Problem 1 (10%)

The spacecraft in the movie 2001: A Space Odyssey has a rotating cylinder to create the illusion of gravity, inside of which the crew walks and exercises.

(a) If the radius of the cylinder is about three times a crew member's height, what should the rate of revolution of the cylinder be in order to replicate Earth's gravity?

(b) For a person standing in this cylinder, how much do the gravitational acceleration at the top of her head and the gravitational acceleration at her feet differ?

(a) Human height is roughly 2m. $r = 3 \times 2\text{ m} = 6\text{ m}$. $\omega^2 r = g$, so $\omega = \sqrt{\frac{g}{r}}$.

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{r}} = 0.203\text{ Hz}$$

(b) $a = \omega^2 r$.

$$a_{\text{feet}} = (2\pi f)^2(6\text{ m}) = 9.8\text{ m/s}^2$$
$$a_{\text{head}} = (2\pi f)^2(4\text{ m}) = 6.5\text{ m/s}^2$$

So the ratio of accelerations is 2/3, or the difference is 3.3 m/s^2. One would feel more at home in a larger spaceship.

Problem 2 (5%)

You are holding a sheet of paper at rest between your thumb and index finger as shown at right. The sheet has a mass of 0.005 kg and its dimensions are 0.20 m by 0.30 m.

(a) Is the earth (via gravity) exerting a torque on the sheet about the top-left corner? If so, what is the magnitude of this torque?

(b) Are you exerting a torque on the sheet about the top-left corner? If so, what is the magnitude of this torque?

(a) Yes. $\tau = mgr_{\perp} = (0.005\text{ kg})(9.8\text{ m/s}^2)(0.10\text{ m}) = 0.005\text{ N m}$
(b) Yes. 0.005 Nm.

Problem 3 (15%)

The graph at right shows the velocities of two vehicles traveling along the same straight line. Around 3 s the two vehicles collide. The mass m_1 of vehicle 1 is 1200 kg.

(a) Determine the mass m_2 of vehicle 2. Explain how you determined your answer.

(b) Is the collision between the vehicles elastic, inelastic, or totally inelastic? Explain how you determined your answer. If the collision is elastic, describe the motion of the two vehicles. If it is inelastic, determine how much energy is dissipated in the collision.

(c) Determine the magnitude of the average force exerted by vehicle 1 on vehicle 2 during the collision and the magnitude of the average force exerted by vehicle 2 on vehicle 1 during the collision.

(a) $m_1\Delta v_1 = m_2\Delta v_2$, so $m_2 = m_1\frac{\Delta v_1}{\Delta v_2} = (1200\text{ kg})(\frac{5}{3}) = 2000$ kg.
(b) Inelastic: the relative speed of the two carts Δv_{12} is different before and after the collision. Before collision, $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = 18.4$ kJ. After collision, $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = 6.4$ kJ. So the dissipated energy is 12 kJ.
(c) $F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{m_2\Delta v_2}{1\text{ s}} = \frac{(2000\text{ kg})(3\text{ m/s})}{1\text{ s}} = 6000$ N. F_{21} and F_{12} have same magnitude.

Problem 4 (5%)

A 2.0 kg ball is suspended from a spring, stretching the spring by 0.50 m from its relaxed length. The ball is then pulled down an additional 0.20 m from its equilibrium position and then released. How long after being released does the ball pass its equilibrium position?

$F = kx = mg$. So $k = mg/x = 39.2$ N/m. $\omega = \sqrt{\frac{k}{m}}$. So period $T = 2\pi\sqrt{\frac{m}{k}} = 1.42$ s. Pass equilibrium position after $\frac{1}{4}$ period: $t = \frac{T}{4} = 0.355$ s.

Problem 5 (15%)

A car with wheels of 0.60 m diameter travels North at 10 m/s.

(a) What are the magnitude and direction of the angular velocity of the wheels? How long does it take for a wheel to complete one rotation?

(b) In the reference frame of the Earth, what are the magnitude and direction of the velocity at the following points on the wheel? (i) at the top of the wheel? (ii) at the bottom of the wheel? (iii) at the back of the wheel?

(a) $r = d/2 = 0.30$ m. $\omega = v/r = 33.3\text{ s}^{-1}$. Using right-hand rule, $\vec{\omega}$ points west. One rotation completes in $T = \frac{2\pi}{\omega} = 0.18$ s.
(b) Let's consider top, bottom, and back of wheel first in car frame and then in earth frame. To get to earth frame from car frame, add 10 m/s north.

		in car frame	in earth frame
i	top	10 m/s north	20 m/s north
ii	bottom	10 m/s south	0 m/s
iii	back	10 m/s up	14.1 m/s at 45° up from north

(c) $I = \frac{1}{2}mr^2 = \frac{1}{2}(20\text{ kg})(0.30\text{ m})^2 = 0.90\text{ kg}\cdot\text{m}^2$.

$$K = 4 \times \left(\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2\right) = 6000\text{ J}$$

Problem 6 (5%)

You have been hired to check the technical correctness of an upcoming made-for-TV murder mystery. The mystery takes place in the space shuttle. In one scene, an astronaut's safety line is sabotaged while she is on a space walk, so she is no longer connected to the space shuttle. She checks and finds that her thruster pack has also been damaged and no longer works. She is 200 meters from the shuttle and moving with it. That is, she is not moving with respect to the shuttle. There she is drifting in space with only 4 minutes of air remaining. To get back to the shuttle, she decides to unstrap her 10 kg tool kit and throw it away with all her strength, so that it has a speed of 8 m/s. According to the script, she makes it back to the shuttle before running out of air. Is this correct? Her mass, including space suit (but without the tool kit) is 80 kg.

Conserve momentum to get astronaut's speed v:

$$(10\ \text{kg})(8\ \text{m/s}) = (80\ \text{kg})(v) \quad \Rightarrow \quad v = 1\ \text{m/s}$$

Time to travel 200 m at 1 m/s is 200 s. 4 minutes is 240 s. 200 s $<$ 240 s. So the script is OK. Whew!

Problem 7 (15%)

A janitor is pushing an 11 kg trashcan across a level floor at constant speed. The coefficient of friction between can and floor is 0.10.

(a) If he is pushing horizontally, what is the magnitude of the force he is exerting against the can?

(b) If he pushes not horizontally but rather at an angle of 30° down from the horizontal, what must the magnitude of his pushing force be to keep the can moving at constant speed?

(c) What is the magnitude of the normal force between the trashcan and the floor in part (a)? And in part (b)?

(a) To keep the trashcan moving at constant velocity, the pushing force must balance the force of kinetic friction:

$$F^{\text{Push}} = F^K = mg\mu_K = (11\ \text{kg})(9.8\ \text{m/s}^2)(0.10) = 10.8\ \text{N}$$

(b) Now the normal force of the floor on the trashcan must balance the sum of the downward force of gravity and the downward component of the janitor's push:

$$F^N = mg + F^{\text{Push}} \sin 30°$$

so then the force of kinetic friction is

$$F^K = \mu_K F^N = \mu_K (mg + F^{\text{Push}} \sin 30°)$$

which must now balance the horizontal component of the janitor's push in order to keep the trashcan sliding at constant velocity:

$$F^{\text{Push}} \cos 30° = F^K = \mu_K (mg + F^{\text{Push}} \sin 30°)$$

Now solve for F^{Push} :

$$\mu_K mg = F^{\text{Push}} \cos 30° - \mu_K F^{\text{Push}} \sin 30°$$

$$\mu_K mg = F^{\text{Push}} (\cos 30° - \mu_K \sin 30°)$$

$$F^{\text{Push}} = \frac{\mu_K mg}{\cos 30° - \mu \sin 30°} = 13.2\ \text{N}$$

Notice that if you replace 30° with 0°, you get back the answer from part (a).
(c) In part (a), $F^N = mg = 108$ N. In part (b), $F^N = mg + F^{\text{Push}} \sin 30° = 114$ N.

Problem 8 (10%)
A satellite that is always over the same spot on the earth is called "geostationary." Geostationary orbits are located above the equator and have an orbital period of 24 hours. How far above the center of the earth is a geostationary orbit? ($G = 6.67 \times 10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}$, and $M_{\text{earth}} = 6.0 \times 10^{24}$ kg. For checking that your answer makes sense, it may also help to know that $R_{\text{earth}} = 6.4 \times 10^6$ m.)

$m\omega^2 R = GmM/R^2$, so then

$$R^3 = \frac{GM}{\omega^2} = \frac{GM}{(2\pi/T)^2} = \frac{GM}{(2\pi)^2}\, T^2$$

$$T = 24 \times 3600 \text{ s} = 86400 \text{ s}$$

$$R = \sqrt[3]{\frac{(6.67 \times 10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2})(6.0 \times 10^{24}\text{ kg})}{(2\pi)^2}\ (86400\text{ s})^2} = 4.3 \times 10^7 \text{ m}$$

which is about 6.7 earth radii, which is the number we worked out in class one day.

Problem 9 (5%)
Explain why, when a truck makes a sharp turn on an unbanked road (i.e. a perfectly horizontal road), the wheels on the inside of the turn tend to come off the ground. You may find it easier to explain with the help of a picture.

Consider a right-hand turn. The wheels push the truck to the right at road height. So the truck (as seen from the back) wants to rotate counterclockwise about its center of mass, which is well above the road. So the right wheels carry less of the truck's weight, and the left wheels carry more. If the centripetal acceleration is large enough, then the right wheels leave the ground.

Problem C1 (15% total for C1–C4)
The two balls have the same initial velocity and the same downward acceleration once they leave the table. They follow the same trajectory as seen from the side. The answer is (1).

Problem C2
Once the centripetal force is removed, the ball continues at constant velocity—the velocity it had at the instant the string broke. The answer is (2).

Problem C3
The initial velocity is that of the airplane, and the downward acceleration is g. The answer is (4).

Problem C4
Since the elevator moves at constant speed (acceleration is zero) and we are told to neglect friction, the force of gravity must balance the force of the cable. The answer is (2).

Bill's total time: 36 minutes.
Zoey's total time: 42 minutes.
Rony's total time: 34 minutes. (Rony is another grad student.)