

This two-hour, closed-book exam has 20% weight in your course grade. You can use one sheet of your own handwritten notes and a calculator.

Work alone. Keep in mind that here at Penn, every member of the University community is responsible for upholding the highest standards of honesty at all times: offering or accepting help with this exam would be a serious violation of Penn's Code of Academic Integrity.

Please show your work on these sheets. Continue your work on the reverse side if needed. The last page of the exam contains a list of equations that you might find helpful. Feel free to approximate $g = 10 \text{ m/s}^2$ if you wish.

1. (20%) Conceptual force questions.

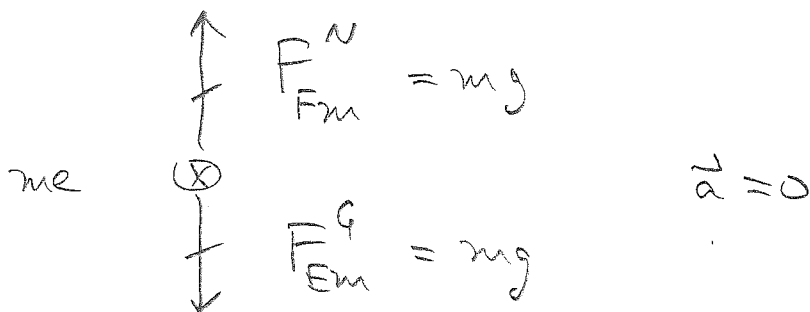
(a) When you stand still on the floor, how large a force does the floor exert on you? Why doesn't this force make you rise up into the air? Include with your answer a Mazur-style free-body diagram (FBD, not EFBD) for yourself, indicating the forces exerted on you as you stand still on the floor.

Floor exerts upward contact force mg on me.

Earth exerts downward gravitational force mg on me.

Vector sum of forces on me is zero

$$\Rightarrow \vec{a}_{me} = 0$$

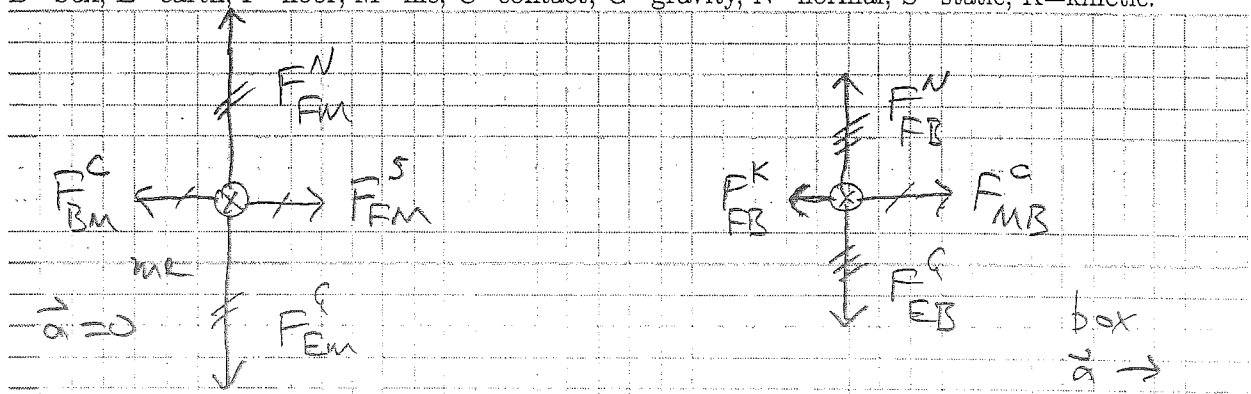


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(b) A worker pushes boxes in a factory. In each case decide which force has the greater magnitude: the force exerted by the worker on the box or the force exerted by the box on the worker. (i) The box is heavy and does not move no matter how hard she pushes. (ii) Some contents are removed, and now when pushed the box slides across the floor at constant speed. (iii) The worker pushes harder, and the box accelerates.

$\vec{F}_{BW} = -\vec{F}_{WB}$ always, by Newton's 3rd law.
 \vec{F}_{BW} and \vec{F}_{WB} are always equal in magnitude.

(c) You and a box are both at rest on a factory floor. You push (horizontally) on the box, and the box starts to slide but you remain at rest. Draw a Mazur-style free-body diagram for you and one for the box. Include all relevant forces, both vertical and horizontal. Make it clear from the lengths of your arrows which forces have equal magnitudes. Then use the diagrams and your understanding of forces to explain why the box accelerates but you don't. You might want to label your forces with B=box, E=earth, F=floor, M=me, C=contact, G=gravity, N=normal, S=static, K=kinetic.

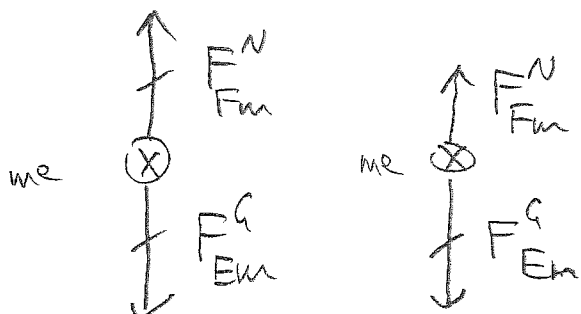


Vector sum of forces acting on me is zero

Vector sum of forces acting on box points \rightarrow

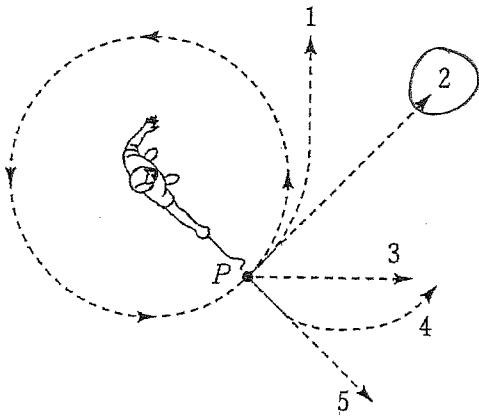
(d) You are in a stationary elevator, so that the contact force exerted by the elevator's floor on you is equal in magnitude to the force of gravity acting on you. When the elevator accelerates downward (and you accelerate downward with the elevator), which force changes? What happens to its magnitude?

Normal force exerted by elevator on me decreases in magnitude



\Rightarrow I accelerate downward because vector sum of forces acting on me points \downarrow .

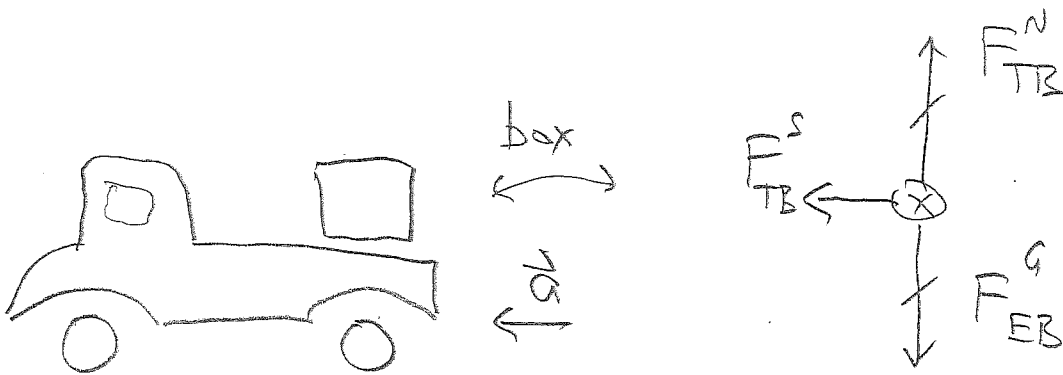
(e) A steel ball is attached to a string and is swung in a circular path in a horizontal plane as illustrated in the figure below. At point P , the string suddenly breaks near the ball. If these events are observed from directly above, which of the 1-5 paths below would the ball most closely follow after the string breaks?



Once string is no longer exerting force on ball, ball continues at constant velocity, which is tangent to circle.

(f) A heavy crate rests on the bed of a flatbed truck. When the truck accelerates, the crate remains where it is on the truck, so it too accelerates. What force causes the crate to accelerate? Draw a Mazur-style free-body diagram for the crate. Be sure to indicate the direction of acceleration.

B = box T = truck



Static friction exerted by truckbed on box causes box to accelerate horizontally.

2. (20%) A janitor is pushing a 15.0 kg trashcan across a level floor at constant velocity. The coefficient of kinetic friction between the can and the floor is $\mu_k = 0.20$.

(a) If he is pushing horizontally, what is the magnitude of the force he is exerting against the can?

$$0 = ma_x = F_{JT}^{push} - F_{FT}^k \Rightarrow F_{JT}^{push} = F_{FT}^k = \mu_k F_{FT}^N = \mu_k mg$$

$$F^{push} = \mu_k mg = (0.20)(15 \text{ kg})(10 \frac{\text{m}}{\text{s}^2}) = \boxed{30 \text{ N}}$$

(b) If he pushes not horizontally but rather at an angle of 30° down from the horizontal, what must be the magnitude of his pushing force be to keep the can moving at constant velocity?

$$0 = ma_y = -F^{push} \sin \theta + F^N - mg \Rightarrow F^N = mg + F^{push} \sin \theta$$

$$0 = ma_k = F^{push} \cos \theta - \mu_k F^N = F^{push} \cos \theta - \mu_k mg - \mu_k F^{push} \sin \theta$$

$$F^{push} (\cos \theta - \mu_k \sin \theta) = \mu_k mg$$

$$F^{push} = \frac{\mu_k mg}{\cos \theta - \mu_k \sin \theta} = \frac{(0.20)(15 \text{ kg})(10 \frac{\text{m}}{\text{s}^2})}{0.866 - (0.20)(0.5)} = \frac{30 \text{ N}}{0.766}$$

$$F^{push} = \boxed{39 \text{ N}}$$

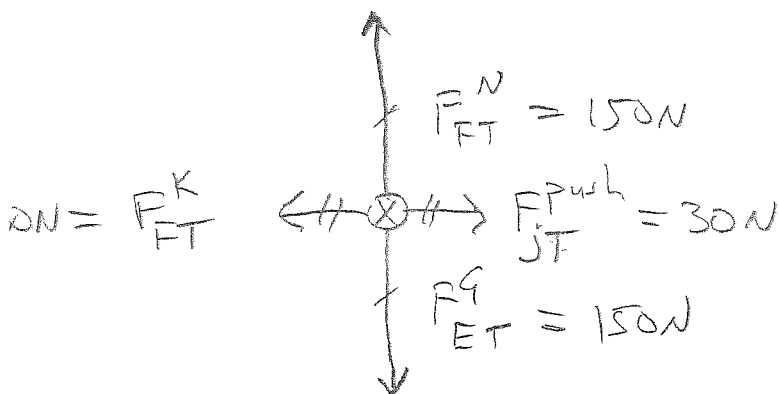
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What is the magnitude of the normal force between the trashcan and the floor in part (a)? And (b)?

(a) 150 N

(b) $150\text{ N} + (39\text{ N})(0.5) = 170\text{ N}$

(d) For the situation described in part (a), draw a Mazur-style free-body diagram (FBD, not EFBD) for the trash can. For every force exerted on the trash can, tell me what kind of force it is, what is exerting the force on what, and the force's magnitude and direction.



F_{FT}^N = upward normal force by floor on trashcan

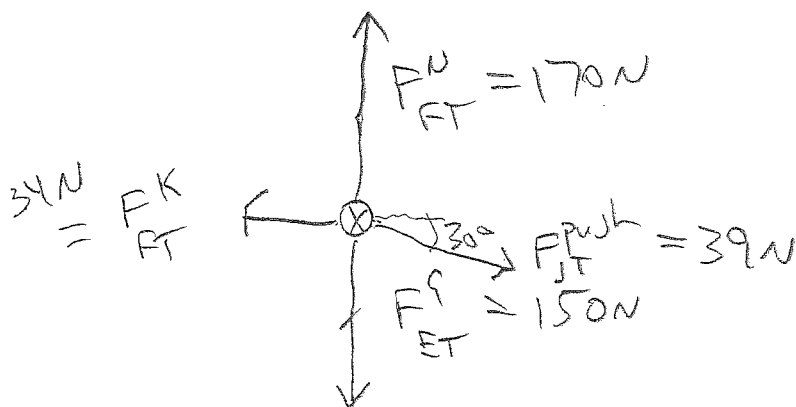
F_{ET}^G = downward gravitational force by Earth on trashcan

F_{JT}^{push} = forward contact force by janitor on trashcan

F_{FT}^K = backward kinetic friction by floor on trashcan

trashcan $\vec{a} = 0$

(e) For the situation described in part (b), draw a Mazur-style free-body diagram (FBD, not EFBD) for the trash can. Indicate the magnitude and direction of every force exerted on the trash can. (No need to repeat the words you wrote in part (d).)



trashcan $\vec{a} = 0$

3. (20%) (a) Use the Method of Joints to find the internal bar forces T_{AB} , T_{AC} , and T_{CB} in the truss shown below. You can work in any order you wish, but what worked out quickly for me was to start with the vertical forces at C , then horizontal forces at C , then horizontal forces at B . Be sure to indicate whether each bar is in tension or in compression.

$$0 = \sum F_y @ C = -T_{AC} \sin 60^\circ - T_{BC} \sin 60^\circ$$

$$T_{AC} = -T_{BC}$$

$$0 = \sum F_x @ C = 3 \text{ kN} - T_{AC} \cos 60^\circ + T_{BC} \cos 60^\circ$$

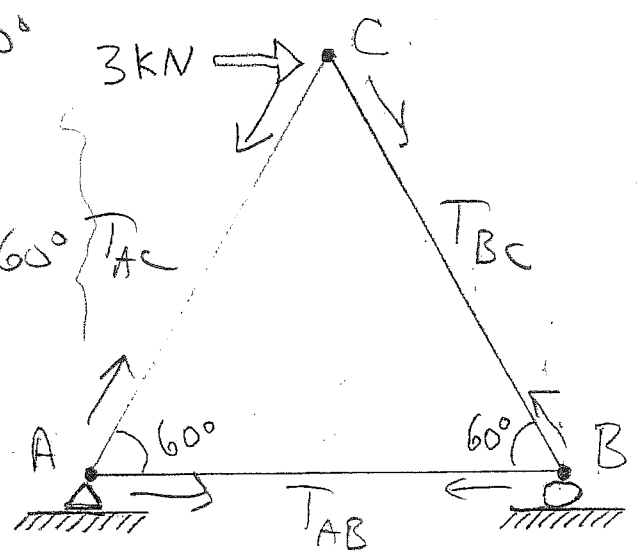
$$0 = 3 \text{ kN} + \frac{1}{2} T_{BC} + \frac{1}{2} T_{BC}$$

$$T_{BC} = -3 \text{ kN}$$

compression

$$T_{AC} = +3 \text{ kN}$$

tension



$$0 = \sum F_x @ B = -T_{AB} - T_{BC} \cos 60^\circ$$

$$T_{AB} = -\frac{1}{2} T_{BC} = +1.5 \text{ kN} = T_{AB}$$

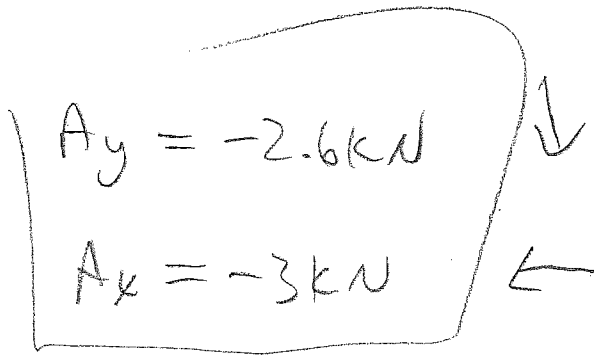
tension

(Problem continues on next page.)

(b) Find the support forces A_x , A_y , and B_y exerted on the truss by the pin support at A and by the roller support at B . Be sure to indicate whether each of these forces points up, down, left, or right. The easiest way to get B_y is to continue to use the Method Of Joints to write the vertical forces at B . After that, it's straightforward to find A_x and A_y .

$$B_y + T_{BC} \sin(60^\circ) = 0 \Rightarrow B_y = -0.866 T_{BC} = -(0.866)(3\text{kN})$$

$$B_y = +2.6\text{kN} \uparrow$$

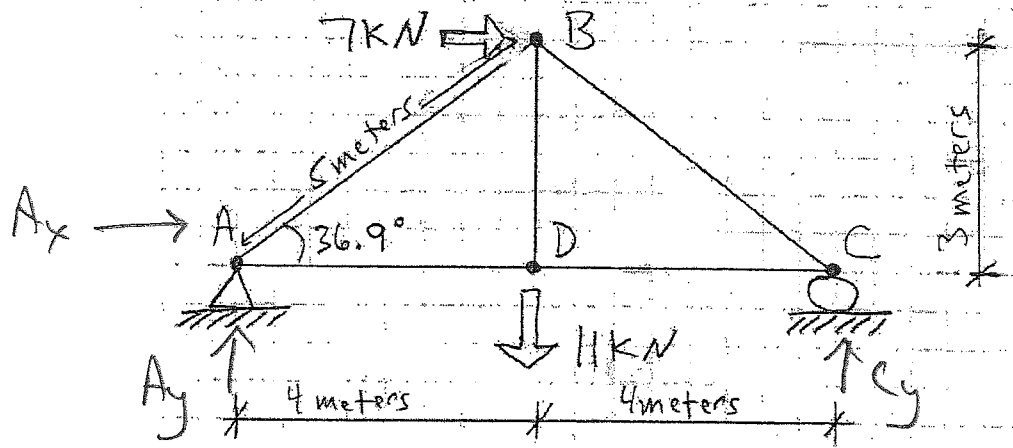


(c) As a double-check on your answer for B_y , evaluate the moment (torque) equation, for equilibrium of the truss as a whole, using joint A as a pivot. Take the distance from A to B to be L , so then the height of the triangle is $0.866L = (L/2) \tan(60^\circ)$.

$$0 = \sum M_A = L B_y - (0.866L)(3\text{kN})$$

$$B_y = (3\text{kN})(0.866) = 2.6\text{kN} \checkmark$$

4. (15%) The truss shown below carries two loads: a 7.0 kN horizontal force pushing joint B to the right, and an 11.0 kN vertical force pulling joint D downward.



(a) Using static equilibrium of the truss as a whole, solve for the support forces A_x and A_y exerted on the truss by the hinge support at A and for the support force C_y exerted on the truss by the roller support at C. Be sure to indicate whether A_x points left or right and whether A_y and C_y point up or down.

$$0 = \sum M_A = (8\text{m})C_y - (4\text{m})(11\text{kN}) - (3\text{m})(7\text{kN})$$

$$C_y = \frac{44 + 21}{8} \text{ kN} = \boxed{8.125 \text{ kN}} = C_y \uparrow$$

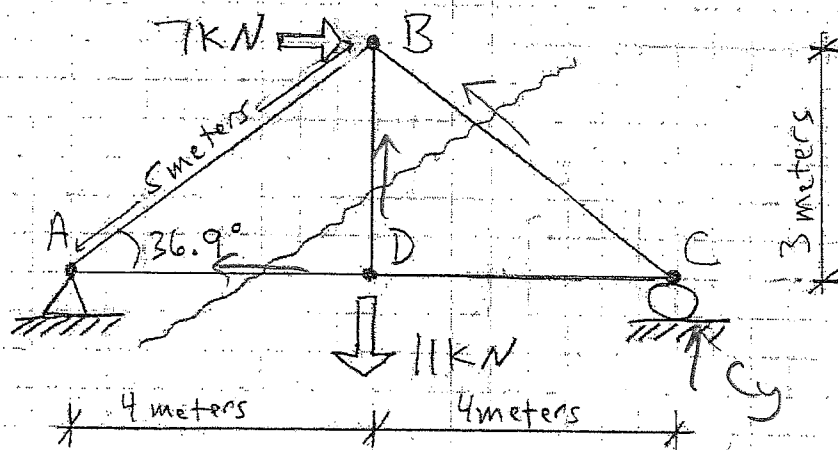
$$0 = \sum F_y = A_y + C_y - 11 \text{ kN} = 0 \Rightarrow \boxed{A_y = 2.875 \text{ kN}} \uparrow$$

$$0 = \sum F_x = A_x + 7 \text{ kN} = 0 \Rightarrow \boxed{A_x = -7 \text{ kN}} \leftarrow$$

So A_x actually
points left with
magnitude 7 kN.

(Problem continues on next page.)

(b) Use the Method of Sections, with the truss sectioned into two pieces as indicated, to solve for internal bar forces T_{BC} , T_{BD} , and T_{AD} . Be sure to indicate whether each of these bars is in tension or in compression. Try to make it obvious to me which half of the truss you've chosen to analyze and what point you're using as a pivot.



forces acting on
analyze a right half

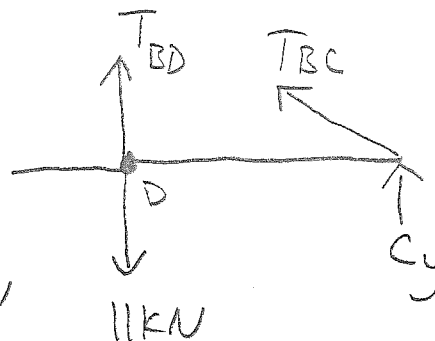
$$0 = \sum M_c = 11 \text{ kN}(4 \text{ m}) - T_{BD}(4 \text{ m})$$

$$\Rightarrow T_{BD} = 11 \text{ kN} \text{ (tension)}$$

$$0 = \sum F_y = 8.125 \text{ kN} + \frac{3}{5} T_{BC} + T_{BD} - 11 \text{ kN}$$

$$T_{BC} = -\frac{5}{3} (8.125 \text{ kN})$$

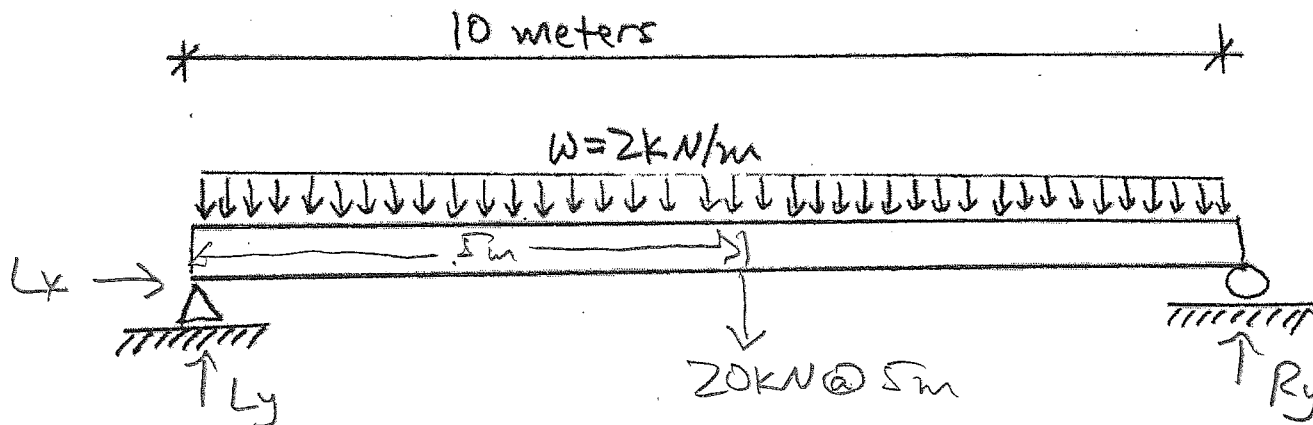
$$T_{BC} = -13.5 \text{ kN} \text{ (compression)}$$



$$0 = \sum F_x = -T_{AD} - \frac{4}{5} T_{BC} \Rightarrow T_{AD} = -\frac{4}{5} T_{BC}$$

$$T_{AD} = +10.8 \text{ kN} \text{ (tension)}$$

5. (10%) The "simply supported" beam shown below is 10 meters long. It carries a uniform distributed load of 2.0 kN/m along its entire length.



(a) Find the support forces L_x , L_y , and R_y exerted on the beam by the left ("L") and right ("R") supports.

$$0 = \sum F_x = L_x \Rightarrow \boxed{L_x = 0}$$

$$0 = \sum M_L = (10 \text{ m})(R_y) - (5 \text{ m})(20 \text{ kN})$$

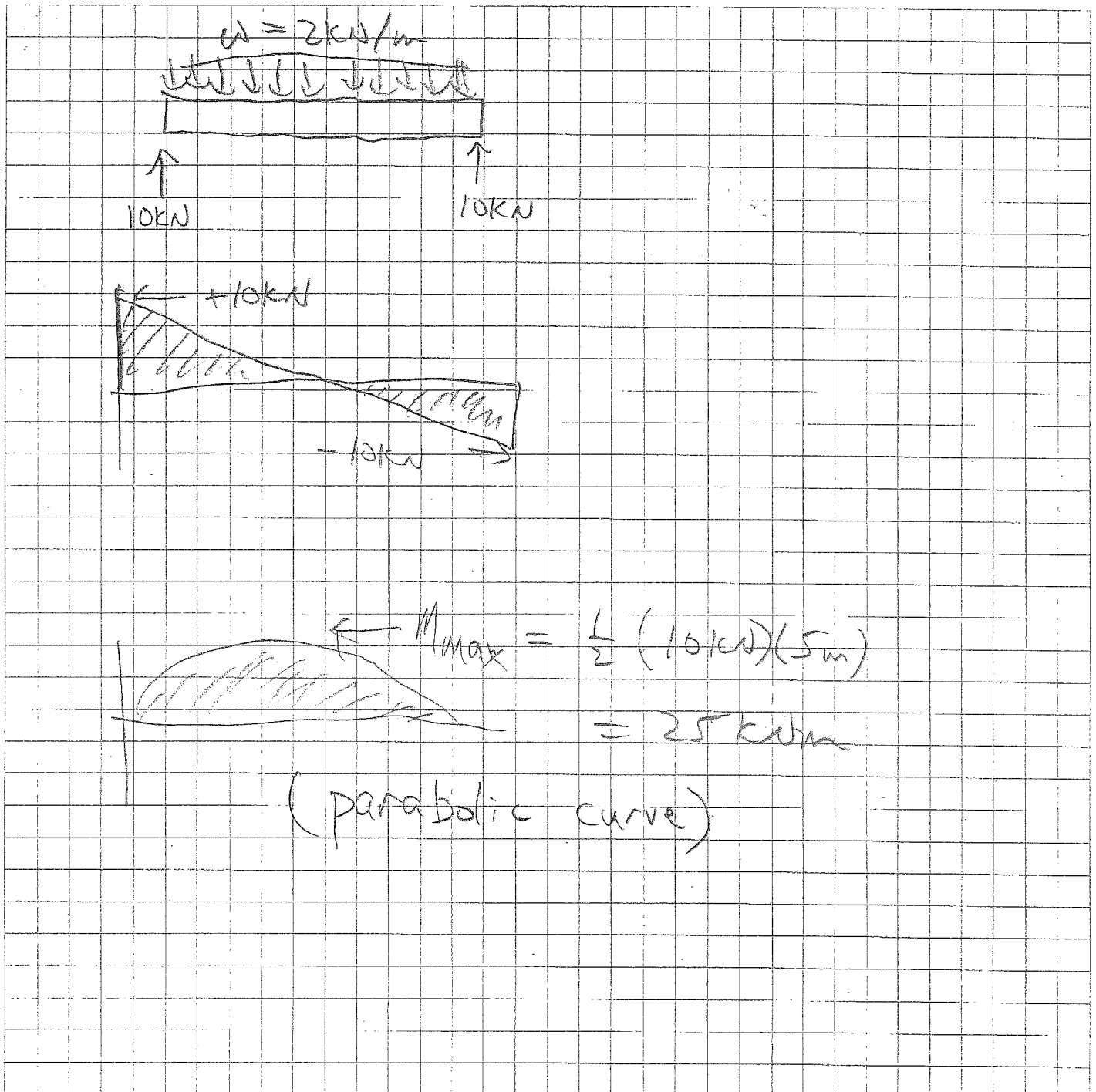
$$\Rightarrow \boxed{R_y = 10 \text{ kN}} \uparrow$$

$$0 = \sum F_y = L_y + R_y - 20 \text{ kN}$$

$$\Rightarrow \boxed{L_y = 10 \text{ kN}} \uparrow$$

(Problem continues on next page.)

(b) Draw load (EFBD), shear (V), and bending moment (M) diagrams for the beam.



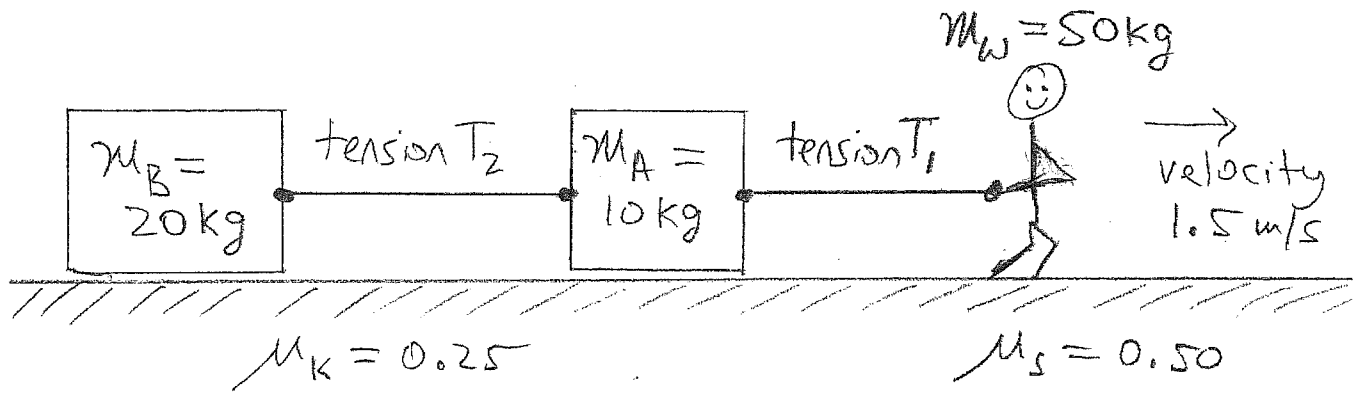
(c) What are the largest magnitude of the shear V (in kilonewtons) and the largest magnitude of the bending moment M (in kilonewton-meters)?

$$10 \text{ kN} = V_{\text{max}}$$

$$25 \text{ kNm} = M_{\text{max}}$$

(kNm NOT $\frac{\text{kN}}{\text{m}}$)

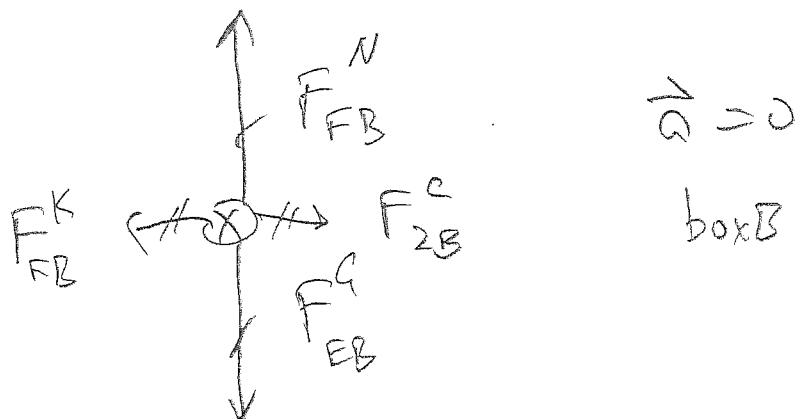
6. (15%) A worker chooses an unusual way to pull two boxes across a factory floor. The worker pulls one tow rope, which she has tied to the front of Box A. A second tow rope connects the back of Box A to the front of Box B. Both ropes are taut, horizontal, and of negligible mass; the floor is horizontal; and the worker and both boxes move to the right at constant velocity $v = 1.5 \text{ m/s}$. The boxes slide across the floor, but the worker's shoes do not slip as she walks. The coefficient of static friction between the worker's shoes and the floor is $\mu_s = 0.50$, while the coefficient of kinetic friction between each box and the floor is $\mu_k = 0.25$.



(a) List all forces (in all directions, both horizontal and vertical) acting on Box B. For each force, indicate what kind of force it is, indicate "by what" and "on what" the force is exerted, and note the force's magnitude and direction.

gravity by Earth on B \downarrow 200N
 normal by floor on B \uparrow 200N
 contact by A on B \rightarrow 50N
 kinetic friction by floor on B \leftarrow 50N

(b) Draw a Mazur-style free-body diagram (FBD, not EFBD) for Box B.

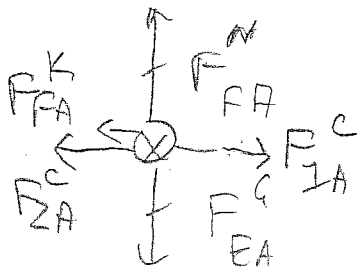


(Problem continues on next page.)

(c) List all forces (in all directions, both horizontal and vertical) acting on Box A. For each force, indicate what kind of force it is, indicate "by what" and "on what" the force is exerted, and note the force's magnitude and direction.

contact by 2 on A \leftarrow 50 N
 contact by 1 on A \rightarrow 75 N
 kinetic friction by floor on A \leftarrow 25 N
 gravity by Earth on A \downarrow 100 N
 normal by floor on A \uparrow 100 N

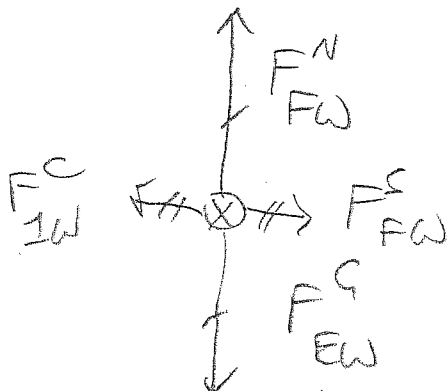
(d) Draw a Mazur-style free-body diagram (FBD, not EFBD) for Box A.



(e) List all forces (in all directions, both horizontal and vertical) acting on the worker. For each force, indicate what kind of force it is, indicate "by what" and "on what" the force is exerted, and note the force's magnitude and direction.

contact by 1 on W = 75 N \leftarrow
 static friction by floor on W = 75 N \rightarrow
 gravity by earth on W = 500 N \downarrow
 normal by floor on W = 500 N \uparrow

(f) Draw a Mazur-style free-body diagram (FBD, not EFBD) for the worker.



(Problem continues on next page.)

(g) What are the tensions T_2 and T_1 in the two tow ropes?

$$T_2 = 50\text{N}$$

$$T_1 = 75\text{N}$$

(h) Suppose that all of the details of this problem stay unchanged, but the boxes are made more massive. (The worker's mass is unchanged, and the friction coefficients are unchanged.) What is the largest combined mass, $m_A + m_B$, for the two boxes that the worker could pull, at constant velocity, before her shoes begin to lose their grip on the floor?

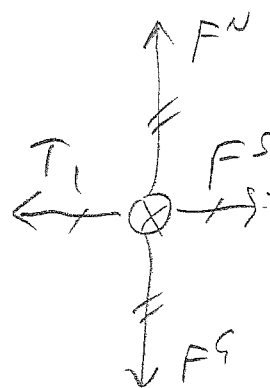
$$\mu_k (m_A + m_B) g = \mu_s m_w g$$

$$m_A + m_B = \frac{\mu_s}{\mu_k} m_w = \frac{0.50}{0.25} 50\text{kg} = \boxed{100\text{kg}}$$

$$(m_{A+B})_{\text{max}} = \boxed{100\text{kg}}$$

$$T_1 = \mu_k (m_A + m_B) g$$

Maximum possible $F^s = \mu_s m_w g$



Have a safe and happy winter break!