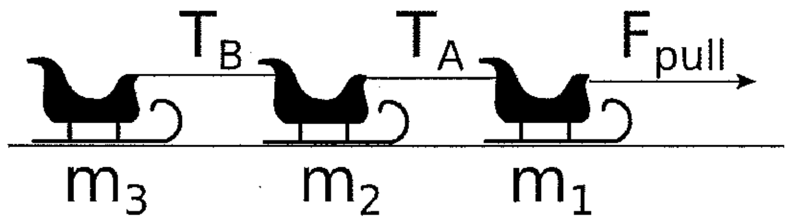
Physics 8, Fall 2017, Practice Exam. **Name:** ____________________

This open-book take-home exam is 10% of your course grade. (The in-class final exam will be 20% of your course grade. For the in-class exam, you can bring one sheet of handwritten notes and a calculator. The in-class exam will be much shorter than this practice exam.) You should complete this exam on your own, without working with other people. It is fine to discuss general topics from the course with your classmates, but it is not OK to share your solutions to these specific problems. Feel free to approximate $g = 10\ \mathrm{m/s^2} = 10\ \mathrm{N/kg}$ if you wish. **In fact, I strongly prefer that you use** $\boxed{g = 10\ \mathrm{m/s^2} = 10\ \mathrm{N/kg}}$ **here, as it simplifies many numerical results.**

Due by 6pm on Monday, December 11, 2017, in DRL 1W15.
No penalty if you turn it in by noon on Dec 14, but I will grade & return on-time exams by Dec 15.

**Please show your work on these sheets. Use spare sheets at back of exam if needed.**

**1. (10%)** Three sleds are pulled to the right across a horizontal sheet of ice using horizontal cables. Friction between the ice and the sleds is negligible. The three sleds (numbered from right to left) have masses $m_1 = 10.0\,\mathrm{kg}$, $m_2 = 20.0\,\mathrm{kg}$, and $m_3 = 30.0\,\mathrm{kg}$ respectively. The pull exerted by the tow cable on sled 1 is $F_{\mathrm{pull}} = 120\,\mathrm{N}$ to the right. Sleds 1 and 2 are connected by a taut cable of tension $T_A$. Sleds 2 and 3 are connected by a taut cable of tension $T_B$.




(a) Find the acceleration $a_x$ of the three-sled system, where the $x$ axis points to the right.

$$(m_1 + m_2 + m_3)\, a_x = \sum F_x = F_{pull} \Rightarrow \boxed{a_x = +2.0\ \frac{\mathrm{m}}{\mathrm{s}^2}}$$

acting on 1+2+3 system

(b) Find the tensions $T_A$ and $T_B$.

$$(m_2 + m_3)\, a_x = \sum F_x = T_A \Rightarrow \boxed{T_A = 100\ N}$$

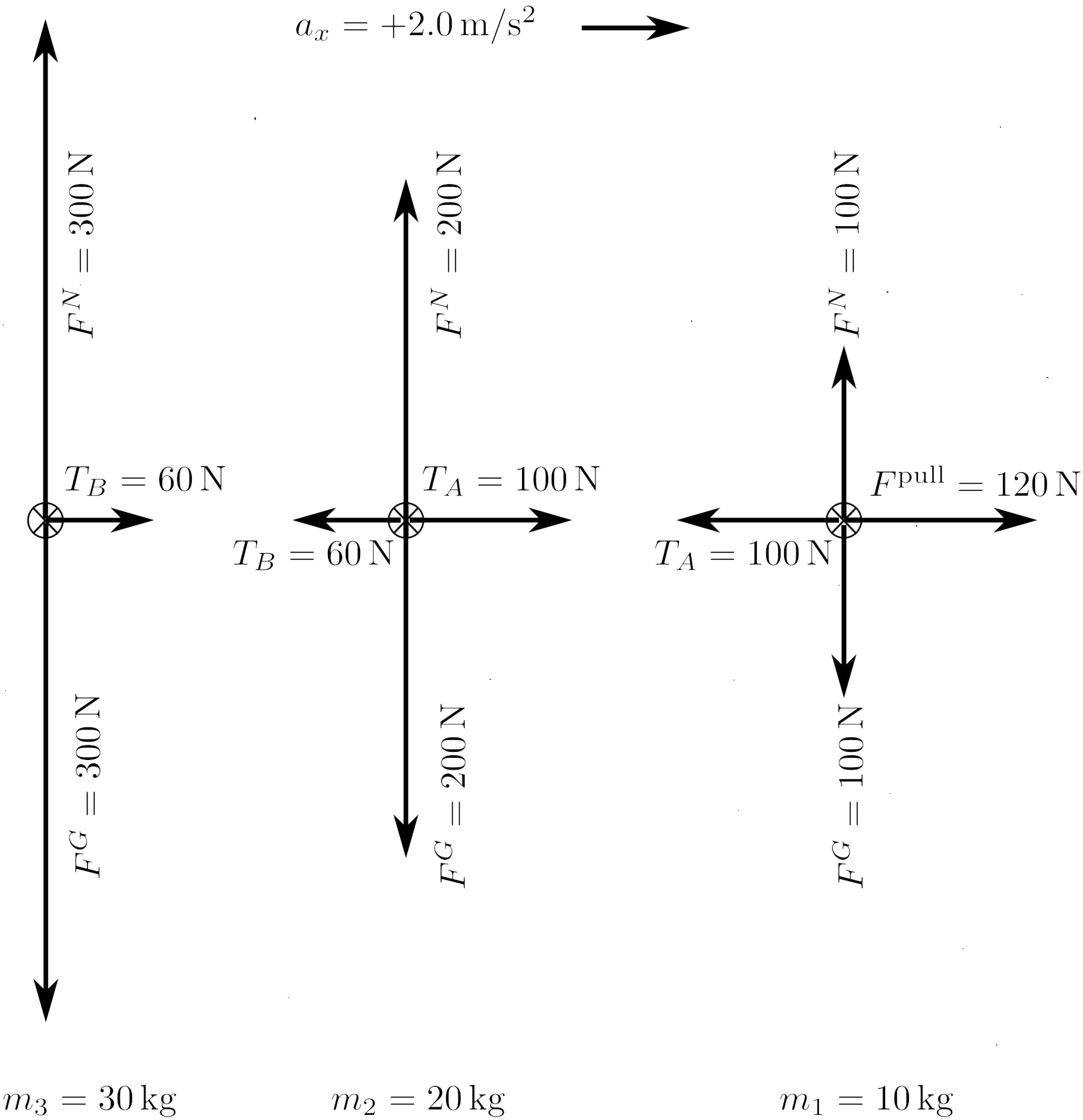
acting on 2+3 system

$$m_3\, a_x = \sum F_x = T_B \Rightarrow \boxed{T_B = 60\ N}$$

acting on sled #3

(Problem continues on next page.)

(c) Draw a free-body diagram for sled 3, then a free-body diagram for sled 2, then a free-body diagram for sled 1. Include both horizontal and vertical forces. Indicate the numerical magnitude of every force (including proper units).



# Mathematica commands for Problem 1 calculations.

## Problem 1

```
ClearAll["Global`*"];
kilogram = Quantity["kg"];
second = Quantity["s"];
newton = Quantity["N"];
meter = Quantity["m"];
m1 = 10.0 kilogram;
m2 = 20.0 kilogram;
m3 = 30.0 kilogram;
fpull = 120.0 newton;
ax = ax /. ToRules[Reduce[(m1 + m2 + m3) ax == fpull]]
```

$2.\ m/s^2$

```
ta = ta /. ToRules[Reduce[(m3 + m2) ax == ta]]
```

100. N

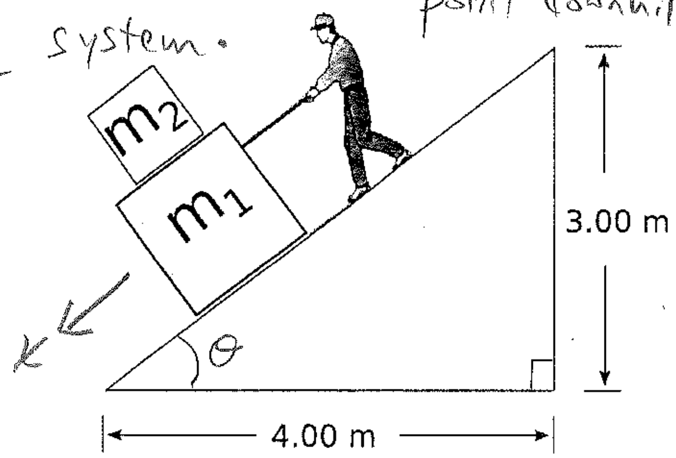
```
tb = tb /. ToRules[Reduce[m3 ax == tb]]
```

60. N

**2.** (10%) You are lowering two boxes, one on top of the other, down a ramp as shown, by pulling on a taut cable that is parallel to the surface of the ramp. The lower box has mass $m_1 = 20.0\,\mathrm{kg}$, and the upper box has mas $m_2 = 10.0\,\mathrm{kg}$. The two boxes move together at constant speed $0.125\,\mathrm{m/s}$: the upper box does not move with respect to the lower box. The coefficient of kinetic friction between the ramp and the lower box is $\mu_k = 0.250$, and the coefficient of static friction between the two boxes is $\mu_s = 0.900$.

(a) Find the magnitude of the force that the cable exerts on the lower box. let x axis point downhill.

Write $ma_x = \sum F_x$ for $m_1 + m_2$ system.




$\sin\theta = \frac{3}{5}$ $\qquad \cos\theta = \frac{4}{5}$

$0 = (m_1 + m_2) a_x = \sum F_x$

$0 = \underbrace{(m_1+m_2) g \sin\theta}_{F^g} - \underbrace{\mu_K \underbrace{(m_1+m_2) g \cos\theta}_{F^N}}_{F^K} - T \Rightarrow \boxed{T = 120\,N}$

(b) Find the magnitude of the static frictional force that the lower box exerts on the upper box. Keep in mind that it is static friction that prevents the upper box from slipping off of the lower box. **Check** that your calculated value is smaller than the maximum possible value for static friction.

Write $ma_x = 0 = \sum F_x$ for box #2.

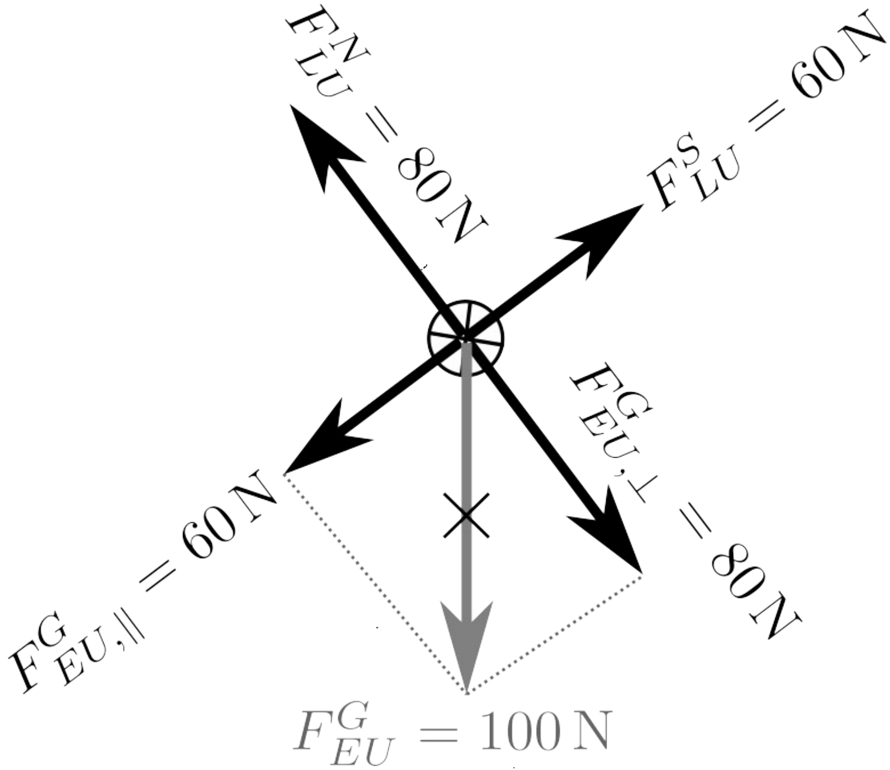
$0 = m_2 g \sin\theta - F^S \Rightarrow \boxed{F^S = 60\,N}$

Maximum possible $F^S = \mu_S F^N = \mu_S m_2 g \cos\theta = \boxed{72\,N}$

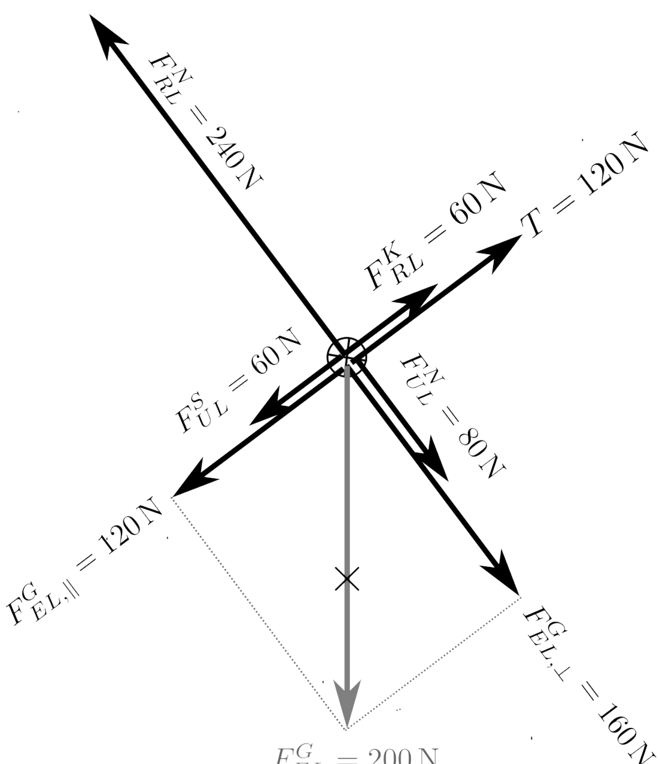
$\boxed{60\,N < 72\,N \quad ✓}$

(Problem continues on next page.)

(c) Draw a free-body diagram for the upper box, showing the direction and numerical magnitude of each force. Decompose the gravitational force into components, as a check that your calculated force values add up as expected. Please label your forces G=gravity, S=static, N=normal, E=Earth, L=lower, U=upper.




(d) Draw a free-body diagram for the lower box, showing the direction and numerical magnitude of each force. Again decompose the gravitational force into components, as a check that your calculated force values add up as expected. Please label your forces G=gravity, S=static, K=kinetic, N=normal, E=Earth, L=lower, U=upper, R=ramp, T=tension (in Cable). [I count 6 forces acting on the lower crate; or 7 if you decompose gravity into two components.]



Mathematica commands for Problem 2 calculations.

## Problem 2

```
ClearAll["Global`*"];
kilogram = Quantity["kg"];
second = Quantity["s"];
newton = Quantity["N"];
meter = Quantity["m"];
g = 10.0 meter / second^2;
m1 = 20.0 kilogram;
m2 = 10.0 kilogram;
μk = 0.250;
μs = 0.900;
θ = ArcTan[3.0 / 4.0];
resultsA =
 Reduce[{ax == 0.0 meter / second^2, (m1 + m2) ax == (m1 + m2) g Sin[θ] - fkRL - tension,
    fkRL == μk fnormalRL, fnormalRL == (m1 + m2) g Cos[θ]}] // ToRules
resultsA[[
 1]]
```

{tension → 120. N , fnormalRL → 240. N , fkRL → 60. N , ax → 0. m/$s^2$}

tension → 120. N

```
resultsB =
 Reduce[{ax == 0.0 meter / second^2, m2 ax == m2 g Sin[θ] - fsLU,
    fsmaximum == μs fnormalLU, fnormalLU == m2 g Cos[θ] }] // ToRules
resultsB[[3]]
resultsB[[1]]
```

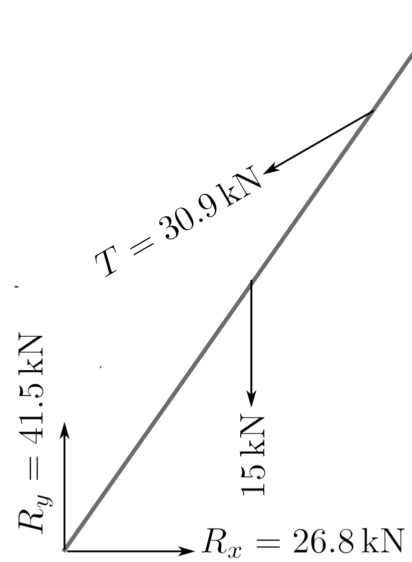
{fsmaximum → 72. N , fnormalLU → 80. N , fsLU → 60. N , ax → 0. m/$s^2$}

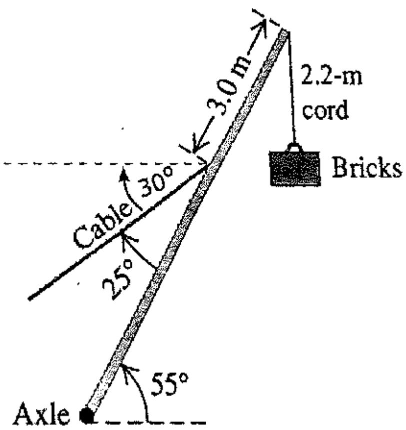
fsLU → 60. N

fsmaximum → 72. N

**3.** **(10%)** The boom of a crane pivots around a frictionless axle at its base and is supported by a cable that makes a 25° angle from the boom. (Equivalently, the cable makes a 30° angle below the horizontal, as indicated.) The boom is 16 m long and is uniform, so the boom's center of gravity is 8.0 m from the axle as measured along the boom; the weight of the boom is 15 kN. The cable is attached 3.0 m from the upper end of the boom (13 m from the axle). The boom is raised to 55° above the horizontal and holds, by a cord, a pallet of bricks whose weight is 11 kN. The cord itself has negligible weight and is 2.2 m long.

(a) Draw an extended free-body diagram of the crane boom, showing all forces acting on the boom and their lines of action.




(b) What is the tension in the cable?

$$0 = \sum_{\substack{\circlearrowleft \\ \text{about axle}}} M = (13\,m)(T\sin 25^\circ) - (8\,m)(\cos 55^\circ)(15\,kN) - (16\,m)(\cos 55^\circ)(11\,kN)$$

$$\Rightarrow \boxed{T = 30.9\,kN}$$

(c) What are the horizontal and vertical components of the "reaction" force that the axle exerts on the boom?

$$0 = \sum F_x = R_x - T\cos 30^\circ \Rightarrow \boxed{R_x = 26.8\,kN}$$

$$0 = \sum F_y = R_y - T\sin 30^\circ - 15\,kN - 11\,kN$$

$$\Rightarrow \boxed{R_y = 41.5\,kN}$$

## Problem 3

Mathematica calculations for Problems 3 and 4

```
ClearAll["Global`*"];
kilogram = Quantity["kg"];
second = Quantity["s"];
newton = Quantity["N"];
meter = Quantity["m"];
g = 10.0 meter / second^2;
resultsB = Reduce[{0.0 newton meter == momentsAboutAxle,
    momentsAboutAxle == (13 meter) (tension) Sin[25 Degree] - (8 meter) (15000 newton)
       Cos[55 Degree] - (16 meter) (11000 newton) Cos[55 Degree]}] // ToRules
tension = tension /. resultsB
```

```
{tension → 30902.3 N , momentsAboutAxle → 0. J }

30902.3 N
```

```
resultsC = Reduce[
   {sumFx == 0.0 newton, sumFx == rx - tension Cos[30 Degree],
    sumFy == 0.0 newton,
    sumFy == ry - tension Sin[30 Degree] - 15000 newton - 11000 newton}] // ToRules
resultsC[[4]]
resultsC[[2]]
```

```
{sumFy → 0. N , ry → 41451.2 N , sumFx → 0. N , rx → 26762.2 N}

rx → 26762.2 N

ry → 41451.2 N
```

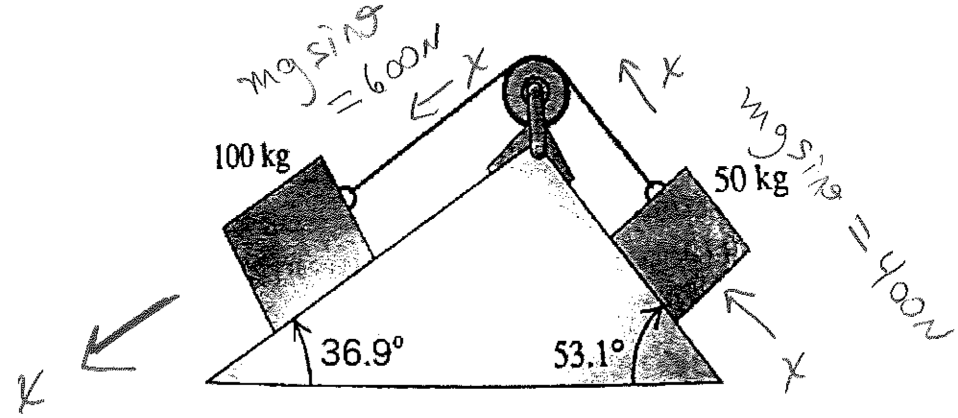
## Problem 4

```
ClearAll["Global`*"];
kilogram = Quantity["kg"];
second = Quantity["s"];
newton = Quantity["N"];
meter = Quantity["m"];
g = 10.0 meter / second^2;
m1 = 100 kilogram;
m2 = 50 kilogram;
resultsB = Reduce[{
    m1 ax == m1 g Sin[36.9 Degree] - tension,
    m2 ax == tension - m2 g Sin[53.1 Degree]}]
```

tension == 466.702 N && ax == 1.33719 $m/s^2$

4. **(10%)** Two blocks are connected by a taut cable that passes over a small pulley (of negligible inertia). Friction in the pulley is negligible, and both blocks rest on planes of negligible friction.

(a) Which way will the system move when the blocks are released from rest? Indicate this direction on the diagram and define this to be the direction in which the $x$ coordinate increases. (The same $x$ coordinate will describe both the uphill motion of one block and the downhill motion of the other block, since the cable stays taut. You only need one coordinate for this problem.)




(b) Write Newton's second law, $ma_x = \sum F_x$, separately for each of the blocks. Since the cable stays taut, $a_x$ is the same for both blocks.

$$m_1 a_x = \underset{\text{acting on 1}}{\sum F_x} = m_1 g \sin 37° - T$$

$$m_2 a_x = \underset{\text{acting on 2}}{\sum F_x} = T - m_2 g \sin 53°$$

$$\Rightarrow a_x = \frac{(3m_1 - 4m_2)g}{5(m_1+m_2)}$$

$$\boxed{a_x = +1.33\ \text{m/s}^2}$$

(c) What is the acceleration, $a_x$ of the blocks?

$$\boxed{a_x = +1.33\ \text{m/s}^2}$$

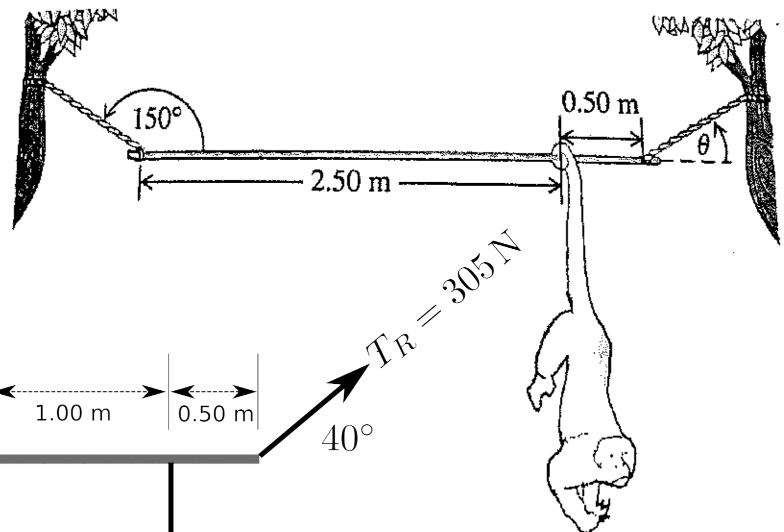
⊕ sign indicates that our guess for direction was correct in (a).

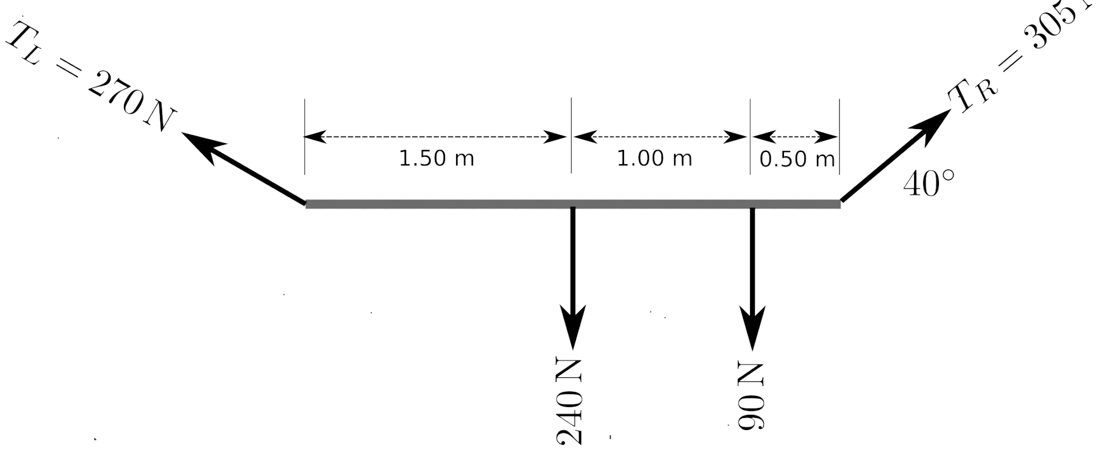
(d) What is the tension $T$ in the cord?

$$T = m_2 a_x + \frac{4}{5} m_2 g = \boxed{467\,N = T}$$

5. (10%) A 3.00 m long uniform rod, whose mass is 24.0 kg, is held in a horizontal position by two cables (of negligible mass) at its ends. The left cable makes a 150° angle with the rod, and the right cable makes an angle $\theta$ with the horizontal. A 9.0 kg monkey hangs motionless by his tail, a horizontal distance 0.50 m from the right end of the rod.

(a) Draw an extended free-body diagram for the rod.




(b) Find the tensions, $T_L$ and $T_R$, in the left and right cables, and find the angle $\theta$. (If you find $T_R \cos\theta$ and $T_R \sin\theta$, you can combine these to find $T_R$ and $\theta$.)

$$0 = \sum_{\substack{\circlearrowleft \\ \text{about left end of rod}}} M = -(2.50\,\text{m})(90\,\text{N}) - (1.50\,\text{m})(240\,\text{N}) + (3.0\,\text{m})\,T_{Ry}$$

$$\Rightarrow \boxed{T_{Ry} = 195\,\text{N}}$$

$$0 = \sum F_y = T_L \sin 30^\circ - 240\,\text{N} - 90\,\text{N} + T_{Ry} \Rightarrow \boxed{T_L = 270\,\text{N}}$$

$$0 = \sum F_x = -T_L \cos 30^\circ + T_{Rx} \Rightarrow \boxed{T_{Rx} = 234\,\text{N}}$$

$$\boxed{T_R = 304\,\text{N}} = \sqrt{T_{Rx}^2 + T_{Ry}^2}$$

$$\boxed{\theta = 40^\circ} = \operatorname{atan}\left(T_{Ry}/T_{Rx}\right)$$

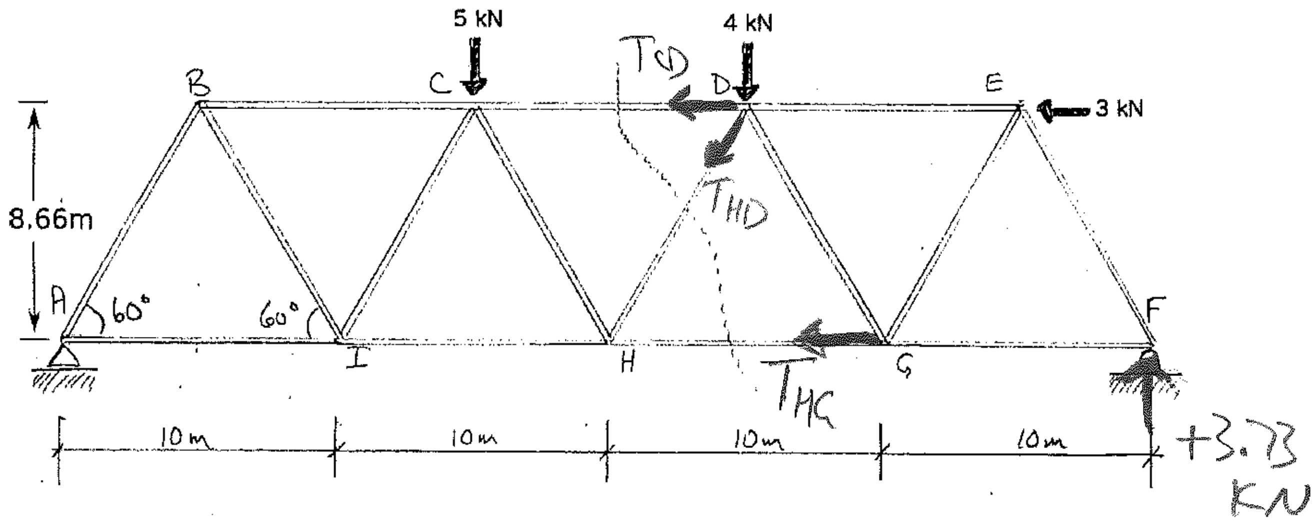
# Mathematica calculations for Problem 5

## Problem 5

```
ClearAll["Global`*"];
kilogram = Quantity["kg"];
second = Quantity["s"];
newton = Quantity["N"];
meter = Quantity["m"];
g = 10.0 meter / second ^ 2;
mrod = 24.0 kilogram;
mmonkey = 9.0 kilogram;
resultsB = Reduce[{
    momentsLeftEnd == 0.0 newton meter,
    momentsLeftEnd ==
     - (mrod g) (1.50 meter) - (mmonkey g) (2.50 meter) + (tRightY) (3.00 meter),
    sumFx == 0.0 newton,
    sumFx == -tLeft Cos[30 Degree] + tRightX,
    sumFy == 0.0 newton,
    sumFy == tLeft Sin[30 Degree] - mrod g - mmonkey g + tRightY,
    tRight == Sqrt[tRightX ^ 2 + tRightY ^ 2],
    θdegrees == ArcTan[tRightY / tRightX] 180 / Pi}] // ToRules
resultsB[[4]]
resultsB[[2]]
resultsB[[1]]
resultsB[[3]]
resultsB[[8]]
```

```
{tRightY → 195. N , tRightX → 233.827 N , tRight → 304.467 N , tLeft → 270. N ,
 sumFy → 0 N , sumFx → 0 N , momentsLeftEnd → 0 J , θdegrees → 39.8264}

tLeft → 270. N

tRightX → 233.827 N

tRightY → 195. N

tRight → 304.467 N

θdegrees → 39.8264
```

**6.** **(15%)** Consider the truss shown below.




(a) Find the force $F_y$ exerted on the truss by the roller support at $F$.

$$0 = \sum_{\circlearrowleft} M_A = (40\text{m})F_y + (8.66\text{m})(3\text{kN}) - (15\text{m})(5\text{kN}) - (25\text{m})(4\text{kN})$$

$$\Rightarrow \boxed{F_y = +3.73\text{kN}}$$

(b) Using the Method of Sections, solve for the forces (tensions or compressions) in truss members $CD$, $HD$, and $HG$. Indicate whether each of these members is in tension or in compression.

$$0 = \sum_{\circlearrowleft} M_D = (+3.73\text{kN})(15\text{m}) - T_{HG}(8.66\text{m}) \Rightarrow \boxed{T_{HG} = 6.45\text{kN} \text{ (tension)}}$$

$$0 = \sum F_y = -T_{HD}\sin 60° - 4\text{kN} + 3.73\text{kN} \Rightarrow \boxed{T_{HD} = -0.32\text{kN} \text{ (compression)}}$$

(forces in y)

$$0 = \sum F_x = -T_{CD} - 3\text{kN} - T_{HD}\cos 60° - T_{HG} = 0$$

$$\Rightarrow \boxed{T_{CD} = -9.29\text{kN} \text{ (compression)}}$$

# Mathematica calcuilations for Problem 6

## Problem 6

```
ClearAll["Global`*"];
kN = Quantity[1000.0, "N"];
m = Quantity[1.0, "m"];
resultsA = Reduce[{
    momentsA == 0.0 kN m,
    momentsA == (40 m) fy + (8.66 m) (3 kN) - (15 m) (5 kN) - (25 m) (4 kN)}] // ToRules
fy = fy /. resultsA
```

```
{momentsA → 0. J , fy → 3725.5 N }
```

```
3725.5 N
```

```
resultsB = Reduce[{
    momentsD == 0.0 kN m,
    momentsD == fy (15 m) - tHG (8.66 m),
    sumFy == 0.0 kN,
    sumFy == -tHD Sin[60 Degree] - 4 kN + fy == 0.0 kN,
    sumFx == 0.0 kN,
    sumFx == -tCD - 3 kN - tHD Cos[60 Degree] - tHG == 0.0 kN}] // ToRules
resultsB[[1]]
resultsB[[2]]
resultsB[[3]]
```

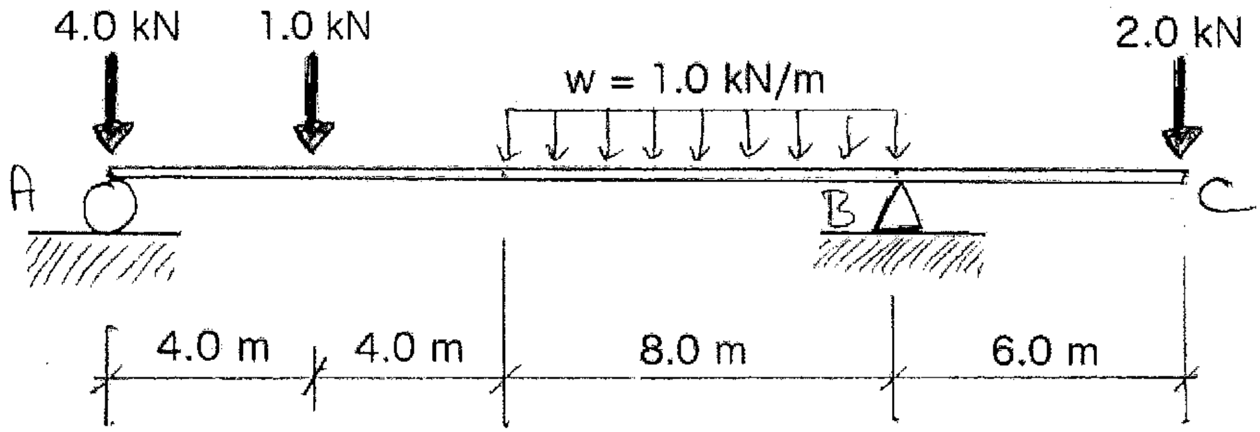
```
{tHG → 6452.94 N , tHD → -316.965 N ,
 tCD → -9294.46 N , sumFy → 0. N , sumFx → 0. N , momentsD → 0. J }
```

```
tHG → 6452.94 N
```

```
tHD → -316.965 N
```

```
tCD → -9294.46 N
```

7. (15%) For the overhang beam shown below.




(a) Find the support forces $A_y$, $B_x$, and $B_y$ exerted on the beam by supports $A$ and $B$.

$$0 = \sum F_x = \boxed{B_x = 0}$$

$$0 = \sum_{A} M = -(1\text{kN})(4\text{m}) - (8\text{kN})(12\text{m}) + B_y(16\text{m}) - (2\text{kN})(22\text{m})$$

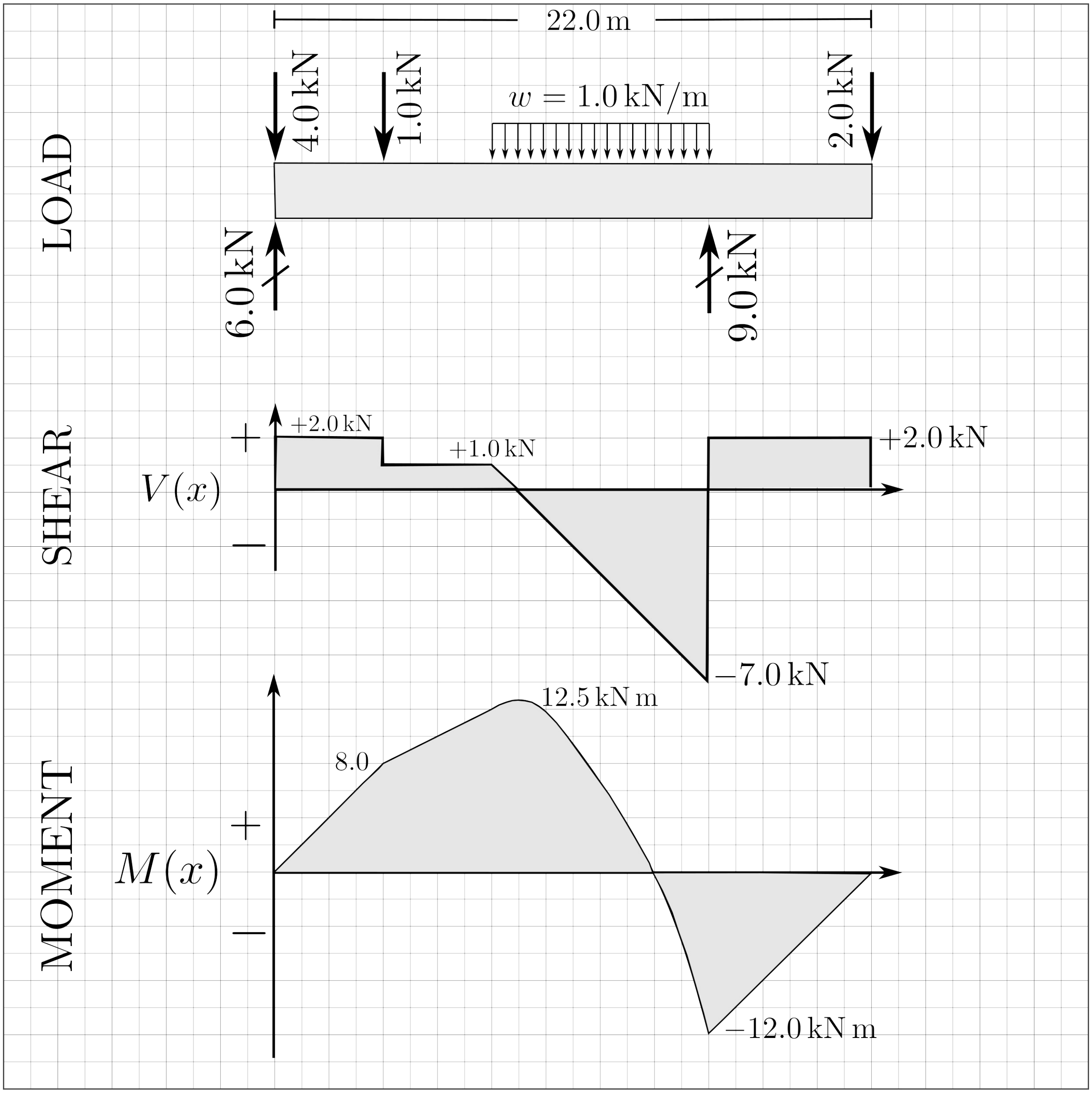
$$\Rightarrow \boxed{B_y = +9\text{kN}}$$

$$0 = \sum F_y = A_y - 4\text{kN} - 1\text{kN} - 8\text{kN} - 2\text{kN} + B_y$$

$$\Rightarrow \boxed{A_y = +6\text{ kN}}$$

(Problem continues on next page.)

(b) Draw load (EFBD), shear ($V$), and bending moment ($M$) diagrams for the beam.




(c) What are the largest magnitude of the shear $V$ (in kilonewtons) and the largest magnitude of the bending moment $M$ (in kilonewton-meters)?

$|V_{max}| = |-7\,\text{kN}| = \boxed{7\,\text{kN}}$

$|M_{max}| = |12.5\,\text{kN}\cdot\text{m}| = \boxed{12.5\,\text{kN}\cdot\text{m}}$

Mathematica calculations for Problem 7

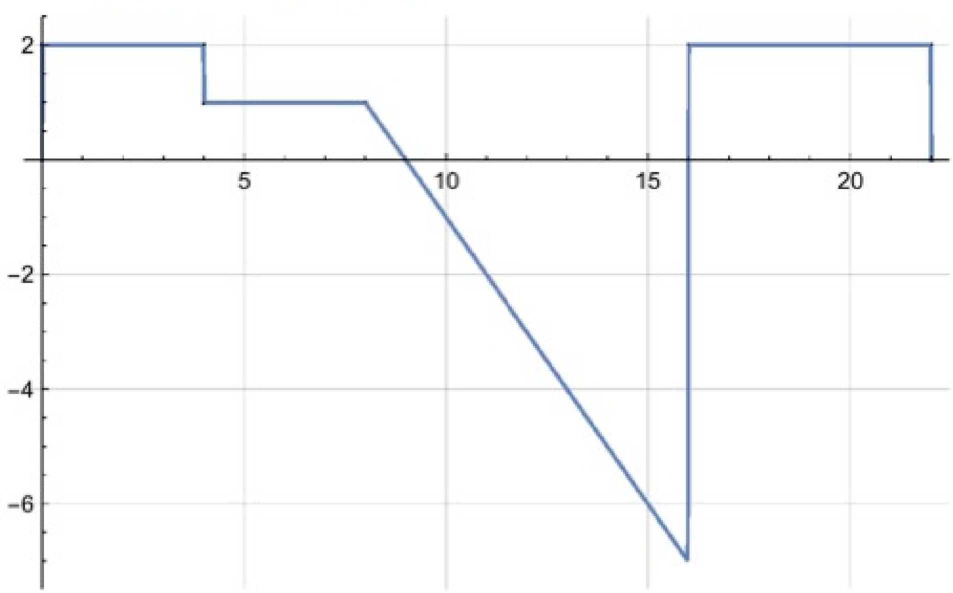
## Problem 7

```
ClearAll["Global`*"];
kN = Quantity["kN"];
m = Quantity["m"];
resultsA = Reduce[{
    sumFx == 0 kN,
    sumFx == bx,
    sumFy == 0 kN,
    sumFy == ay - 4 kN - 1 kN - 8 kN + by - 2 kN,
    momentsA == 0 kN m,
    momentsA == - (1 kN) (4 m) - (8 kN) (12 m) + by (16 m) - (2 kN) (22 m) }]
resultsA[[{6, 2, 5}]]
```

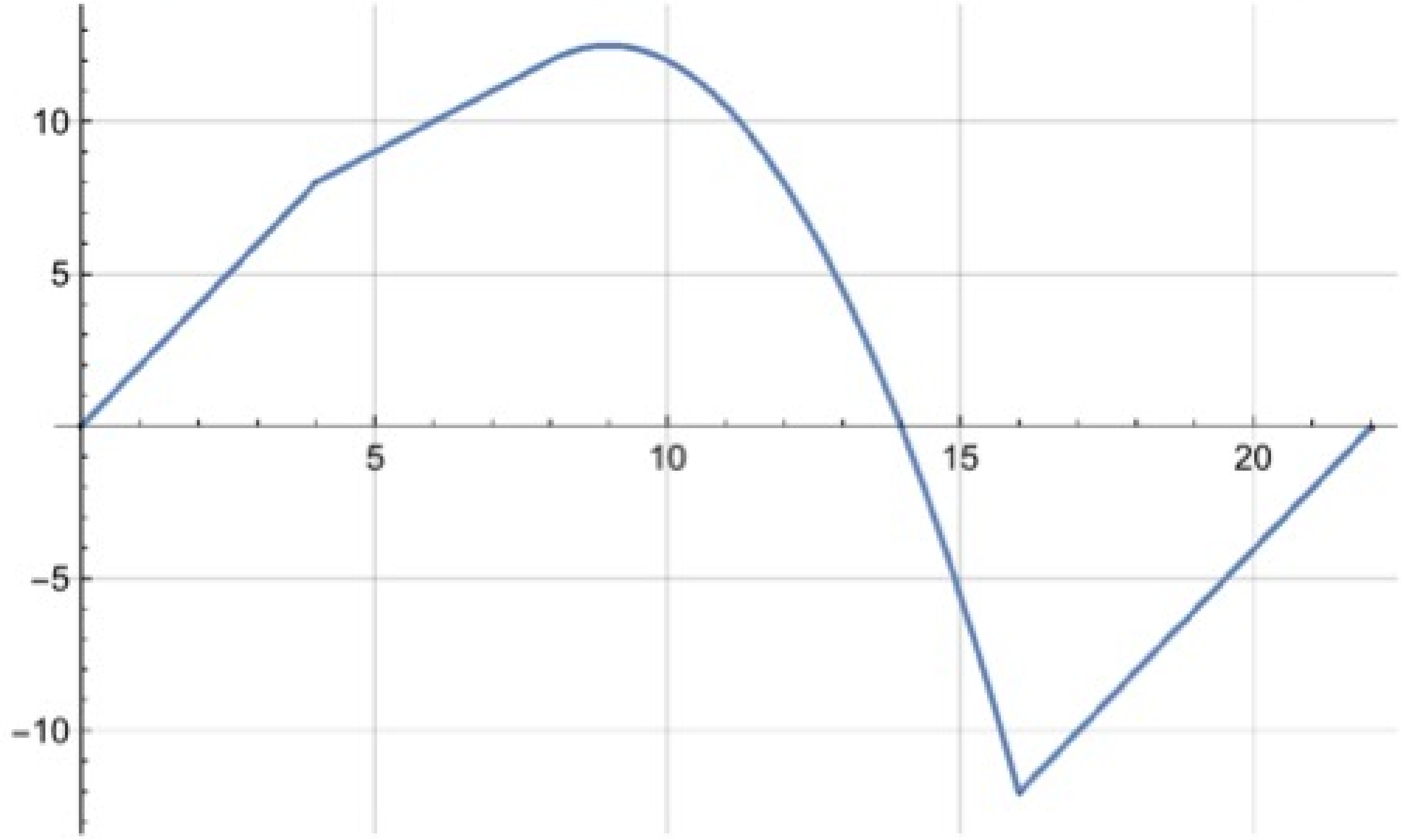
```
sumFx == 0 N && bx == 0 N && sumFy == 0 N && momentsA == 0 J && by == 9000 N && ay == 6000 N

ay == 6000 N && bx == 0 N && by == 9000 N
```

```
:= ClearAll["Global`*"];
v = Interpolation[{{0, 0}, {0.01, 2}, {3.99, 2}, {4.01, 1}, {8, 1},
    {15.99, -7}, {16.01, 2}, {21.99, 2}, {22.0, 0}}, InterpolationOrder → 1];
Plot[v[x], {x, 0, 22}, GridLines → Automatic]
```

]=



```
m[x_] := Integrate[v[xx], {xx, 0, x}];
Plot[m[x], {x, 0, 22}, GridLines → Automatic]
```




```
Map[m, {0, 4, 8, 9, 16, 22}]
```

{0, 7.9875, 11.99, 12.4894, -12.0275, -0.06}

```
MaxValue[{Abs[v[x]], x > 0, x < 22}, x]
```

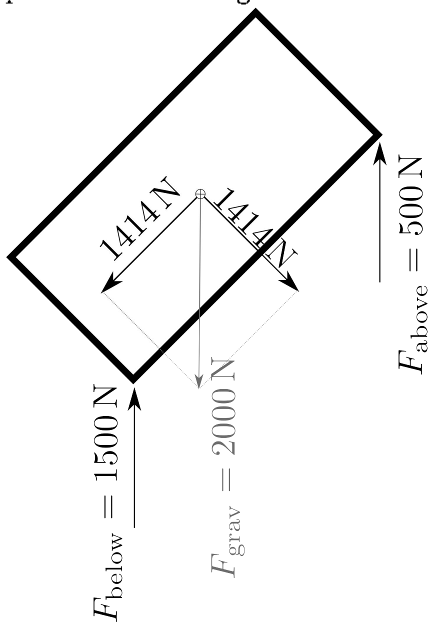
7.

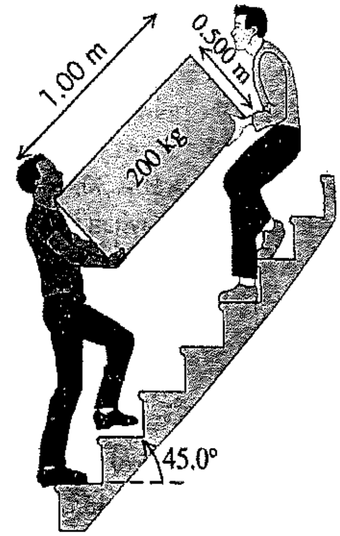
```
MaxValue[{Abs[m[x]], x > 0, x < 22}, x]
```

12.4894

8. **(10%)** You and your friend are carrying a 200 kg box up a flight of stairs. The box is 1.00 m long and 0.500 m high, and its center of gravity is at its center. The stairs make a 45.0° angle to the floor. The crate is carried at a 45.0° angle, so that its bottom side is parallel to the stairs. Assume that each person applies, with his or her hands, a purely vertical force to the corresponding corner of the box.

(a) Draw an extended free-body diagram for the box, showing each force exerted on the box and its line of action. To simplify computing the moments (torques) in part (b) considerably, decompose the gravity vector into components parallel to the long and short axes of the box.




(b) What are the magnitudes, $F_{\text{below}}$ and $F_{\text{above}}$ of the vertical forces applied by the person below and the person above, respectively?

$$0 = \sum_{\substack{\circlearrowleft \text{ about} \\ \text{bottom} \\ \text{corner}}} M = F_{\text{above}}\left(\frac{1.00\,\text{m}}{\sqrt{2}}\right) - \left(\frac{2000\,N}{\sqrt{2}}\right)\left(\frac{1.00\,\text{m}}{2}\right) + \left(\frac{2000\,N}{\sqrt{2}}\right)\left(\frac{0.500\,\text{m}}{2}\right)$$

$$\boxed{F_{\text{above}} = 500\,N}$$

$$0 = \sum F_y = F_{\text{below}} + F_{\text{above}} - 2000\,N \Rightarrow \boxed{F_{\text{below}} = 1500\,N}$$

(c) Is it easier to be the person above or the person below on the stairs?

It's easier to be the person above on the stairs.

# Mathematica calculations for Problems 8 and 9

## Problem 8

```
kilogram = Quantity["kg"];
second = Quantity["s"];
newton = Quantity["N"];
meter = Quantity["m"];
g = 10.0 meter / second^2;
m = 200 kilogram;
width = 1.00 meter;
height = 0.500 meter;
resultsB = Reduce[{
    mgTowardStairs == m g / Sqrt[2],
    mgDownStairs == m g / Sqrt[2],
    sumFy == 0.0 newton,
    sumFy == fBelow + fAbove - m g,
    sumMomentsBottomCorner == 0 newton meter,
    sumMomentsBottomCorner == (fAbove) (width) / Sqrt[2] -
      (mgTowardStairs) (width / 2) + (mgDownStairs) (height / 2)}]
resultsB[[
 {-1,
  -2}]]
```

```
sumMomentsBottomCorner == 0. J && sumFy == 0. N && mgTowardStairs == 1414.21 N &&
 mgDownStairs == 1414.21 N && fBelow == 1500. N && fAbove == 500. N
```

```
fAbove == 500. N && fBelow == 1500. N
```

## Problem 9

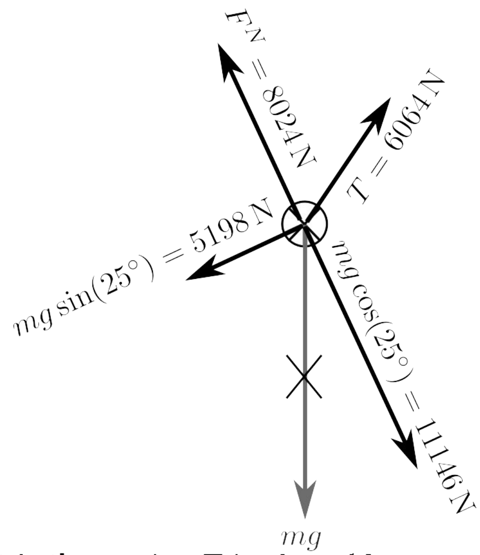
```
kilogram = Quantity["kg"];
second = Quantity["s"];
newton = Quantity["N"];
meter = Quantity["m"];
g = 10.0 meter / second^2;
m = 1230 kilogram;
resultsB = Reduce[{
    sumFx == 0.0 newton,
    sumFx == m g Sin[25 Degree] - tension Cos[31 Degree],
    sumFy == 0.0 newton,
    sumFy == fNormal + tension Sin[31 Degree] - m g Cos[25 Degree]}]
resultsB[[{1, -1}]]
```

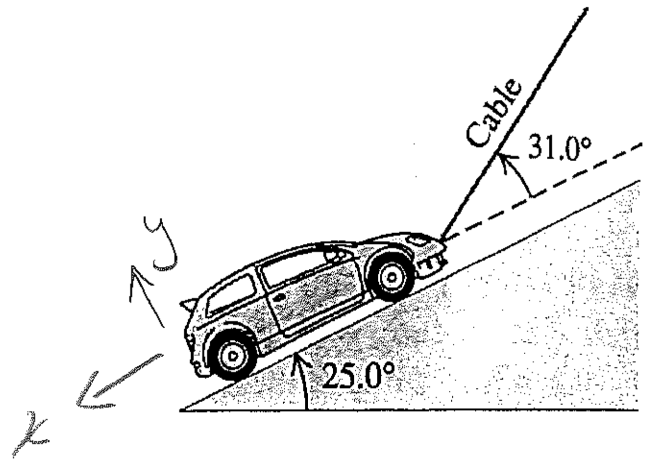
```
tension == 6064.4 N && sumFy == 0. N && sumFx == 0. N && fNormal == 8024.19 N
```

```
tension == 6064.4 N && fNormal == 8024.19 N
```

**9. (10%)** A car of mass 1230 kg is held in place by a light cable on a very smooth (negligible friction) ramp. The cable makes an angle 31.0° above the surface of the ramp. The ramp itself is inclined at 25.0° above the horizontal.

(a) Draw a free-body diagram for the car. (Just an ordinary FBD, not an extended one.)




(b) What is the tension $T$ in the cable.

$$ma_x = 0 = \underbrace{\sum F_x}_{\text{acting on car}} = -T\cos 31° + mg\sin 25°$$

$$\Rightarrow \boxed{T = 6064\,N}$$

(c) What is the normal force $F_N$ exerted by the ramp on the car.

$$ma_y = 0 = \underbrace{\sum F_y}_{\text{acting on car}} = F^N + T\sin 31° - mg\cos 25°$$

$$\Rightarrow \boxed{F^N = 8024\,N}$$