Physics 8, Fall 2015, Practice Exam. **Name:** Bill

This open-book take-home exam is 10% of your course grade. (The in-class final exam will be 20% of your course grade. For the in-class exam, you can bring one sheet of handwritten notes and a calculator.) You should complete this exam on your own, without working with other people. It is fine to discuss general topics from the course with your classmates, but it is not OK to share your solutions to these specific problems. Feel free to approximate $g = 10\ \mathrm{m/s^2}$ if you wish.

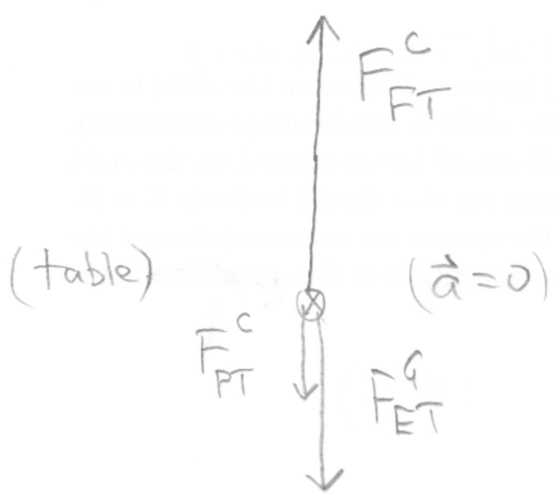
Due by 5pm on Tuesday, December 8, 2015, in DRL 1W15.

Please show your work on these sheets. Continue on other side if needed.

1. (15%) Conceptual force questions.

(a) A table sits on the floor, and a small scale model of the Parthenon sits on top of the table. For the sake of specificity, assume that the mass of the scale model equals one-half the mass of the table. All objects are at rest. Identify all forces exerted on the table and describe how these forces are related. Then draw a free-body diagram for the table, making the lengths of the arrows represent the relative magnitudes of the forces.

Let m be the mass of the table.



F^G_{ET} is gravitational force exerted by Earth on table. Points downward & has magnitude mg.

F^c_{PT} is contact force exerted by Parthenon model on table. Points downward & has magnitude $\frac{1}{2}mg$.

F^c_{FT} is contact force (a.k.a. normal force) exerted by floor on table. Points upward & has magnitude $\frac{3}{2}mg$.

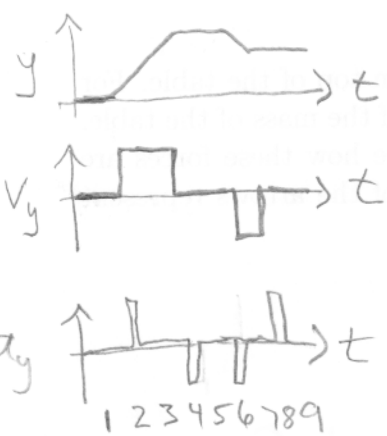
$$F^c_{FT} = F^G_{ET} + F^c_{PT}$$

$$F^c_{PT} = \frac{1}{2} F^G_{ET}$$

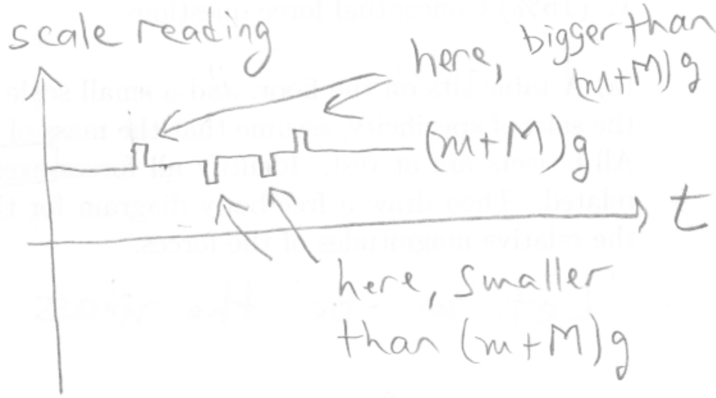
(Problem continues on next page.)

(b) You and a child are standing on a bathroom scale, which uses compression of a spring to measure the force exerted by your feet on the scale. Draw a graph (force vs. time) that shows (qualitatively) what happens to the scale reading as you lift the child onto your shoulders. Assume that first you're both standing still. Then you raise the child above your head and pause. Then you lower the child onto your shoulders, where he or she stays. You just need to indicate (and clearly label), for each relevant part of the journey, whether the scale reading is equal to, larger than, or smaller than your combined weights. Don't omit any relevant features!

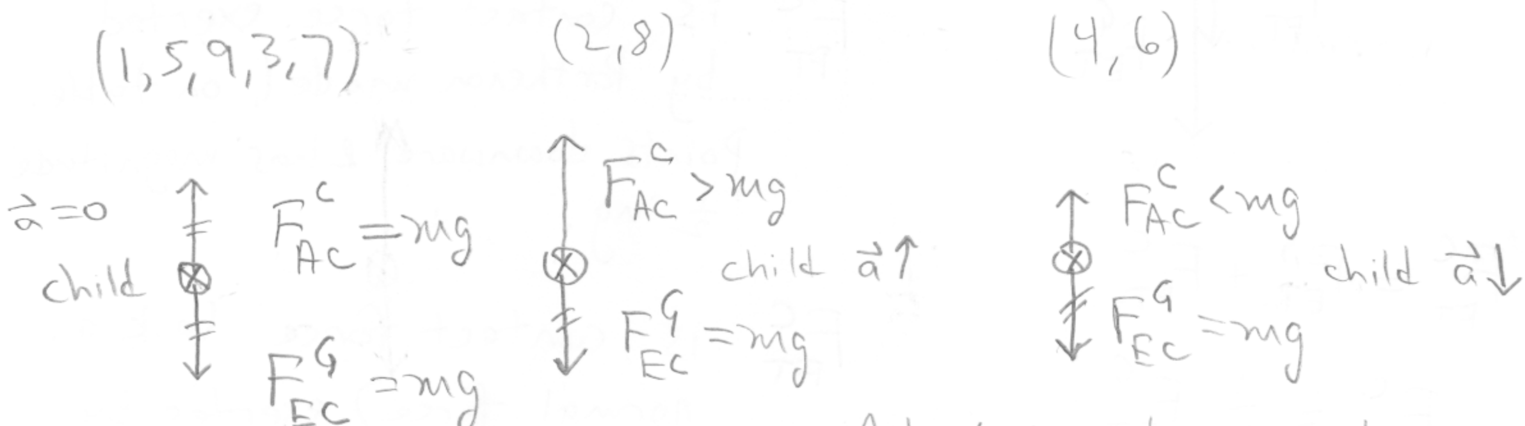
It may help first to draw $y_{child}(t)$, then $v_y(t)$, then $a_y(t)$ for the child. The upward force exerted by the scale equals $Mg + mg + ma_y$ where M = adult, m = child.



1,5,9 = at rest
2,8 = accelerating upward
4,6 = accelerating downward
3,7 = constant velocity (3 = upward, 7 = downward)



(c) For three representative segments of your graph from part (b) (representing when the child is not accelerating, when the child is accelerating upward, and when the child is accelerating downward), draw a free-body diagram for the child. Each diagram should indicate all forces exerted on the child and should indicate the direction in which the child is accelerating (or else should indicate $\vec{a} = 0$). (So that's three free-body diagrams.) Make sure the lengths of the arrows are representative of the relative magnitudes of the forces. Also, if there is any force whose magnitude is the same in all three diagrams, make that clear on your diagrams.



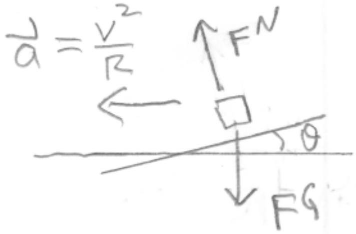
(In interval ①, you may replace Adult with scale itself.)

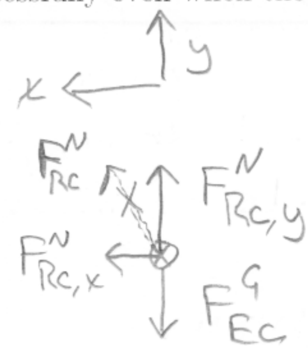
("A" = adult, "C" = child, "E" = Earth)

Adult is always pushing up on child, but pushes up with smaller magnitude than mg to accelerate child downward.

2. (10%) You have a weekend job selecting speed limit signs to put at road curves. The speed limit is determined by the radius of the curve and the bank angle.

(a) For a turn of radius 400 m and a 7.0° bank angle, what speed limit should you post so that a car traveling at that speed negotiates the turn successfully even when the road is wet and slick?





"R" = road
"C" = car
"E" = Earth

$\leftarrow \vec{a}$ has magnitude v^2/R, points along $\hat{x}$.

Car is not accelerating vertically:

$$0 = ma_y = F^N_{RC,y} - F^G_{EC} = F^N \cos\theta - mg \Rightarrow F^N = \frac{mg}{\cos\theta}$$

$$\frac{mv^2}{R} = ma_x = F^N_{RC,x} = F^N \sin\theta = mg\tan\theta \Rightarrow v = \sqrt{gR\tan\theta} = \boxed{21.9\ \text{m/s}}$$

(b) Now suppose that road conditions are good, so that even a car whose speed is quite a bit faster or slower than your posted speed limit successfully navigates the turn without slipping. Draw three free-body diagrams showing the forces exerted on the car as it travels around the curve: (i) one for the car moving at exactly your posted speed limit; (ii) one for the car moving somewhat faster than your posted speed limit; (iii) one for the car moving somewhat slower than your posted speed limit.

$v = 21.9\ \frac{m}{s}$

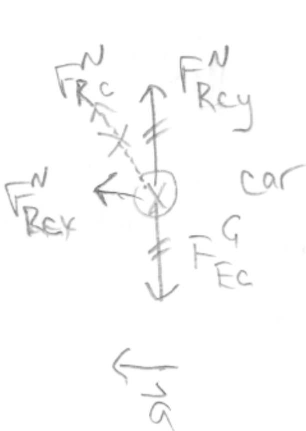
no friction is needed to keep car moving in circle

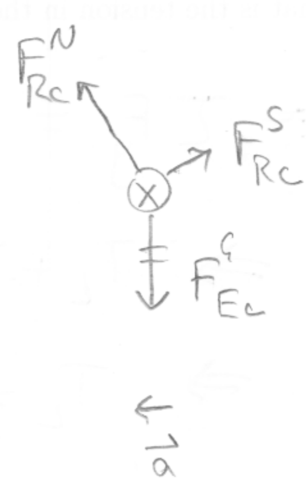
$v > 21.9\ \frac{m}{s}$

static friction points downhill

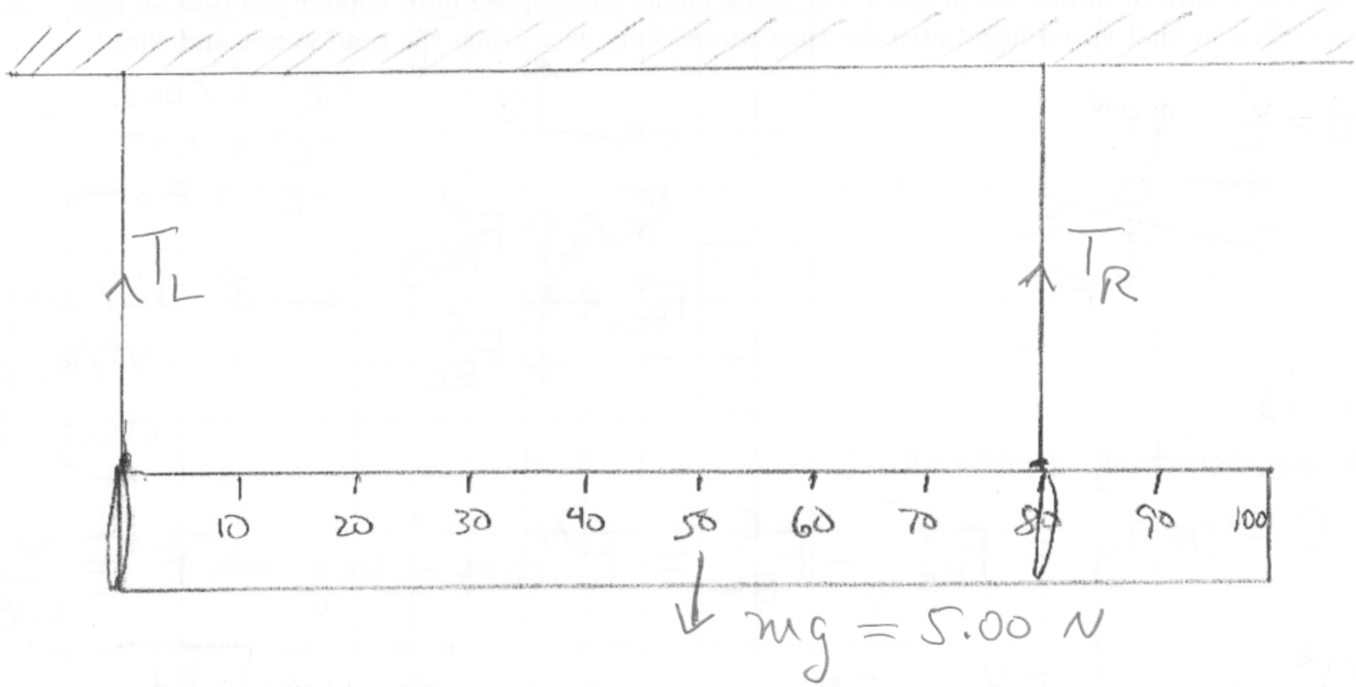
$v < 21.9\ \frac{m}{s}$

static friction points uphill





3. (10%) A meter stick of mass 0.510 kg is supported, in a horizontal orientation, by two vertical strings, one at the 0 cm mark and the other at the 80 cm mark.



(a) What is the tension in the string at 80 cm? (I mean the tension in the straight part of the string that is above the ruler; don't worry about the small portion that encircles the ruler.)

$$0 = \sum M_O = T_R(80) - mg(50)$$

$$\Rightarrow T_R = \frac{5}{8}(5.00\ N) = \boxed{3.125\ N}$$

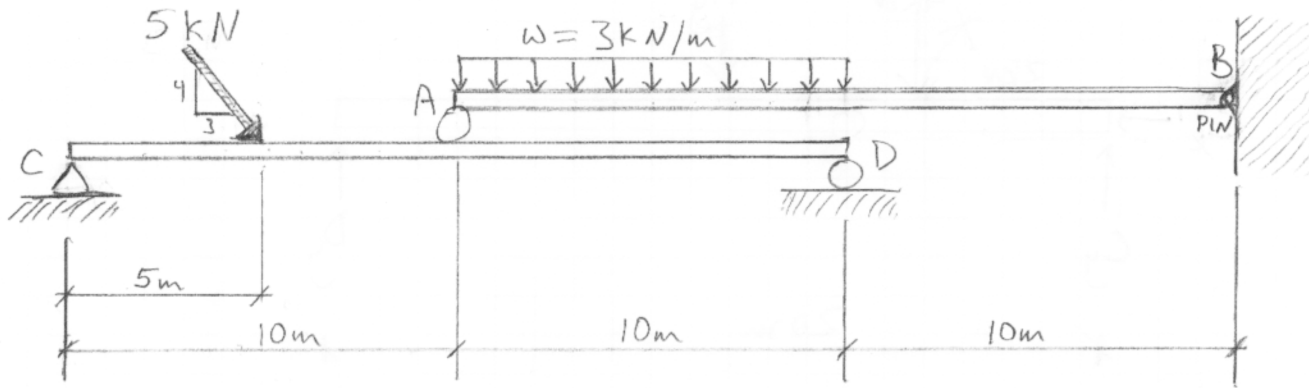
(b) What is the tension in the string at 0 cm?

$$0 = \sum F_y = T_L + T_R - mg$$

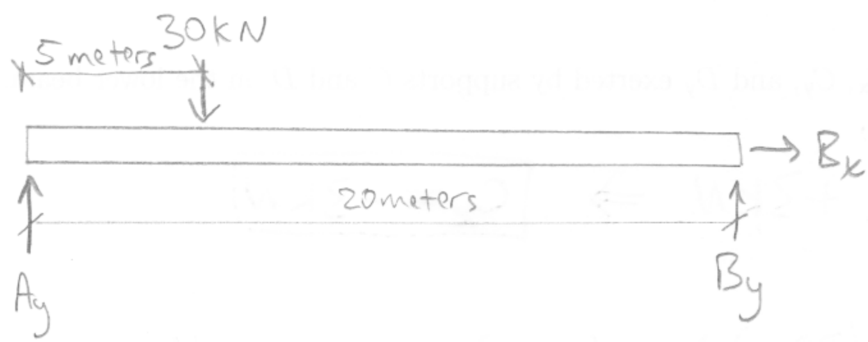
$$\Rightarrow T_L + T_R = 5.00\ N$$

$$\Rightarrow T_L = \boxed{1.875\ N}$$

4. (10%) The figure shows two horizontal beams. Each beam is 20 m long. The upper beam is roller-supported at A (its left end) and pin-supported at B (its right end). It carries a uniform load $w = 3.0\,\mathrm{kN/m}$ over the left half of its length. The lower beam is pin-supported at C (its left end) and roller-supported at D (its right end). A 5.0 kN concentrated load is applied to the lower beam, at the angle shown (it's 53.13□ from the horizontal, i.e. tan(53.13□) = 4/3), a distance 5.0 m from support C. The roller A that supports the upper beam rests on the midpoint of the lower beam. The weight of each beam itself can be neglected.



(a) Draw an extended free-body diagram for the upper beam, indicating all forces acting on the beam, their directions, and their lines of action. In this diagram, replace the distributed load with an equivalent point load. (It's "equivalent" for the purpose of finding the support forces.)



(b) Find the support forces A_y, B_x, and B_y exerted by supports A and B on the upper beam.

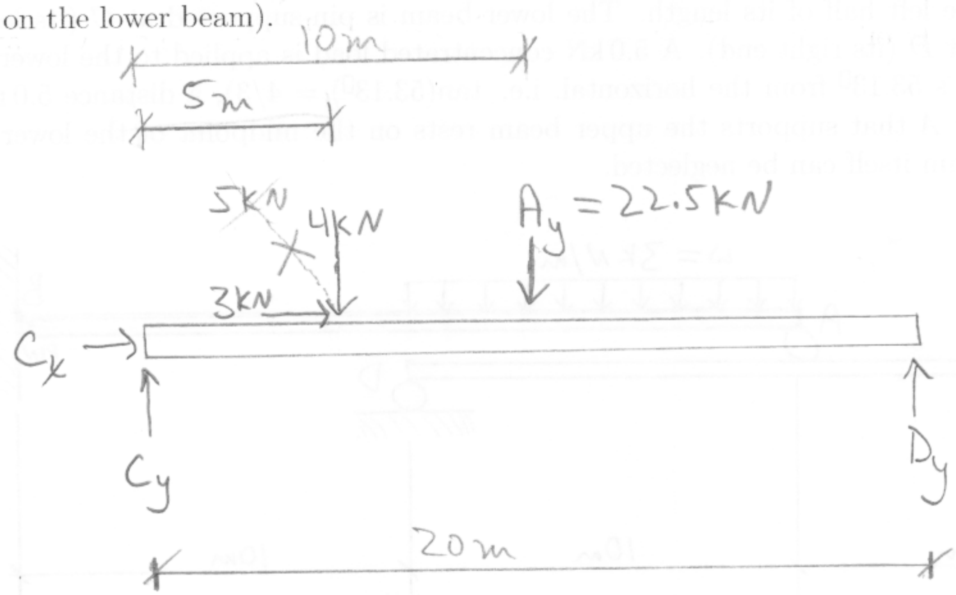
$$0 = \sum F_x = \boxed{B_x = 0}$$

$$0 = \sum M_A = (20\,\mathrm{m})B_y - (5\,\mathrm{m})(30\,\mathrm{kN}) \Rightarrow \boxed{B_y = 7.5\,\mathrm{kN}}$$

$$0 = \sum F_y = A_y + B_y - 30\,\mathrm{kN} \Rightarrow \boxed{A_y = 22.5\,\mathrm{kN}}$$

(Problem continues on next page.)

(c) Draw an extended free-body diagram for the lower beam, indicating all forces acting on the beam, their directions, and their lines of action. Don't forget the force exerted by roller A (which counts as a load on the lower beam).



(d) Find the support forces C_x, C_y, and D_y exerted by supports C and D on the lower beam.

$$0 = \sum F_x = C_x + 3\text{kN} \Rightarrow \boxed{C_x = -3\text{kN}}$$

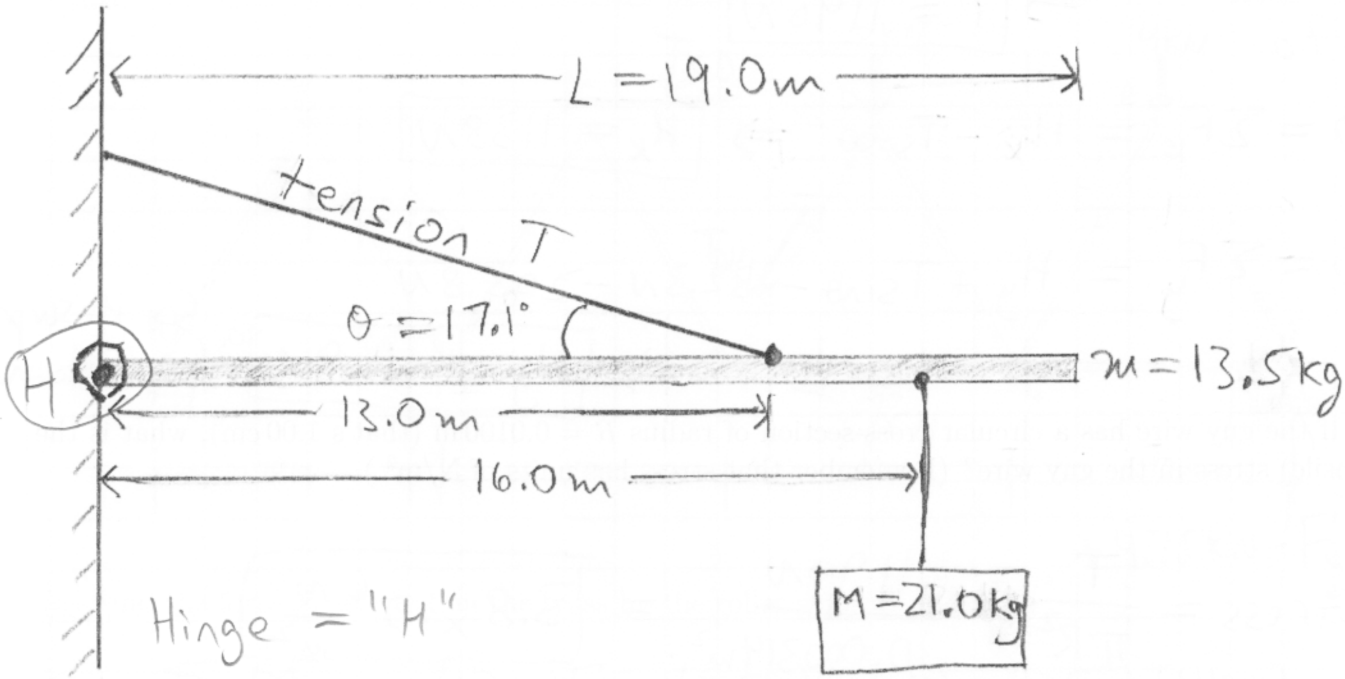
$$0 = \sum M_C = (20\text{m})\, D_y - (10\text{m})(22.5\text{kN}) - (5\text{m})(4\text{kN})$$

$$\Rightarrow \boxed{D_y = 12.25\text{kN}}$$

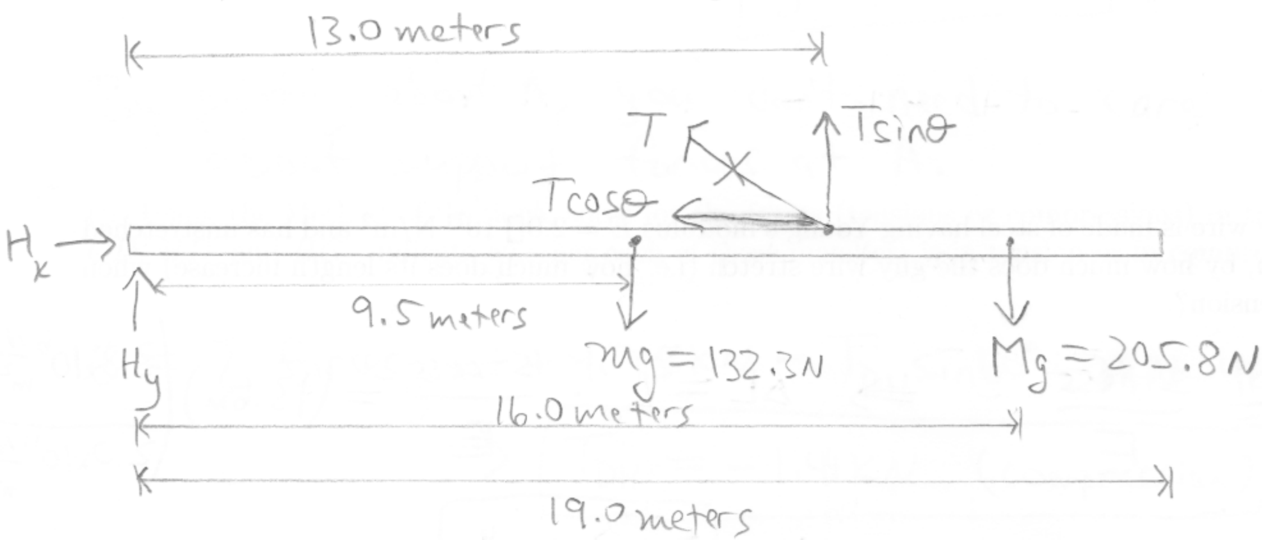
$$0 = \sum F_y = C_y + D_y - 4\text{kN} - 22.5\text{kN}$$

$$\Rightarrow \boxed{C_y = 14.25\text{ kN}}$$

5. (15%) A shop sign of mass $M = 21.0\,\mathrm{kg}$ is suspended from a uniform beam of mass $m = 13.5\,\mathrm{kg}$ and length $L = 19.0\,\mathrm{m}$. The horizontal beam is supported on the left by a hinge; the beam is also supported, a distance $13.0\,\mathrm{m}$ from the hinge, by a guy wire that makes an angle ⃗ = 17.1□ w.r.t. the beam. The sign is supported $16.0\,\mathrm{m}$ from the hinge. Neglect the mass of the guy wire and the thickness of the beam.



(a) Draw an extended free-body diagram for the beam, indicating all forces acting on the beam, their directions, and their lines of action. Don't forget the mass of the beam itself.



(Problem continues on next page.)

(b) Find the tension T in the diagonal guy wire and the forces F_x and F_y exerted by the hinge on the beam. (Remember that forces are in newtons, while masses are in kilograms.)

$$0 = \sum_{5} M_H = (13.0\,m)T\sin\theta - (9.5\,m)(132.3\,N) - (16.0\,m)(205.8\,N)$$

$$\Rightarrow \boxed{T = 1190\,N}$$

$$0 = \sum F_x = H_x - T\cos\theta \Rightarrow \boxed{H_x = 1138\,N}$$

$$0 = \sum F_y = H_y + T\sin\theta - 132.3\,N - 205.8\,N$$

$$\Rightarrow \boxed{H_y = -11.9\,N}$$

So it surprisingly points downward

(c) If the guy wire has a circular cross-section of radius $R = 0.0100\,\text{m}$ (that's $1.00\,\text{cm}$), what is the (tensile) stress in the guy wire? (Remember that stress has units of N/m^2.)

$$\text{stress} = \frac{T}{\pi R^2} = \frac{1190\,N}{0.000314\,m^2} = \boxed{3.8\times 10^6\,\frac{N}{m^2}}$$

(d) If the guy wire is made of steel having Young's modulus $E = 2.0 \square\ 10^{11}\,\text{N/m}^2$ and has unstretched length $13.6\,\text{m}$, by how much does the guy wire stretch (i.e. how much does its length increase) when it is under tension?

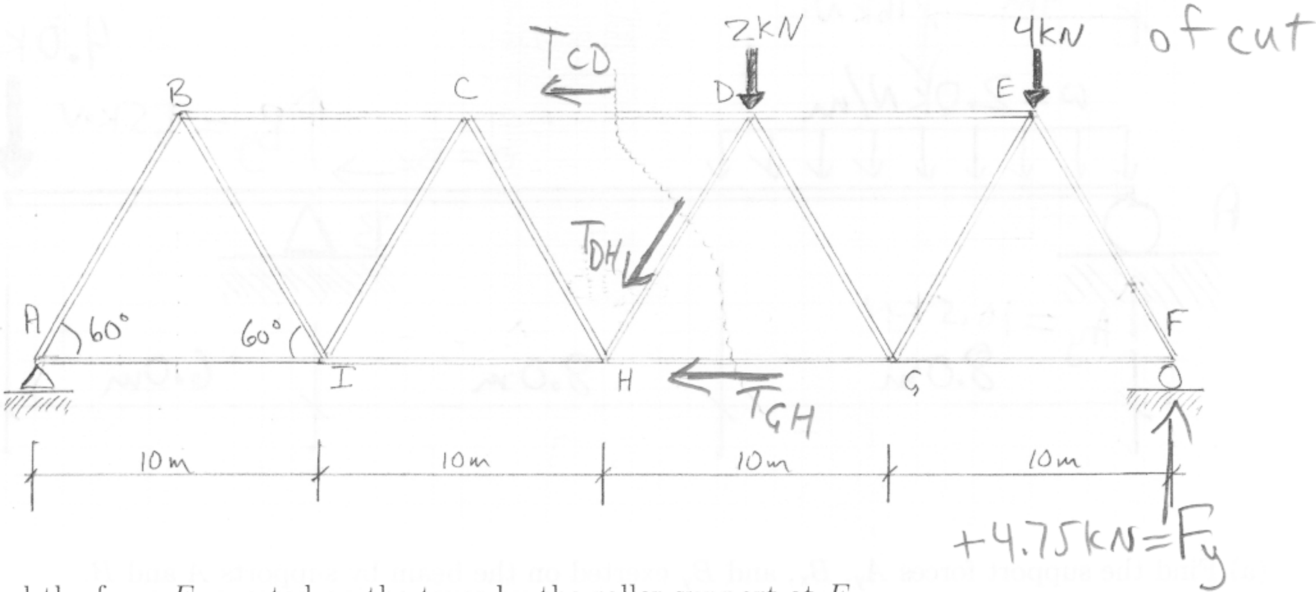
$$\frac{\Delta L}{L_0} = \frac{\text{stress}}{E} \Rightarrow \Delta L = L_0 \cdot \frac{\text{stress}}{E} = (13.6\,m)\left(\frac{3.8\times 10^6\,\frac{N}{m^2}}{2.0\times 10^{11}\,\frac{N}{m^2}}\right)$$

$$\Delta L = \boxed{2.6\times 10^{-4}\,m} = 0.26\,mm$$

wire stretches by 0.26 mm

(or strain $= \frac{\Delta L}{L_0} = 1.9\times 10^{-5}$, i.e. strain is 0.002%.)

6. (15%) Consider the truss shown below. draw forces exerted <u>ON</u> RHS of cut



(a) Find the force F_y exerted on the truss by the roller support at F.

$$0 = \sum_{\circlearrowleft} M_A = (40\text{m})F_y - (25\text{m})(2\text{kN}) - (35\text{m})(4\text{kN})$$

$$\Rightarrow \boxed{F_y = 4.75\text{kN}}$$

By pivoting about A, you don't need to care about support forces at A.

(b) Using the Method of Sections, solve for the forces (tensions or compressions) in truss members CD, HD, and HG. Indicate whether each of these members is in tension or in compression.

$$0 = \sum \text{forces}_y = +4.75\text{kN} - T_{DH}\sin 60° - 2\text{kN} - 4\text{kN}$$

$$\Rightarrow \boxed{T_{DH} = -1.44\text{kN} \ \ (\text{compression})}$$

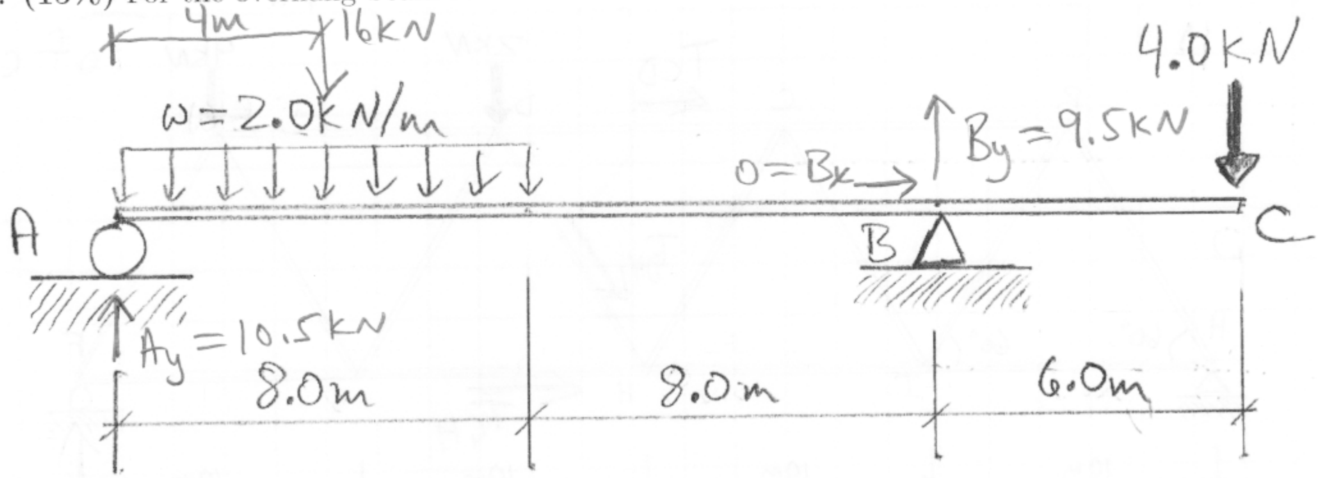
$$0 = \sum_{\circlearrowleft} M_H = (5\text{m}\tan 60°)T_{CD} + (20\text{m})(4.75\text{kN}) - (5\text{m})(2\text{kN}) - (15\text{m})(4\text{kN})$$

$$\Rightarrow \boxed{T_{CD} = -2.89\text{kN} \ \ (\text{compression})}$$

$$0 = \sum \text{forces}_x = -T_{GH} - T_{CD} - T_{DH}\cos 60° = -T_{GH} + 2.89\text{kN} + 0.72\text{kN}$$

$$\Rightarrow \boxed{T_{GH} = +3.61\text{kN} \ \ (\text{tension})}$$

7. (15%) For the overhang beam shown below.



(a) Find the support forces A_y, B_x, and B_y exerted on the beam by supports A and B.

$$0 = \sum F_x = \boxed{B_x = 0}$$

$$0 = \sum M_A = (16\,\text{m})B_y - (4\,\text{m})(16\,\text{kN}) - (22\,\text{m})(4\,\text{kN})$$

$$\Rightarrow \boxed{B_y = 9.5\,\text{kN}}$$

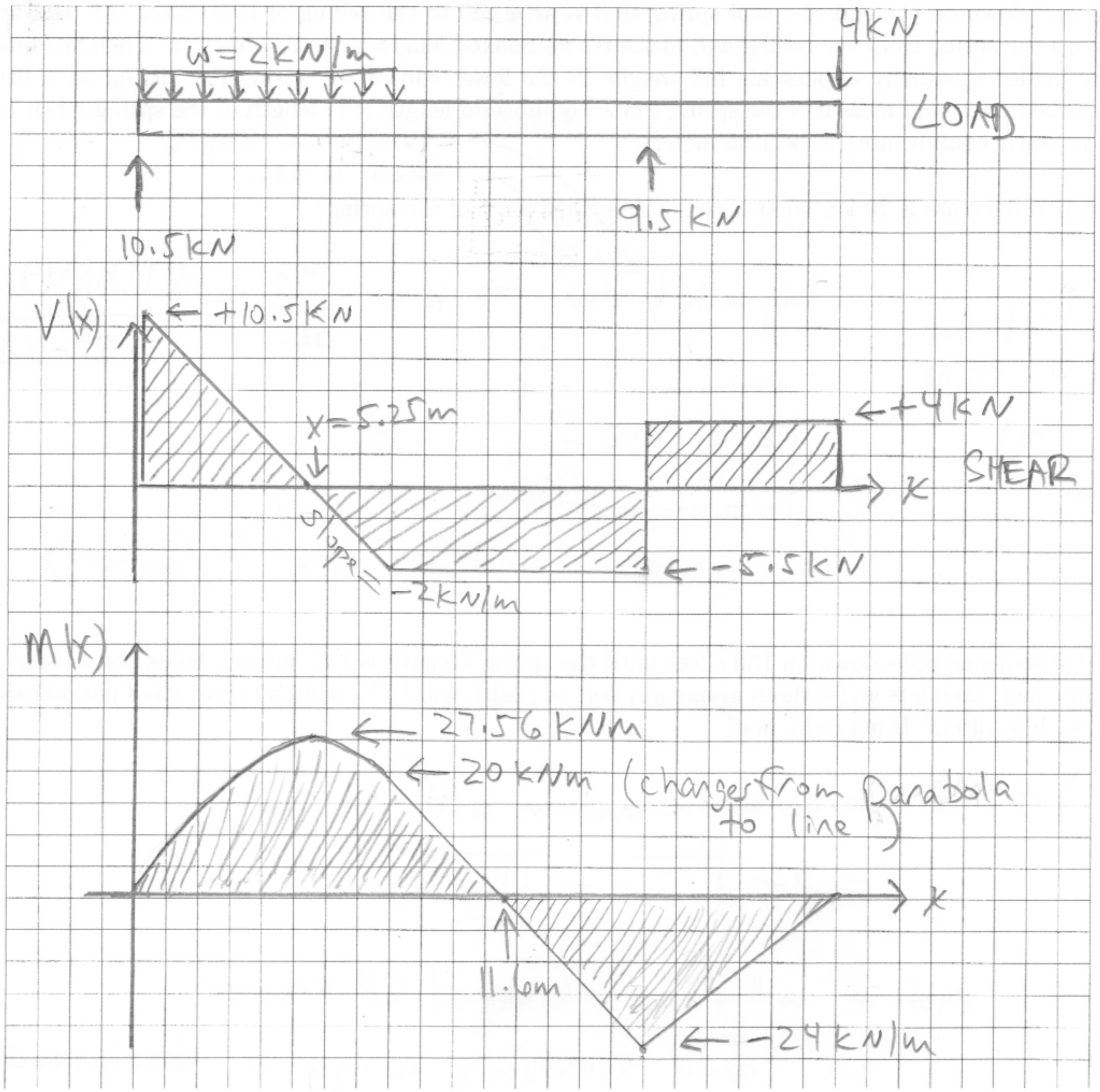
$$0 = \sum F_y = A_y + B_y - 16\,\text{kN} - 4\,\text{kN}$$

$$\Rightarrow \boxed{A_y = 10.5\,\text{kN}}$$

(Problem continues on next page.)

Try online @ bendingmomentdiagram.com

(b) Draw load (EFBD), shear (V), and bending moment (M) diagrams for the beam.



(c) What are the largest magnitude of the shear V (in kilonewtons) and the largest magnitude of the bending moment M (in kilonewton-meters)?

$|V|_{max} = \boxed{10.5\,kN}$ (occurs at $x=0$)

$|M|_{max} = \boxed{27.56\,kNm}$ (occurs at $x = 5.25\,m$)

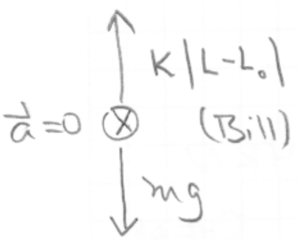
(area of left triangle on $V(x)$ diagram is $\frac{1}{2}(5.25\,m)(10.5\,kN) = 27.56\,kNm$)

8. (10%) Your physics teacher Bill gets the crazy idea that he himself will be the "mass" bobbing up and down on the end of a stiff spring that is attached to the ceiling of DRL room A2. Bill first hangs the spring from the ceiling and measures its relaxed length to be 0.85 meters. Then he climbs the ladder, gradually applies his full weight to the lower end of the spring (by sitting on a little attached bar), and measures the spring's new equilibrium length (the length of the spring when Bill is in static equilibrium) to be 1.55 meters.

equilibrium length

relaxed length

(a) If Bill's mass is 70 kg, what is the spring constant k of the spring?



$$mg = k(L-L_0) \Rightarrow k = \frac{mg}{\Delta L} = \frac{(70\,\mathrm{kg})(9.8\,\frac{N}{kg})}{(1.55-0.85)\,m}$$

$$\boxed{k = 980\ \frac{N}{m}}$$

(units = newtons/meter)

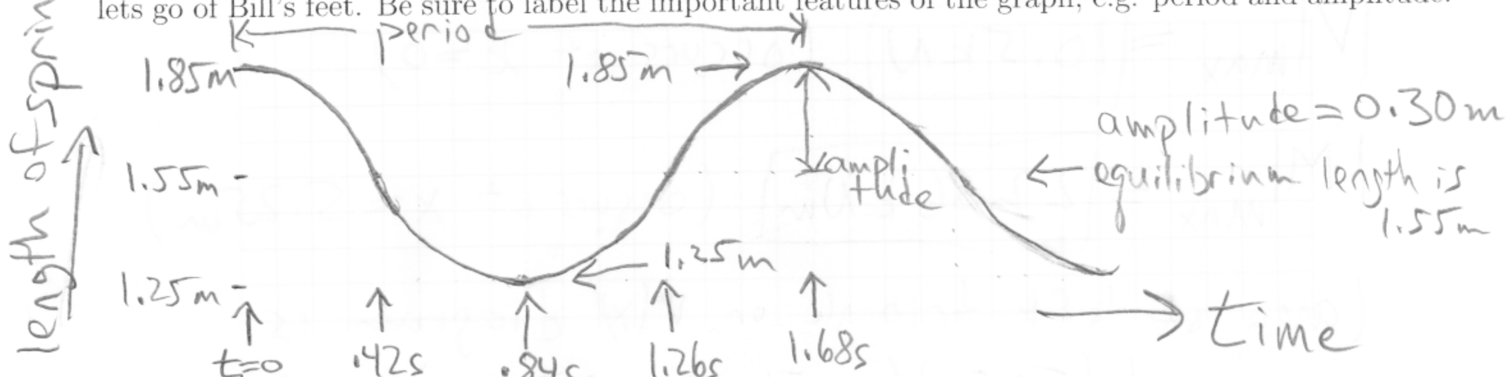
(b) If someone pulls down on Bill's feet until the spring's length is 1.85 meters, holds them there for a moment, then lets go (without giving any sort of push), will Bill's motion repeat itself periodically? If so, how often? If not, why not?

Yes: simple harmonic motion with period

$$T = 2\pi\sqrt{\frac{m}{k}} = \boxed{1.68\ s}$$

motion will repeat every 1.68 seconds (until eventually energy is dissipated into heat)

(c) Sketch a graph of the length of the spring as a function of time, where $t = 0$ is where the person lets go of Bill's feet. Be sure to label the important features of the graph, e.g. period and amplitude.



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(d) If the person instead pulls down on Bill's feet until the spring's length is 1.70 meters, then lets go, how will the period of the motion be affected? (State what the period will be.)

Period will be unaffected by this amplitude change. Period will still be $\boxed{1.68s} = 2\pi\sqrt{\frac{m}{k}}$

(This is the same spring. We are just pulling down less far on Bill's feet than we did the first time.)

(e) How will the amplitude of the motion be affected? (State what the amplitude will be.)

new amplitude is $A = \boxed{0.15 \text{ meter}}$

(old amplitude was 0.30 meter)

$1.70m - 1.55m = 0.15m$

$1.85m - 1.55m = 0.30m$

(f) If Bill somehow managed to hold a 70 kg medicine ball while sitting on this same spring, thus effectively doubling his mass, would the natural period of the motion be affected? (State what the period would be.)

Yes: new period is $T = 2\pi\sqrt{\frac{140kg}{k}}$

$T = 2\pi\sqrt{\frac{140kg}{980 N/m}} = \boxed{2.38s} = \sqrt{2}\,(1.68s)$