Physics 8, Fall 2013, Practice Exam. **Name:** Bill

This open-book take-home exam is 10% of your course grade. (The in-class final exam will be 20% of your course grade. For the in-class exam, you can bring one sheet of handwritten notes and a calculator.) You must complete this exam on your own, without working with other people. It is fine to discuss general topics from the course with your classmates, but not your solutions to these problems. The time listed for each problem is a guideline to budget your time for the 120-minute in-class final. Feel free to approximate $g = 10\ \mathrm{m/s^2}$ if you wish.

Due by 5pm on Tuesday, December 10, 2013, in DRL 1W15.

Please show your work on these sheets. Continue on other side if needed.

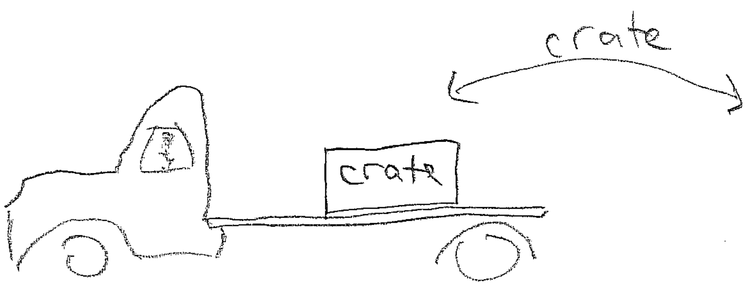
1. (7 minutes, 6%) A heavy crate rests on the bed of a flatbed truck. When the truck accelerates, the crate remains where it is on the truck, so it too accelerates.

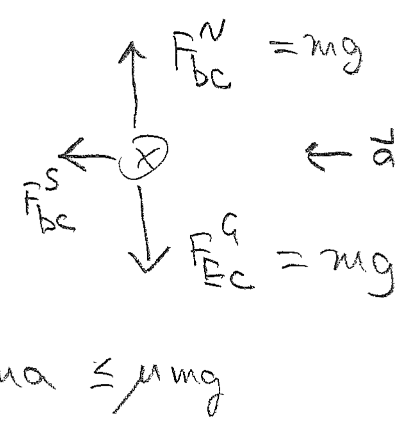
(a) What force causes the crate to accelerate?

Static friction between the truck bed and the lower surface of the crate.

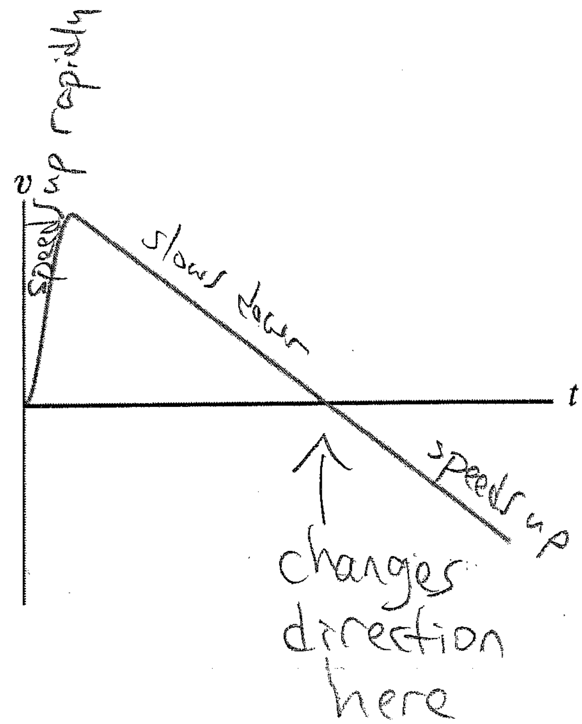
(b) Draw a free-body diagram for the crate. Be sure to indicate the direction of acceleration.

Assume truck accelerates ← to the left.





2. **(8 minutes, 7%)** The graph at right shows the velocity vs. time curve for the first part of the motion of an object traveling along a line. Which of the motion(s) described below could be represented by the graph?



A) a person sprinting 100 m from rest

B) a ball thrown in the air

C) a ball kicked at a wall from which it rebounds

D) a ball, released from rest, rolling down a uniform slope

E) a bus journey from one stop to the next

F) none of the above

Briefly explain your reasoning.

A would not change direction

C would have constant velocity after kick & after recoil

D would always speed up, never change direction

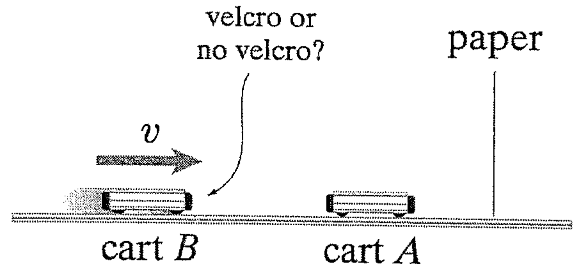
E would never change direction (at least not reverse)

B speeds up rapidly during the throw, while in contact with hand.

After release, its acceleration is a constant $a_y = -g = \frac{dv_y}{dt}$

It will turn around at the top of its trajectory (if thrown straight up).

3. **(7 minutes, 6%)** You want to drive cart **A**, initially at rest, through a piece of paper by launching cart **B** against it. Both carts have the same mass, and you've determined that the larger the kinetic energy of an object, the more easily it goes through a piece of paper. One side of cart **B** is equipped with velcro pads so that it sticks to cart **A**; the other side is smooth and collides elastically. Which side of cart **B** do you use? Explain briefly.



If I use the elastic side, then after the collision, $V_{Af} = V$, and $V_{Bf} = 0$.

So $K_{Af} = \frac{1}{2} m v^2$.

If I use the velcro side, then after the collision, $V_{Af} = V_{Bf} = \frac{V}{2}$.

So $K_{Af} = \frac{1}{2} m \left(\frac{V}{2}\right)^2 = \frac{1}{8} m v^2$.

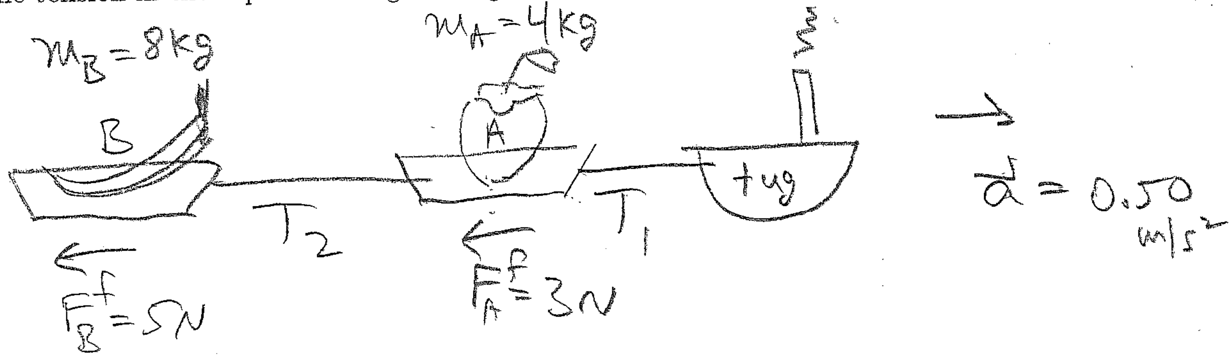
Even if I consider "the object" to be the fused A+B, then $K_{(A+B)f} = \frac{1}{2}(2m)\left(\frac{V}{2}\right)^2$
$= \frac{1}{4} m v^2$.

So either way you look at it, $(K.E.)_{final}$ is larger for the $\boxed{\text{elastic}}$ collision.

$\therefore$ Use the $\boxed{\text{Smooth}}$ side.

4. (13 minutes, 11%) A toy tugboat pulls two toy barges (connected in series, like a train) across a swimming pool. The barge connected to the tugboat, carrying apples, has mass $m_A = 4.0$ kg. The other barge, carrying bananas, has mass $m_B = 8.0$ kg. The frictional force between the apple barge and the water is 3.0 N, and that between the banana barge and the water is 5.0 N. The common acceleration of all three boats is 0.50 m/s^2. Neglect the masses of the tow-ropes, which are taut and perfectly horizontal.

(a) What is the tension in the rope conecting the tugboat to the apple barge?



T_1 must accelerate the A+B system at $a = 0.50\ \frac{m}{s^2}$

$$(m_A + m_B)\, a_x = \sum (F_{A+B})_x = T_1 - F_A^f - F_B^f$$

$$(12\,\text{kg})(0.5\ \tfrac{m}{s^2}) = T_1 - 3N - 5N \Rightarrow \boxed{T_1 = 14\,N}$$

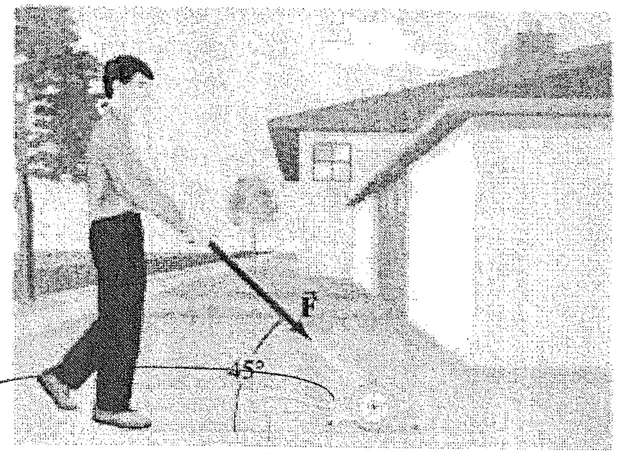
(b) What is the tension in the rope connecting the apple barge to the banana barge?

T_2 must accelerate barge B at $a = 0.50\ \frac{m}{s^2}$

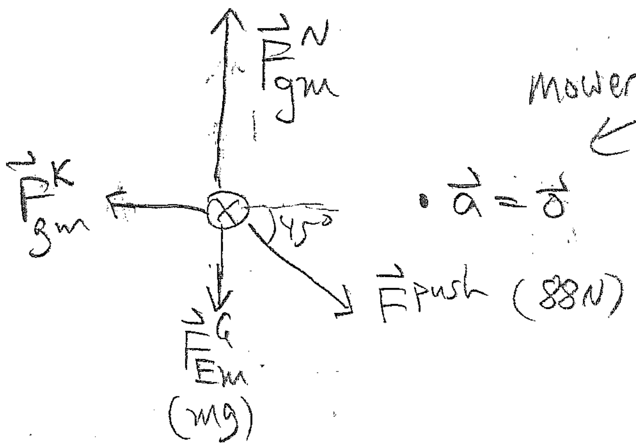
$$m_B\, a_x = \sum (F_B)_x = T_2 - F_B^f$$

$$(8\,\text{kg})(0.5\ \tfrac{m}{s^2}) = T_2 - 5N \Rightarrow \boxed{T_2 = 9\,N}$$

5. (18 minutes, 15%) A person pushes a 14.0 kg lawn mower at constant speed with a force $F = 88.0$ N directed along the handle, which is at an angle of 45.0° to the horizontal.



(a) Draw the free-body diagram showing all forces acting on the mower.



(b) Calculate the horizontal friction force on the mower.

$$F^{K}_{gm} = F^{push}\cos 45° = (88\,N)(0.707) = \boxed{62.2\,N}$$

(c) Calculate the normal force exerted vertically upward on the mower by the ground.

$$F^{N}_{gm} = mg + F^{push}\sin 45° = (14.0\,kg)(9.8\,\tfrac{m}{s^2}) + (88N)/\sqrt{2}$$

$$= \boxed{199\,N}$$

(d) What force (still directed along the handle) must the person exert on the lawn mower to accelerate it from rest to 1.5 m/s in 2.5 seconds, assuming the same friction force? (It's more realistic to assume the same coefficient μ_K, but to keep this problem manageable just assume the same force F^K.)

$$a_x = \frac{\Delta v}{\Delta t} = \frac{1.5\,m/s}{2.5\,s} = 0.6\,\frac{m}{s^2}$$

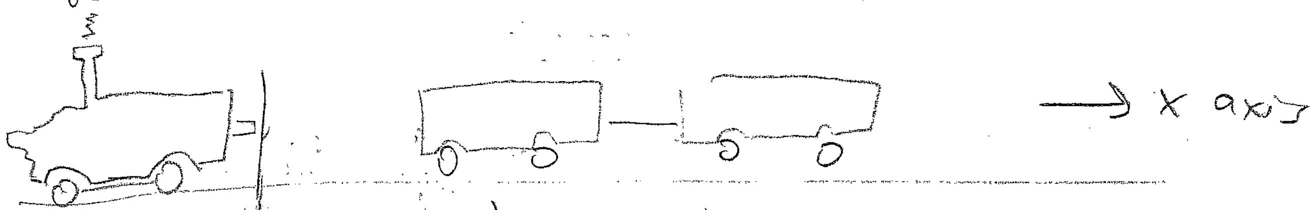
$$ma_x = F^{push}\cos 45° - F^{K}_{gm} = (F^{push})(0.707) - 62.2N = 8.4$$

$$\boxed{F^{push} = 99.8\,N}$$

6. (8 minutes, 7%) You are standing on the roof of a locomotive that has just been decoupled from its passenger cars. The locomotive travels at a steady speed of 10 m/s, while the speed of the cars decreases by 1 m/s each second once they are decoupled. When the locomotive and the nearest car are 10 m apart, you are handed a package and told to throw it to someone standing on the nearest car. You know you can throw the package such that it will stay aloft for about 3.0 s and such that its horizontal speed is about 4.0 m/s.

Can you make it?

Work in reference frame of locomotive.



$\vec{V}_L = \vec{0}$ $\quad x=0$

$\vec{a}_L = \vec{0}$

$\vec{V}_c = 0$ when $X_c = 0$

$\vec{a}_c = 1\,m/s^2 \rightarrow$

$X_c = \frac{1}{2} a_c t^2 \Rightarrow t = \sqrt{\frac{2X_c}{a_c}}$

throw when $X_c = 10\,m \rightarrow t = \sqrt{\frac{2(10m)}{1\,m/s^2}} = 4.47s$

lands 3s later at $t = 7.47s$

$X_{lands} = (3.0s)(4.0\,\frac{m}{s}) = 12m$

at which time, $X_c = \frac{1}{2} a_c t^2 = \frac{1}{2}(1\,\frac{m}{s^2})(7.47s)^2 = 27.9m$

so there is no way you can make it!

the package falls short by $\approx 16m$

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* more straightforwardly, at $t = 4.47s$, the cars are moving away (in locomotive frame) at $V_c = 4.47\,m/s$, and speeding up!

They recede faster than I can throw!

7. (17 minutes, 13%) (a) On a frictionless track, a small object of mass m initially moving at speed v collides with a somewhat larger object of mass M that is at rest, and sticks to it. Calculate the change in kinetic energy of the two-object system in terms of v, m, and M.

$m \longrightarrow M$

momentum: $mv = (m+M)v_f$

$$v_f = \frac{mv}{m+M}$$

$$K_i = \frac{1}{2}mv^2 \qquad K_f = \frac{1}{2}(m+M)v_f^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

$$K_f = \frac{(mv)^2}{2(m+M)}$$

$$K_f - K_i = \frac{(mv)^2}{2(m+M)} - \frac{mv^2}{2} = \frac{(mv)^2 - mv^2(m+M)}{2(m+M)} = \frac{v^2}{2}\cdot\frac{m^2-m^2-mM}{m+M} = \boxed{-\frac{mMv^2}{2(m+M)}}$$

(b) Suppose now the small object of part (a) is initially at rest and is struck by the larger object initially moving at speed v (the same v as in part (a)), and again they stick together. Calculate the change in the kinetic energy of the two-object system in terms of v, m, and M.

momentum: $Mv = (m+M)v_f \Rightarrow v_f = \frac{Mv}{m+M}$

$$K_i = \frac{1}{2}Mv^2 \qquad K_f = \frac{1}{2}(m+M)\left(\frac{Mv}{m+M}\right)^2 = \frac{(Mv)^2}{2(m+M)}$$

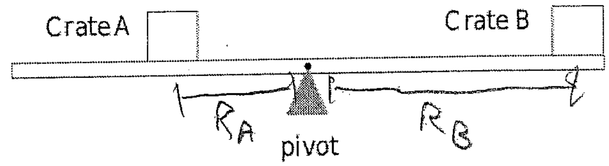
$$K_f - K_i = \frac{(Mv)^2}{2(M+m)} - \frac{Mv^2}{2} = \frac{M^2v^2 - Mv^2(M+m)}{2(M+m)} = \frac{v^2}{2}\cdot\frac{M^2-M^2-Mm}{M+m}$$

$$K_f - K_i = \boxed{-\frac{Mmv^2}{2(M+m)}}$$

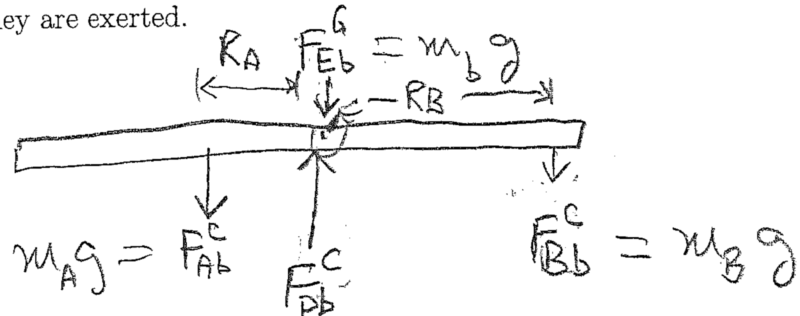
(c) Compare your answers to parts (a) and (b). Why are they the same or different? Explain briefly.

The two ΔK values are the same. This is because the change in energy of a colliding two-body system is the same when viewed from two inertial reference frames. The two collisions are the same collision, seen from different reference frames.

8. (13 minutes, 11%) Two crates sit on a uniform wooden beam as shown at right. The crates and beam remain at rest. The beam is supported by a *frictionless* pivot. The crates are *not* identical.



(a) Draw an extended free-body diagram of the beam. Label your forces, drawing them at the point where they are exerted.



(b) Is the magnitude of the torque produced by crate B about the pivot greater than, less than, or equal to that of crate A? Explain.

About the pivot, there are only two nonzero torques. For equilibrium, they must sum to zero. So the two torques due to A & B must be equal in magnitude.

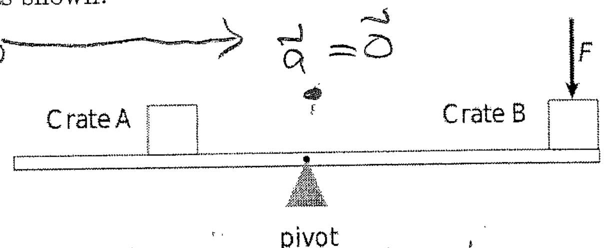
(c) Is the mass of crate B greater than, less than, or equal to the mass of crate A? Explain.

$|\tau_A| = |\tau_B| \Rightarrow m_A g R_A = m_B g R_B$, but $R_B > R_A$.

So $\boxed{m_A > m_B}$. Specifically, $m_A = \frac{R_B}{R_A} m_B$.

(d) Suppose a downward force F is applied to crate B as shown.

(i) Indicate the direction of the vector sum of the forces exerted on the beam in this situation. If the vector sum of the forces is zero, state so explicitly. Explain. $\rightarrow \vec{a} = \vec{0}$



Surprisingly, the vector sum of forces is still zero, at least initially (until the beam tilts too far), because the CoM is not accelerating. Crate A goes up a factor $\frac{R_A}{R_B}$ as far as crate B goes down, so the CoM stays put.

(ii) Is the net torque about the pivot zero or nonzero? Explain your reasoning.

The net torque is nonzero b/c we added a nonzero torque at R_B to a system whose torques were balanced before. The only other force that changes in response is F_{pivot}, which has zero lever arm. So $\sum \tau \neq 0$ about the pivot. Once the beam has tipped a non-negligible angle, it becomes more complicated.

9. (9 minutes, 8%) While driving cross-country over holiday break, you become bored with the music you are playing and decide to change CDs. Alas, your CD case is sitting on the far-right side of the passenger seat, beyond the reach of your right arm. You decide to use your knowledge of physics to slide the CD case closer to you — so you'll make a sharp turn. Conveniently, just ahead on the highway are one exit ramp turning right and another exit ramp turning left.

(a) Which direction should you turn the car so as to make the CD case slide closer to you?

The CD case will continue with its initial velocity as long as the car doesn't exert a force on the CD case. I turn the car $\boxed{\text{right}}$, the CD case keeps going straight (in Earth frame), so in my frame I see the CD case slide left, toward me.

(b) If the coefficient of static friction between the CD case and the seat of the car is 0.40, and the exit ramp is circular with a radius of 50 m, what is the minimum constant speed at which you could make your turn and still have the CD case slide your way?

The CD case follows the car (& does not slide toward me) as long as the force of static friction exerted by the seat on the CD case is as large as the required centripetal force:

$$\frac{mv^2}{R} = F^s_{SC} \leq \mu_s mg$$

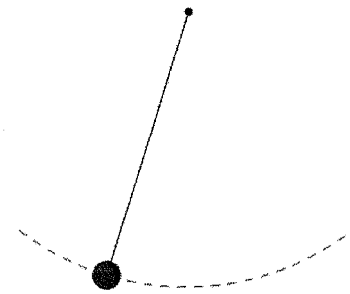
$v^2 \leq \mu_s g R$ to keep CD from sliding

$v^2 > \mu_s g R$ to make CD case slide

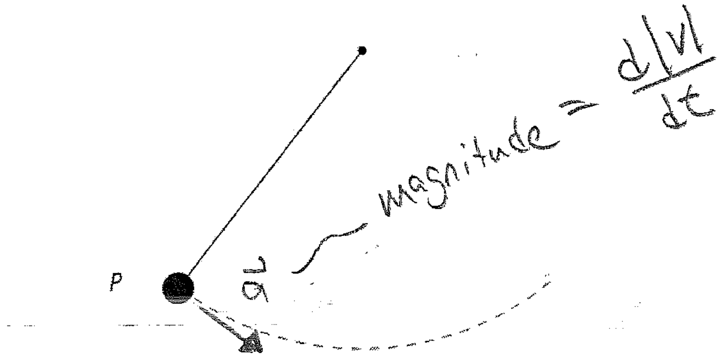
$$v > \sqrt{\mu_s g R} = \sqrt{(0.40)(9.8\,\tfrac{m}{s^2})(50\,m)} = \boxed{14\,\tfrac{m}{s}}$$

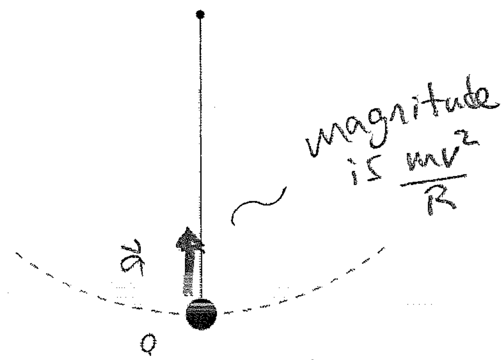
minimum speed

10. (10 minutes, 8%) Consider a pendulum swinging back and forth, as shown in the figure on the right.



(a) In the diagrams below, point **P** is at the turn-around point of the swing and point **Q** is at the bottom of the swing. Draw the acceleration vector of the pendulum bob at locations **P** and **Q**. Explain briefly.

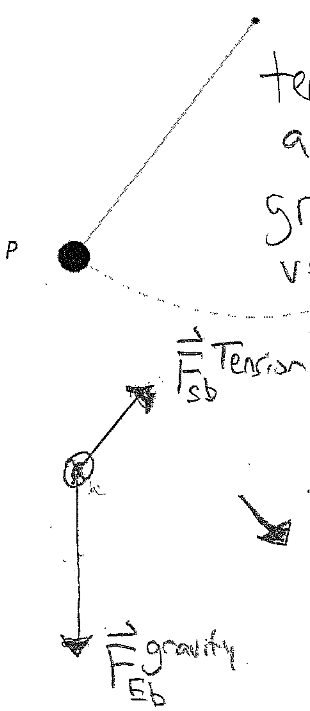




At this instant, speed is zero, so centripetal acceleration is zero. But rate of change of speed is nonzero as swing goes from clockwise to counterclockwise.
⇒ tangential acceleration

maximum speed at bottom of swing ⇒ instantaneous change in speed is zero. only centripetal acceleration is nonzero.

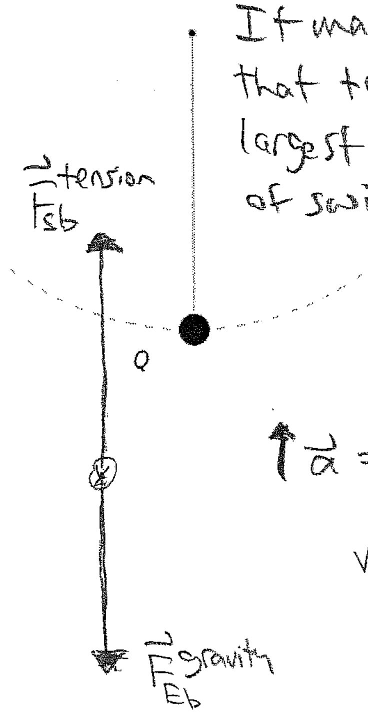
(b) Draw free-body diagrams for the pendulum bob at locations **P** and **Q**. Below each diagram, indicate the direction of the vector sum of the forces. (If the vector sum is zero, state this explicitly.) Explain briefly.



tension must point along string.
gravity must point down.
vector sum must point in direction of acceleration.

↘ $\vec{a} = \frac{1}{m}\sum\vec{F}$

vector sum points down & to right



It makes sense that tension is largest at bottom of swing.

↑ $\vec{a} = \frac{1}{m}\sum\vec{F}$

vector sum points up

11. (10 minutes, 8%) Returning over winter break to the favorite playground of your childhood, you decide to try out the swingset. You notice that the swing seems to go fastest and highest if you kick your legs with just the right rhythm: you hold your legs out forward for 2.5 seconds, then tucked back by your seat for 2.5 seconds, then out forward for 2.5 seconds, then back by your seat for 2.5 seconds, and so on.

(a) What is the length of the chains connecting the top of the swingset to your seat? (Neglect the mass of the chains, and assume that your body's mass is concentrated near the seat.)

period $T = 2 \times 2.5s = 5.0s$. Since this period is optimal for large response of swing to my kick, it equals natural period $T_0 = 2\pi\sqrt{\frac{L}{g}}$.

So $5.0s = 2\pi\sqrt{\frac{L}{9.8m/s^2}} \Rightarrow \boxed{L = 6.2m}$ this is a rather tall swing set!

(b) Explain why moving your legs back and forth more often, e.g. every 1.5 seconds, would be less effective at making this swing go fast and high. (You may find this question easier to answer after you've done the "Chapter G11" reading for December 4th.)

The response of the swing is largest near resonance — when the driving force has a period close to the natural period of oscillation of the system. $T = 3.0s$ is not very close to $T_0 = 5.0s$.

(c) A little kid sitting next to you weighs only half what you do, and wants your advice on how often to kick back and forth. What do you suggest? Why?

Since $T_0 = 2\pi\sqrt{\frac{L}{g}}$ is independent of the mass of the pendulum, I would advise the kid to kick at the same frequency I do. That's $f = 0.2Hz$, or $T = 5s$.

Remember **online response** at `positron.hep.upenn.edu/wja/jitt`