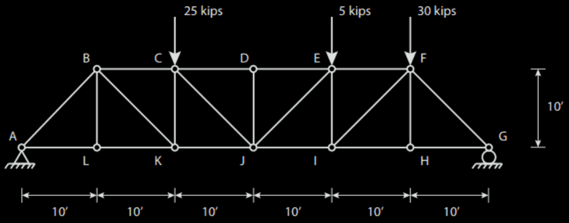
- begin video preceding ws06
- There are very few results from chapter 6 that you will need to remember, so this time watch the video lecture first, then consider whether or not to **skim** Mazur chapter 6. Having first watched the video will, I hope, help to you know where to focus your attention as you skim through the chapter. (Or you might just prefer not to skim it at all.)
- The main reason for chapter 6 is to build some intuition that will help us, once we reach chapter 8, to better understand forces.
- Your patience will soon be rewarded: once we reach chapter 8, we'll be using Newton's three laws. We'll then continue to use them throughout the rest of the course.

- Ch6 and Ch7 take some time to read, but they don't add many new equations ("quantitative results") to learn how to work with. Ch5 (energy) and Ch8 (force) get more class time than Ch6 and Ch7. So there is really one **substantive** chapter this week and one next week.
- We are going quickly through the early chapters & will slow down for the more difficult topics in Ch10,11,12. The faster pace now lets us make time for "structures" applications later.

# Where are we (going)?

3) Find the forces in members BK and CJ. State whether they are in tension or compression.




To analyze structures, you need a thorough understanding of **forces**, **torques** (a.k.a. "moments" of forces), and **vectors**.

# Where are we (going)?

(Richard Wesley: "A course should tell a story.")

- To analyze structures, you need a thorough understanding of **forces**, **torques** (a.k.a. "moments" of forces), and **vectors**.
- To understand forces well, you need a solid grasp of
  - How forces affect motion.
  - How different forces relate to one another.
  - How objects interact with one another via forces.
- In ch2–3, we studied the key concepts of motion: position, velocity, acceleration, etc.
- In ch4–5, we studied two key conservation laws (momentum, energy) and some of the restrictions they place on how colliding objects can interact with one another.
- Finally in ch8 we'll discuss forces! We're preparing your mind for forces in ch6–7 by learning a few more of the restrictions imposed, as a consequence of momentum & energy conservation, on how objects can interact with one another.

"Inertial reference frames:" ch06 topic that baffles most students.

- Imagine yourself trying to pour a cup of coffee while standing up on an airplane that is cruising smoothly at constant velocity. No problem.
- Now imagine trying to pour coffee while the airplane is taking off, landing, turning sharply, or experiencing turbulence. Your eye and hand are working from the perspective of a non-inertial reference frame — a set of coordinate axes that is accelerating w.r.t. "the fixed stars." The usual rules of physics don't work. To use the usual rules of physics, you have to analyze the situation from the perspective of an inertial frame.
- If you want more detail on frames of reference, watch this 30-minute educational video from 1960. Email me a few sentences detailing what you learned for extra credit.


Physics: Frames of Reference 1960 PSSC Physical Science Study Committee; Reference & Relativity

https://youtu.be/bJMYoj4hHqU

Textbook Chapter 6 includes a few key ideas, some of which are obscured by the notation and equations.

Law of inertia — this is big deal! (a.k.a. Newton's law #1.)

- In an inertial reference frame, an isolated object at rest remains at rest, and an isolated object in motion keeps moving at a constant velocity.
- You can't "feel" the difference between being at rest in Earth's frame vs. being at rest in some other inertial reference frame.
- Imagine that you're sitting on an airplane, pouring a cup of coffee, juggling, or maybe just tossing a single ball into the air and catching it. If the airplane is cruising at constant velocity, is all of this activity feasible?
- What if you try the same thing while the airplane is rapidly screeching to a halt on the runway immediately after landing?
- (Illustrate with "ball popper.")

The **law of inertia** states that in an inertial reference frame, any isolated object that is at rest remains at rest, and any isolated object in motion keeps moving at a constant velocity.

Imagine that you are in a jet airplane that has just landed and is in the midst of screeching to a stop on the runway. (You are wearing your seatbelt!) Is the frame of reference in which the airplane is at rest an inertial frame? Will a marble, initially sitting at rest on the floor of the airplane, as observed from the frame in which the airplane is at rest (i.e. as observed by a passenger, with the window shades down), remain at rest as the airplane screeches to a stop?

(A) Yes (inertial frame) and Yes (marble)
(B) Yes (inertial frame) and No (marble)
(C) No (inertial frame) and Yes (marble)
(D) No (inertial frame) and No (marble)

Suppose I'm a passenger on a train that is speeding toward NYC at 40 m/s (heading "north"). In search of coffee, I walk toward the back of the train at 2 m/s, just as the train whizzes past Princeton Junction. From the perspective of a passenger watching me from the train platform, my velocity is

(A) 2 m/s northward
(B) 2 m/s southward
(C) 42 m/s northward
(D) 40 m/s northward
(E) 38 m/s northward

$$\vec{v}_{\mathrm{Earth,me}} = \vec{v}_{\mathrm{Earth,Train}} + \vec{v}_{\mathrm{Train,me}}$$

"(My velocity w.r.t. Earth) =
(Train's velocity w.r.t. Earth) + (my velocity w.r.t. Train)"

I'm driving east at 50 kph. A little kid looks out the window of a westbound car that is going 40 kph. From the kid's point of view, what is my velocity?

(A) 10 kph east
(B) 40 kph east
(C) 50 kph east
(D) 90 kph east
(E) 10 kph west
(F) 40 kph west
(G) 50 kph west
(H) 90 kph west

I'm driving east at 50 kph. A truck driving east at 60 kph overtakes me. As I look out my window, how fast does the truck appear to be moving?

(a) 10 kph
(b) 50 kph
(c) 60 kph
(d) 110 kph

# More chapter 6 key ideas

- Center of mass: basically a weighted-average of positons.

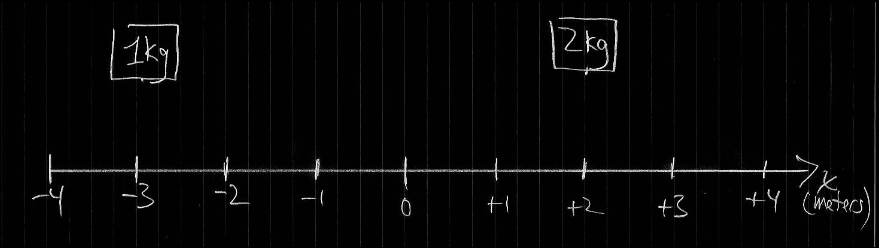
$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

- CoM of an object lies along axis of symmetry (if there is one).
- When analyzing the motion of a complicated object (composed of many pieces), it is often useful to consider separately the motion of its CoM and the motion of the various internal parts w.r.t. the CoM.
- Illustrate by tossing complicated object in the air.

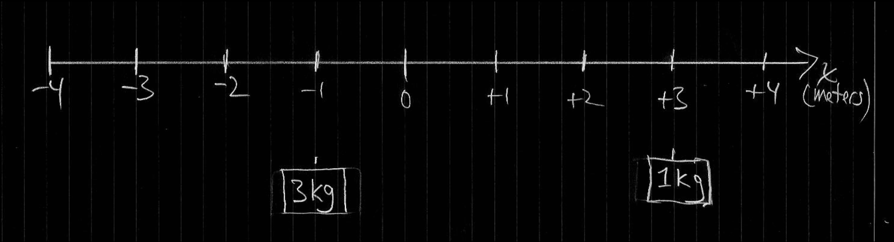
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

At what value of $x$ is the CoM of this pair of masses?



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

At what value of $x$ is the CoM of this pair of masses?



# More chapter 6 key ideas

- "Center-of-mass velocity" is the velocity of the CoM of a system of objects:

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$v_{x,cm} = \frac{m_1 v_{x1} + m_2 v_{x2} + \cdots}{m_1 + m_2 + \cdots}$$

- An isolated system's CoM velocity cannot change!
- You can see this by noticing that the numerator in $\vec{v}_{cm}$ is the system's total momentum, which you know is constant for an isolated system.
- If you observe this system from a camera that is moving at $\vec{v}_{cm}$, the system's CoM will appear to be at rest. This camera's frame-of-reference is called the "ZM frame," because the system's momentum is zero as seen from that frame (i.e. as seen by that moving camera).

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity 30 $\mathrm{m/s}$), with our little track aligned with the train axis.

I push a 1 kg cart north toward my friend at 1 m/s. She pushes a 1 kg cart south toward me at 1 m/s.

As seen by a camera mounted on the ceiling of the train, what is the velocity (north) of the center-of-mass of the two-cart system?

(A) $+30\,\mathrm{m/s}$ (B) $-30\,\mathrm{m/s}$ (C) $+1\,\mathrm{m/s}$ (D) $-1\,\mathrm{m/s}$ (E) $0\,\mathrm{m/s}$

As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

Is one of these two cameras watching from the "zero-momentum" frame of the two-cart system?

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity 30 $\mathrm{m/s}$), with our little track aligned with the train axis.

I push a 1 kg cart north toward my friend at 3 m/s. She pushes a 2 kg cart south toward me at 6 m/s.

As seen by a camera mounted on the ceiling of the train, what is the velocity of the center-of-mass of the two-cart system? (Let the $+x$ axis point north.)

As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

Is one of these two cameras watching from the “zero-momentum” frame of the two-cart system?

If I observe a system from its zero-momentum reference frame, what can I say about its center-of-mass velocity?

(A) The center-of-mass velocity (as seen from the ZM frame) is the same as the velocity of the ZM reference frame (as seen from the Earth frame).

(B) The center-of-mass velocity (as seen from the ZM frame) is zero.

(C) When observing from **Earth's** frame of reference, a system's center-of-mass velocity will be the same as the velocity (w.r.t. Earth) of the ZM reference frame.

(D) (A) and (B)

(E) (A) and (C)

(F) (B) and (C)

(G) (A), (B), and (C)

# More chapter 6 key ideas

- An isolated system's CoM velocity cannot change!
- A somewhat obscure consequence of this fact is that even in a totally inelastic collision, it is not necessarily possible to convert 100% of the initial kinetic energy into heating up, mangling, etc. the colliding objects.
- Momentum conservation requires that the CoM velocity cannot change, so if the CoM is moving initially, it has to keep moving after the collision.
- Textbook: "convertible" vs. "translational" parts of a system's kinetic energy.
- That idea is worth remembering, **but the math is not.**
- (Can illustrate using colliding carts.)

You really only need these first two equations from Ch6. The third one is in the "obscure" category. Don't worry about it.

**(Chapter 6: relative motion)**

Center of mass:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Center of mass velocity (equals velocity of ZM frame):

$$v_{ZM,x} = \frac{m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Convertible kinetic energy: $K_{\text{conv}} = K - \frac{1}{2} m v_{CM}^2$

# Zero-Momentum (ZM) frame for two-object collisions

- Very useful (but difficult to visualize) tool: ZM frame.
- Elastic collision analyzed in ZM ("$*$") frame:

$$v^*_{1i,x} = v_{1i,x} - v_{ZM,x}, \quad v^*_{2i,x} = v_{2i,x} - v_{ZM,x}$$

$$\boxed{v^*_{1f,x} = -v^*_{1i,x}, \quad v^*_{2f,x} = -v^*_{2i,x}}$$

$$v_{1f,x} = v^*_{1f,x} + v_{ZM,x}, \quad v_{2f,x} = v^*_{2f,x} + v_{ZM,x}$$

- Inelastic collision analyzed in ZM frame (restitution coeff. $e$):

$$\boxed{v^*_{1f,x} = -e v^*_{1i,x}, \quad v^*_{2f,x} = -e v^*_{2i,x}}$$

- Step 1: shift velocities into ZM frame, by subtracting $v_{ZM,x}$
- Step 2: write down (very simple!!) answer in ZM frame
- Step 3: shift velocities back into Earth frame, by adding $v_{ZM,x}$

You can try this on some XC problems. Otherwise, skip it.

There are actually three pretty neat situations that you can analyze quite easily using the “ZM frame” trick:

- When a stationary golf ball is hit by a much more massive golf club, the golf ball’s outgoing speed is $2\times$ the incoming speed of the (end of the) club.
- It’s easier to hit a home run off of a fastball than a slow pitch.
- When you drop a basketball with a tennis ball resting atop the basketball, the result is quite remarkable.

I was planning to skip these as “obscure,” and leave them as topics for extra-credit problems. But if there is overwhelming demand, we could work them out one day in class?

(A) Leave it for XC (B) Do it in class