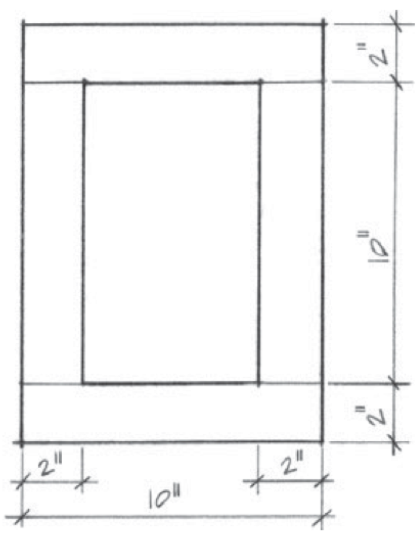
Physics 8, Fall 2019, Homework #11.

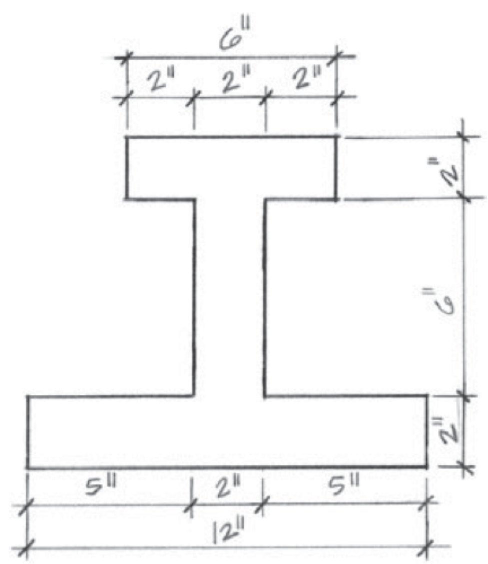
Due on Friday, November 22, 2019
but feel free to turn it in in class on Monday, November 25

Problems marked with (*) must include your own drawing or graph representing the problem and at least one complete sentence describing your reasoning.

Remember **online response** at `positron.hep.upenn.edu/q008/?date=2019-11-22`

1. Determine the second moment of area $I_x = \int y^2 \, \mathrm{d}A$ for the box-beam cross-section shown below (left). The dimensions are given in inches, so I_x should be given in in^4. Take $y = 0$ to lie at the centroid of the cross-section. Take as given the result $I_x = bh^3/12$ for a rectangle of width b and height h whose centroid is at $y = 0$. Use the method of Onouye/Kane §6.3, i.e. the parallel-axis theorem, to find I_x for the composite shape. The key result you need is $I_x = \sum I_{xc} + \sum A d_y^2$, where each sum is over the constituent simple shapes that compose the final shape. (In this example, the sum is over the 4 rectangular shapes that compose the box-beam cross-section.) For each simple shape, I_{xc} is that shape's own I_x value about its own centroid, A is that shape's area, and d_y is the vertical displacement of that shape's centroid from $y = 0$. (Note that taking 4th root of your answer for I_x is one way to check whether it is of a plausible magnitude.)





2. First compute the centroid $\overline{x}$ and $\overline{y}$ for the cross-section shown above (right). (It's OK to say that $\overline{x}$ is obvious from symmetry.) Then determine the second moment of area $I_x = \int y^2 \, \mathrm{d}A$, taking $x = 0$ and $y = 0$ to lie at your calculated centroid. The dimensions are given in inches, so I_x should be given in in^4. Your solution should include a redrawn figure that shows your calculated centroid. The method is the same as in the previous problem, except that here you need to start by finding the

centroid $\overline{y}$. Because of the lack of vertical symmetry, even the middle shape will have a non-zero d_y value in this problem.

(Onouye/Kane Chapter 7 problems)

3. A simply supported beam (i.e. supported with a pin beneath one end and a roller beneath the other end) is 4.0 m long and carries two vertical (downward) point loads: 1.5 kN at distance 1.0 m from the left support and 2.5 kN at distance 2.0 m from the left support. (a) Draw the load diagram for the beam, including all vertical forces acting on the beam—both the load and the vertical support ("reaction") forces. (b) Beneath the load diagram, draw the shear diagram $V(x)$ for the beam. (c) Beneath the shear diagram, draw the bending-moment diagram $M(x)$ for the beam. (d) Comment on whether the beam is smiling or frowning and how this relates to the sign of the bending moment.

4. A simply supported beam is 4.0 m long and carries a uniformly distributed (downward) load of 1.0 kN/m. (a) Draw the load diagram for the beam, including all vertical forces acting on the beam—both the load and the vertical support ("reaction") forces. (b) Beneath the load diagram, draw the shear diagram $V(x)$ for the beam. (c) Beneath the shear diagram, draw the bending-moment diagram $M(x)$ for the beam. (d) What is the meaning of the $V(x)$ and $M(x)$ curves? For instance, if you were to section the beam a distance x from the left side, what would $V(x)$ and $M(x)$ tell you about the interactions (forces and torques) between the left and right sides of the section?

(Two more beam-related problems)

5. The formula for the maximum vertical deflection (i.e. the deflection at mid-span) of a simply supported beam under uniform load w (vertical force per unit length) is $\Delta_{\rm max} = (5wL^4)/(384EI)$, where L is the length of the beam, E is Young's modulus, and I is the second moment of area (a.k.a. "area moment of inertia"). Typically the maximum allowable deflection of a beam of length L is $\frac{L}{360}$, to prevent plaster ceilings from cracking under excessive deflection, etc. (a) Using the $L/360$ rule, what is the maximum allowed vertical deflection, $\Delta_{\rm max}$, of a beam of length $L = 4.5$ m? (b) If the beam is designed to carry a load of 100 kg/m, what is w in N/m? (I chose these numbers to correspond to 50 pounds per square foot at a spacing between beams ("joists") of 16 inches.) (c) If the beam is made from southern pine timber having Young's modulus $E = 1.1 \times 10^{10}$ N/m^2, what minimum value of I (second moment of area) is required? (d) If the beam has a rectangular cross-section of width $b = 0.038$ m (1.5 inches), what minimum vertical depth h is required to obtain this value of I? Remember $I = bh^3/12$ for a rectangular cross-section. (e) Would a 2×10 wooden beam (cross section 1.5 in $\times$ 9.5 in) suffice for the required depth you

calculated? (Note: in real life, deflection is one of several criteria that a beam must satisfy; other criteria include maximum bending stress, maximum shearing stress, and lateral stability, as discussed in O/K chapter 8.)

6. In problem 5, we went step-by-step through a beam design whose only criterion was allowable deflection (which is a "stiffness" criterion). Now let's try a beam-design problem in which we evaluate allowable bending stress (which is a "strength" criterion), as in O/K §8.2. Imagine a set of floor joists, of length $L = 4.5$ m, spaced at 0.40 m intervals, designed to support a uniform load of 2400 N/m^2. (I chose these numbers to correspond roughly to 50 pounds per square foot load, 16 inch joist spacing, 15-foot span.) (a) What is the load, in N/m, carried by each floor joist? (Multiply load per unit area by joist spacing to get load per unit length of each joist.) (b) Considering each floor joist to be a simply-supported beam, draw the usual load, shear (V), and bending moment (M) diagrams for one floor joist. (c) From your bending-moment diagram, what is the maximum bending moment that the beam (joist) must resist? For the given loading and support conditions, this maximum should occur at mid-span. The answer should be in newton-meters. (d) Our floor joists will be made of Southern Pine timber having allowable bending stress $F_b = 10700$ kN/m^2 (that's 1.07×10^7 N/m^2, which is about 1550 psi in US customary units). Given that $S_{\rm required} = M_{\rm max}/F_b$, what is the required section modulus S for this floor-joist design? Your answer should be in m^3, but a meter is quite large compared to the transverse dimensions of a floor joist, so you will get a number that is a small fraction of a cubic meter. (e) For a rectangular beam, the second moment of area is $I = bh^3/12$, and the distance from neutral axis to extreme fibers is $c = h/2$. So the section modulus for a rectangular beam is $S = I/c = bh^2/6$. If $b = 0.038$ m (that's 1.5 inches, the width of "2×6," '2×8," "2×10," "2×12" etc. dimensional lumber that you would buy at Home Depot), what minimum value of h is required, to get the necessary minimum section modulus? (f) Convert your answer for part (e) to inches. Would you need a "2×6" ($h = 5.5$ in), a "2×8" ($h = 7.5$ in), a "2×10" ($h = 9.5$ in), or a 2×12 ($h = 11.5$ in) for each floor joist? (g) Remember that for identical conditions in problem 5, the "$L/360$" deflection rule required us to use "2×10" floor joists. Which design criterion (allowable bending stress vs. allowable deflection) turned out to be more stringent in this case? (Quote from my copy of the ARCH 435 notes, quoting Prof. Farley's lecture: "Sizing of a beam will almost always be dependent on the deflection equation; rarely shear or bending.")

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