Physics 8, Fall 2011, Homework Solutions #12.

Problem 1 (10%)
(a) Yes. $\omega = \sqrt{k/m}$. Neither k nor m changes on Mars.
(b) No. $\omega = \sqrt{g/\ell}$. g is different on Mars.
(c) $g_{\rm mars} = GM_{\rm mars}/R^2_{\rm mars}$, while $g_{\rm earth} = GM_{\rm earth}/R^2_{\rm earth}$

$$\frac{g_M}{g_E} = (M_M/M_E)(R_E/R_M)^2 = (1/10)(2)^2 = 0.4$$

Pendulum clock is slower on Mars by a factor $\boxed{\sqrt{0.4} = 0.63}$. So on Mars, the spring clock is faster than the pendulum clock by a factor $\boxed{1.6}$.

Problem 2 (5%)
Total weight is 300 N, so we know that $T_P + T_R = 300$ N. Consider torques about center of beam:

$$-(L/2)T_P + (L/4)T_R = 0 \quad \Rightarrow \quad T_R = 2T_P$$

So $\boxed{T_R = 200\ \text{N}}$ and $\boxed{T_P = 100\ \text{N}}$.

Problem 3 (10%)
(a) $F = 30$ N, $m = 15$ kg, so $a = 2$ m/s^2.

$$x = \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{2x/a} = \sqrt{\frac{2 \times 4\ \text{m}}{2\ \text{m/s}^2}} = \boxed{2\ \text{s}}$$

(b) $W = F \cdot \Delta x = (30\ \text{N})(4\ \text{m}) = \boxed{120\ \text{J}}$
(c) $v^2 = v_0^2 + 2ax$, so $v = \sqrt{2ax} = \sqrt{2 \times 2 \times 4}$ m/s $= \boxed{4\ \text{m/s}}$
(d) $W = \int F \mathrm{d}x = \boxed{-5\ \text{J}}$ (by counting the boxes to add up the area under the curve)

Problem 4 (5%)
$f_1 = 2\pi\sqrt{k/m_1}$, while $f_2 = 2\pi\sqrt{k/(m_1+m_2)}$

$$f_2/f_1 = 1/2 = \sqrt{m_1/(m_1+m_2)}$$

So $m_1/(m_1+m_2) = 1/4$, so $4m_1 = m_1 + m_2$, so $m_2 = 3m_1 = \boxed{3.0\ \text{kg}}$.

Problem 5 (15%)
(a) Hollow cylinder: $I_{hc} = mR^2$. Solid cylinder: $I_{sc} = \frac{1}{2}mR^2$. Billiard ball: $I_{bb} = \frac{2}{5}mR^2$.

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I(v/R)^2 = mgh$$

The second term increases as I/mR^2 increases. So

$$\boxed{HC > SC > BB}$$

(b)

$$v^2\left(1+\frac{I}{mR^2}\right) = 2gh \quad\Rightarrow\quad v = \sqrt{\frac{2gh}{1+\frac{I}{mR^2}}}$$

$$v_{bb} \propto \frac{1}{\sqrt{1+\frac{2}{5}}} = \sqrt{\frac{5}{7}} = 0.845$$

$$v_{sc} \propto \frac{1}{\sqrt{1+\frac{1}{2}}} = \sqrt{\frac{2}{3}} = 0.816$$

$$v_{hc} \propto \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

So the speeds scale as $\boxed{BB : SC : HC \sim 1.195 : 1.154 : 1.000}$

Problem 6 (10%)

$$(m+9m)(800\text{ m/s}) = (m)(v_{\text{rocket}} + 100\text{ m/s}) + (9m)(v_{\text{rocket}})$$

$$(10)(800\text{ m/s}) = v_{\text{rocket}} + 100\text{ m/s} + 9v_{\text{rocket}}$$

So $\boxed{v_{\text{rocket}} = +790\text{ m/s}}$ and $\boxed{v_{\text{shuttle}} = +900\text{ m/s}}$.

Problem 7 (20%)

(a) As we saw in the classroom demo, the **only** angle that works is to aim directly for the target. The easiest way to see this is to notice that the same $\frac{1}{2}gt^2$ is subtracted from both y_{bullet} and y_{target} at any instant. So

$$\tan\theta = \frac{50\text{ m}}{100\text{ m}} = \frac{1}{2} \quad\Rightarrow\quad \boxed{\theta = 26.5^\circ}$$

(b) Horizontally, the bullet crosses the target's path at $t = (100\text{ m})/v_{0x}$. Vertically, the slowest possible case is that the bullet hits the target at ground level, i.e. the bullet just goes up and back down to the ground after time t. So t is the time that it takes for v_y to go from v_{0y} all the way down to $-v_{0y}$: $t = 2v_{0y}/g$. Because we know $v_{0x} = 2v_{0y}$, $t = v_{0x}/g$. Combining,

$$\frac{100\text{ m}}{v_{0x}} = \frac{v_{0x}}{g} \quad\Rightarrow\quad v_{0x} = \sqrt{(100\text{ m})(9.8\text{ m/s}^2)} = 31.3\text{ m/s}$$

So then $v_{0y} = 15.6$ m/s, and $\boxed{v_0 = 35\text{ m/s}}$ (minimum possible speed to hit target).

(c) $v_0 = 50$ m/s, so $v_{0x} = 44.7$ m/s. Then $t = \frac{100\text{ m}}{v_{0x}} = 2.23$ s. So $\Delta y = \frac{1}{2}gt^2 = \boxed{24.4\text{ m}}$.

Problem 8 (5%)

$$m\omega^2 R = \frac{GmM}{R^2}$$

$$R^3 = \frac{GM}{\omega^2} = \frac{GM}{(2\pi f)^2} = \frac{GM}{(2\pi)^2}\, T^2$$

$$\boxed{R = 3.8 \times 10^8 \text{ m}}$$

which is about a quarter of a million miles, which is a familiar number.

Problem 9 (5%)
(a) For maximum torque, $\sin\theta = 1$, and $\tau = rF = mgr = (51 \text{ kg})(9.8 \text{ m/s}^2)(0.20 \text{ m}) = \boxed{100 \text{ Nm}}$
(b) pull up on handle bars while standing and pedaling; or wear toe clips so that you can pull up on one pedal while you push down on the other; or install longer pedals; or gain weight!

Problem C1 (3%)
Answer is **(E)** because $\vec{F}_{a,b} = -\vec{F}_{b,a}$: action and reaction have same magnitude and opposite direction.

Problem C2 (6%)
(a) Answer is **(A,B,E)**. Downward force of gravity is balanced by upward contact force. Centripetal acceleration is provided by contact force pointing toward center of circle.
(b) Answer is **(2)**. Since there are no unbalanced forces, the velocity is constant, and the trajectory is a straight line.

Problem C3 (3%)
Answer is **(2)**. There is a constant downward acceleration g, so the slope starts out at zero and continuously decreases. The other curves all show constant slope at least part of the time, which cannot be correct.

Problem C4 (3%)
Answer is **(5)**. Constant force acting on constant mass yields constant acceleration in the y direction, while no force yields constant velocity in the x direction. So the curve should be a parabolic arc for which v_y starts out at zero and increases at a constant rate, while v_x is constant.

Bill's total time: 49 minutes.
Zoey's total time: 50 minutes.