Physics 8, Fall 2019, Practice Exam. **Name:** Bill

This open-book take-home exam is 10% of your course grade. (The in-class final exam will be 25% of your course grade. For the in-class exam, you can bring one 3×5 card of handwritten notes and a calculator. You will turn in your 3×5 card of notes [if any] with your final exam.) You should **complete this exam on your own, without working with other people.** It is fine to discuss general topics from the course with your classmates, but it is not OK to share your solutions to these specific problems. Feel free to use either $g = 9.80\ \mathrm{m/s^2}$ or $g = 10.0\ \mathrm{m/s^2}$ — whichever you prefer. The in-class exam will be shorter than this practice exam and will consist mainly of problems very similar to problems you have already solved in the weekly homework; the topics covered will be similar to this practice exam. FYI, you can find four previous years' exams and practice exams at `http://positron.hep.upenn.edu/p8/files/oldexams`

If I do not receive your take-home exam by **5pm on Monday, December 9** (in my office, DRL 1W15, or in class), then your score will be **zero**, without exception, as I need to return graded take-home exams promptly. If you turn in your take-home exam by **5pm on Friday, December 6**, then I will grade and email your exam back to you on Monday evening (December 9). Otherwise, I will return your graded exam to you at the review session on Wednesday, December 11.

Please show your work on these sheets. Add blank sheets if needed.

1. (10%) You and your little sister are out in the snow with a sled that has a mass of 11 kg. Your sister, who weighs 29 kg, is sitting on the sled and you want to push her along. You start applying a horizontal force and initially the sled doesn't move but you slowly increase your force until, suddenly, the sled does move. You maintain the same force that you were applying when the sled started moving for the next 5.0 seconds after which you let go. Use coefficient of kinetic friction $\mu_k = 0.020$ and coefficient of static friction $\mu_s = 0.080$.

(a) How far do you have to run if you apply the force for 5.0 s?

We consider forces acting on the sled+sister system, whose total mass is $m = 40\,\mathrm{kg}$. Let the x axis point "forward." Let F^P be the pushing force by me on the sled, F_N be the normal force by the ground on the sled, F_K be kinetic friction by the ground on the sled. The sled starts to move once F^P reaches the limit of static friction: $F_x^P = \mu_S F^N = \mu_S mg$. While I push the moving sled forward, F^K pushes the sled backward, and Newton's 2nd law gives $ma_x = F_x^P + F_x^K = \mu_S mg - \mu_K mg$. During the push, $a_x = (\mu_S - \mu_K)g = +0.588\,\mathrm{m/s^2}$ [$+0.600\,\mathrm{m/s^2}$]. Thus after $t = 5\,\mathrm{s}$, $x_f = \frac{1}{2}a_x t^2 = \boxed{7.35\,\mathrm{m}}$ $\boxed{[7.50\,\mathrm{m}]}$ (using $g = 9.80\,\mathrm{m/s^2}$ [$g = 10.00\,\mathrm{m/s^2}$])

(b) What is your sister's speed at $t = 5.0\,s$?

$v_{xf} = a_x t = \boxed{2.94\,\mathrm{m/s}}$ $\boxed{[3.00\,\mathrm{m/s}]}$

(Problem continues on next page.)

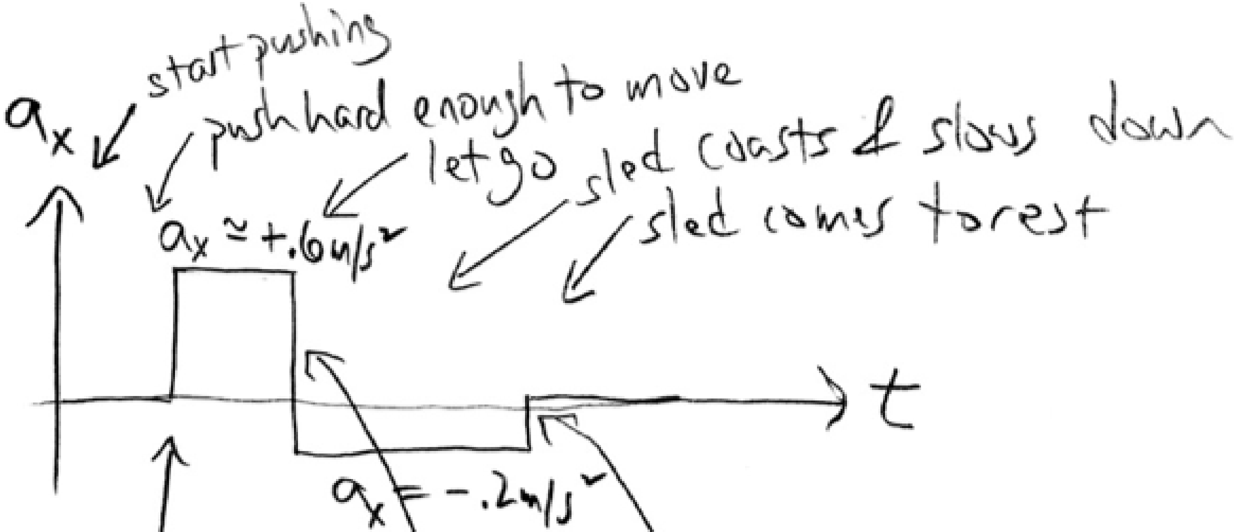
(c) After letting go, how far do your sister and her sled move (with respect to the point where you let go) until she is stationary again?

Once I let go, the sled accelerates backward (slows down) under the influence of F^K

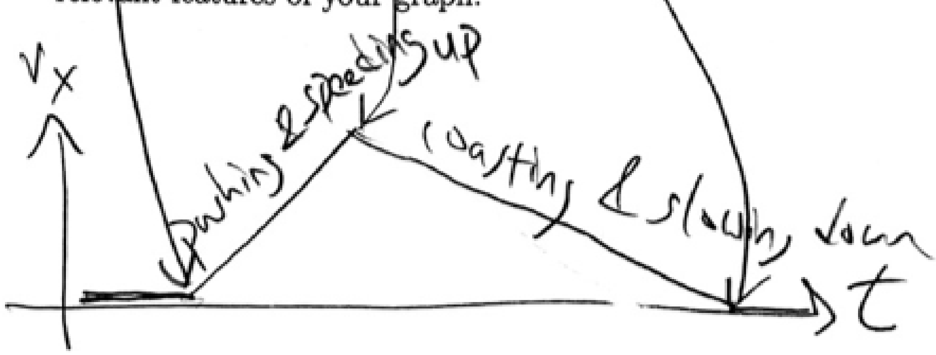
$$ma_x = -F^K \;\Rightarrow\; a_x = -\mu_K g = -0.196\,\mathrm{m/s^2} \quad [-0.200\,\mathrm{m/s^2}]$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x\Delta x \;\Rightarrow\; \Delta x = -\frac{v_{xi}^2}{2a_x} = \boxed{22.05\,\mathrm{m}} \quad [22.50\,\mathrm{m}]$$

(d) Draw and label a qualitative graph of the acceleration of the sled as a function of time. Qualitative means that it explains the overall behavior without using exact numbers. Annotate all relevant features of your graph.



(e) Draw and label a qualitative graph of the velocity of the sled as a function of time. Annotate all relevant features of your graph.



2. (10%) A 1000 kg car traveling due east at speed 20.0 m/s collides head-on with a 1500 kg light truck traveling due west at speed 10.0 m/s.

(a) If 50% of the car+truck system's kinetic energy is converted to internal energy during the collision, what are the final velocities of the car and truck? (If you find two solutions, then you should eliminate the solution that does not make sense and report as your result the solution that does make sense.)

Let the x axis point east. Let the car be object 1 and the truck be object 2. Notice that initially $v_{1xi} - v_{2xi} > 0$. Momentum conservation gives

$$m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

If the final K.E. is 50% of the initial K.E. then

$$\frac{1}{2} m_1 v_{1xf}^2 + \frac{1}{2} m_2 v_{2xf}^2 = 0.50 \left(\frac{1}{2} m_1 v_{1xi}^2 + \frac{1}{2} m_2 v_{2xi}^2 \right)$$

Of the two solutions (quadratic formula or Wolfram Alpha etc.), the one we want has $v_{1xf} - v_{2xf} < 0$, because the relative velocity should change sign after a collision.

The solution is $\boxed{v_{x1f} = -10.61\,\mathrm{m/s}}$ (car) and $\boxed{v_{x2f} = +10.41\,\mathrm{m/s}}$ (truck).

For what it's worth, the solution we reject is $v_{x1f} = +14.61\,\mathrm{m/s}$ and $v_{x2f} = -6.41\,\mathrm{m/s}$, which doesn't make sense because $v_{1xf} - v_{2xf} > 0$. This problem is much, much harder than anything I would put on an in-class exam, and I it's arguably harder than anything you did in the homework. In retrospect, for this practice exam I should have given you the coefficient of restitution and asked you to determine how much of the initial K.E. had been converted into other forms — a much less tedious problem. I had to find problems you hadn't seen before — sorry!

(b) What is the coefficient of restitution for the collision you analyzed in part (a)?

$$e = -\frac{v_{1xf} - v_{2xf}}{v_{1xi} - v_{2xi}} = \boxed{0.701}$$

(Problem continues on next page.)

(c) If the collision had instead been perfectly elastic, what would have been the final velocities of the car and truck?

Momentum conservation gives

$$m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

and for an elastic collision we have

$$(v_{1xf} - v_{2xf}) = -(v_{1xi} - v_{2xi})$$

The solution is $\boxed{v_{x1f} = -16.0\,\mathrm{m/s}}$ (car) and $\boxed{v_{x2f} = +14.0\,\mathrm{m/s}}$ (truck).

(d) If the collision had instead been totally inelastic, what would have been the final velocities of the car and truck?

Momentum conservation gives

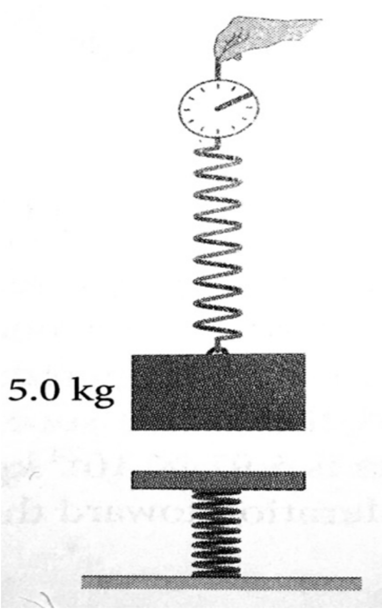
$$m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

and for a totally inelastic collision we have

$$v_{1xf} = v_{2xf}$$

The solution is $\boxed{v_{x1f} = v_{x2f} = +2.00\,\mathrm{m/s}}$.

3. (10%) A 5.0 kg block suspended from a spring scale is very slowly lowered onto a vertical spring, as shown.



(a) What does the scale read before the block touches the vertical spring?

Let F^{scale} be the magnitude of the (upward) force exerted by the scale on the block. Let F^{spring} be the magnitude of the (upward) force exerted by the spring on the block. We know that mg is the magnitude of the (downward) gravitational force exerted by Earth on the block. Since the block is being "very slowly lowered," we infer that the block's acceleration is negligible, i.e. the block is in equilibrium.

$$0 = F^{\text{scale}} - mg \ \Rightarrow \ F^{\text{scale}} = \boxed{49\,\text{N}} \quad \boxed{[50\,\text{N}]}$$

(b) If the scale reads 40 N when the bottom spring is compressed 3.0 cm (0.030 m), what is k for the bottom spring?

Let D be the distance the spring has compressed from its relaxed length.

$$0 = F^{\text{scale}} + F^{\text{spring}} - mg = F^{\text{scale}} + kD - mg$$

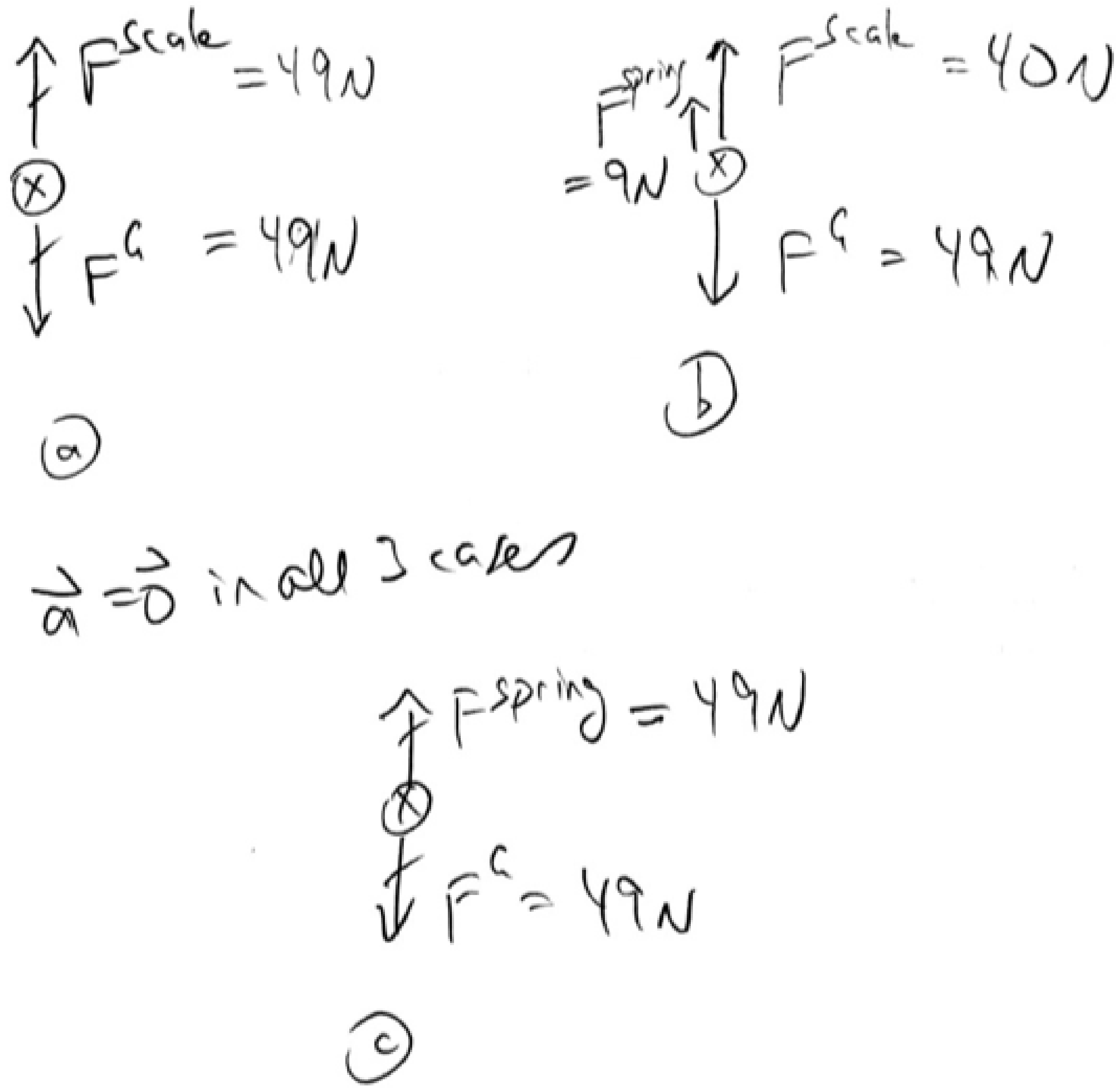
$$k = \frac{mg - F^{\text{scale}}}{D} = \boxed{300\,\text{N/m}} \quad \boxed{[333\,\text{N/m}]}$$

(Problem continues on next page.)

(c) How far does the block compress the bottom spring when the scale reads 0 N?

$$0 = F^{\text{scale}} + F^{\text{spring}} - mg = 0 + kD - mg \;\Rightarrow\; D = \boxed{0.163\,\text{m}} \quad \boxed{[0.150\,\text{m}]}$$

(d) Draw three Mazur-style free-body diagrams for the block: one for part (a), one for part (b), and one for part (c).



4. (10%) A 10 kg dog jumps up in the air to catch a ball. The dog's center of mass is normally 0.20 meter above the ground, and he is 0.50 meter long (which means 0.50 meter tall when he is up on his hind legs). The lowest he can get his center of mass is 0.10 meter above the ground, and the highest he can get his center of mass before he can no longer push against the ground is 0.30 meter. [You may not need to use all of the numbers given in the problem statement. If you happen to draw a good illustration to guide your thinking about this problem, that is worth a bonus point.]

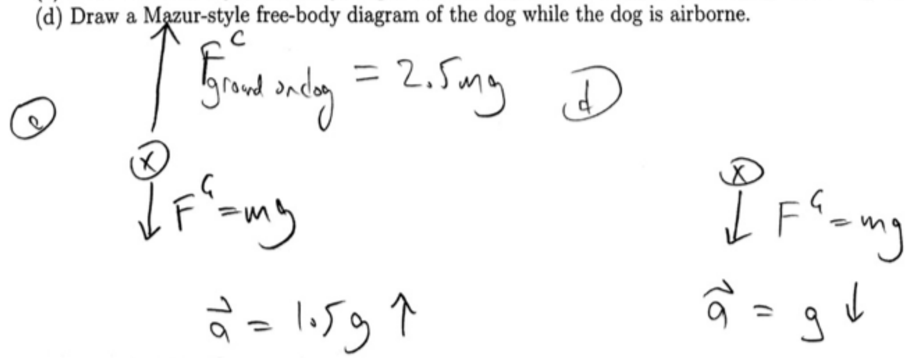
(a) If the maximum force the dog can exert on the ground in pushing off is 2.5 times his own weight, how high can he jump? [To answer unambiguously, state the greatest height above the ground that the dog's center of mass can reach.]
Let $y_1 = 0.10\,\mathrm{m}$ be the height of the dog's CoM when he starts to push off. Let $y_2 = 0.30\,\mathrm{m}$ be the height of the dog's CoM when he first loses contact with the ground. Let y_3 be the height of the dog's CoM at the highest point he reaches, at the peak of his jump. Let $F^P = 2.5mg$ be the upward push exerted by the floor on the dog (equal in magnitude to the downward push exerted by the dog on the floor). The vector sum of forces acting on the dog while he is pushing off is $\sum F_y = F^P - mg$. While these two forces act, the dog's CoM is displaced $y_2 - y_1$. Thus the dog's K.E. increases from zero to $(F^P - mg)(y_2 - y_1)$. Then over the distance $y_3 - y_2$ the dog loses that same K.E. under the influence of a single force mg. Equating the energy gained to energy lost:

$$(F^P - mg)(y_2 - y_1) = mg(y_3 - y_2) \;\Rightarrow\; 1.5mg(y_2 - y_1) = mg(y_3 - y_2)$$

The maximum height of the dog's CoM is $\boxed{y_3 = 0.60\,\mathrm{m}}$. Technically this is Mazur (9.14), but if you ignore that subtlety and treat the dog's CoM as a particle being pushed up by its back legs and down by gravity, you write $(F_P - mg)(y_2 - y_1) = W = \Delta K = mg(y_3 - y_2)$ and get the right result. Or you could get the exact same math using upward constant acceleration 1.5g followed by downward constant acceleration g, using the equations for constant acceleration.

(b) State clearly in words (sentences) your reasoning for part (a).
option 1: During the push, the net force $(F^P - mg)$ accelerates the dog's CoM upward at $1.5g$. Then once he leaves the ground, the net force $-mg$ accelerates the dog's CoM downward at g. Equating increase in speed (during push) to decrease in speed (airborne) gives y_3.
option 2: The "work" done on the dog's CoM by the net force $(F^P - mg)$ over displacement $y_2 - y_1$ increases his K.E. That K.E. is then given back as Earth's gravity does negative work of magnitude $mg(y_3 - y_2)$. The dog's speed is zero both before the push and at his highest point.
option 3: Same as (2) but stated more rigorously using Mazur (9.14). I didn't notice this subtlety when I typed up the problem, but I thought of it while solving the problem. Anyone who gets a correct answer while noticing the subtlety about force displacement of F^P gets a bonus point!
option 4: Work done by dog's legs in displacing dog's CoM up by 0.2 meter with $2.5mg$ push equals total change in dog+Earth G.P.E.$=mgh$.

(c) Draw a Mazur-style free-body diagram of the dog while the dog is pushing against the ground.
(d) Draw a Mazur-style free-body diagram of the dog while the dog is airborne.



5. (10%) A janitor is pushing an 18.0 kg trashcan across a level floor at constant velocity. The coefficient of friction between the can and the floor is 0.150.

(a) If the janitor is pushing horizontally, what is the magnitude of the force she exerts on the can?

Let F^N be the (upward) normal force exerted by the floor on the can. Let $F^K = \mu_K F^N$ be the (backward) kinetic friction exerted by the floor on the can. Let mg be the (downward) gravitational force exerted by Earth on the can. Let F^P be the contact force (pushing along the axis of her arm or whatever stick she uses to push the can) exerted by the janitor on the can. The janitor is in equilibrium. Summing vertical forces gives $0 = F^N - mg$. Summing horizontal forces gives $0 = F^P - F^K = F^P - \mu_K mg \;\Rightarrow\; F^P = \mu_K mg = \boxed{26.5\,\mathrm{N}}$ $\boxed{[27.0\,\mathrm{N}]}$.

(b) If she pushes at an angle of 36.9° down from the horizontal, what must the magnitude of her pushing force be to keep the can moving at constant velocity?

Let $\theta = 36.9°$ be the angle w.r.t. horizontal at which she pushes. Summing vertical forces gives

$$0 = F^N - mg - F^P \sin\theta \;\Rightarrow\; F^N = mg + F^P \sin\theta$$

Summing horizontal forces gives

$$0 = F^P \cos\theta - F^K = F^P \cos\theta - \mu_K F^N = F^P \cos\theta - \mu_K (mg + F^P \sin\theta)$$

Solving for F^P gives

$$F^P = \frac{\mu_K mg}{\cos\theta - \mu_K \sin\theta} = \boxed{37.3\,\mathrm{N}} \quad \boxed{[38.0\,\mathrm{N}]}$$

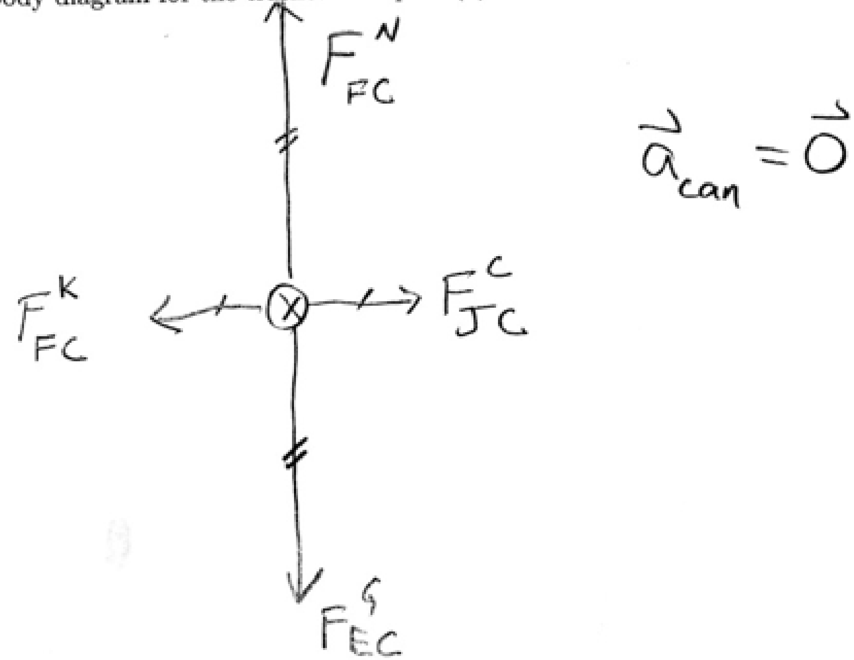
(Problem continues on next page.)

(c) For part (b), what is the work done by the janitor as she moves the trashcan 25.0 meters across the floor?

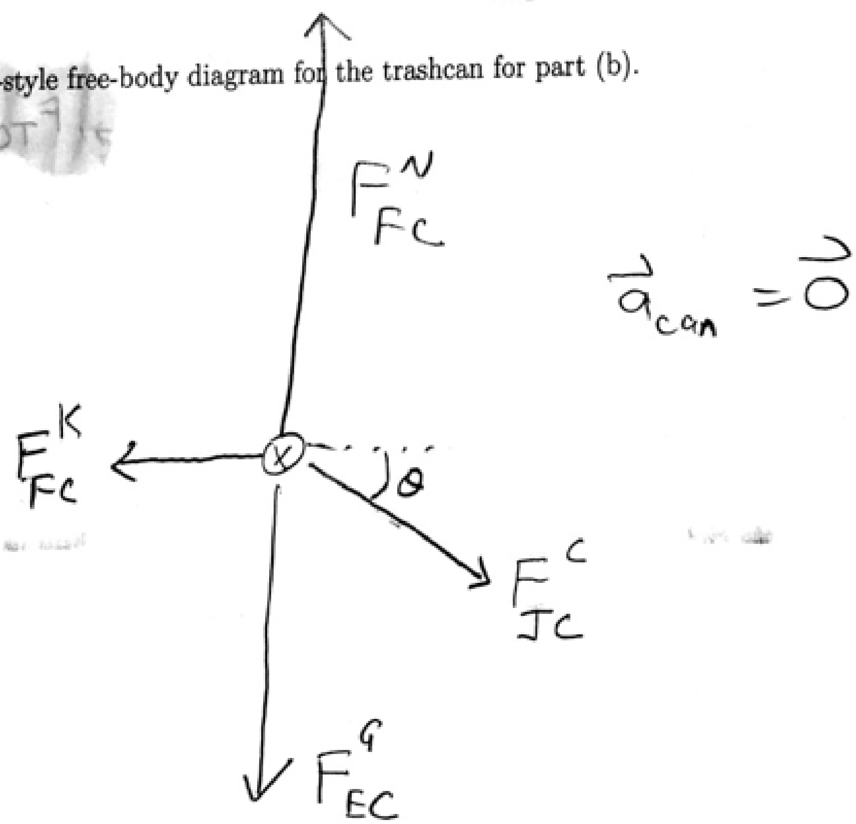
The displacement of the can is entirely horizontal, so we want to multiply this displacement by the horizontal component of $\vec{F}^P$.

$$W = \vec{F}^P \cdot \Delta\vec{r} = (F^P \cos\theta)\Delta x = \boxed{745\ \mathrm{J}}\ \ \boxed{[761\ \mathrm{J}]}$$

(d) Draw a Mazur-style free-body diagram for the trashcan for part (a).



(e) Draw a Mazur-style free-body diagram for the trashcan for part (b).

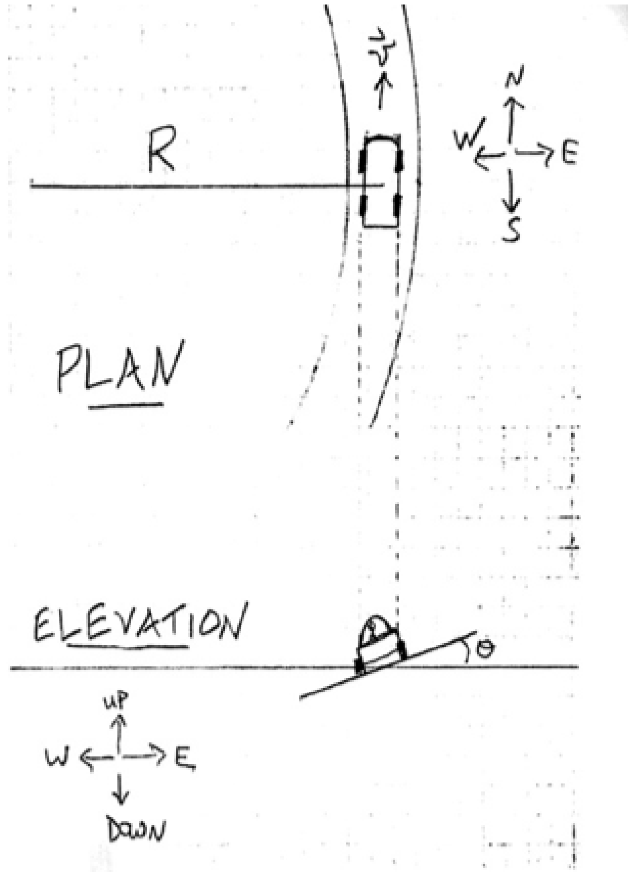


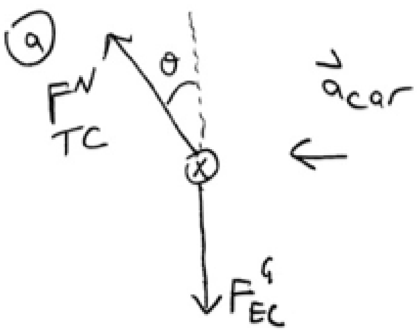
6. (10%) A race car is negotiating a curve, of radius R, on a track which is banked at an angle θ with respect to horizontal. There is a certain speed v_{critical} at which friction is not needed to keep the car on the track in the curve. (This is the speed the car should travel if there is an oil slick on the track, for example.) Draw a Mazur-style free-body diagram showing the forces exerted on the car when the car is moving at speed

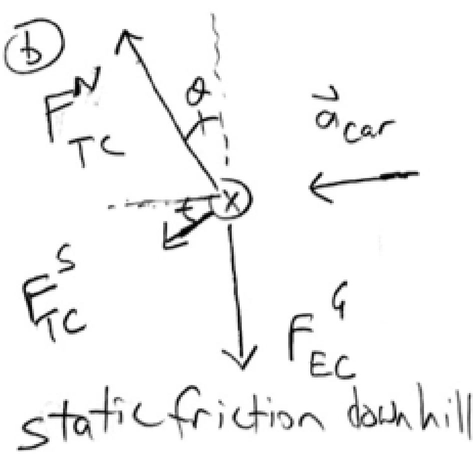
(a) $v = v_{\text{critical}}$,

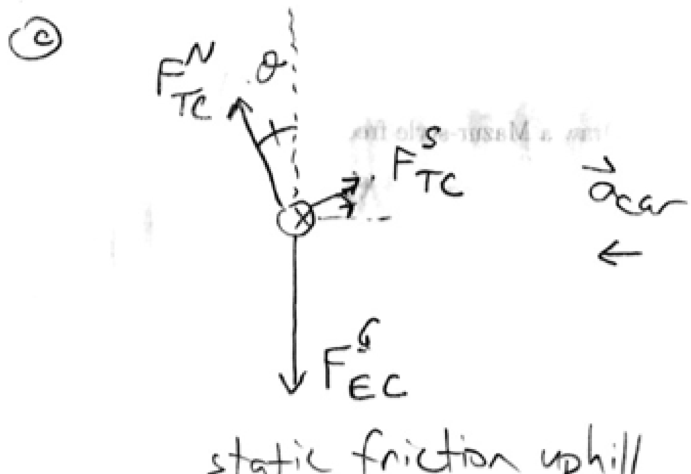
(b) $v > v_{\text{critical}}$, and

(c) $v < v_{\text{critical}}$.





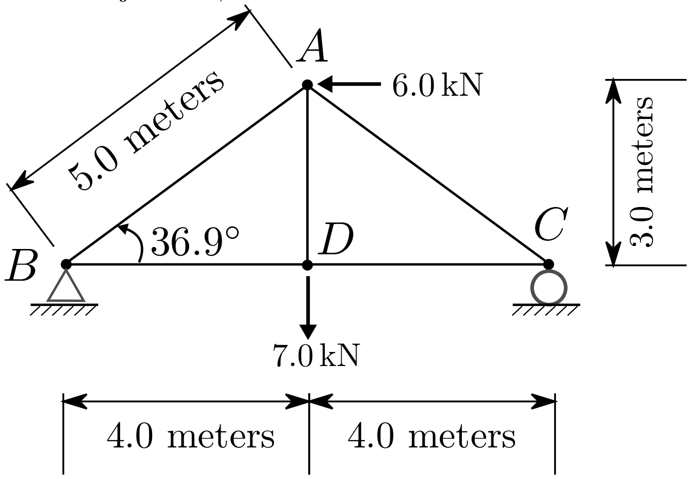


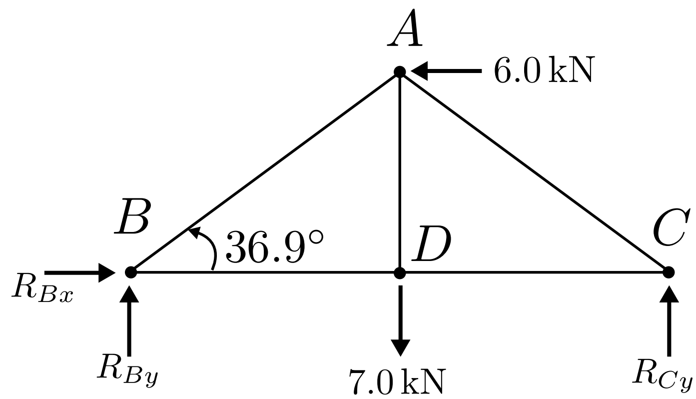


(d) Using your drawing for (a) to guide your reasoning, write v_{critical} in terms of the radius of curvature R, the bank angle θ, and the gravitational acceleration constant g.

The car's acceleration equals v^2/R westward. Let y point up and x point east. When $v = v_{\text{critical}}$, the only two forces acting on the car are the normal force by the road on the car and the gravitational force by Earth on the road. Newton's second law in the y direction reads $0 = \sum F_y = F^N \cos\theta - mg$ and Newton's second law in the x direction reads $-mv^2/R = -F^N \sin\theta$ Eliminating F^N and solving for v yields $v_{\text{critical}} = v = \sqrt{gR\tan\theta}$.

7. (20%) The truss shown below is simply supported at joints B and C, carries a 6.0 kN horizontal load at joint A, and carries a 7.0 kN vertical load at joint D.





(a) I've drawn, above-right, an Extended Free Body Diagram for the truss as a whole. Using joint B as a pivot, write the moment equation about joint B to solve for the vertical "reaction" force R_{Cy} exerted on the truss by the roller support at joint C. As a check against careless mistakes, you should expect your answer to be one of the following: ±1.250 kN, ±1.667 kN, ±2.083 kN, ±5.750 kN, ±6.000 kN, ±7.000 kN, ±9.583 kN.

$$0 = (6.0\,\text{kN})(3.0\,\text{m}) + R_{Cy}(8.0\,\text{m}) - (7.0\,\text{kN})(4.0\,\text{kN})$$

$$R_{Cy} = \boxed{+1.250\,\text{kN}}$$

(b) Now use $\sum F_x$ and $\sum F_y$ for the truss as a whole to find the two reaction forces, R_{Bx} and R_{By} exerted on the truss by the hinge support at joint B. As a check against careless mistakes, you should expect each answer to be one of the following: ±1.250 kN, ±1.667 kN, ±2.083 kN, ±5.750 kN, ±6.000 kN, ±7.000 kN, ±9.583 kN.

$$0 = -6.0\,\text{kN} + R_{Bx}$$

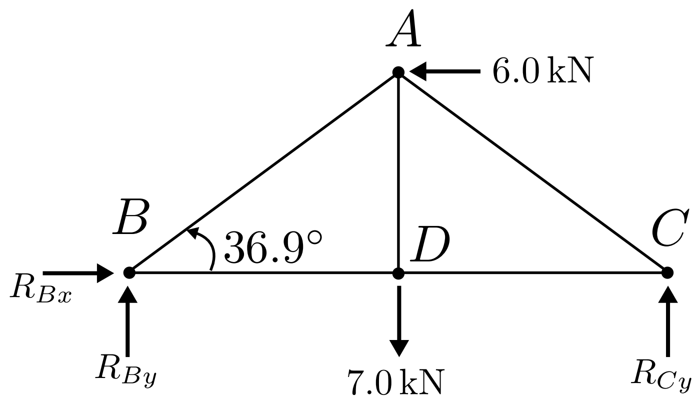
$$R_{Bx} = \boxed{+6.000\,\text{kN}}$$

$$0 = R_{By} - 7.0\,\text{kN} + R_{Cy}$$

$$R_{By} = \boxed{+5.750\,\text{kN}}$$

(Problem continues on next page.)

(c) Using the method of joints, write $\sum F_y$ for joint D to solve for the bar tension $\boxed{T_{AD}}$. Indicate whether bar AD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ±1.250 kN, ±1.667 kN, ±2.083 kN, ±5.750 kN, ±6.000 kN, ±7.000 kN, ±9.583 kN.



$$0 = T_{AD} - 7.0\,\text{kN}$$

$$T_{AD} = \boxed{+7.000\,\text{kN}}\ \text{(tension)}$$

(d) Using the method of joints, write $\sum F_y$ for joint C to solve for the bar tension $\boxed{T_{AC}}$. Indicate whether bar AC is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ±1.250 kN, ±1.667 kN, ±2.083 kN, ±5.750 kN, ±6.000 kN, ±7.000 kN, ±9.583 kN.

$$0 = T_{AC} \sin\theta + R_{Cy}$$

$$T_{AC} = \boxed{-2.083\,\text{kN}}\ \text{(compression)}$$

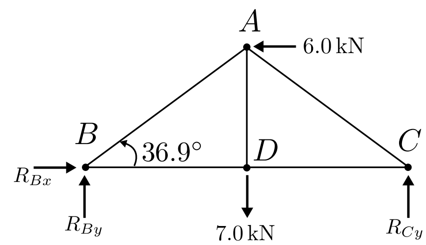
(e) Using the method of joints, write $\sum F_y$ for joint B to solve for the bar tension $\boxed{T_{AB}}$. Indicate whether bar AB is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ±1.250 kN, ±1.667 kN, ±2.083 kN, ±5.750 kN, ±6.000 kN, ±7.000 kN, ±9.583 kN.

$$0 = T_{AB} \sin\theta + R_{By}$$

$$T_{AB} = \boxed{-9.583\,\text{kN}}\ \text{(compression)}$$

(Problem continues on next page.)

(f) Using the method of joints, write $\sum F_y$ for joint A to check that your results for T_{AB}, T_{AD}, and T_{AC} are consistent.



$$-T_{AB}\sin\theta - T_{AD} - T_{AC}\sin\theta = 0 \quad \checkmark$$

(g) Using the method of joints, write $\sum F_x$ for joint C to solve for the bar tension $\boxed{T_{CD}}$. Indicate whether bar CD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

$$0 = -T_{CD} - T_{AC}\cos\theta$$
$$T_{CD} = \boxed{+1.667\,\mathrm{kN}}\ \text{(tension)}$$

(h) Using the method of joints, write $\sum F_x$ for joint D to solve for the bar tension $\boxed{T_{BD}}$. Indicate whether bar BD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

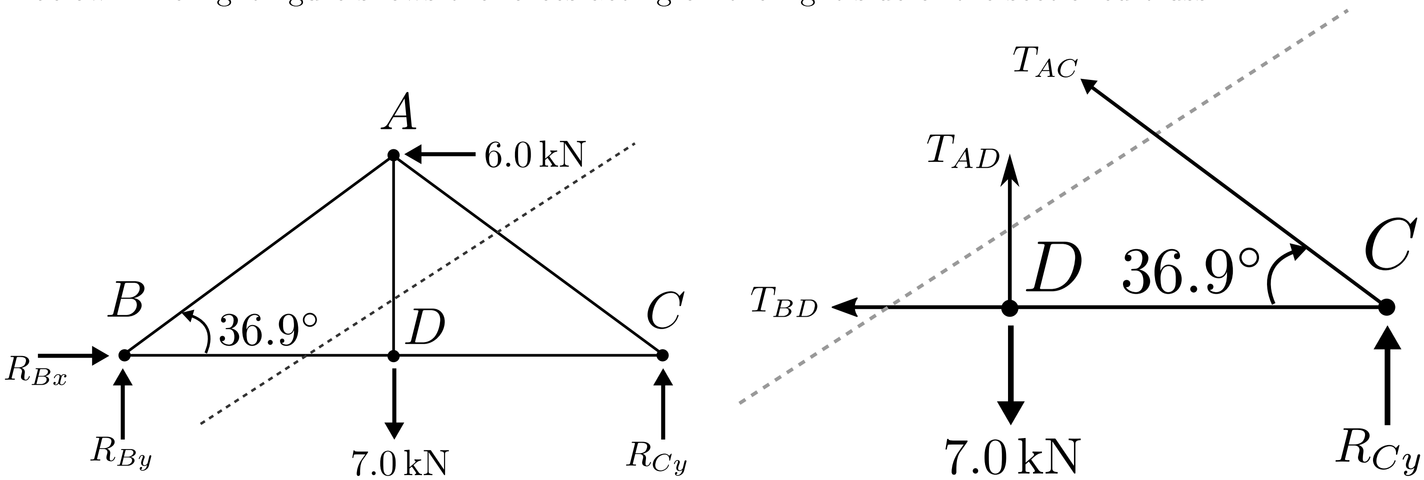
$$0 = -T_{BD} + T_{CD}$$
$$T_{BD} = \boxed{+1.667\,\mathrm{kN}}\ \text{(tension)}$$

(i) Using the method of joints, write $\sum F_x$ for joint B to check that your results for T_{AB}, T_{BD}, and R_{Bx} are consistent.

$$R_{Bx} + T_{BD} + T_{AB}\cos\theta = 0 \quad \checkmark$$

(Problem continues on next page.)

Now let's use the method of sections to analyze the right-hand side of the section shown in the figure below. The right figure shows the forces acting on the right side of the sectioned truss.



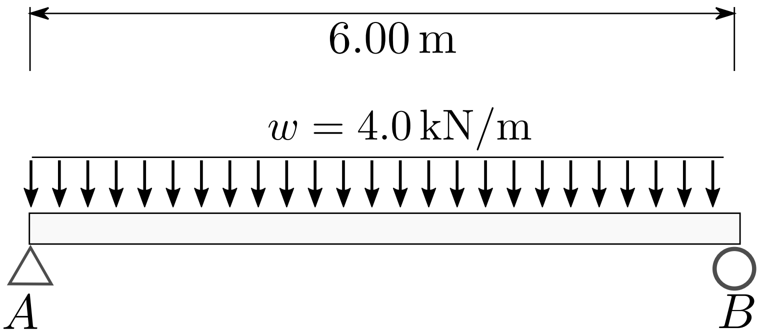
(j) Using moments about joint D, write an equation that lets you check the consistency of your results for T_{AC} and R_{Cy}. (Alas, this equation may feel a bit redundant to you.)

$$(4.0\,\mathrm{m})T_{AC}\sin\theta + (4.0\,\mathrm{m})R_{Cy} = 0 \quad \checkmark$$

(k) Using moments about joint A (which is invisible here, but you know where it is), write an equation that lets you check the consistency of your results for T_{BD} and R_{Cy}.

$$(4.0\,\mathrm{m})R_{Cy} - (3.0\,\mathrm{m})T_{BD} = 0 \quad \checkmark$$

8. (20%) (a) A simply-supported beam of length $L = 6.00\,\mathrm{m}$ carries a uniform distributed load $w = 4.00\,\mathrm{kN/m}$ along its entire length. Solve for the three "reaction" forces exerted by the supports: the upward force $\boxed{R_{Ay}}$ exerted by the support at A, the horizontal force $\boxed{R_{Ax}}$ exerted by the support at A, and the upward force $\boxed{R_{By}}$ exerted by the support at B.



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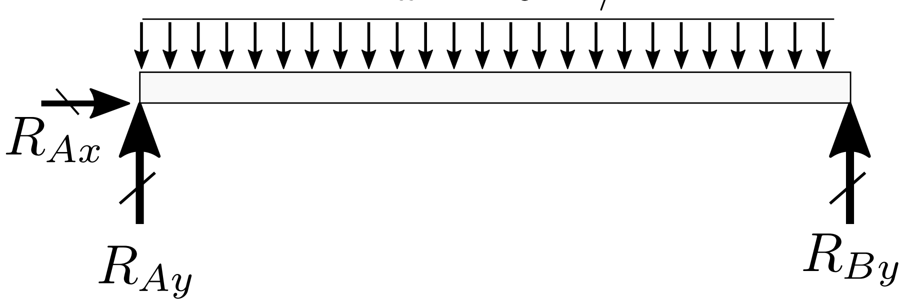


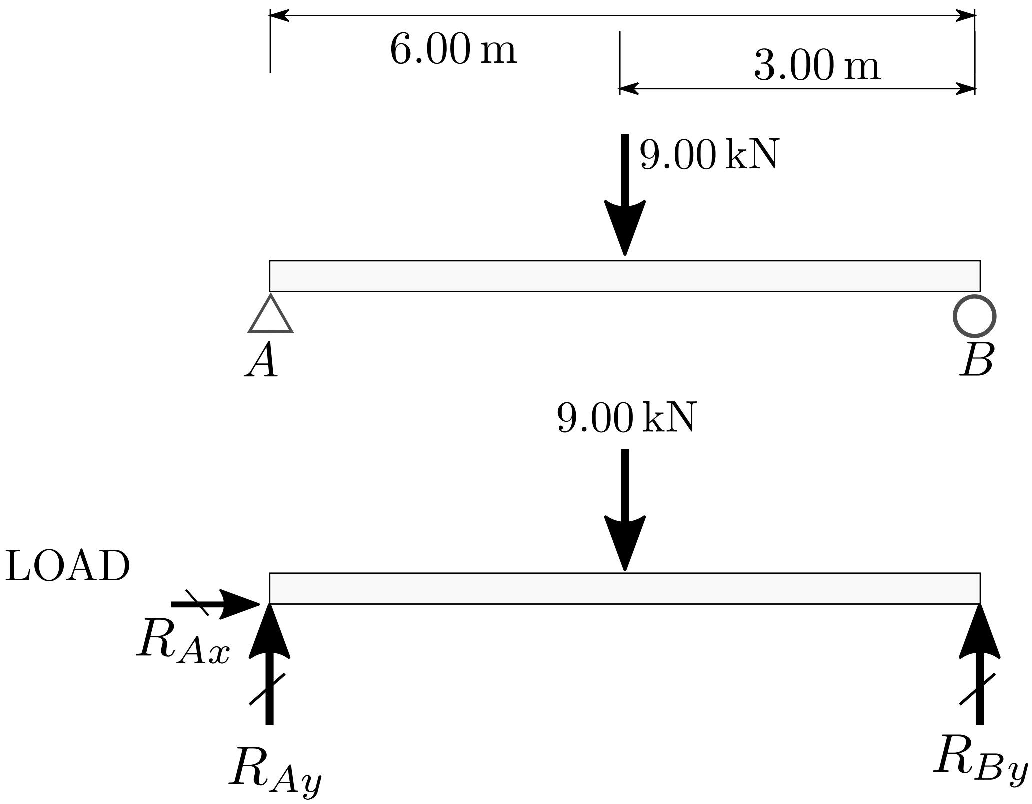
$R_{Ax} = 0.0\,\mathrm{kN}$

$R_{Ay} = +12.0\,\mathrm{kN}$

$R_{By} = +12.0\,\mathrm{kN}$

(Problem continues on next page.)

(b) A simply-supported beam of length $L = 6.00\,\text{m}$ carries a single concentrated load $9.00\,\text{kN}$ at mid-span. Solve for the three "reaction" forces exerted by the supports: the upward force $\boxed{R_{Ay}}$ exerted by the support at A, the horizontal force $\boxed{R_{Ax}}$ exerted by the support at A, and the upward force $\boxed{R_{By}}$ exerted by the support at B.



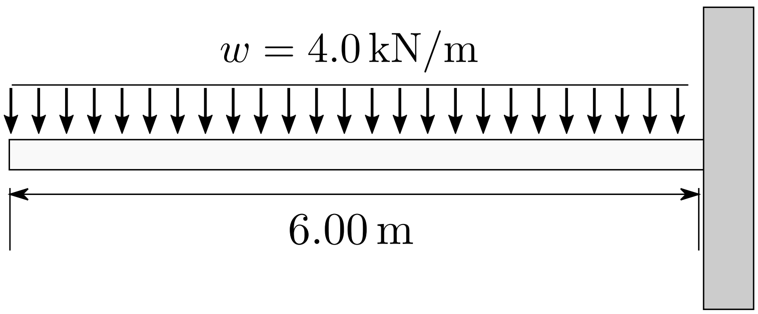
$R_{Ax} = 0.0\,\text{kN}$

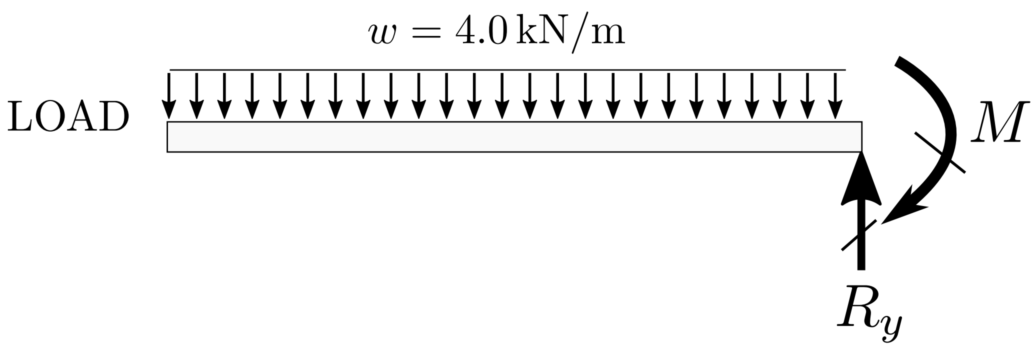
$R_{Ay} = +4.5\,\text{kN}$

$R_{By} = +4.5\,\text{kN}$

(Problem continues on next page.)

(c) A cantilever beam of length $L = 6.00\,\mathrm{m}$ carries a uniform distributed load $w = 4.00\,\mathrm{kN/m}$ along its entire length. Solve for the upward “reaction” force $\boxed{R_y}$ exerted by the wall on the right end of the beam. Also solve for the moment (torque) $\boxed{M}$ exerted by the wall on the beam: to do this, use the right end of the beam (at the wall) as a pivot, and let M be whatever torque is needed to make all torques (moments) add up to zero for equilibrium.



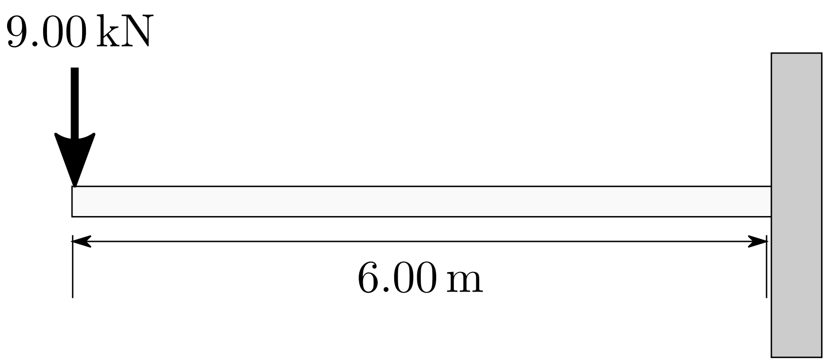


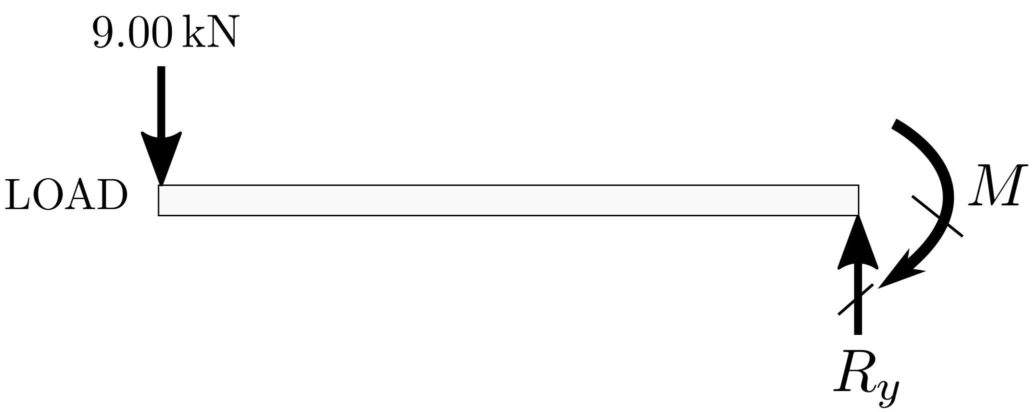
$R_y = 24.0\,\mathrm{kN}$

$M = 72.0\,\mathrm{kN \cdot m}$

(Problem continues on next page.)

(d) A cantilever beam of length $L = 6.00\,\mathrm{m}$ carries a single concentrated load of $w = 9.00\,\mathrm{kN}$ at its left end. Solve for the upward "reaction" force $\boxed{R_y}$ exerted by the wall on the right end of the beam. Also solve for the moment (torque) $\boxed{M}$ exerted by the wall on the beam: to do this, use the right end of the beam (at the wall) as a pivot, and let M be whatever torque is needed to make all torques (moments) add up to zero for equilibrium.





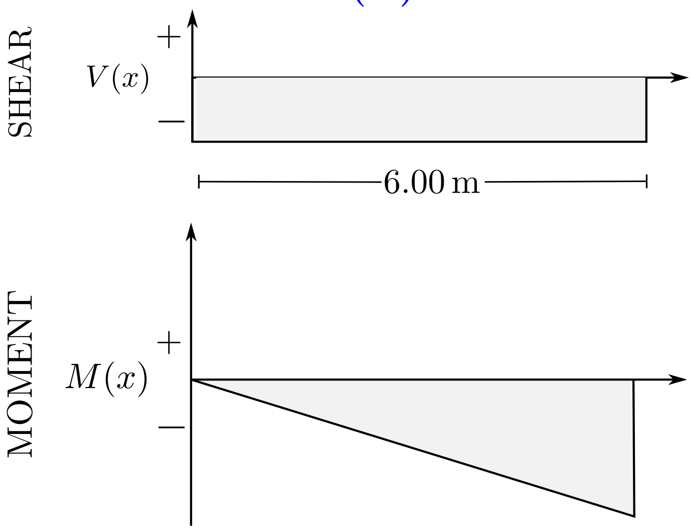
$$R_y = 9.0\,\mathrm{kN}$$

$$M = 54.0\,\mathrm{kN \cdot m}$$

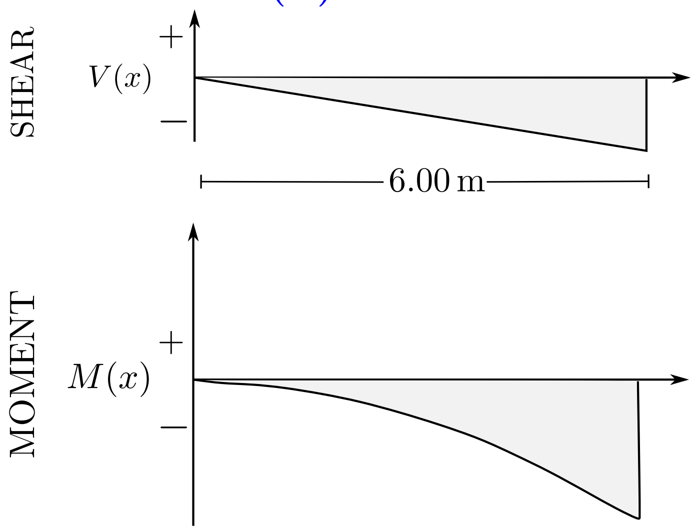
(Problem continues on next page.)

(e) For each of the following four shear/moment diagram pairs, write in the letter **A**, **B**, **C**, or **D** to indicate whether the graphs correspond to the beam shown in part (a), part (b), part (c), or part (d) of this problem (above).

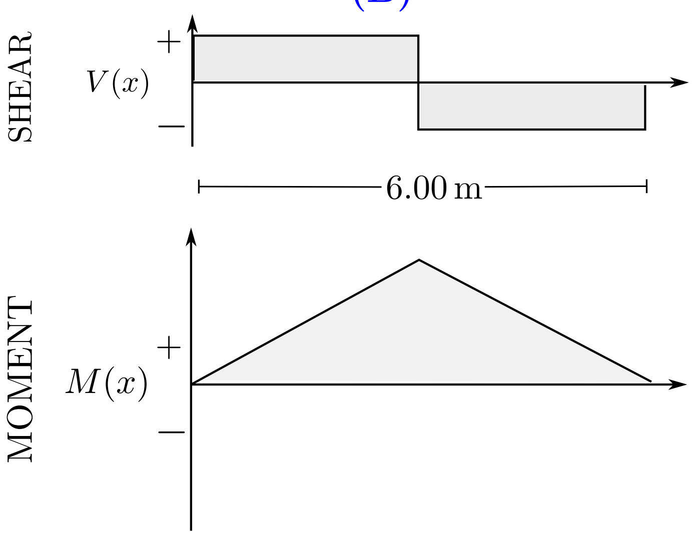
(D)



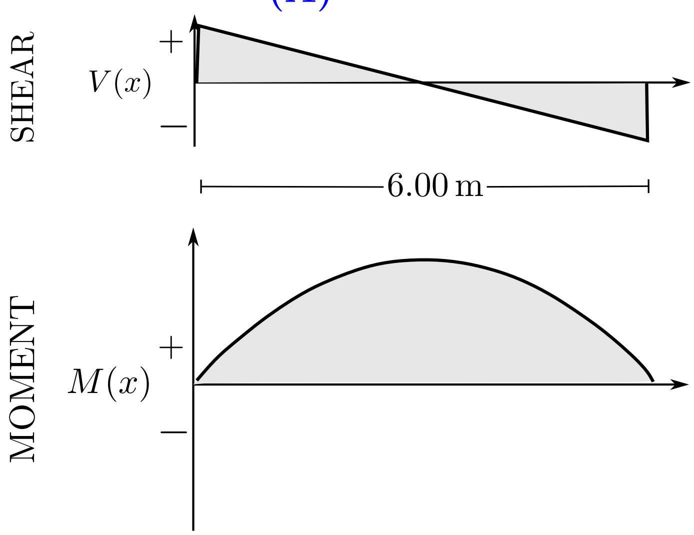
(C)



(B)



(A)



Possibly useful equations

$$\cos(30°) = \frac{\sqrt{3}}{2} \approx 0.866 \qquad \sin(30°) = \frac{1}{2} \qquad \tan(30°) = \frac{1}{\sqrt{3}} \approx 0.577$$

$$\cos(60°) = \frac{1}{2} \qquad \sin(60°) = \frac{\sqrt{3}}{2} \approx 0.866 \qquad \tan(60°) = \sqrt{3} \approx 1.732$$

$$\cos(36.9°) = \frac{4}{5} \qquad \sin(36.9°) = \frac{3}{5} \qquad \tan(36.9°) = \frac{3}{4}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$f_{\text{spring}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad f_{\text{pendulum}} = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$$\sum \vec{F} = m\vec{a} \qquad \sum \vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} \qquad \vec{p} = m\vec{v}$$

$$F_c = \frac{mv^2}{r} \qquad F_c = m\omega^2 r \qquad v = \omega r$$

$$F^K = \mu^K F^N \qquad F^s \le \mu_s F^N$$

$$F_x^{\text{ spring}} = -k\,(x - x_0)$$

$$F_y^{\text{ grav}} = -mg \qquad g = 9.8\ \text{m/s}^2$$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \tau = rF\sin\theta = r_\perp F = rF_\perp$$

$$\frac{F}{A} = (\text{stress}) = (E)\,(\text{strain}) = E\,\frac{\Delta L}{L_0}$$

$$V = \frac{\mathrm{d}M}{\mathrm{d}x} \qquad (M_2 - M_1) = (x_2 - x_1)\overline{V}_{1\to 2} \qquad w = -\frac{\mathrm{d}V}{\mathrm{d}x} \qquad V(x) = \sum_{0\to x} F_y(\text{up minus down})$$