Physics 8, Fall 2019, Homework #7.

Due at start of class on Friday, October 25, 2019

Problems marked with (*) must include your own drawing or graph representing the problem and at least one complete sentence describing your reasoning.

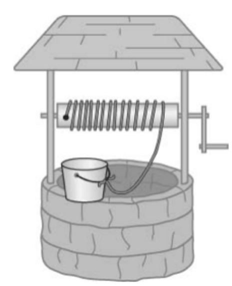
Remember **online response** at `positron.hep.upenn.edu/q008/?date=2019-10-25`

(Chapter 11 problems.)

1. An automobile accelerates from rest starting at t = 0 such that its tires undergo a constant rotational acceleration $\alpha = 6.3\,\mathrm{s}^{-2}$. The radius of each tire is 0.33 m. At t = 13 s after the acceleration begins, find (a) the instantaneous rotational speed ω of the tires, (b) the total rotational displacement $\Delta\vartheta$ of each tire, (c) the linear speed v of the automobile (assuming the tires stay perfectly round), and (d) the total distance the car travels in the 13 s.

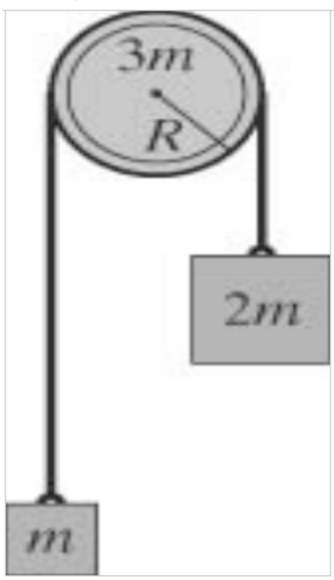
2*. You have a pail of water with a rope tied to the handle. If you whirl it fast enough in a vertical circle (i.e. a circle whose central axis is horizontal), none of the water spills out of the bucket, even when the bucket is upside down. (a) Explain how this works. (b) If the bucket rotates at constant speed v on the end of a rope of length L, what minimum speed is required to keep the water from falling out of the pail? (c) If you plug in $L = 1.0\,\mathrm{m}$, what number of **revolutions per second** does this speed correspond to? (You may remember that I spun the bucket about two or three times faster than this in the in-class demo.)

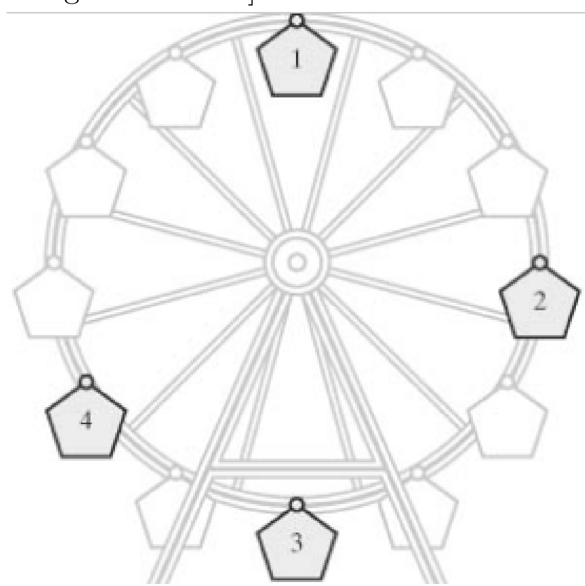
3*. You accidentally knock a full bucket of water off the side of the well shown in the figure at right. The bucket plunges 15 m to the bottom of the well. Attached to the bucket is a light rope that is wrapped around the crank cylinder. (a) How fast is the handle turning (rotational speed ω) when the bucket hits bottom? (b) How fast is the bucket moving (linear speed v) when it hits the bottom? The inertia of the bucket plus water is 12 kg. The crank cylinder is a solid cylinder of radius 0.80 m and inertia 4.0 kg. (Assume the small handle's inertia is negligible in comparison with the crank cylinder.)



4*. One type of wagon wheel consists of a 3.00 kg hoop plus four 1.00 kg thin rods placed along diameters of the hoop so as to make eight evenly spaced spokes. For a hoop of **radius** 1.00 m (diameter 2.00 m), what is the rotational inertia of the wheel about an axis perpendicular to the plane of the wheel and through the center?

5*. A block of inertia m is attached to a block of inertia $2m$ by a very light string hung over a uniform disk of inertia $3m$ and radius R that can rotate on a horizontal axle, as shown below (left). The disk's outer surface is rough, so the string and the outer surface of the disk move together without slipping. The lower block is held so that the string is taut, and then the blocks are released from rest. What is the speed of the block of inertia m after it has risen a distance h? Ignore any friction between disk and axle. [Hint: since we consider dissipative forces to be negligible here, you can use the fact that total mechanical energy (translational kinetic, rotational kinetic, plus gravitational potential) is constant during the motion.]

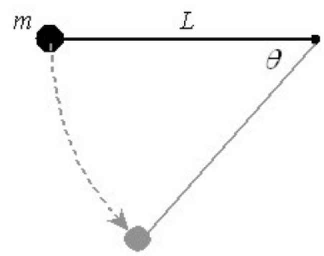


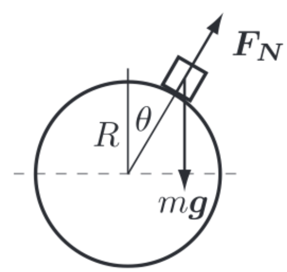


6*. The Ferris wheel shown above (right) is rotating at constant speed. Suppose that each of the pentagonal carriages is attached to the wheel by its own metal rod (where the little circle is drawn at the top of the pentagon). Draw and label free-body diagrams showing the two forces exerted on the carriage at each of the four numbered positions. Consider the tops of the carriages to be at 12:00, 3:00, 6:00, and 8:00 on the face of a clock. In each case, there is a gravitational force, whose magntitude never changes and whose direction is always downward; and there is a contact force exerted by the rod on the carriage. Since the top of each carriage moves at constant speed in a circle of constant radius, the acceleration vector for each carriage always points directly toward the center of the Ferris wheel and always has the same magnitude. For scale, you might as well draw $m\vec{a}$ on your FBD instead of the usual $\vec{a}$. Then you have two vectors, $\vec{F}^{\text{gravitational}}$ and $\vec{F}^{\text{contact}}$, whose vector sum must equal $m\vec{a}$. Try to get the relative lengths of the vector arrows about right. For a realistic Ferris wheel, ma will be much smaller than mg, but to make a decent drawing, I recommend making ma be about half the size of mg.

7*. You have a weekend job selecting speed-limit signs to put at road curves. The speed limit is determined by the radius of the curve and the bank angle of the road with respect to horizontal. Your first assignment today is a turn of radius 350 m at a bank angle of 5.7°. (a) What speed limit sign should you choose for that curve such that a car traveling at the speed limit negotiates the turn successfully even when the road is wet and slick? (So at this speed, the car stays on the road even when friction is negligible.) (b) Draw a free-body diagram showing all of the forces acting on the car when it is moving at this maximum speed. (Your diagram should also indicate the direction of the car's acceleration vector.)

8. You attach one end of a string of length L to a small ball of inertia m. You attach the string's other end to a pivot that allows free revolution. You hold the ball out to the side with the string taut along a horizontal line, as the in figure (below, left). (a) If you release the ball from rest, what is the tension in the string as a function of angle ϑ swept through? To do this, first use the radial component of $m\vec{a} = \sum \vec{F}$ to relate the speed v, the centripetal acceleration, the gravitational force, the string tension, and the angle ϑ. Then use energy conservation to relate v to ϑ. Then eliminate v in favor of ϑ. (b) At what angle, in the range $0 \leq \vartheta \leq \pi$, is the string tension largest? (c) What should the minimum tensile strength of the string be (tensile strength is the maximum tension the string can sustain without breaking) if you want the string not to break through the ball's entire motion (from $\vartheta = 0$ to $\vartheta = \pi$)?





9*. A block of mass m slides down a sphere of radius R, starting from rest at the top (with perhaps a tiny push to start it sliding). (See figure, above right.) The sphere is immobile, and friction between the block and the sphere is negligible. In terms of m, g, R, and θ, determine (a) the kinetic energy of the block, (b) the centripetal acceleration of the block, and (c) the normal force exerted by the sphere on the block. (d) At what value of θ does the block lose contact with the sphere? (Be proud of yourself once you've solved this problem: students in Physics 150 find it challenging!)

10. A long, thin rod is pivoted from a hinge such that it hangs vertically from one end. (The hinge is at the top.) The length of the rod is 1.23 m. You want to hit the lower end of the rod just hard enough to get the rod to swing all the way up and over the pivot (i.e. to swing more than 180°). How fast do you have to make the end go? (Hint: consider gravitational potential energy and rotational kinetic energy. For

rotational inertia, remember that the rod rotates about its end here, not about its center, so you'll need to use the parallel-axis theorem to get I about the end.)

Remember **online response** at `positron.hep.upenn.edu/q008/?date=2019-10-25`

XC1*. Optional / extra-credit. Assume that Earth's orbit around the Sun is a perfect circle (it's really an ellipse, but to a good approximation it's a circle). Earth's inertia is 5.97×10^{24} kg, the radius of its orbit is 1.50×10^{11} m, and its orbital period is 365.26 days. (a) What is the magnitude of Earth's centripetal acceleration as it revolves about the Sun? (b) What are the magnitude and direction of the force necessary to cause this acceleration.

XC2*. Optional/extra-credit. Imagine that an asteroid 1 km in diameter collides with Earth. Estimate the maximum fractional change in length of the day due to this collision. Take the density of the asteroid to be somewhere between the density of concrete and the density of steel, both of which are structural materials whose approximate densities you should know. For purposes of estimation, assume that the relative speed of Earth and the asteroid is just the speed at which Earth travels around the Sun (as if Earth ran into a stationary asteroid). Where (and from what direction) should the asteroid hit Earth to cause the biggest change in the length of a day? Use Problem XC1 for Earth's mass and the radius of its orbit around the Sun. Earth's radius is 6.4×10^6 m.

XC3*. Optional/extra-credit. A 5.0 kg bowling ball is thrown down the alley with a speed of 10.0 m/s. At first the ball slides with no rotation. The coefficient of kinetic friction between the ball and the alley surface is 0.10. (a) How much time does it take for the ball to achieve pure rolling motion? (b) What is its translational speed at this time? [I think this is a fun problem!]

XC4*. Optional/extra-credit. A marble that has a radius of 10 mm (that's 0.010 m) is placed at the very top of a (stationary) globe of radius 1.00 m. When released, the marble rolls without slipping down the globe. Find the angle from the top of the globe to the point where the marble flies off the globe (where the top of the globe is $\theta = 0°$, the equator is $\theta = 90°$, etc.).