- begin video covering Mazur ch09, preceding day10/ws10
- If you have time, you might consider looking over Mazur ch09.

Ch9: work. Two definitions of work

- Work equals the change in energy of a system due to external forces. If the energy of a system increases, the (arithmetic sum of) work done by external forces on the system is positive; if the energy of a system decreases, the (sum of) work done by external forces on the system is negative.

$$\Delta E_{\text{system}} = W_{\text{done ON system}}$$

- The work done by an external force $\vec{F}$ on a system (in one dimension) is $\boxed{W = \int F_x(x)\,\mathrm{d}x}$ or just $\boxed{W = F_x\,\Delta x}$ for a constant force. When the force and the "displacement of the point of application of the force" point in the same (opposite) direction, the work done by $\vec{F}$ is positive (negative).

Let's initially focus on the second, more familiar, definition.

Chapter 9: first reading question

1. If you graph the work, $W(x)$, done by a force on an object as a function of the object's position, x, what graphical feature represents the force, $F(x)$, exerted on the object?

(A) The force is the area under the work curve.

(B) The force is the slope of the work curve.

(C) The vertical axis, i.e. the height of the work curve.

(D) The second derivative.

Chapter 9: first reading question

1. If you graph the work done by a force on an object as a function of the object's position, what graphical feature represents the force exerted on the object?

Since work equals the integral of force w.r.t. displacement, $W = \int F_x \mathrm{d}x$ or $W = F_x \Delta x$, the force is equal to the work per unit displacement. On a graph of W vs. x, the slope, $\mathrm{d}W/\mathrm{d}x$, is equal to the force.

Suppose you want to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. (Your bicycle has only one gear.) Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is

(A) one-fourth as large.

(B) one-third as large.

(C) one-half as large.

(D) the same.

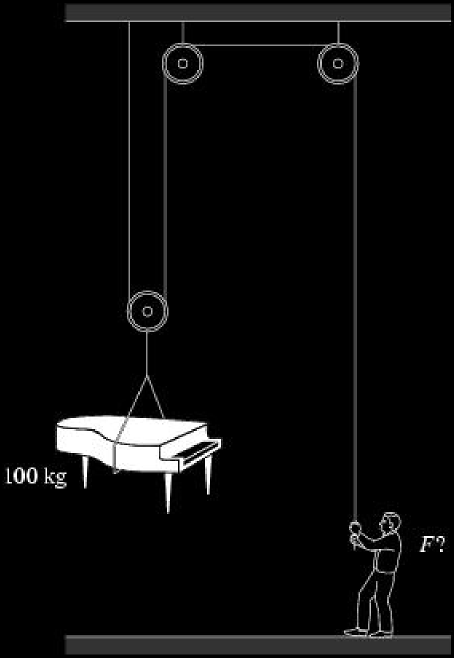
(E) twice as large.

(F) undetermined — it depends on the time taken

(Imagine how hard you have to press down on the pedals, on average, to make the bike go up one path vs. the other. As a kid, did you ever zig-zag up a really steep hill on your one-speed bike, or if your multi-speed bike's lowest gear was still not low enough?)

Imagine me towing Alfie up a steep hill behind my bicycle . . . .

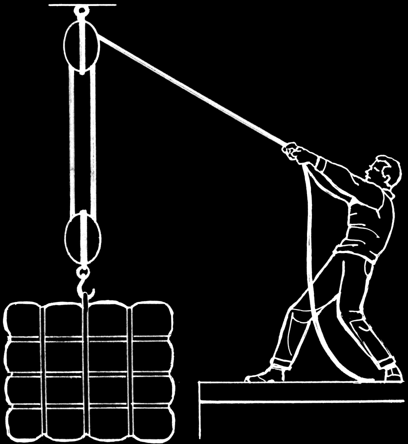
`https://youtu.be/Yigqi7zGCfQ`
`https://youtu.be/ewvet0I1YiM`



A piano mover raises a 100 kg piano at a constant speed using the pulley system shown here. With how much force is she pulling on the rope? (Ignore friction and assume $g \approx 10\ \mathrm{m/s^2}$.)

(A) 2000 N
(B) 1500 N
(C) 1000 N
(D) 750 N
(E) 500 N
(F) 200 N
(G) 50 N
(H) impossible to determine.

Block and tackle: "mechanical advantage"



This graphic shows a 2:1 mechanical advantage. The block & tackle in the classroom shows a 4:1 advantage. How would you get a HUGE mechanical advantage, like 1000:1 ? (Phys 009 topic.)

A spring-loaded toy dart gun is used to shoot a dart straight up in the air, and the dart reaches a maximum height of 8 m. The same dart is shot straight up a second time from the same gun, but this time the spring is compressed only half as far before firing. How far up does the dart go this time (neglecting friction)?

(A) 1 m

(B) 2 m

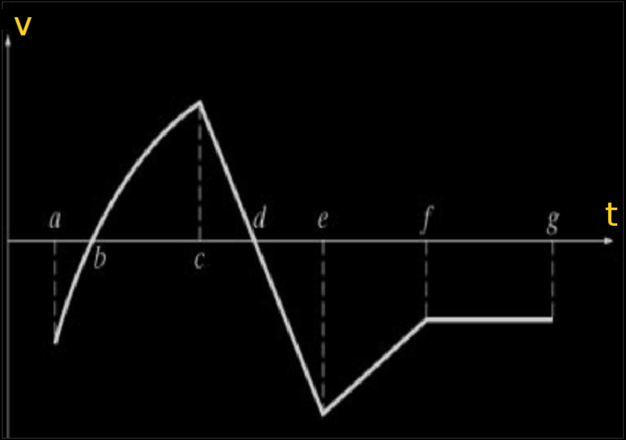
(C) 4 m

(D) 8 m

(E) 16 m

(F) 32 m

2. The velocity of an object as a function of time is shown in the figure below. Over what intervals is the work done on the object (a) positive, (b) negative, (c) zero? (Hint: make a table showing sign of acceleration (hence sign of net force), sign of displacement, and sign of their product, for each segment.)



(Consider work done by whatever external force is causing the object's velocity to change.)

Stretching a certain spring 0.10 m from its equilibrium length requires 10 J of work. How much more work does it take to stretch this spring an additional 0.10 m from its equilibrium length?

(A) No additional work

(B) An additional 10 J

(C) An additional 20 J

(D) An additional 30 J

(E) An additional 40 J

A block initially at rest is allowed to slide down a frictionless ramp and attains a speed v at the bottom. To achieve a speed $2v$ at the bottom, how many times as high must a new ramp be?

(A) 1

(B) 1.414

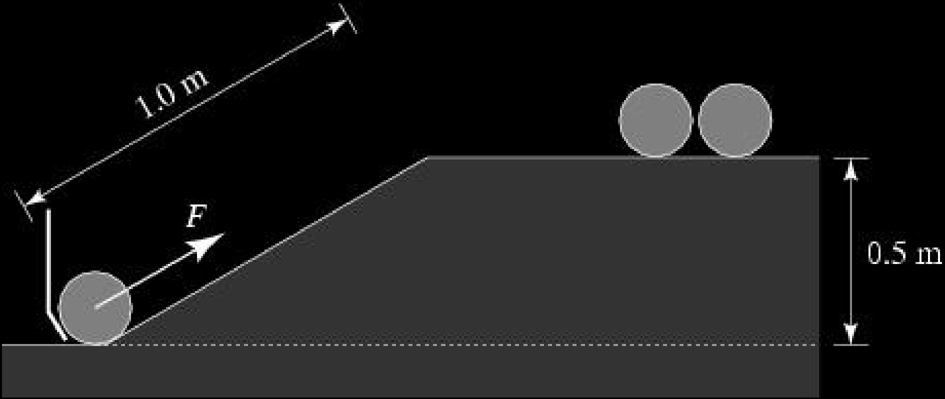
(C) 2

(D) 3

(E) 4

(F) 5

(G) 6



At the bowling alley, the ball-feeder mechanism must exert a force to push the bowling balls up a 1.0 m long ramp. The ramp leads the balls to a chute 0.5 m above the base of the ramp. About how much force must be exerted on a 5.0 kg bowling ball?

(A) 200 N

(B) 100 N

(C) 50 N

(D) 25 N

(E) 5.0 N

(F) impossible to determine.

Suppose you drop a 1 kg rock from a height of 5 m above the ground. When it hits, how much force does the rock exert on the ground? (Take $g \approx 10 \ \mathrm{m/s^2}$.)

(A) 0.2 N

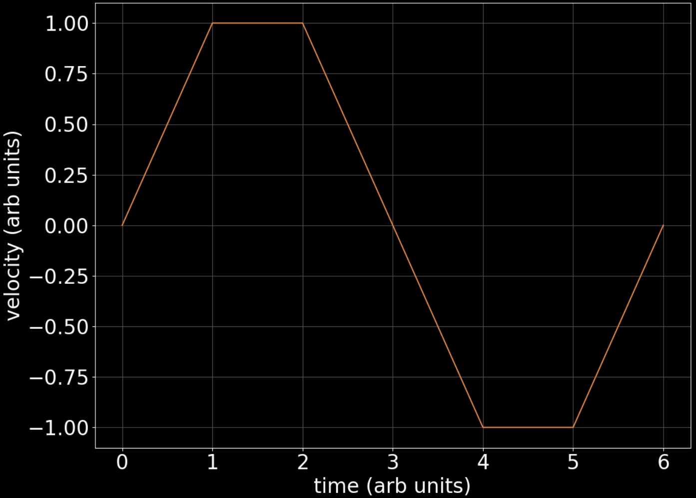
(B) 5 N

(C) 50 N

(D) 100 N

(E) impossible to determine without knowing over what distance the rock slows when it impacts the ground.

The velocity of an object as a function of time is shown. Over what time intervals is the work done on the object (a) positve, (b) negative, (c) zero? Hint: make a table showing sign of acceleration (hence sign of net force), sign of displacement, and sign of their product, for each segment. (Consider work done by whatever external force is causing the object's velocity to change.)



From a bridge at initial height h above the water, I release from rest an object of mass m which is attached to a "bungee cord" (a spring) of relaxed length ℓ_0 spring constant k. Which equation correctly expresses, assuming that no mechanical energy is dissipated into heat, the speed v_f of the object when it reaches the water surface? (One end of the bungee cord is tied to the bridge. The cord is initially slack does not begin to stretch until the object has fallen a distance equal to the cord's relaxed length.)

(A) $mg = kh$

(B) $mg = k\,(h - \ell_0)$

(C) $mgh + \frac{1}{2}mv_f^2 = \frac{1}{2}kh^2$

(D) $mgh + \frac{1}{2}mv_f^2 = \frac{1}{2}k\,(h - \ell_0)^2$

(E) $mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}kh^2$

(F) $mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}k\,(h - \ell_0)^2$

A motor lifts an object of mass m at constant upward **velocity** $v_y = \mathrm{d}y/\mathrm{d}t$. How much power (work per unit time) does the motor supply?

(A) $\text{power} = mgv_y$

(B) $\text{power} = mgy$

(C) $\text{power} = \frac{1}{2}mv_y^2$

(D) $\text{power} = \frac{1}{2}mv_y^2 + mgy$

(E) $\text{power} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv_y^2 + mgy\right)$

(F) (A) and (E) are both correct.

(G) (B) and (E) are both correct.

A motor lifts an object of mass m at constant upward **acceleration** $a_y = \mathrm{d}v_y/\mathrm{d}t$. How much power (work per unit time) does the motor supply?

(A) $\mathrm{power} = mgv_y$

(B) $\mathrm{power} = m(a_y + g)v_y$

(C) $\mathrm{power} = \frac{1}{2}mv_y^2$

(D) $\mathrm{power} = \frac{1}{2}mv_y^2 + mgy$

(E) $\mathrm{power} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv_y^2 + mgy\right)$

(F) (A) and (E) are both correct.

(G) (B) and (E) are both correct.

An object is said to be in *stable equilibrium* if a displacement in either direction requires positive work to be done on the object by an external force. Let's suppose that there is some potential energy associated with every position of the object, i.e. there is a potential energy curve $U(x)$, where x is the object's position. How do you expect $U(x)$ to change as you move the object away (in either direction) from its position of stable equilibrium?

(A) When displacing the object away from its equilibrium position, the positive work done (on the object plus its environment) by the external force causes a positive change in the potential energy function $U(x)$. So $U(x)$ must have a local minimum at the object's stable equilibrium position.

(B) $U(x)$ must have a local maximum at the object's stable equilibrium position.

(C) The derivative $\mathrm{d}U(x)/\mathrm{d}x$ must have a local minimum at the object's stable equilibrium position.

(D) The derivative $\mathrm{d}U(x)/\mathrm{d}x$ must be zero at the object's stable equilibrium position.

(E) Both (A) and (D) are true.

Chapter 9 reading question

2. When you stand up from a seated position, you push down with your legs. So then do you do negative work when you stand up?

"In this situation, we have 2 systems. Firstly, in the system of just the person, the action of standing up will result in a loss of internal or chemical energy, thereby resulting in a loss of system energy and hence positive work (BY the system) [which implies negative work done ON the system, by Earth's gravitational force]. For the system of the person and Earth, the action of standing up increases the [system's] potential energy at the expense of [the person's] internal [food] energy. In this situation, there is no change in system energy and therefore no work is done."

Reading question 2 had no really simple answer

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose "system" $=$ me $+$ Earth $+$ floor $+$ chair

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = -mg\ (\Delta x)_{\text{my c.o.m.}} < 0$

- $W = 0$

There are no external forces. Everything of interest is inside the system boundary.

Let's try choosing a different "system."

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose "system" = me + floor + chair

- $\Delta K = 0$
- $\Delta U = 0 \quad$ (U^G undefined if Earth not in system)
- $\Delta E_{\mathrm{thermal}} = 0 \quad$ (debatable but irrelevant)
- $\Delta E_{\mathrm{source}} = -mg\,(\Delta x)_{\mathrm{my\ c.o.m.}} < 0$
- $W = -mg\,(\Delta x)_{\mathrm{my\ c.o.m.}} < 0$

External gravitational force, exerted by Earth on me, does negative work on me. Point of application of this external force is my body's center of mass. Force points downward, but displacement is upward. $W < 0$. System's total energy decreases.

Let's try answering a slightly different question.

When **a friend** *stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?*

Suppose "system" $=$ me $+$ Earth $+$ floor $+$ chair

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = 0$

- $W = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$

My friend applies an upward force beneath my arms. The point of application of force is displaced upward.

Let's include my friend as part of "the system."

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

Suppose "system" = me + Earth + floor + chair + friend

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = -mg\ (\Delta x)_{\text{my c.o.m.}} < 0$

- $W = 0$

There is no external force. Everything is within the system.

Back to the original reading question

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

I think the work done **ON the system BY my legs** is either positive (if my legs are considered "external" to the me+Earth+floor system and are supplying the energy to lift me) or zero (if my legs are part of the system).

Remember the one way we got a negative answer: In the case in which Earth was not part of the system, we found that the external force of Earth's gravity did negative work on me. But I was pushing Earth downward, away from me. I lost energy. So even in this case (where the work done **on me** was negative), the work done **by me** was positive.

Key point: what you call "work" depends on how you define "the system."

A few key ideas from Chapters 8 (force) and 9 (work)

Impulse (i.e. momentum change) delivered by external force:

$$\mathrm{force} = \frac{\mathrm{d(momentum)}}{\mathrm{d}t} \quad \Leftrightarrow \quad \vec{J} = \int \vec{F}_{\mathrm{external}}\,\mathrm{d}t$$

External force exerted ON system:

$$\mathrm{force} = \frac{\mathrm{d(work)}}{\mathrm{d}x} \quad \Leftrightarrow \quad W = \int F_x\,\mathrm{d}x$$

Force exerted BY spring, gravity, etc.:

$$\mathrm{force} = -\frac{\mathrm{d}(\textit{potential energy})}{\mathrm{d}x}$$

$\Delta E_{\mathrm{system}}$ = flow of energy into system = work done ON system:

$$\mathrm{work} = \Delta(\mathrm{energy}) = \Delta K + \Delta U + \Delta E_{\mathrm{source}} + \Delta E_{\mathrm{thermal}}$$

Notice that work **:** energy **::** impulse **:** momentum

Some equation sheet entries for Chapters 8+9

`http://positron.hep.upenn.edu/physics8/files/equations.pdf`

Work (external, nondissipative, 1D):

$$W = \int F_x(x)\ dx$$

which for a constant force is

$$W = F_x\ \Delta x$$

Power is rate of change of energy:

$$P = \frac{\mathrm{d}E}{\mathrm{d}t}$$

Constant external force, 1D:

$$P = F_x v_x$$

G.P.E. near earth's surface:

$$U_{\mathrm{gravity}} = mgh$$

Force of gravity near earth's surface (force is $-\frac{\mathrm{d}U_{\mathrm{gravity}}}{\mathrm{d}x}$):

$$F_x = -mg$$

Potential energy of a spring:

$$U_{\mathrm{spring}} = \frac{1}{2}k(x - x_0)^2$$

Hooke's Law (force is $-\frac{\mathrm{d}U_{\mathrm{spring}}}{\mathrm{d}x}$):

$$F_{\mathrm{by\ spring\ ON\ load}} = -k(x - x_0)$$