Physics 8, Fall 2019, Final Exam. **Name:** BILL

This two-hour, closed-book exam has 20% weight in your course grade. You can use one 3×5 card of your own handwritten notes and a calculator. **Turn in your 3×5 card of notes with your exam.**

Please show your work on these sheets. Use the spare sheet at the back of the exam if needed. I also have a pad of graph paper up front if you need a sheet or two. The last page of the exam contains a list of equations that you might find helpful. **Please approximate** $\boxed{g = 10.00\ \mathrm{m/s^2}}$ **to simplify numerical values, so that results are easier to compare.**

Work alone. Keep in mind that here at Penn, every member of the University community is responsible for upholding the highest standards of honesty at all times: offering or accepting help with this exam would be a serious violation of Penn's Code of Academic Integrity.

My signature below certifies that I am familiar with Penn's Code of Academic Integrity and that I agree to comply with the Code during this exam.

This is not a casual or perfunctory statement: honesty and academic integrity are core values of the Penn community. Please sign or write out your full name below.

Signature: Benjamin Franklin

Problem 1. (10%) [AA] You have been hired to check the technical correctness of an upcoming made-for-TV murder mystery. The mystery takes place in the International Space Station. In one scene, an astronaut's safety line is sabotaged while she is on a space walk, so she is no longer connected to the space station. She checks and finds that her thruster pack has also been damaged and no longer works. She is 100 meters from the station and moving with it. That is, she is not moving with respect to the station. There she is drifting in space with only a few minutes of air remaining. To get back to the shuttle, she decides to unstrap her 10.0 kg tool kit and throw it away with all her strength, so that the toolkit has a speed of 10.0 m/s. According to the script, she makes it back to the shuttle before running out of air. Her mass, including space suit (but without the tool kit) is 100.0 kg.

(a) In what direction should the astronaut throw her tool kit?

$\boxed{\text{Away}}$ from the space station, since the astronaut's change in momentum will be equal & opposite the tool kit's change in momentum.

$\Delta \vec{p}_1 = -\Delta \vec{p}_2$ since $\vec{p}_1 + \vec{p}_2$ is constant for an isolated system.

(b) After the astronaut throws away her tool kit, what is her velocity (with respect to the space station)? 1 = astronaut, 2 = tool kit. Let x axis point toward station.

$$m_1 v_{1xf} + m_2 v_{2xf} = m_1 \cancel{v_{1xi}} + m_2 \cancel{v_{2xi}} = 0$$

$$v_{1xf} = -\frac{m_2}{m_1} v_{2xf} = -\left(\frac{10\,\text{kg}}{100\,\text{kg}}\right)\left(-10\,\frac{\text{m}}{\text{s}}\right) = +1\,\frac{\text{m}}{\text{s}}$$

Her velocity is $\boxed{1.0\ \text{m/s toward}}$ station.

(c) How long does it take her to reach the space station? What do you conclude about whether the murder-mystery script (story) is plausible?

$$x_f = \cancel{x_i} + v_{xi} t$$

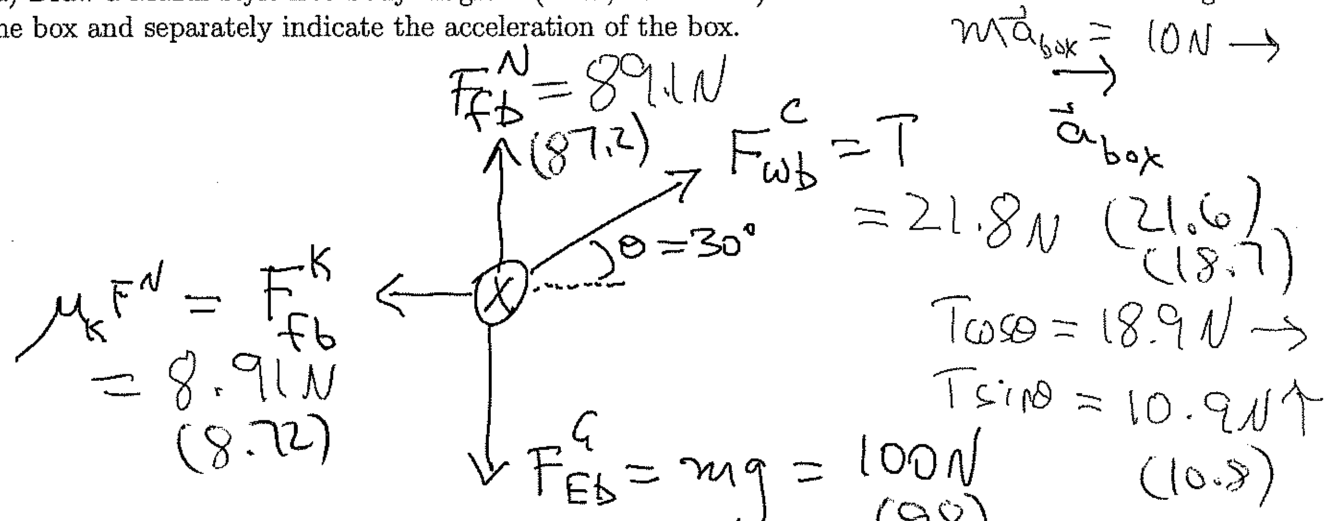
$$(100\,\text{m}) = (1.0\,\text{m/s})\,t \Rightarrow \boxed{t = 100\,\text{s}}$$

$\boxed{\text{It seems plausible}}$ that she can make it back to safety with just "a few minutes of air" remaining.

(If you use $g = 9.8 \frac{N}{kg}$.)

Problem 6. (15%) [BB] A woman applies a constant force to pull a 10.0 kg box across a floor. The force is large enough to cause the box to accelerate horizontally forward (toward the woman) at 1.00 m/s^2. The woman applies this force by pulling on a rope that makes an angle of 30.0° above the horizontal, and for the box-floor interface, the coefficient of kinetic friction is $\mu_k = 0.100$.

(a) Draw a Mazur-style free-body diagram (FBD, not EFBD) for the box. Show all forces acting on the box and separately indicate the acceleration of the box.



(b) Find the tension in the rope.

$$0 = ma_y = \sum_{\substack{on\\box}} F_y = F^N + T\sin\theta - mg = 0$$

$$\Rightarrow F^N = mg - T\sin\theta$$

$$ma_x = \sum_{on\,box} F_x = T\cos\theta - \mu_k(mg - T\sin\theta) = T\cos\theta + \mu_k T\sin\theta - \mu_k mg$$

$$\boxed{T} = \frac{ma_x + \mu_k mg}{\cos\theta + \mu_k \sin\theta} = 21.8N \quad (21.6)$$

(c) Label your FBD with numerical values for all of the forces that you indicated on your diagram.

(d) How far does the box travel in 5.0 seconds?

$$X_f = \cancel{X_i} + \cancel{v_{xi}}t + \frac{1}{2}a_x t^2 = \frac{1}{2}\left(1\frac{m}{s^2}\right)(5s)^2$$

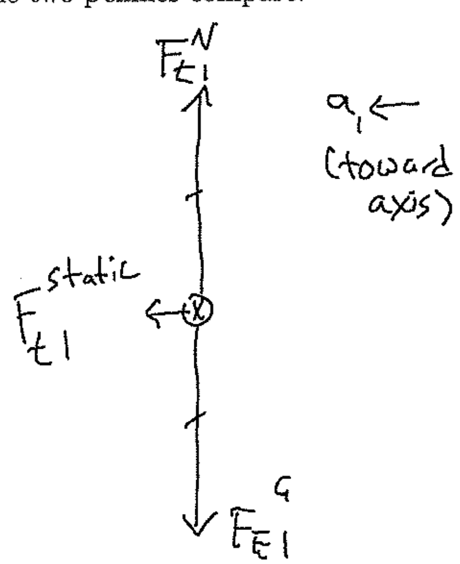
$$\boxed{X_f = 12.5m}$$

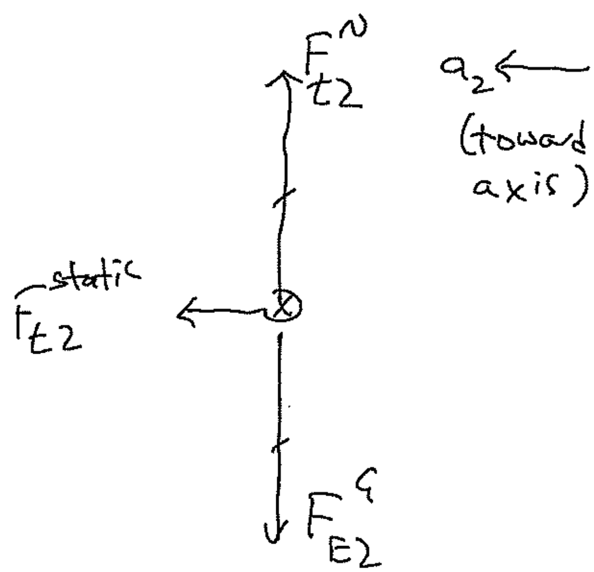
Problem 2. **(10%)** [CC] For an in-class physics demonstration (just like one you saw), I place two pennies (coins) on a turntable. The first penny is at radius $R_1 = 0.100\,\text{m}$ from the center axis of the turntable. The second penny is at radius $R_2 = 0.200\,\text{m}$ from the center axis of the turntable. The turntable initially spins at constant rotational speed $\omega = 1.00$ radian/second. Remember that the speed of each penny, as it moves in a circle, is given by $v = \omega R$. Only static friction prevents the pennies from sliding off the turntable, with $\mu_S = 0.100$.

(a) What are the speeds v_1 and v_2 of the two pennies?

$$v_1 = \omega R_1 = 0.100\ \text{m/s}$$
$$v_2 = \omega R_2 = 0.200\ \text{m/s}$$

(b) Draw a Mazur-style free-body diagram (FBD, not EFBD) for penny 1, and draw a FBD for penny 2. Be sure to indicate the corresponding acceleration direction for each FBD. Try to draw the two FBDs roughly (approximately) to scale, so that one can see approximately how the forces acting on the two pennies compare.





(Problem continues on next page.)

(c) Now I very gradually increase the rotational speed ω of the turntable. At what value of ω does each penny slide off of the table? Which penny slides off first? (You saw this happen in class.)

[Note related to yesterday's review session: just taking a square-root does not count as "solving a quadratic equation!"]

Penny remains on turntable while $F_{tp}^{static} \le \mu_s F_{tp}^{N}$

$$m a_x = \sum F_x = -F_{tp}^{static}$$

(I let x axis point away from turntable axis.)

$$-m\omega^2 R = -F_{tp}^{static}$$

$$F_{tp}^{N} = mg$$

So penny leaves table once $m\omega^2 R > \mu_s m g$

$$\omega_{max} = \sqrt{\frac{g\mu_s}{R}}$$

$$\omega_{max1} = \sqrt{\frac{g\mu_s}{R_1}} = 3.16 \frac{\text{radians}}{s} \qquad (3.13)$$

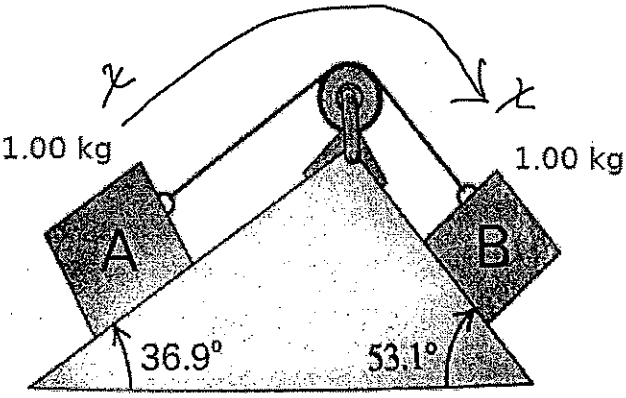
$$\omega_{max2} = \sqrt{\frac{g\mu_s}{R_2}} = 2.24 \frac{\text{radians}}{s} \qquad (2.21)$$

So penny #2 (larger R) slides off first.

Problem 3. (15%) [DD] Two blocks of equal mass are connected by a taut cable that passes over a small pulley (of negligible inertia). Friction in the pulley is negligible, and both blocks rest on planes of negligible friction.

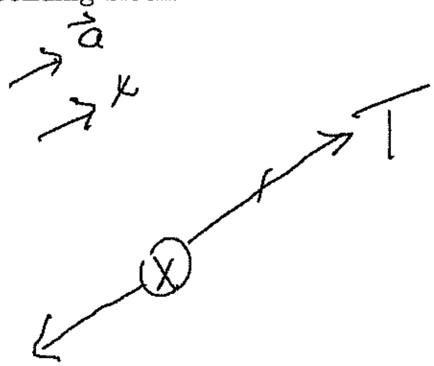
(a) Which way will the system move when the blocks are released from rest? Indicate this direction on the diagram and define this to be the direction in which the x coordinate increases. (The same x coordinate will describe both the uphill motion of one block and the downhill motion of the other block, since the cable stays taut. You only need one coordinate for this problem.)

Reasoning: $m_B g \sin\theta_B > m_A g \sin\theta_A$



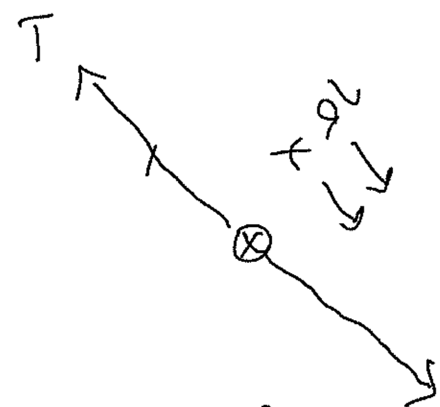
(b) Draw a Mazur-style free-body diagram (FBD, not EFBD) for block A, and draw a Mazur-style FBD for block B. Be sure to indicate, adjacent to each FBD, the direction of acceleration for the corresponding block.

A



$F^G_{EA,x} = -m_A g \sin\theta_A$

B



$F^G_{EB,x} = +m_B g \sin\theta_B$

(Problem continues on next page.)

(c) Write Newton's second law, $ma_x = \sum F_x$, separately for each of the blocks. Since the cable stays taut, ~~a_x is the same for both blocks.~~

$$\boxed{\begin{aligned} m_A a_x &= -m_A g \sin\theta_A + T \\ m_B a_x &= -T + m_B g \sin\theta_B \end{aligned}}$$

$m_A = m_B = m$

$\sin\theta_A = \frac{3}{5}$

$\sin\theta_B = \frac{4}{5}$

$$T = m_A (a_x + g\sin\theta_A) = m(a_x + \tfrac{3}{5} g)$$

$$m a_x = -m(a_x + \tfrac{3}{5} g) + m(\tfrac{4}{5} g)$$

$$m a_x = -m a_x - \tfrac{3}{5} mg + \tfrac{4}{5} mg$$

(d) What is the acceleration, a_x of the blocks?

$$2ma_x = \frac{1}{5} mg \Rightarrow \boxed{a_x = +\frac{g}{10} = +1\ \frac{m}{s^2}}$$

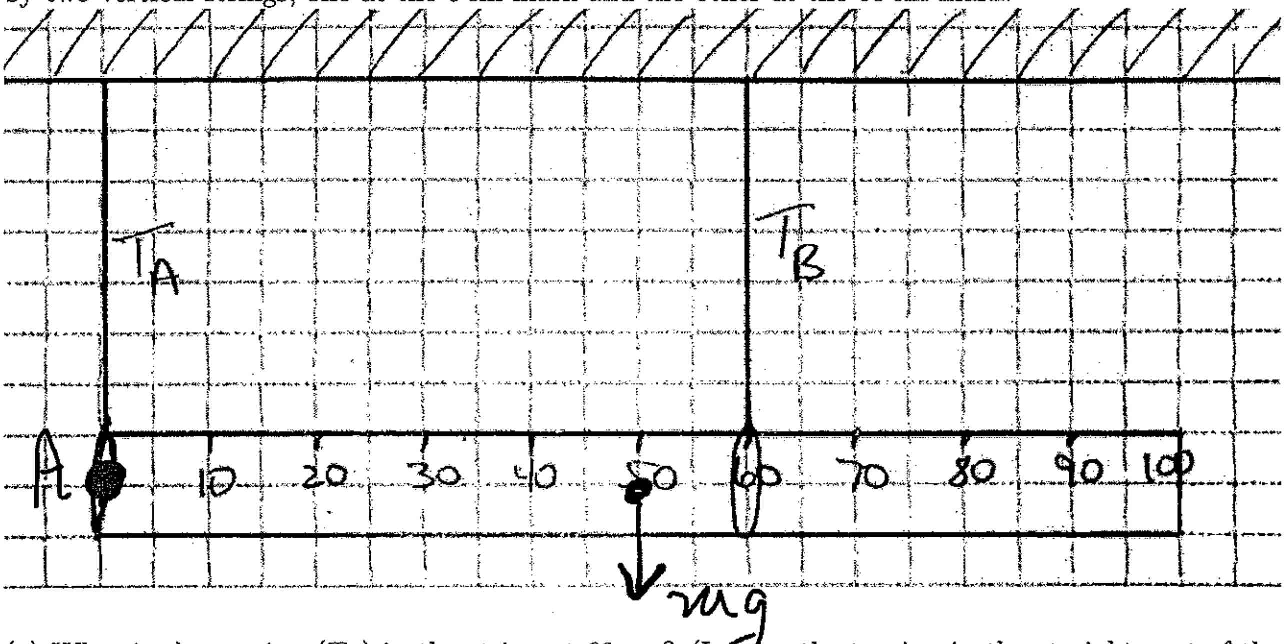
(0.98)

$$T = m\left(\frac{g}{10} + \frac{3}{5} g\right) = \frac{7}{10} mg$$

(e) What is the tension T in the cord?

$$\boxed{T = 7N} \quad (6.86)$$

Problem 7. **(10%)** **[EE]** A meter stick of mass 1.00 kg is supported, in a horizontal orientation, by two vertical strings, one at the 0 cm mark and the other at the 60 cm mark.



(a) What is the tension (T_B) in the string at 60 cm? (I mean the tension in the straight part of the string that is above the ruler; don't worry about the small portion that encircles the ruler.)

Pivot about A: $0 = \sum M_A = T_B\,(0.6\text{m}) - mg\,(0.5\text{m})$

$\boxed{T_B} = \frac{5}{6} mg = \frac{5}{6}(10\,N) = \boxed{8.33N}$

(8.17)

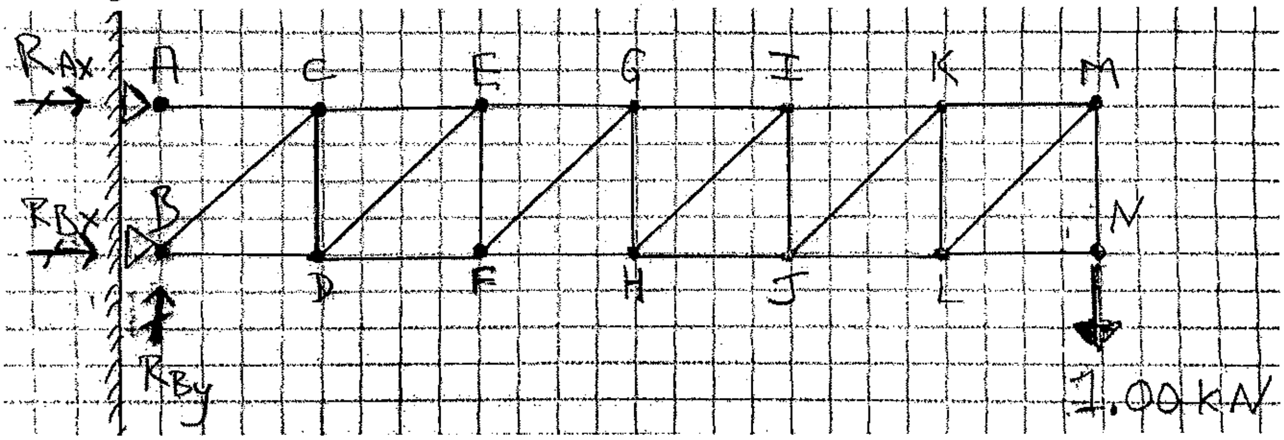
(b) What is the tension (T_A) in the string at 0 cm?

$0 = \underset{\text{on ruler}}{\sum F_y} = T_A + T_B - mg \Rightarrow T_A = mg - T_B$

$\boxed{T_A} = \frac{1}{6} mg = \boxed{1.67N}$

(1.63)

Problem 8. **(25%)** **[FF]** A cantilever truss is supported from the wall, on the left-hand side, at joints **A** and **B**. All horizontal and vertical bars of the truss have length 1.000 m. All diagonal bars have length $\sqrt{2}\,\text{m} \approx 1.414\,\text{m}$. The truss carries a single vertical load, 1.00 kN downward, at joint **N**.



(a) Using the moment equation for the truss as a whole about joint **B**, determine the horizontal support "reaction" force R_{Ax}. What does the sign of your answer imply about the direction that this support force actually points?

$$0 = \sum_{\substack{\circlearrowleft \text{ on truss}}} M_B = -R_{Ax}(1\text{m}) - (1\text{kN})(6\text{m}) \Rightarrow \boxed{R_{Ax} = -6\text{kN}}$$

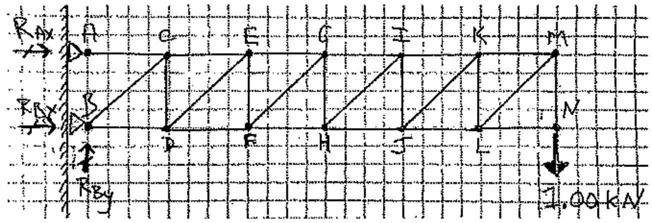
So R_{Ax} actually points to the left, i.e. the wall at A pulls the truss left.

(b) Using equilibrium for the truss as a whole, determine the horizontal and vertical support "reaction" forces R_{Bx} and R_{By}.

$$0 = \sum_{\text{on truss}} F_x = R_{Ax} + R_{Bx} \Rightarrow \boxed{R_{Bx} = +6\text{kN}}$$

$$0 = \sum_{\text{on truss}} F_y = R_{By} - 1\text{kN} \Rightarrow \boxed{R_{By} = +1\text{kN}}$$

(Problem continues on next page.)



(c) Using the method of joints for joint **A**, determine bar force T_{AC}. Is this bar in tension or in compression?

$$0 = \sum_{\substack{\text{on joint} \\ A}} F_x = R_{Ax} + T_{AC} \Rightarrow T_{AC} = -R_{Ax} = +6\text{kN}$$

$$\boxed{T_{AC} = +6\text{kN}} \text{ (tension)}$$

(d) Using the method of joints for joint **B**, first determine bar force T_{BC}, then determine bar force T_{BD}. Note whether each of these two bars is in tension or in compression.

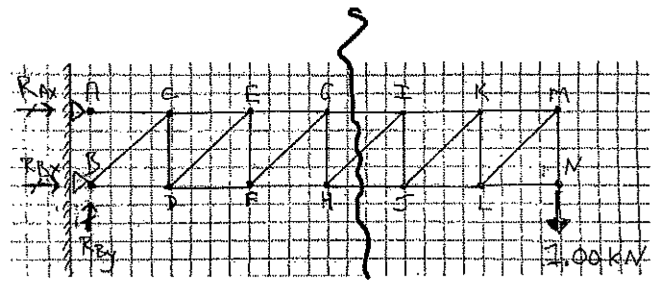
$$0 = \sum_{\substack{\text{on joint} \\ B}} F_y = R_{By} + \frac{T_{BC}}{\sqrt{2}} \Rightarrow T_{BC} = -\sqrt{2}\, R_{By}$$

$$\boxed{T_{BC} = -1.41\text{ kN}} \text{ (compression)}$$

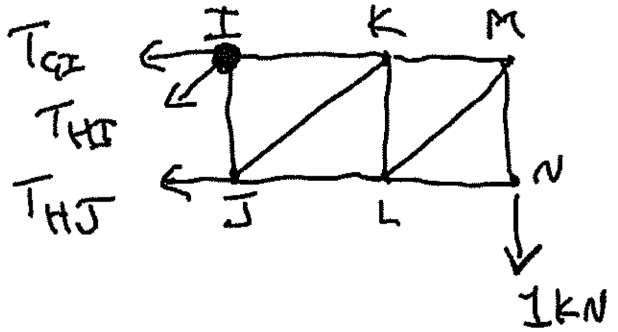
$$0 = \sum_{\substack{\text{on joint} \\ B}} F_x = R_{Bx} + T_{BD} + \frac{T_{BC}}{\sqrt{2}} \Rightarrow T_{BD} = -\left(R_{Bx} + \frac{T_{BC}}{\sqrt{2}}\right)$$

$$\boxed{T_{BD} = -5\text{ kN}} \text{ (compression)}$$

(Problem continues on next page.)



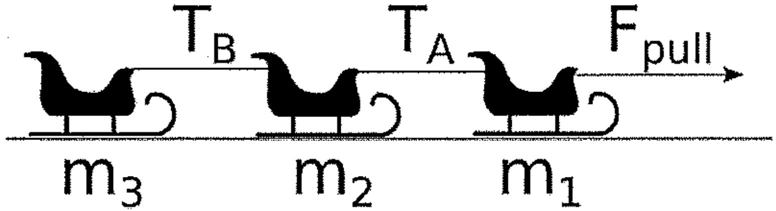
(e) Now determine the single bar force T_{HJ}. To do this, use the method of sections with a cut passing through bars **GI**, **HI**, and **HJ**. Redraw the part of the truss that is on the right-hand side of your cut line, with the cut bars' forces now drawn as external forces acting on the visible side of the truss. [Imagine what force the missing left-hand side would have had to exert on each cut bar, which by convention we assume to be in tension.] The original 1.00 kN load is also an external force. Write the moment equation about joint **I** to solve for bar force T_{HJ}. Your result should have the same sign as you found earlier for T_{BD}, but should be smaller in magnitude.



$$0 = \sum_{\text{on RHS of truss}} M_I = -(1\,\mathrm{m})\,T_{HJ} - (2\,\mathrm{m})(1\,\mathrm{kN})$$

$$\boxed{T_{HJ} = -2\,\mathrm{kN}} \quad (\text{compression})$$

Problem 5. (10%) [GG] Three sleds are pulled to the right across a horizontal sheet of ice using horizontal cables. Friction between the ice and the sleds is negligible. The three sleds (numbered from right to left) have masses $m_1 = 10.0\,\text{kg}$, $m_2 = 20.0\,\text{kg}$, and $m_3 = 30.0\,\text{kg}$ respectively. The pull exerted by the tow cable on sled 1 is $F_{\text{pull}} = 120\,\text{N}$ to the right. Sleds 1 and 2 are connected by a taut cable of tension T_A. Sleds 2 and 3 are connected by a taut cable of tension T_B.



(a) Find the acceleration a_x of the three-sled system, where the x axis points to the right.

The only external force acting on the 3-sled system is F_{pull}.

$$(m_1 + m_2 + m_3)\, a_x = F_{pull}$$

(Newton's 2nd law for the 3-sled system.)

$$a_x = \frac{120\,N}{60\,kg} = \boxed{+2.00\,\frac{m}{s^2}}$$

(b) Find the tensions T_A and T_B. I found it easier to find T_B first, then T_A.

Writing Newton's 2nd law for sled 3:

$$m_3 a_x = T_B \Rightarrow \boxed{T_B} = (30\,kg)(2\,\tfrac{m}{s^2}) = \boxed{60.0\,N}$$

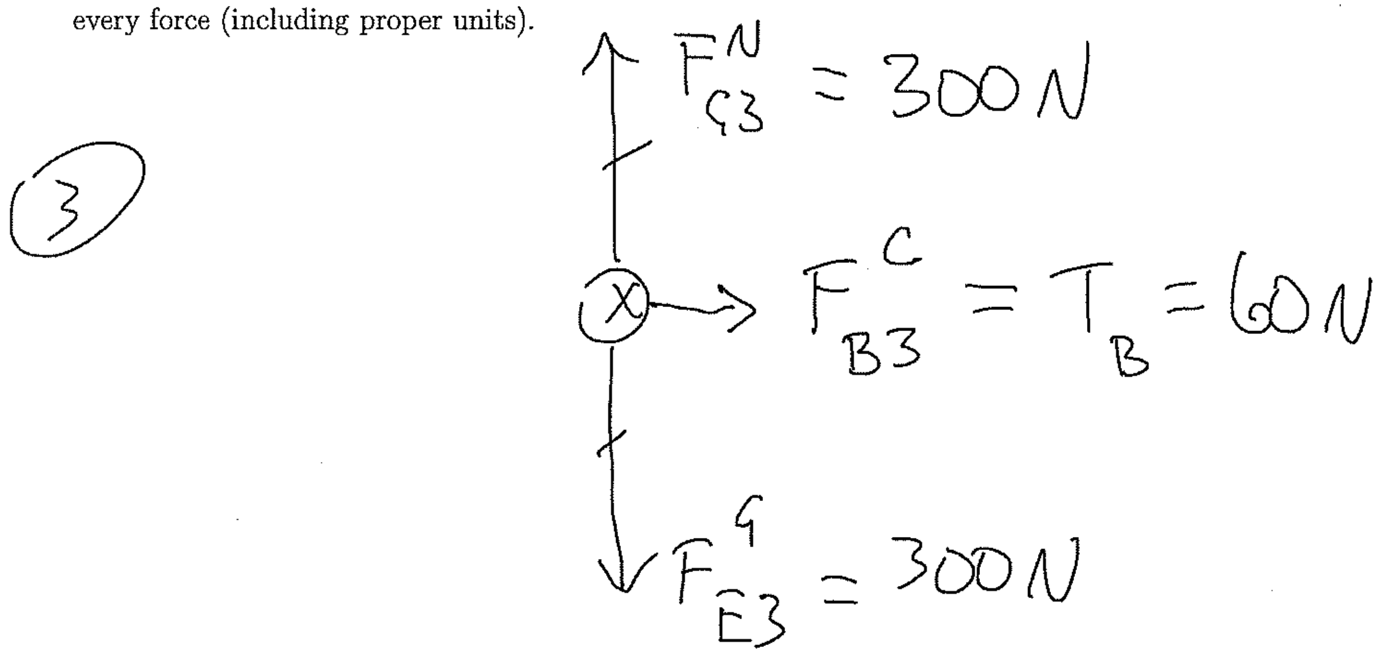
Writing Newton's 2nd law for sled 2:

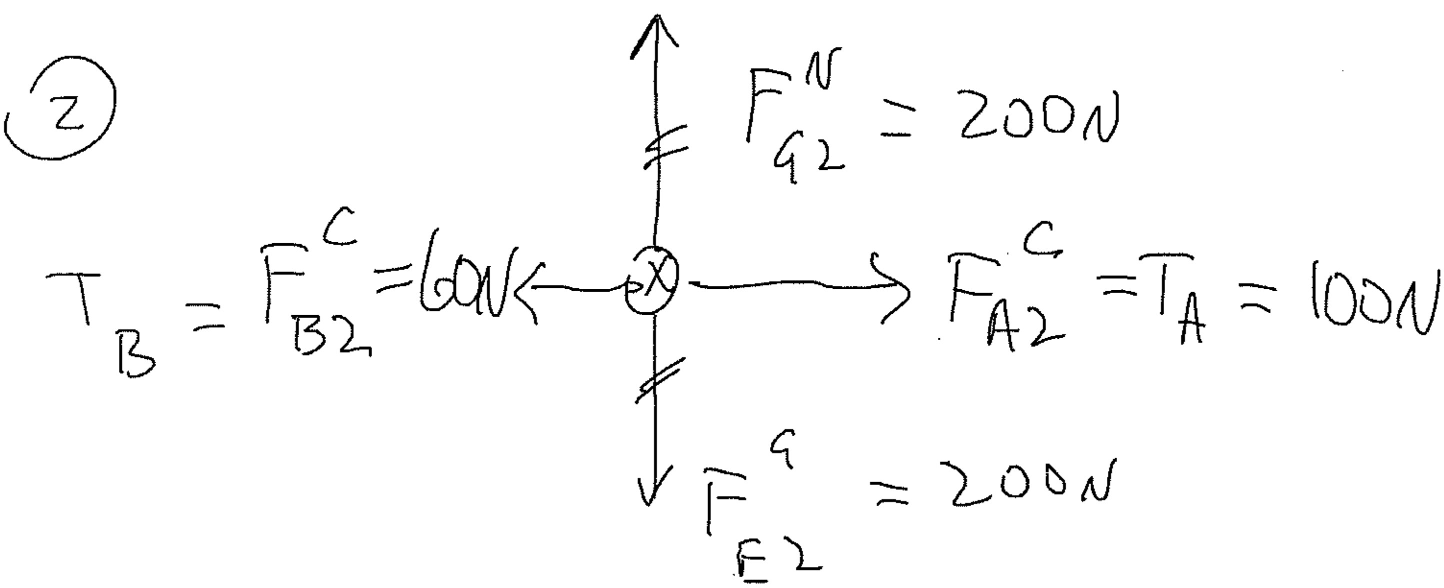
$$m_2 a_x = T_A - T_B \Rightarrow T_A = T_B + m_2 a_x$$

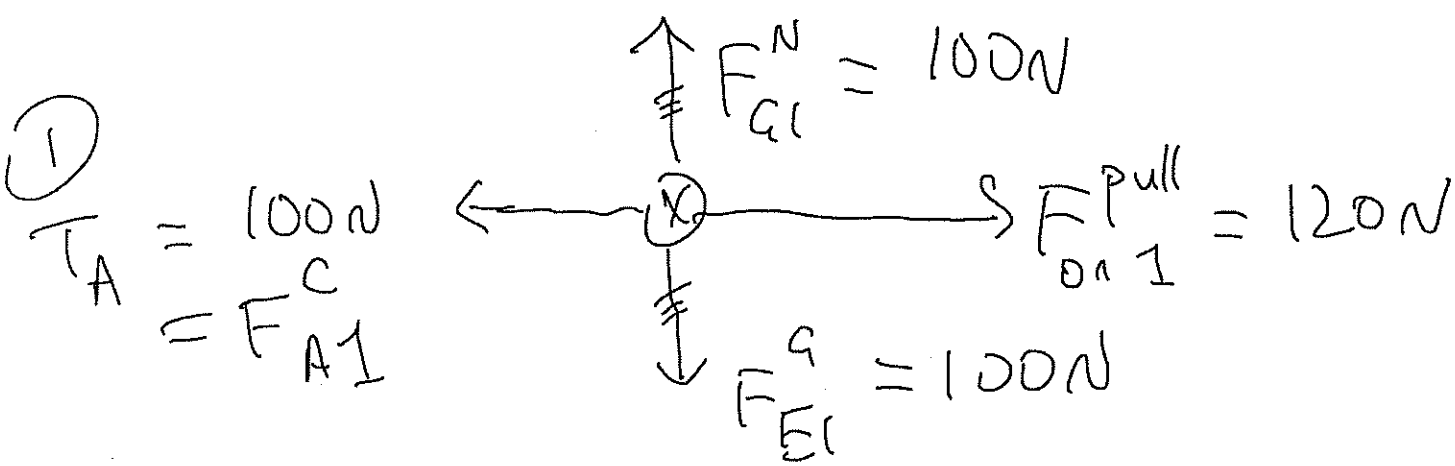
$$\boxed{T_A} = 60\,N + (20\,kg)(2\,\tfrac{m}{s^2}) = \boxed{100\,N}$$

(Problem continues on next page.)

(c) Draw a free-body diagram for sled 3, then a free-body diagram for sled 2, then a free-body diagram for sled 1. Include both horizontal and vertical forces. Indicate the numerical magnitude of every force (including proper units).







Problem 4. **(5%)** **[HH]** Two male reindeer charge head-on at each other with the same speed and meet on an icy patch of tundra. The smaller reindeer is charging to the right, and the larger reindeer is charging to the left. As they collide, their antlers lock together and the two animals slide together with one-quarter of their original speed.

(a) What is the ratio of their masses? let x axis point right

let small reindeer be #1
large reindeer be #2

initial momenta: ① → ← ②
final momentum: ← (1+2)

$$m_1 v_{1xi} + m_2 v_{2xi} = (m_1 + m_2) v_{xf}$$

$$m_1 v_i + m_2 (-v_i) = (m_1 + m_2)\left(-\frac{v_i}{4}\right)$$

$$(m_1 - m_2) v_i = -\frac{1}{4}(m_1 + m_2) v_i$$

$$4(m_2 - m_1) = m_1 + m_2$$

$$4m_2 - 4m_1 = m_1 + m_2$$

$$3m_2 = 5m_1$$

$$\boxed{m_2 = \frac{5}{3} m_1}$$

(b) In which direction do they slide after colliding?

They slide in the direction in which the larger reindeer was initially traveling, which I called the $-x$ direction.

Possibly useful equations

$$\cos(30°) = \frac{\sqrt{3}}{2} \approx 0.866 \qquad \sin(30°) = \frac{1}{2} \qquad \tan(30°) = \frac{1}{\sqrt{3}} \approx 0.577$$

$$\cos(60°) = \frac{1}{2} \qquad \sin(60°) = \frac{\sqrt{3}}{2} \approx 0.866 \qquad \tan(60°) = \sqrt{3} \approx 1.732$$

$$\cos(36.9°) = \frac{4}{5} \qquad \sin(36.9°) = \frac{3}{5} \qquad \tan(36.9°) = \frac{3}{4}$$

$$\cos(45°) = \sin(45°) = \frac{1}{\sqrt{2}} \approx 0.707$$

$$\sum \vec{F} = m\vec{a} \qquad \sum \vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} \qquad \vec{p} = m\vec{v}$$

$$F_c = \frac{mv^2}{r} \qquad F_c = m\omega^2 r \qquad v = \omega r$$

$$F^K = \mu^K F^N \qquad F^s \le \mu_s F^N$$

$$F_x^{\text{spring}} = -k\,(x - x_0)$$

$$F_y^{\text{grav}} = -mg \qquad g = 9.8\ \mathrm{m/s^2}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \tau = rF\sin\theta = r_\perp F = rF_\perp$$

$$\frac{F}{A} = (\text{stress}) = (E)\,(\text{strain}) = E\,\frac{\Delta L}{L_0}$$

$$V = \frac{\mathrm{d}M}{\mathrm{d}x} \qquad (M_2 - M_1) = (x_2 - x_1)\overline{V}_{1\to 2} \qquad w = -\frac{\mathrm{d}V}{\mathrm{d}x} \qquad V(x) = \sum_{0\to x} F_y(\text{up minus down})$$