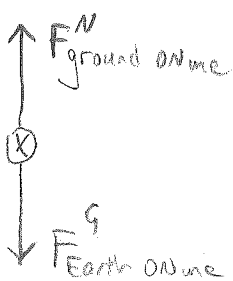
Physics 8, Fall 2013, Final Exam. **Name:** BILL & ZOEY

This closed-book exam has 20% weight in your course grade. You can use one sheet of your own handwritten notes and a calculator. Work alone. Keep in mind that every member of the University community is responsible for upholding the highest standards of honesty at all times: offering or accepting help with this exam would be a serious violation of Penn's Code of Academic Integrity. Please show your work on these sheets. Continue your work on the reverse side if needed. The time listed for each problem is a guideline to budget your time. The last page of the exam contains a list of equations that you might find helpful.

1. (17 minutes, 14%) Conceptual questions. (Your answers don't need to be as thorough as ours — don't worry.)

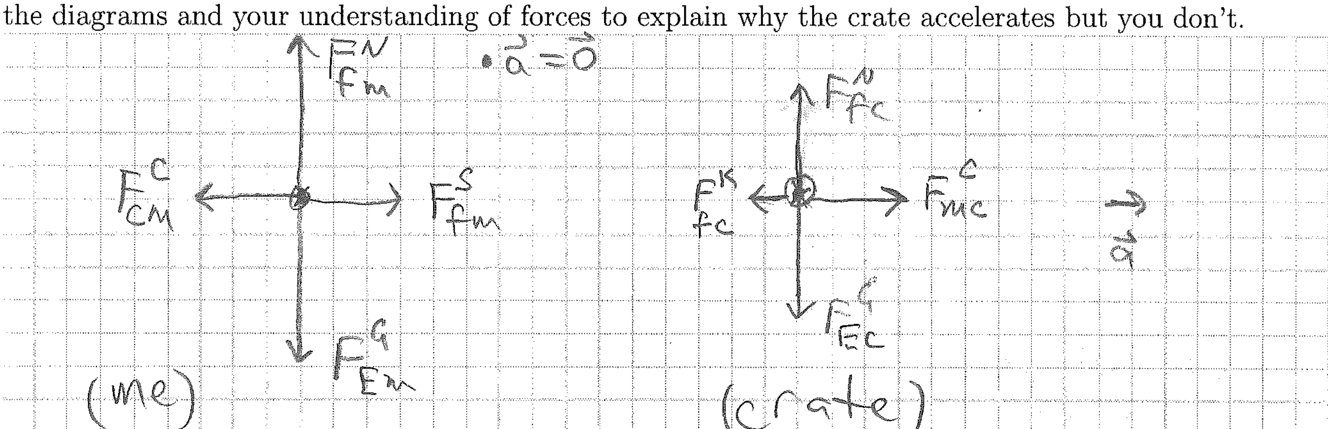
(a) When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?



The ground exerts an upward force on my feet, of magnitude mg.

Earth's gravity exerts a downward force on my center of mass, of magnitude mg.
The vector sum of these two forces is zero.

(b) You push on a crate, and it starts to move but you don't. Draw a free-body diagram for you and one for the crate. Make it clear from the lengths of your arrows which forces have equal magnitudes. Then use the diagrams and your understanding of forces to explain why the crate accelerates but you don't.



m = me
f = floor
c = crate
E = Earth

$|F^c_{mc}| = |F^c_{cm}|$ is contact force between me and crate.

I push on the crate, it pushes back on me, but static friction from my good shoes keeps me from moving. The vector sum of forces on me = 0.
The vector sum of forces on the crate ≠ 0.
Friction force on crate < friction force on me.

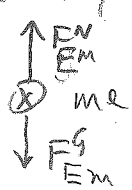
(Problem 1 continues on next page.)

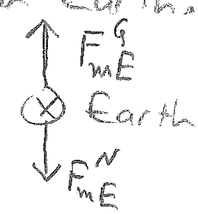
(c) A worker pushes boxes in a factory. In each case decide which force has the greater magnitude — the force exerted by the worker on the box or the force exerted by the box on the worker. (i) The box is heavy and does not move no matter how hard she pushes. (ii) Some contents are removed, and now when pushed the box slides across the floor at constant speed. (iii) The worker pushes harder, and the box accelerates.

In all cases, the two forces are equal in magnitude and opposite in direction.

(d) When you are standing motionless on the ground, your feet are exerting a force on Earth. Why doesn't Earth move away from you?

I am exerting a gravitational force mg on Earth, pulling Earth toward me. It balances the force exerted by my feet on Earth.





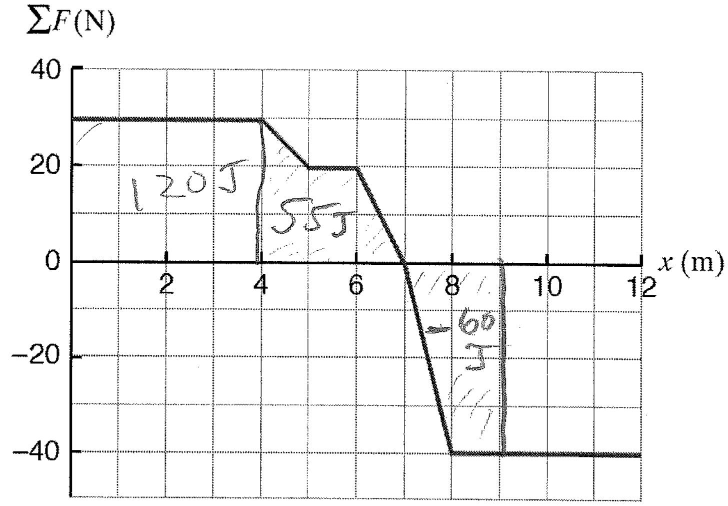
The FBD for me shows two forces. Each of these two forces has a third-law complement: $|F^g_{Em}| = |F^g_{mE}|$ etc.

(e) You are in a stationary elevator, so that the contact force exerted by the floor on you is equal in magnitude to the force of gravity acting on you. When the elevator accelerates downward, which force changes? What happens to its magnitude?

The normal force exerted by elevator floor on my feet decreases in magnitude. Gravitational force is unchanged. I accelerate downward.

$$\left.\begin{aligned} F^N - mg &= m a_y \\ F^N &= m(g + a_y) \end{aligned}\right\} \text{ where } a_y < 0 \text{ in this case.}$$

2. (17 minutes, 14%) A 15 kg object starts from rest at $x = 0$ m. The x-component of the vector sum of the forces exerted on the object varies with *position* as shown in the graph. (The net force is zero in the y and z directions.)



(a) How much time does it take the object to get from $x = 0$ m to $x = 4$ m?

Force is constant for $0 \leq x \leq 4$m.

$$a = \frac{F}{m} = \frac{30N}{15kg} = 2\frac{m}{s^2}$$

$$\Delta x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{\frac{(2)(4m)}{(2\,m/s^2)}} = \boxed{2s = t}$$

(b) What is the object's speed at $x = 4$ m?

$$V_f^2 = \cancel{V_i^2} + 2a\Delta x = 2\left(2\frac{m}{s^2}\right)(4m) = 16\frac{m^2}{s^2} \Rightarrow \boxed{V_f = 4\frac{m}{s}}$$

check: $V_f = \cancel{V_i} + at = \left(2\frac{m}{s^2}\right)(2s) = 4\,m/s.$ ✓

(c) How much work is done on the object in moving it from $x = 0$ m to $x = 4$ m?

constant force in x direction:

$$W = F_x\Delta x = (30N)(4m) = 120J$$

(d) How much work is done on the object in moving it from $x = 0$ m to $x = 9$ m?

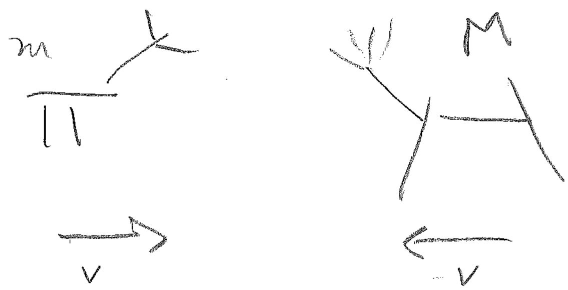
Variable force: $W = \int_0^{9m} F_x\,dx$ = area under curve

count boxes @ 10 J per box

$$W = 120J + 55J - 60J = \boxed{115J}$$

3. **(9 minutes, 7%)** Two male moose charge head-on at each other with the same speed and meet on an icy patch of tundra. The smaller moose is charging to the right, and the larger moose is charging to the left. As they collide, their antlers lock together and the two animals slide together with one-quarter of their original speed.

(a) What is the ratio of their masses?



$$P_{initial} = P_{final}$$

$$V_{1final} = V_{2final} = V_f = -\frac{v}{4}$$

$$mv + M(-v) = (m+M)\left(-\frac{1}{4}v\right)$$

$$mv - Mv = -\frac{mv}{4} - \frac{Mv}{4}$$

$$4mv - 4Mv = -mv - Mv \Rightarrow 5m = 3M$$

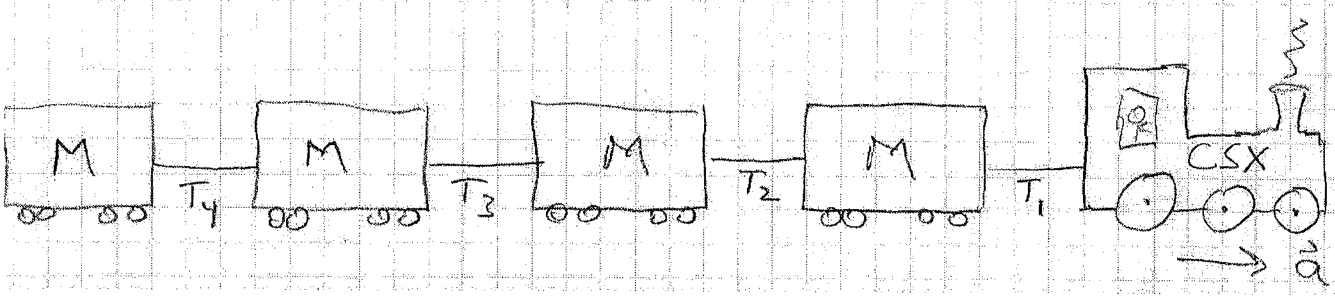
$$\boxed{M = \frac{5}{3}m}$$

(b) In which direction do they slide after colliding?

They slide to the left, in the direction in which the larger moose was charging.

4. (4 minutes, 4%) A train engine is pulling four boxcars, each of mass M. As the train accelerates, the engine exerts a force of magnitude F on the load that it is pulling.

(a) Assuming that friction can be ignored, what is the tension in each of the four couplers? (Coupler 1 connects the engine to the first boxcar. Coupler 2 connects the first boxcar to the second. Coupler 3 connects the second boxcar to the third. Coupler 4 connects the third boxcar to the fourth.)



$$T_1 = (4M)a = F$$

$$T_2 = (3M)a = \frac{3}{4}F$$

$$T_3 = (2M)a = \frac{1}{2}F$$

$$T_4 = Ma = \frac{1}{4}F$$

(b) Write the acceleration of the train in terms of F and M.

$$F = (4M)a \Rightarrow a = \frac{F}{4M}$$

5. (16 minutes, 13%) A janitor is pushing a 10 kg trashcan across a level floor at constant speed. The coefficient of friction between the can and the floor is 0.20.

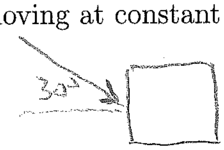
(a) If he is pushing horizontally, what is the magnitude of the force he is exerting against the can?

Constant speed, so presumably constant velocity,

so $\sum \vec{F} = 0$, so $F_x^{push} - F^K = 0$

$$F_x^{push} = F^K = \mu_K F^N = \mu_K m g = (0.20)(10\,kg)(9.8\,\tfrac{m}{s^2})$$
$$= \boxed{19.6\,N}$$

(b) If he pushes not horizontally but rather at an angle of 30° down from the horizontal, what must be the magnitude of his pushing force to keep the can moving at constant speed?



$F^N = mg + F^{push} \sin 30°$ (because $\sum F_y = m a_y = 0$)

$F^K = \mu F^N = F_x^{push} = F^{push} \cos 30°$ (because $\sum F_x = m a_x = 0$)

$\mu(mg + F \sin 30°) = F \cos 30° \Rightarrow \mu m g + \mu F \sin 30° = F \cos 30°$

$\mu m g = F(\cos 30° - \mu \sin 30°)$

$$F^{push} = \frac{\mu m g}{\cos 30° - \mu \sin 30°} = \frac{19.6\,N}{0.766} = \boxed{25.6\,N = F^{push}}$$

check:
- makes sense that it's somewhat bigger than 19.6N
- makes sense that for $\theta = 0°$ I get answer (a).

6. (10 minutes, 8%) A ball is rolled off of a horizontal table and falls to the ground. The ball's speed while rolling across the table is 4.9 m/s. The ball hits the ground 0.50 s after passing the edge of the table.

(a) How high is the table?

$$h = \frac{1}{2} g t^2 = \left(\frac{1}{2}\right)\left(9.8 \frac{m}{s^2}\right)(0.50s)^2 = \boxed{1.23m = h}$$

$$(V_y \text{ initial} = 0, \quad a_y = -g)$$

(b) What is the horizontal distance from the edge of the table to the point where the ball hits the ground?

$$X = V_x t = \left(4.9 \frac{m}{s}\right)(0.50s) = \boxed{2.45m}$$

(constant velocity for horizontal coordinate) because there are no horizontal forces here

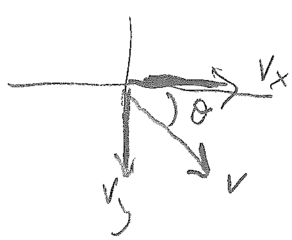
(c) What is the ball's speed at the instant before it hits the ground?

$$V_x = 4.9 \frac{m}{s}$$

$$V_y = -gt = -\left(9.8 \frac{m}{s^2}\right)(0.5s) = -4.9 \frac{m}{s}$$

$$|v| = \sqrt{v_x^2 + v_y^2} = 6.93 \frac{m}{s}$$

(d) At what angle from the horizontal is the ball traveling when it hits the ground?



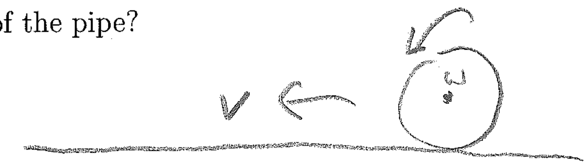
$$\tan\theta = \left|\frac{V_y}{V_x}\right| = 1$$

$$\Rightarrow \boxed{\theta = 45°}$$

7. (7 minutes, 6%) A 20 kg piece of steel pipe that is 0.20 m in radius, 0.80 m long, and approximately 1 mm thick is rolling toward you without slipping at a speed of 3.0 m/s.

(a) What is the magnitude of the rotational velocity of the pipe?

(because it's rolling w/out slipping)



$$v = \omega r \Rightarrow \omega = \frac{v}{r} = \frac{3.0\ \text{m/s}}{0.20\ \text{m}} = 15\ \text{s}^{-1}$$

$$|\omega| = 15\ \frac{\text{radians}}{\text{second}} = \boxed{15\ \text{s}^{-1}}$$

(b) If the pipe is coming toward you, in what direction does the rotational velocity vector point?

$\vec{\omega}$ points to my right, using right-hand rule (my right fingers curl in direction of rotation, and thumb points along $\vec{\omega}$)

(c) What is the total kinetic energy of the pipe (including rotational kinetic energy)?

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$I = mr^2$ (for cylindrical shell)

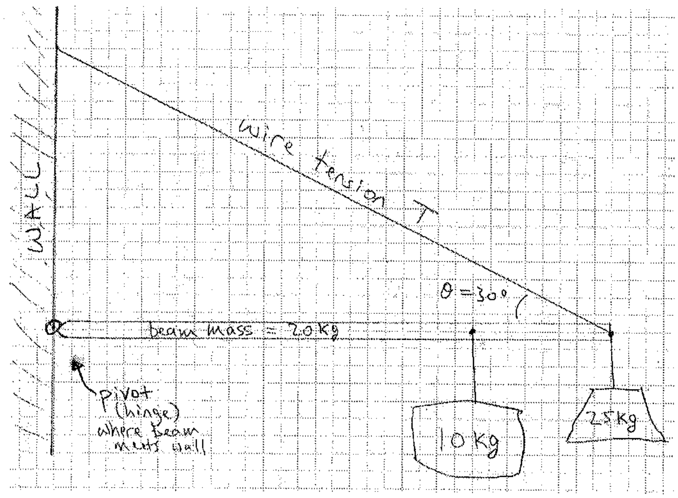
(1mm thickness is negligible)

$$K = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\left(\frac{v}{r}\right)^2$$

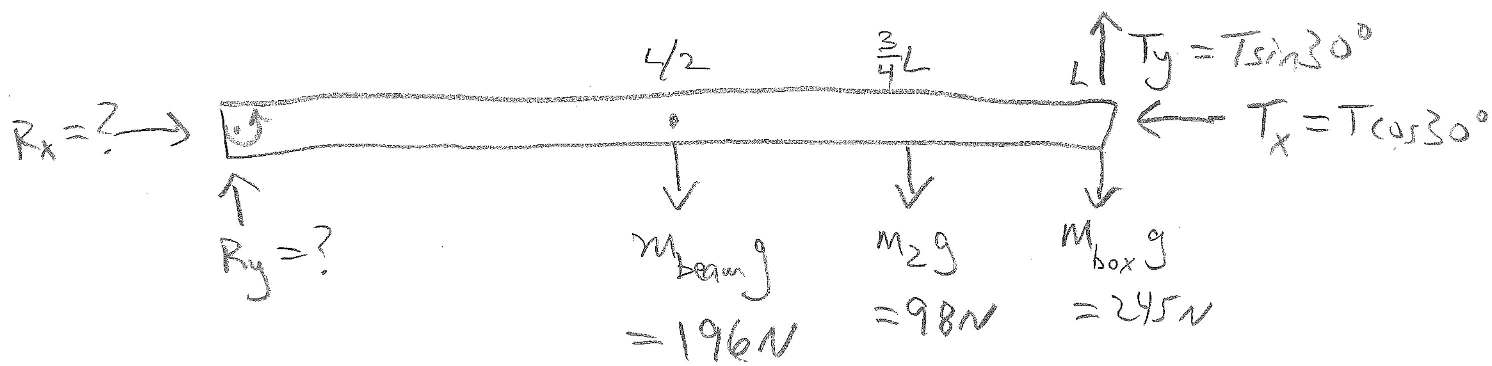
$$K = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

$$K = (20\ \text{kg})(3.0\ \tfrac{\text{m}}{\text{s}})^2 = \boxed{180\ \text{J}}$$

8. (20 minutes, 17%) A box of mass $m_{\text{box}} = 25$ kg is suspended from the right end of a horizontal beam of mass $m_{\text{beam}} = 20$ kg. A second box of mass $m_2 = 10$ kg is suspended $\frac{3}{4}$ of the way out. The left end of the beam is affixed to a wall by a pin. A wire connects the right end of the beam to the wall directly above the pin, making an angle of 30° with the beam, as shown in the figure.



(a) Draw an extended free-body diagram for the horizontal beam, showing each force acting on the beam and its line of action.



b) Find the tension in the wire. (Use correct units for tension!)

$$0 = \Sigma\tau_{pivot} = LT\sin 30° - L(245N) - \frac{3L}{4}(98N) - \frac{L}{2}(196N)$$

$$T_y = T\sin 30° = \frac{1}{2}T = 245N + \frac{3}{4}(98N) + \frac{1}{2}(196N) = 416.5N$$

$$\boxed{T = 833N}$$

(c) Determine the horizontal and vertical components of the "reaction" force that the pivot exerts on the beam. (Again, be sure to label your answers with correct units.)

$$\Sigma F_x = 0 \Rightarrow R_x = T\cos 30° = 833N\frac{\sqrt{3}}{2} = \boxed{721N}$$ to the right

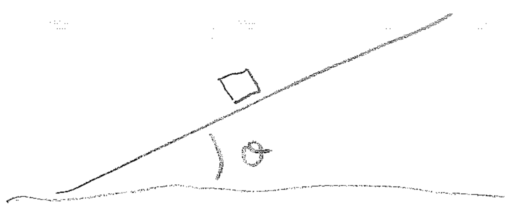
$$\Sigma F_y = 0 \Rightarrow R_y = (196N + 98N + 245N - 416.5N)$$

$$\boxed{R_y = 122N}$$ upward

9. (6 minutes, 5%) You have been hired to help troubleshoot a new airport baggage ramp that is used for unloading luggage from airplanes' cargo compartments. The manager shows you the inclined conveyor belt that is supposed to transport each bag at constant speed from the airplane to the ground. As a small suitcase is placed at the top of the 20° downward incline, the manager yells, "It's that type of slick suitcase that gives us a hard time!" Indeed, the suitcase begins to slide, creating a pile-up at the bottom of the conveyor belt. You immediately roll up your sleeves: using a stationary piece of conveyor belt, you determine that the coefficient of static friction between the small suitcase and the rubber conveyor belt is 0.30. The manager, looking over your shoulders, tells you that most suitcases are much heavier than the one you are holding and that the conveyor belt moves at a steady speed of 1.3 m/s. You think for a moment about what you learned in Physics 8 and then recommend an adjustment in the angle of incline for the ramped conveyor belt, to prevent the suitcases from sliding.

What adjustment do you recommend? Explain briefly, and be quantitative!

constant velocity $\Rightarrow \underset{\text{on suitcase}}{\sum \vec{F} = 0}$



$\boxed{\text{need to set } \theta < 16.7°}$

normal force by belt on suitcase $\quad F^N_{bs} = mg\cos\theta$

downhill component of gravity is $mg\sin\theta$
uphill frictional force is $F^s \le \mu_s F^N$

to maintain constant velocity, $\sum\vec{F} = 0$, so

$$mg\sin\theta = F^s \le \mu_s F^N = \mu_s mg\cos\theta$$

$\Rightarrow$ need $\mu_s \ge \frac{\sin\theta}{\cos\theta} = \tan\theta$ or need $\tan\theta \le \mu_s$

$\tan(20°) = 0.364$ is bigger than $\mu_s = 0.30$.

Adjust angle so that $\theta < \arctan(0.30) = 16.7°$

10. (14 minutes, 12%) Your physics teacher Bill gets the crazy idea that he himself will be the "mass" bobbing up and down on the end of a stiff spring that is attached to the ceiling of DRL room A2. Bill first hangs the spring from the ceiling and measures its relaxed length to be 1.10 meters. Then he climbs the ladder, gradually applies his full weight to the lower end of the spring (by sitting on a little attached bar), and measures the spring's new equilibrium length to be 1.90 meters.

(a) If Bill's mass is 70 kg, what is the spring constant k of the spring?

$$mg = k(x - x_o)$$

$$k = \frac{mg}{\Delta x} = \frac{(70\,\text{kg})(9.8\,\frac{\text{m}}{\text{s}^2})}{0.80\,\text{m}} = \boxed{858\,\frac{\text{N}}{\text{m}}}$$

(b) If Zoey pulls down on Bill's feet until the spring's length is 2.00 meters, holds them there for a moment, then lets go, will Bill's motion repeat itself periodically? If so, how often? If not, why not?

$\Delta x = 0.10\,\text{m}$ (not important here)

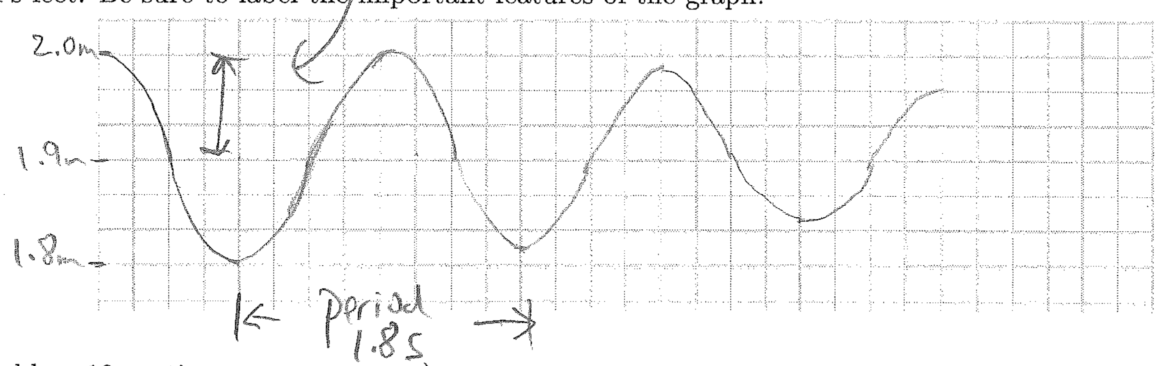
$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{70\,\text{kg}}{858\,\text{N/m}}}$$

$\boxed{T = 1.8\,\text{s}}$ (period)

Yes, periodic motion

amplitude 0.1m. Amplitude will slowly decrease as energy is dissipated, but you don't need to say that.

(c) Sketch a graph of the length of the spring as a function of time, where $t = 0$ is where Zoey lets go of Bill's feet. Be sure to label the important features of the graph.



(Problem 10 continues on next page.)

(d) If Zoey instead pulls down on Bill's feet until the spring's length is 2.1 meters, then lets go, how will the period of the motion be affected? (State what the period will be.)

Period T is unaffected by change in amplitude. $T = 1.8s$.

(e) How will the amplitude of the motion be affected? (State what the amplitude will be.)

Amplitude is doubled: now $A = 0.2m$.

(f) If Bill somehow managed to hold a 70 kg medicine ball while sitting on this same spring, thus effectively doubling his mass, would the natural period of the motion be affected? (State what the period would be.)

Period becomes larger by factor $\sqrt{2}$.

$$T = (1.8s)\sqrt{2} = \boxed{2.54s.}$$

check: $2\pi \sqrt{\frac{140kg}{858\frac{N}{m}}} = 2.54s.$

Have a safe and happy winter break!

Possibly useful equations

$$\cos(30^\circ) = \frac{\sqrt{3}}{2} \approx 0.866 \qquad \sin(30^\circ) = \frac{1}{2} \qquad \tan(30^\circ) = \frac{1}{\sqrt{3}} \approx 0.577$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$f_{\text{spring}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad f_{\text{pendulum}} = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$$F = \frac{GMm}{r^2} \qquad G = 6.67 \times 10^{-11} \ \frac{\text{N m}^2}{\text{kg}^2}$$

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} \qquad \vec{p} = m\vec{v}$$

$$F_c = \frac{mv^2}{r} \qquad F_c = m\omega^2 r \qquad v = \omega r$$

$$F^K = \mu^K F^N \qquad F^s \leq \mu_s F^N$$

$$F_x^{\text{spring}} = -k\,(x - x_0)$$

$$F_y^{\text{grav}} = -mg \qquad g = 9.8\ \text{m/s}^2$$

solid cylinder: $I = \frac{1}{2}mR^2$ cylindrical shell: $I = mR^2$

solid sphere: $I = \frac{2}{5}mR^2$ spherical shell: $I = \frac{2}{3}mR^2$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \tau = rF\sin\theta = r_\perp F = rF_\perp$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x\,(x_f - x_i) \qquad x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$W = \int \vec{F}_{\text{external}} \cdot \mathrm{d}\vec{r} = \int (F_x\ \mathrm{d}x + F_y\ \mathrm{d}y + F_z\ \mathrm{d}z)$$