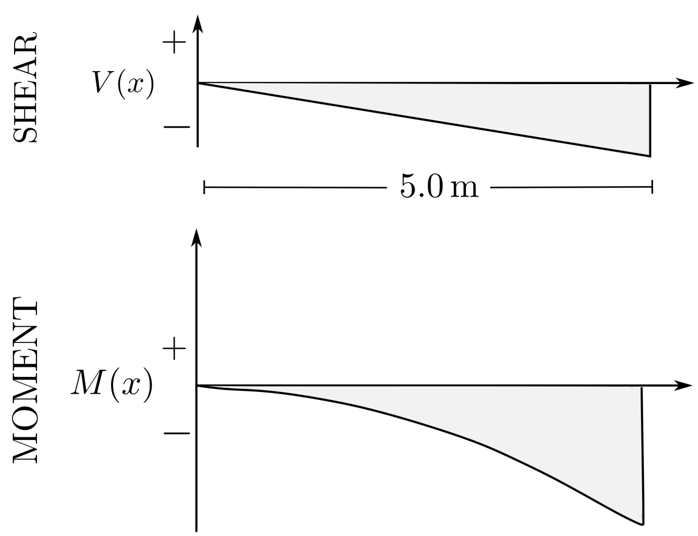
(e) For each of the following four shear/moment diagram pairs, write in the letter **A**, **B**, **C**, or **D** to indicate whether the graphs correspond to the beam shown in part (a), part (b), part (c), or part (d) of this problem (above).





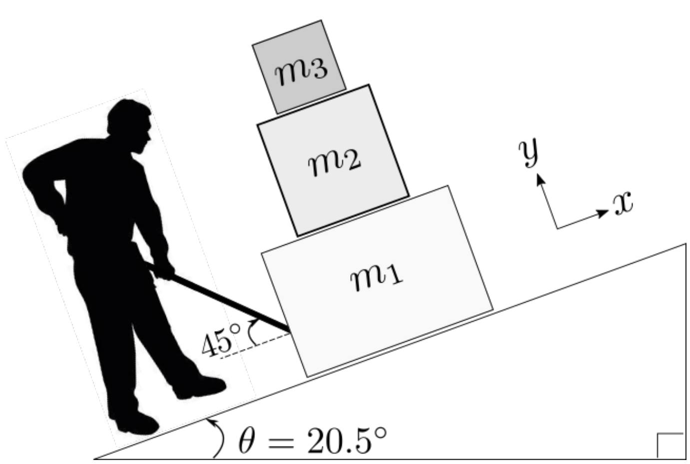


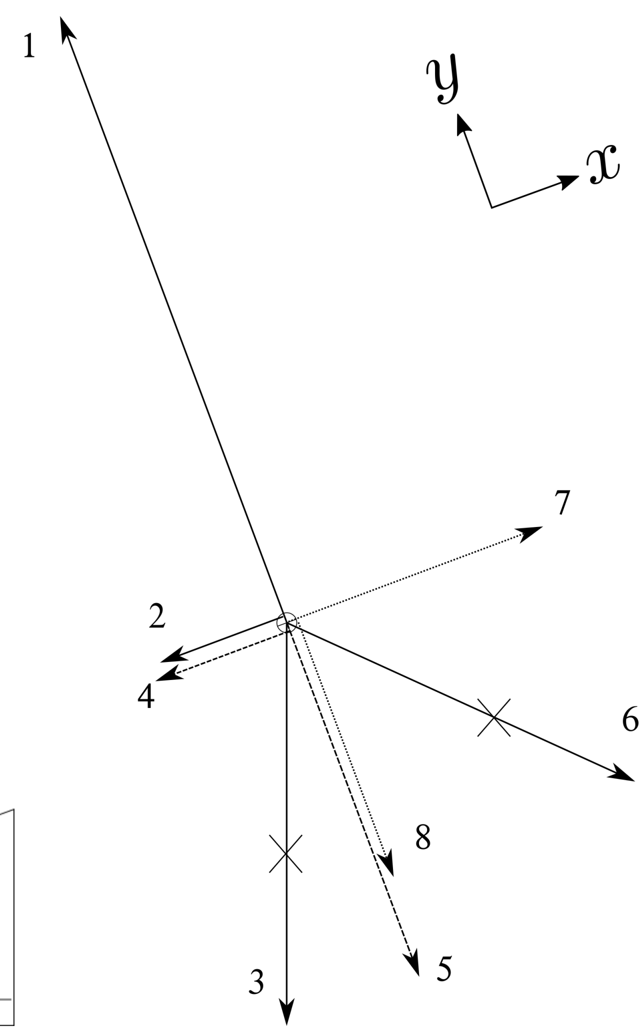


4. (30%) A worker (**W**) is **pushing** three boxes, stacked on top of one another, **up** a 20.5° ramp as shown, by pushing on a broomstick that is firmly attached to box **1** (the lower box). The force applied by the worker to box **1** (via the broomstick) points along the axis of the broomstick. The broomstick is inclined at a 45° angle with respect to the ramp. Box **1** has mass $m_1 = 50.0\,\text{kg}$, box **2** has mass $m_2 = 30.0\,\text{kg}$, and box **3** has mass $m_3 = 20.0\,\text{kg}$. The three boxes move together at **constant speed:** boxes **2** and **3** do not move with respect to box **1**. The coefficient of kinetic friction between the ramp (**R**) and the box **1** is $\mu_k = 0.200$, and the coefficient of static friction between the boxes **1** and **2** and between boxes **2** and **3** is $\mu_s = 0.800$. Note that $\sin(20.5°) \approx 0.350$, $\cos(20.5°) \approx 0.937$, $\sin(45°) = \cos(45°) \approx 0.707$. Let the x axis denote the uphill direction, and let the y axis denote the normal direction pointing away from the ramp surface, as shown. Use $\boxed{g = 10.0\,\text{m/s}^2}$ so that we get the same numbers.

(a) On the right, I have drawn a free-body diagram for the three-box *system* (**S**). For each of the following Mazur-style force labels, write the corresponding arrow number 1 . . . 8. (**C**=contact, **N**=normal, **K**=kinetic, **G**=gravity, **E**=Earth, **W**=worker, **S**=system, **R**=ramp). Include x and y components of decomposed forces, using the x and y axes shown.

F^G_{ES} $\quad$ $F^G_{ES,x}$ $\quad$ $F^G_{ES,y}$ $\quad$ F^N_{RS} $\quad$ F^C_{WS} $\quad$ $F^C_{WS,x}$ $\quad$ $F^C_{WS,y}$ $\quad$ F^K_{RS}





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(b) Write Newton's second law (both components, using my x and y axes) for the three-box system: $m_{\text{system}} a_x = \sum F_x$ and $m_{\text{system}} a_y = \sum F_y$. Solve for the two unknowns: F^N_{RS} and F^C_{WS}. Note that $\sin(20.5°) \approx 0.350$, $\cos(20.5°) \approx 0.937$, $\sin(45°) = \cos(45°) \approx 0.707$.

(c) Write the magnitude (a number, in newtons) of each of the following forces or force components from the FBD in part (a). Don't worry about minus signs for $\pm x$, $\pm y$, etc.

$F^G_{ES} =$

$F^G_{ES,x} =$

$F^G_{ES,y} =$

$F^C_{WS} =$

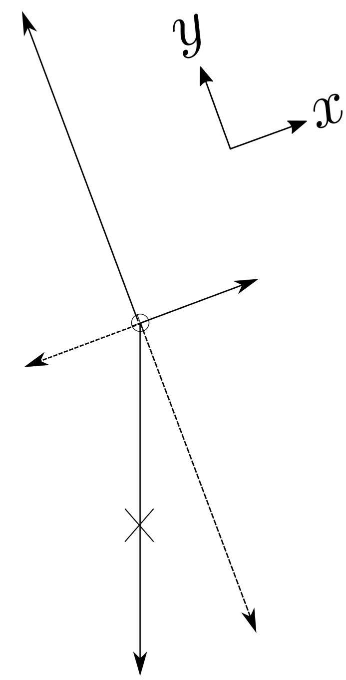
$F^C_{WS,x} =$

$F^C_{WS,y} =$

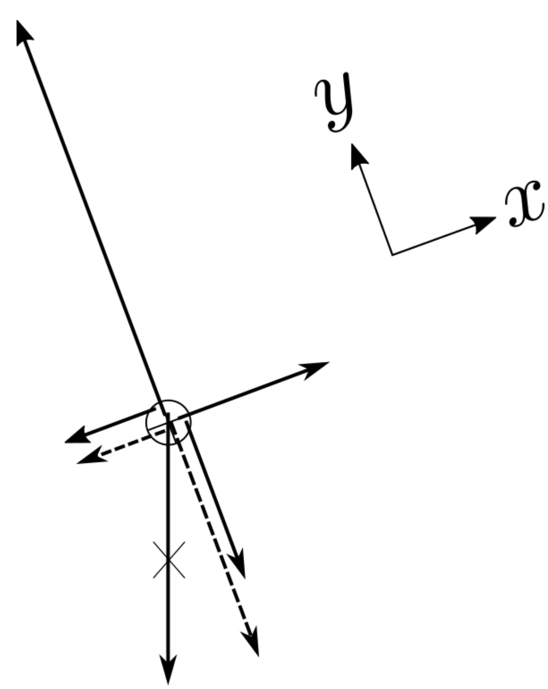
$F^N_{RS} =$

$F^K_{RS} =$

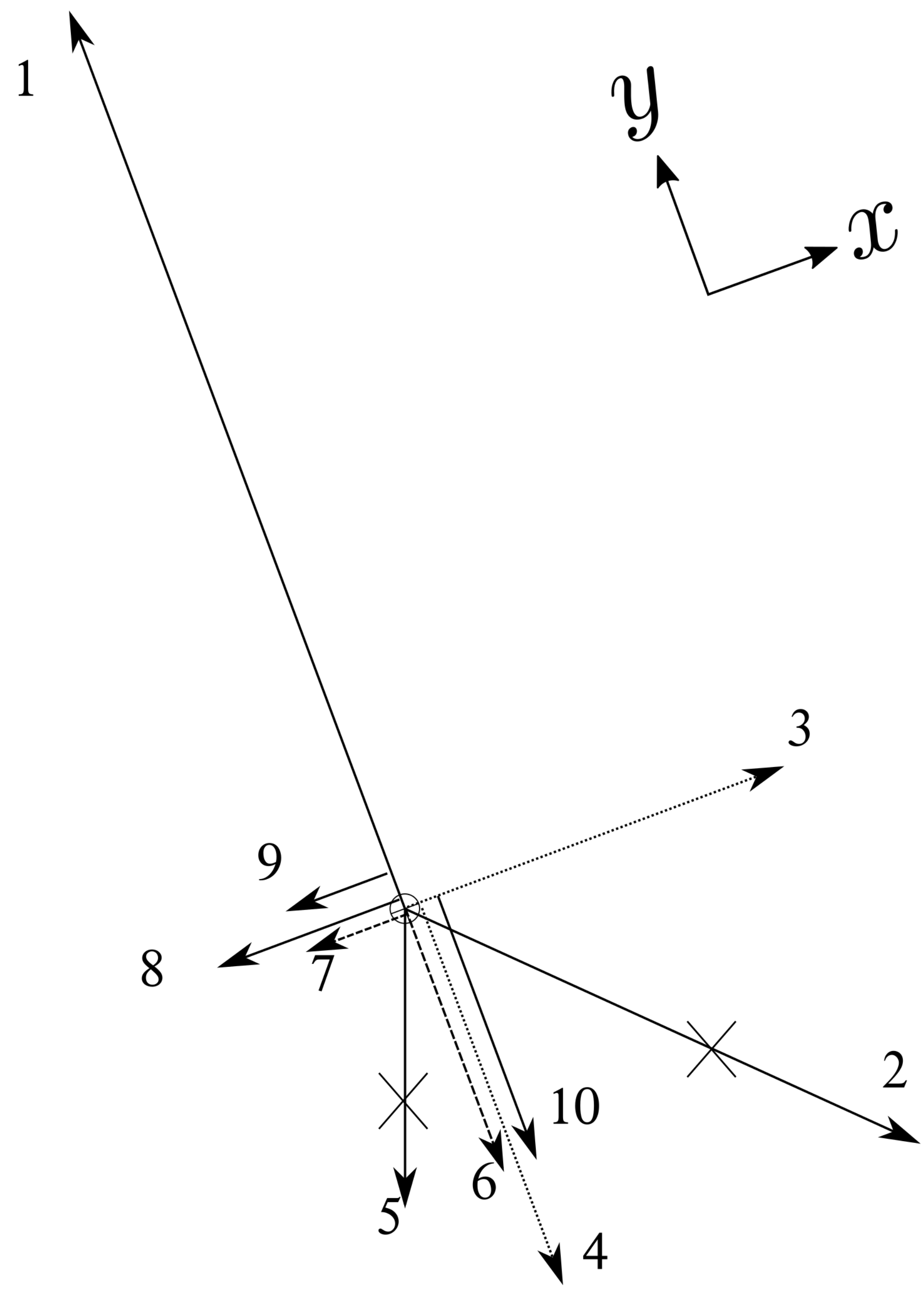
(d) Fill in my free-body diagram for box **3** (which is the top box, $m_3 = 20.0\,\mathrm{kg}$). Give every force a name that makes it clear what kind of force it is, what object is exerting the force, and on what object it is being exerted. **Include the numerical value of every force, e.g.** $200\,\mathrm{N}$**.**



(e) Fill in my free-body diagram for box **2** (which is the middle box, $m_2 = 30.0\,\mathrm{kg}$). Give every force a name that makes it clear what kind of force it is, what object is exerting the force, and on what object it is being exerted. **Include the numerical value of every force, e.g.** $300\,\mathrm{N}$**.**



(f) Fill in my free-body diagram for box **1** (which is the bottom box, $m_1 = 50.0\,\mathrm{kg}$). For each of the force arrows numbered $1 \ldots 10$, give the force a name that makes it clear what kind of force it is, what object is exerting the force, and on what object it is being exerted. **If you have time, also include the numerical value of every force, e.g.** $500\,\mathrm{N}$**,** which lets you check that everything adds up correctly.



Have a safe and happy winter break!

Possibly useful equations

$$\cos(30°) = \frac{\sqrt{3}}{2} \approx 0.866 \qquad \sin(30°) = \frac{1}{2} \qquad \tan(30°) = \frac{1}{\sqrt{3}} \approx 0.577$$

$$\cos(60°) = \frac{1}{2} \qquad \sin(60°) = \frac{\sqrt{3}}{2} \approx 0.866 \qquad \tan(60°) = \sqrt{3} \approx 1.732$$

$$\cos(36.9°) = \frac{4}{5} \qquad \sin(36.9°) = \frac{3}{5} \qquad \tan(36.9°) = \frac{3}{4}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$f_{\text{spring}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad f_{\text{pendulum}} = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$$\sum \vec{F} = m\vec{a} \qquad \sum \vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} \qquad \vec{p} = m\vec{v}$$

$$F_c = \frac{mv^2}{r} \qquad F_c = m\omega^2 r \qquad v = \omega r$$

$$F^K = \mu^K F^N \qquad F^s \le \mu_s F^N$$

$$F_x^{\text{ spring}} = -k\,(x - x_0)$$

$$F_y^{\text{ grav}} = -mg \qquad g = 9.8\ \text{m/s}^2$$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \tau = rF\sin\theta = r_\perp F = rF_\perp$$

$$\frac{F}{A} = (\text{stress}) = (E)\,(\text{strain}) = E\,\frac{\Delta L}{L_0}$$

$$V = \frac{\mathrm{d}M}{\mathrm{d}x} \qquad (M_2 - M_1) = (x_2 - x_1)\overline{V}_{1\to 2} \qquad w = -\frac{\mathrm{d}V}{\mathrm{d}x} \qquad V(x) = \sum_{0\to x} F_y(\text{up minus down})$$