- Before third class meeting:
- first watch this video
- then skim through Mazur chapter 3, focusing mainly on the concepts half, and glossing over most equations

## Defining acceleration

- Last week, we defined velocity as the rate of change of position with respect to time

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$$

(considering only the $x$ component for now), and we learned to identify $v_x$ visually as the slope on a graph of $x(t)$
- Moving at constant velocity is not very interesting! So we need to be able to talk about changes in velocity.
- The rate of change of velocity with respect to time is called acceleration:

$$a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t}$$

- While acceleration can also vary with time (!), there are many situations in which constant acceleration ($a_x = \mathrm{constant}$) gives a good description of the motion. We'll see soon what math lets us conclude, if we start with $a_x = \mathrm{constant}$.

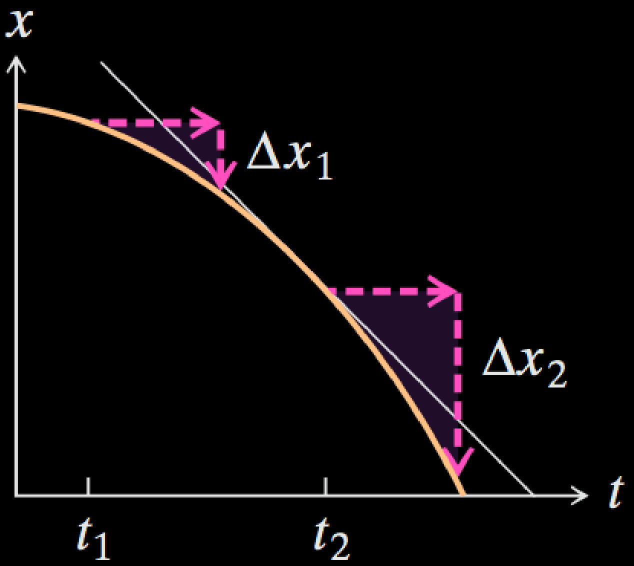
At time $t_2$ in the position-vs-time graph below, the object is

(A) not moving

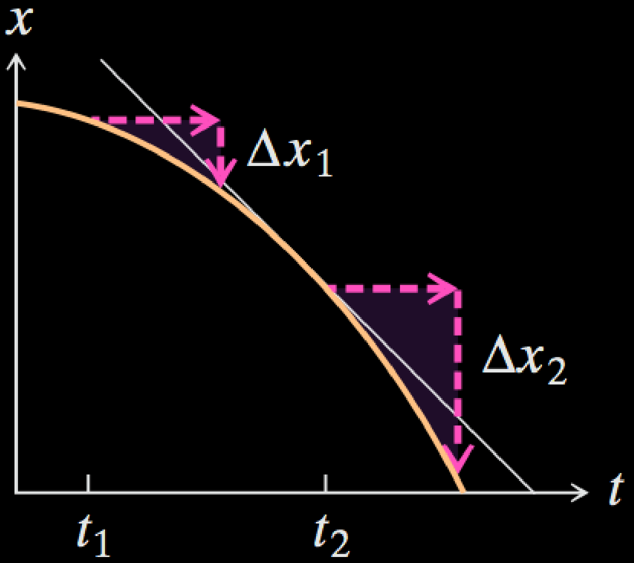
(B) moving at constant speed

(C) speeding up

(D) slowing down

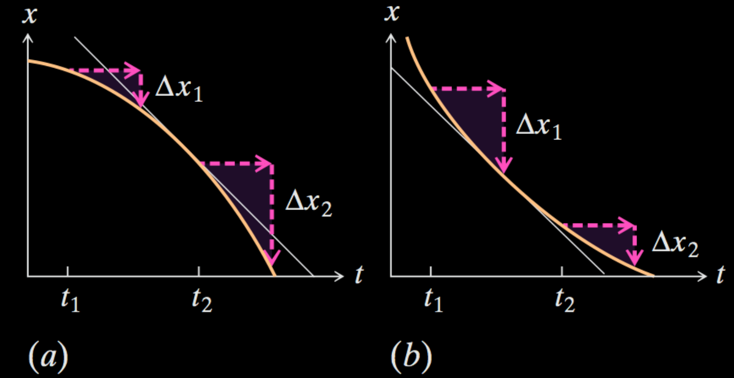


At time $t_2$ in the position-vs-time graph below, is $v_x$ (the $x$ component of velocity) is




(A) zero
(B) not changing
(C) increasing
(D) decreasing

The $x$ component of acceleration in these two graphs is




(A) positive in (a), negative in (b)

(B) negative in (a), positive in (b)

(C) negative in both (a) and (b)

(D) positive in both (a) and (b)

(E) zero in both (a) and (b)

# Accelerating under gravity's influence

- One important situation in which constant acceleration ($a_x = \mathrm{constant}$) gives a good description of the motion is "free fall" near Earth's surface.
- (Until Chapter 10, we will use only one axis in any given problem, and we will call that axis $x$. So for free-fall problems, for now, the $x$ axis will be vertical, pointing upward.)
- *Free fall* is the motion of an object subject only to the influence of gravity.
  - Not being pushed or held by your hand or by the ground
  - When air resistance is small enough to neglect
- Close to Earth's surface, an object in free fall experiences a constant acceleration, of magnitude $|\vec{a}| = 9.8\ \mathrm{m/s^2}$ and pointing in the *downward* direction.
- If we define the $x$ axis to point upward (as we often will, for free-fall problems before Ch10), then $a_x = -9.8\ \mathrm{m/s^2}$.
- Since we see the quantity $9.8\ \mathrm{m/s^2}$ so often, we give it a name: $g = 9.8\ \mathrm{m/s^2}$. Then $a_x = -g$.

# (Checkpoint 3.7)

Let's pause here to go through Checkpoint 3.7 together.

- Does the speed of a falling object (A) increase or (B) decrease?
- If the positive $x$ axis points up, does $v_x$ (A) increase or (B) decrease as the object falls?
- is the $x$ component of the acceleration (A) positive or (B) negative?

*Pause to think for a moment, then we'll compare answers.* To aid your thinking, you may want to graph $v_x(t)$, for a falling object – where we define the $x$ axis to point *upward*.

# Let's do what Galileo could only imagine doing!

- Let's see if different objects really do fall with the same acceleration
  $$a_x = -g$$
  if we are able to remove the effects of air resistance.

# Equations we can derive from $a_x = \mathrm{constant}$

- **You don't need to know how to do these derivations,** but if you like calculus, you might enjoy seeing where these often-used results come from.
- We defined $a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t}$ and $v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$, without worrying so far about whether or not $a_x$ is changing with time.
- Integrating the first equation $\left(\frac{\mathrm{d}v_x}{\mathrm{d}t} = a_x\right)$ over time,

$$v_x(t) = v_{x,i} + \int_0^t a_x \, \mathrm{d}t$$

- If $a_x = \mathrm{constant}$, then this integral becomes easy to do:

$$v_x(t) = v_{x,i} + a_x t$$

- We can also integrate the equation $\left(\frac{\mathrm{d}x}{\mathrm{d}t} = v_x\right)$ over time:

$$x(t) = x_i + \int_0^t v_x \, \mathrm{d}t$$

keeping in mind that $v_x$ (unlike $a_x$) is changing with time

# Equations we can derive from $a_x = \mathrm{constant}$

$$v_x(t) = v_{x,i} + a_x t$$

$$x(t) = x_i + \int_0^t v_x \, \mathrm{d}t$$

- Plugging our $v_x(t)$ result into the second integal:

$$x(t) = x_i + \int_0^t (v_{xi} + a_x t) \, \mathrm{d}t$$

$$x(t) = x_i + v_{x,i}\, t + \frac{1}{2} a_x \, t^2$$

## Equations we can derive from $a_x = \mathrm{constant}$

- That's all there is to it. Just writing down the assumption that $a_x$ is constant allows us to integrate twice to get two results that you will use many times:

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

- If you plug one of these equations into the other, you can eliminate $t$ to get one more very useful result

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x \left(x_f - x_i\right)$$

- This last one is helpful e.g. to know how fast the dropped steel ball is traveling at the instant before it hits the ground.
- My point is that these equations are just the result of taking $a_x = \mathrm{constant}$ and doing some math.

# Inclined planes

- Falling to the ground at $a_x = -g$ happens so quickly that it can be difficult to see exactly what is happening.
- Maybe there is a way to "fall" in slow motion?
- Yes! We can slide down a hill.

$$|g| \;\rightarrow\; |g \sin\theta|$$

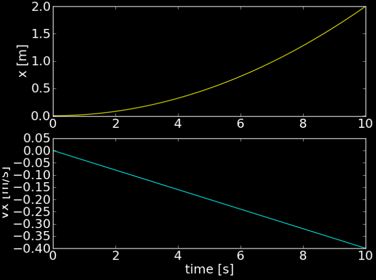
(We'll see in Chapter 10 why it's $\sin\theta$ here. Don't worry.)

- To get the $\pm$ sign right, you have to choose which direction to draw the $x$ axis. Eric chooses the $x$ axis to point *downhill*

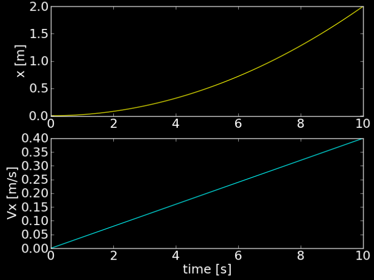
$$a_x = +g \sin\theta$$

- Let's look at the **inclined air track** and figure out which way it defines the $x$ axis to point . . . .
- We'll see that the $x$ axis points *downhill*, and the point on the top of the ramp is called $x = 0$.

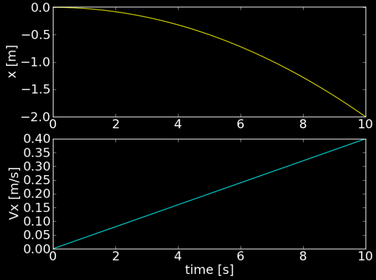
Which of the following shows the expected shapes of $x(t)$ [yellow] and $v_x(t)$ [cyan] if I release the cart (at rest) from $x = 0$?



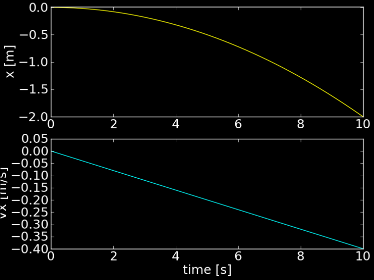

A




B

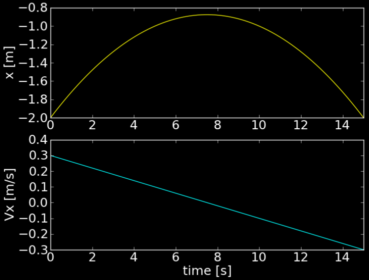



C

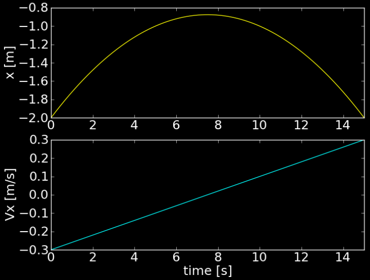



D

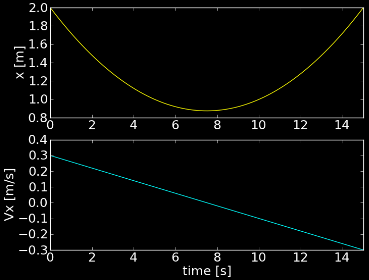
Which of the following shows the expected shapes of $x(t)$ [yellow] and $v_x(t)$ [cyan] if I shove the cart upward starting from $x = +2$ m?



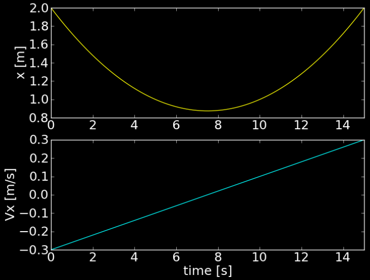

A




B




C




D

I shove the cart uphill and watch it travel up and back. We define the $x$ axis to point *downhill*. At the top of its trajectory (where it turns around), $v_x$ is

(A) positive

(B) negative

(C) zero

(D) infinite

(E) undefined

I shove the cart uphill and watch it travel up and back. We define the $x$ axis to point *downhill*. At the top of its trajectory (where it turns around), $a_x$ is

(A) positive

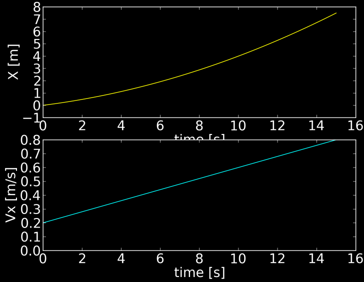
(B) negative

(C) zero

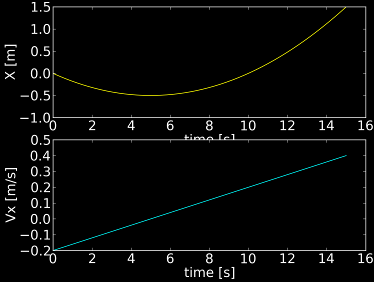
(D) infinite

(E) undefined

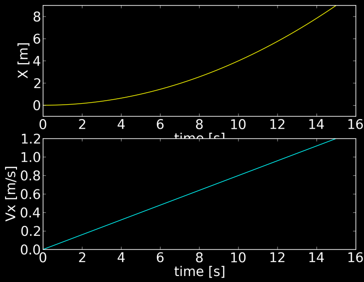
Which of the following shows the expected shapes of $x(t)$ [yellow] and $v_x(t)$ [cyan] if I **shove** the cart gently downward from $x = 0\ \mathrm{m}$?



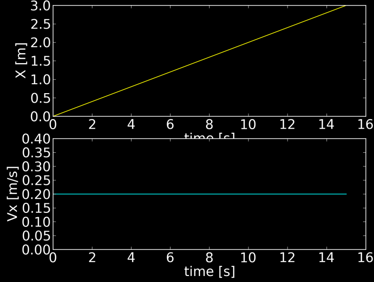

A




B



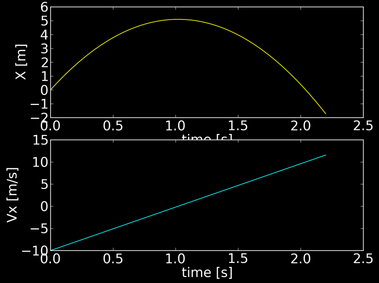

C



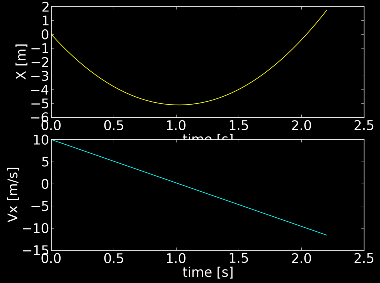

D

In a past year, someone asked an excellent question after class about the difference, in the previous slide, between scenario (A) and scenario (C). Let's ponder that.

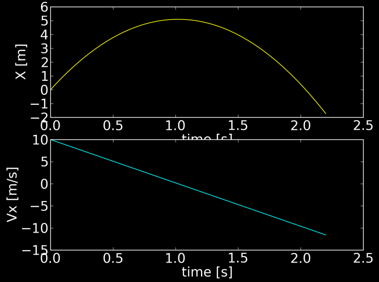
What are the expected shapes of $x(t)$ [yellow] and $v_x(t)$ [cyan] for a **basketball tossed upward**, when the $x$ axis points *upward*?



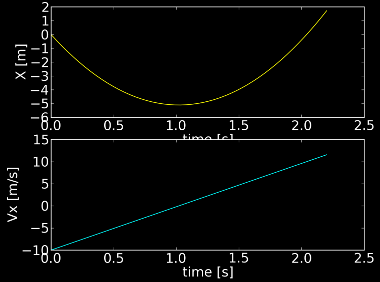

A




B




C




D

# Basketball tossed upward

What are the values of $v_x$ and $a_x$ at the top of the basketball's trajectory (assuming that the $x$ axis points upward)?

(A) $v_x < 0$, $a_x = -9.8\ \mathrm{m/s^2}$

(B) $v_x < 0$, $a_x = 0$

(C) $v_x = 0$, $a_x = -9.8\ \mathrm{m/s^2}$

(D) $v_x = 0$, $a_x = 0$

(E) $v_x = 0$, $a_x$ is undefined

# Ball thrown downward

If you stand up high and release an object with a downward shove, in the absence of air resistance, the motion (after release, but before hitting the ground) is best described by

(A) $v_x < 0$, $a_x = -9.8\ \mathrm{m/s^2}$

(B) $v_x < 0$, $a_x = 0$

(C) $v_x = 0$, $a_x = -9.8\ \mathrm{m/s^2}$

(D) $v_x = 0$, $a_x = 0$

(E) $v_x = 0$, $a_x$ is undefined

(Where we've defined the $x$ axis to point *upward* here.)

I'm going to drop the basketball from a few meters in the air, and I'll let it bounce twice before I catch it. Try for yourself to draw a graph of $v_x(t)$ (velocity) and a graph of $a_x(t)$ (acceleration), spanning the time from release to catch. Let the $x$ axis point upward. Don't worry about labeling the axes with numerical values, but do be clear about positive vs. zero vs. negative values. You will repeat a similar exercise in class with your group.

# Reminder

velocity is rate of change of position: $v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$

acceleration is rate of change of velocity: $a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t}$

**If** acceleration is **constant**, then: (write these on board)

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x \left(x_f - x_i\right)$$

Important cases for which $a_x$ is constant:

free fall: $a_x = -g$
($x$ axis points up)

inclined plane: $a_x = +g \sin\theta$
($x$ axis points downhill)

Q: If I stand $h = 20\,\mathrm{m}$ above the ground and release a steel ball from rest, how long does it take to reach the ground? (Hint: to avoid using a calculator, you can approximate $g \approx 10\ \mathrm{m/s^2}$.)

(A) 2.0 s

(B) 1.5 s

(C) 1.0 s

(D) 0.50 s

(E) 0.25 s

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$0 = h + 0 - \frac{1}{2} g t^2$$

Q: If I stand 20 m above the ground and release a steel ball from rest, what is its velocity at the instant just before it reaches the ground? (Use $g = 10\ \mathrm{m/s^2}$ to simplify math.)

(A) 10 m/s, pointing downward

(B) 15 m/s, pointing downward

(C) 20 m/s, pointing downward

(D) 40 m/s, pointing downward

(E) 40 m/s, pointing upward

If you already solved for $t$ in the previous question then:

$$v_{x,f} = v_{x,i} + a_x t$$

$$v_{x,f} = 0 - gt$$

Or if you don't already know $t$ then:

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\left(x_f - x_i\right)$$

$$v_{x,f}^2 = 0^2 + 2(-g)\left(0 - h\right)$$

A box is at the lower end of a frictionless ramp of length $L = 10\ \mathrm{m}$ that makes a nonzero angle $\theta = 30^\circ$ with the horizontal. A worker wants to give the box a shove so that it just reaches the top of the ramp. How fast must the box be going immediately after the shove (assumed to be instantaneous) for it to reach its goal? Remember $\sin 30^\circ = \frac{1}{2}$ and use $g \approx 10\ \mathrm{m/s^2}$ to keep the math simple.

(A) 1.0 m/s

(B) 5.0 m/s

(C) 7.0 m/s

(D) 10 m/s

(E) 20 m/s

Reminder (on board): results derived from $\boxed{a_x = \mathrm{constant}}$.

A box is at the lower end of a frictionless ramp of length $L = 10\ \mathrm{m}$ that makes a nonzero angle $\theta = 30^\circ$ with the horizontal. A worker wants to give the box a shove so that it just reaches the top of the ramp. She shoves the box, as we worked out on the previous page: the box's initial speed is 10 m/s. (Again, use $g \approx 10\ \mathrm{m/s^2}$ to keep the math simple.)

What is the box's speed when it is halfway up the ramp?

(A) 1.0 m/s

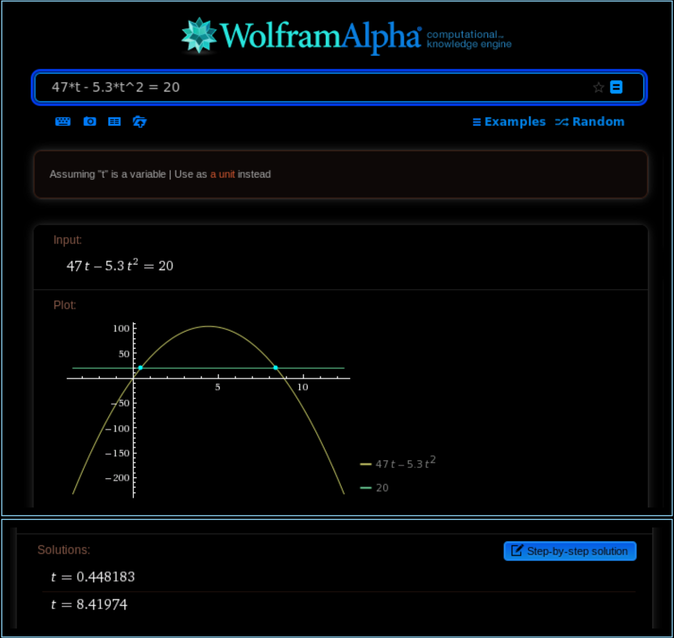
(B) 5.0 m/s

(C) 7.1 m/s

(D) 10.0 m/s

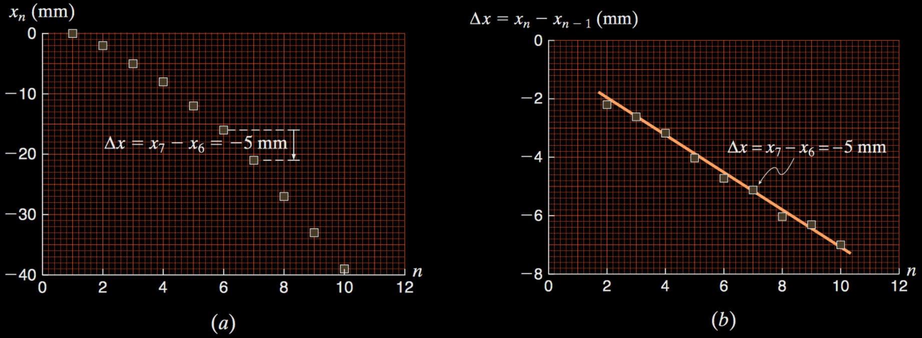
(E) 20.0 m/s

# Another trick: Wolfram Alpha knows the quadratic formula



Potential sources of confusion from **today's** reading (Chapter 3)

- Inclined planes are new to many people.
- How do you draw a motion diagram?
- Don't follow Eric's reasoning about what is happening ($v_x$, $a_x$) at the very top of the motion for a ball tossed upward.
- Some of the mathy parts at the end are hard to follow.
- For checkpoint 3.6, you have to stare at Figure 3.6b for a while before you see that, since the points are all equal steps in time, the quantity being graphed is proportional to $v_x$, the $x$ component of velocity.



# A variation on a worksheet problem

A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (The worksheet will probably ask, "How tall is the building?") Which of the following statements is true? (Let $x$-axis point upward.)

(A) The rock's average velocity $v_{x,\mathrm{av}}$ during the last 1.0 s of its fall is $-21.5\ \mathrm{m/s}$.

(B) The rock's instantaneous velocity $v_x$ one second before it hits the ground is $-21.5\ \mathrm{m/s}$.

(C) The rock's instantaneous velocity $v_x$ at the instant just before it hits the ground is $-21.5\ \mathrm{m/s}$.

(D) Statements (A), (B), (C) are all true.

(E) Statements (A), (B), (C) are all false.

A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (Worksheet will probably ask, "How tall is the building?") At the instant just before hitting the ground, the rock's **speed** is

(A) $21.5\ \mathrm{m/s}$

(B) $-21.5\ \mathrm{m/s}$

(C) Somewhat faster than $21.5\ \mathrm{m/s}$

(D) Somewhat slower than $21.5\ \mathrm{m/s}$

(E) We don't have enough information to decide.

A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (Worksheet will probably ask, "How tall is the building?") Let the building height be $h$. Let the total time the rock falls be $t$. Which is a true statement about the problem?

(A) $h - \frac{1}{2}gt^2 = 0$

(B) $0 - gt = -21.5\ \mathrm{m/s}$

(C) $h - \frac{1}{2}g[t - 1.0\ \mathrm{s}]^2 = 21.5\ \mathrm{m}$

(D) $0 - g[t - 1.0\ \mathrm{s}] = -21.5\ \mathrm{m/s}$

(E) (A) and (B) are both true.

(F) (A) and (C) are both true.

(G) (A), (B), (C), and (D) are all true.

(H) (A), (B), (C), and (D) are all false.