

This open-book take-home exam is 10% of your course grade. (The in-class final exam will be 25% of your course grade. For the in-class exam, you can bring one 3×5 card of handwritten notes and a calculator. You will turn in your 3×5 card of notes [if any] with your final exam.) You should **complete this exam on your own, without working with other people**. It is fine to discuss general topics from the course with your classmates, but it is not OK to share your solutions to these specific problems. Feel free to use either $g = 9.80 \text{ m/s}^2$ or $g = 10.0 \text{ m/s}^2$ — whichever you prefer. The in-class exam will be shorter than this practice exam and will consist mainly of problems very similar to problems you have already solved in the weekly homework; the topics covered will be similar to this practice exam. FYI, you can find four previous years' exams and practice exams at <http://positron.hep.upenn.edu/p8/files/oldexams>

If I do not receive your take-home exam by **5pm on Monday, December 9** (in my office, DRL 1W15, or in class), then your score will be **zero**, without exception, as I need to return graded take-home exams promptly. If you turn in your take-home exam by **5pm on Friday, December 6**, then I will grade and email your exam back to you on Monday evening (December 9). Otherwise, I will return your graded exam to you at the review session on Wednesday, December 11.

Please show your work on these sheets. Add blank sheets if needed.

1. (10%) You and your little sister are out in the snow with a sled that has a mass of 11 kg. Your sister, who weighs 29 kg, is sitting on the sled and you want to push her along. You start applying a horizontal force and initially the sled doesn't move but you slowly increase your force until, suddenly, the sled does move. You maintain the same force that you were applying when the sled started moving for the next 5.0 seconds after which you let go. Use coefficient of kinetic friction $\mu_k = 0.020$ and coefficient of static friction $\mu_s = 0.080$.

(a) How far do you have to run if you apply the force for 5.0 s?

(b) What is your sister's speed at $t = 5.0 \text{ s}$?

(Problem continues on next page.)

(c) After letting go, how far do your sister and her sled move (with respect to the point where you let go) until she is stationary again?

(d) Draw and label a qualitative graph of the acceleration of the sled as a function of time. Qualitative means that it explains the overall behavior without using exact numbers. Annotate all relevant features of your graph.

(e) Draw and label a qualitative graph of the velocity of the sled as a function of time. Annotate all relevant features of your graph.

2. (10%) A 1000 kg car traveling due east at speed 20.0 m/s collides head-on with a 1500 kg light truck traveling due west at speed 10.0 m/s.

(a) If 50% of the car+truck system's kinetic energy is converted to internal energy during the collision, what are the final velocities of the car and truck? (If you find two solutions, then you should eliminate the solution that does not make sense and report as your result the solution that does make sense.)

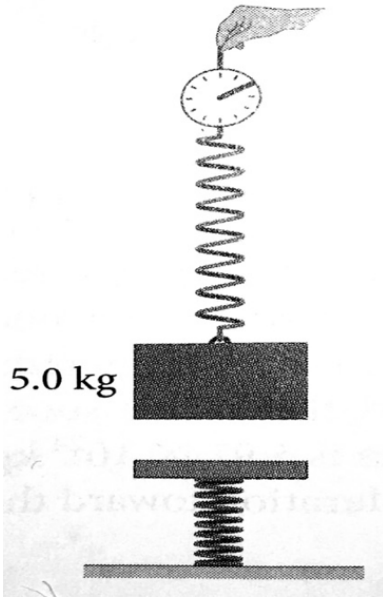
(b) What is the coefficient of restitution for the collision you analyzed in part (a)?

(Problem continues on next page.)

(c) If the collision had instead been perfectly elastic, what would have been the final velocities of the car and truck?

(d) If the collision had instead been totally inelastic, what would have been the final velocities of the car and truck?

3. (10%) A 5.0 kg block suspended from a spring scale is very slowly lowered onto a vertical spring, as shown.



(a) What does the scale read before the block touches the vertical spring?

(b) If the scale reads 40 N when the bottom spring is compressed 3.0 cm (0.030 m), what is k for the bottom spring?

(Problem continues on next page.)

(c) How far does the block compress the bottom spring when the scale reads 0 N?

(d) Draw three Mazur-style free-body diagrams for the block: one for part (a), one for part (b), and one for part (c).

4. (10%) A 10 kg dog jumps up in the air to catch a ball. The dog's center of mass is normally 0.20 meter above the ground, and he is 0.50 meter long (which means 0.50 meter tall when he is up on his hind legs). The lowest he can get his center of mass is 0.10 meter above the ground, and the highest he can get his center of mass before he can no longer push against the ground is 0.30 meter. [You may not need to use all of the numbers given in the problem statement. If you happen to draw a good illustration to guide your thinking about this problem, that is worth a bonus point.]

(a) If the maximum force the dog can exert on the ground in pushing off is 2.5 times his own weight, how high can he jump? [To answer unambiguously, state the greatest height above the ground that the dog's center of mass can reach.]

(b) State clearly in words (sentences) your reasoning for part (a).

(c) Draw a Mazur-style free-body diagram of the dog while the dog is pushing against the ground.

(d) Draw a Mazur-style free-body diagram of the dog while the dog is airborne.

5. (10%) A janitor is pushing an 18.0 kg trashcan across a level floor at constant velocity. The coefficient of friction between the can and the floor is 0.150.

(a) If the janitor is pushing horizontally, what is the magnitude of the force she exerts on the can?

(b) If she pushes at an angle of 36.9° down from the horizontal, what must the magnitude of her pushing force be to keep the can moving at constant velocity?

(Problem continues on next page.)

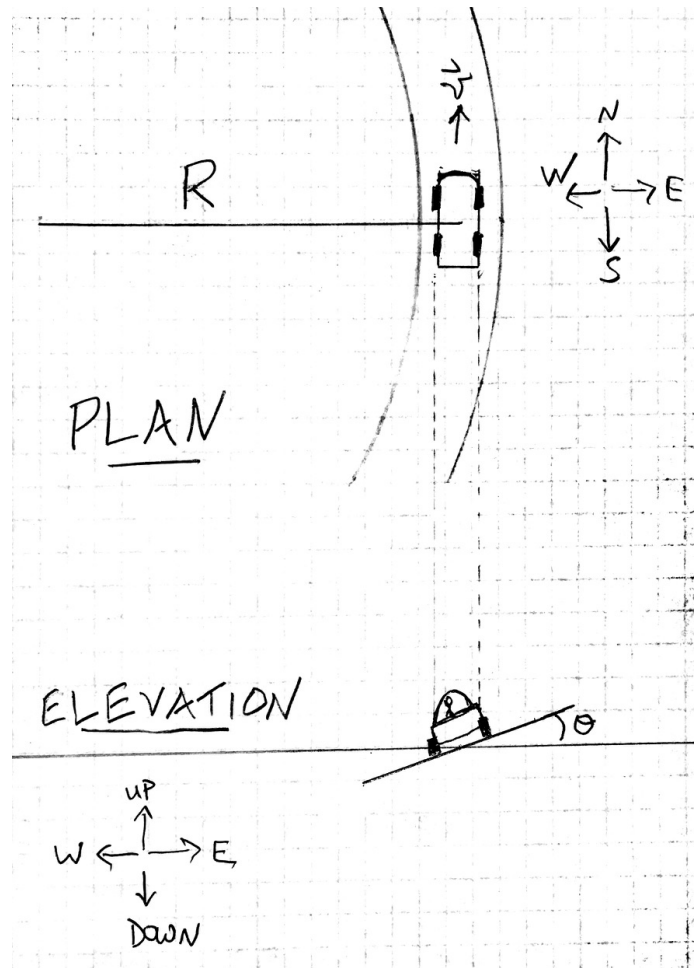
(c) For part (b), what is the work done by the janitor as she moves the trashcan 25.0 meters across the floor?

(d) Draw a Mazur-style free-body diagram for the trashcan for part (a).

(e) Draw a Mazur-style free-body diagram for the trashcan for part (b).

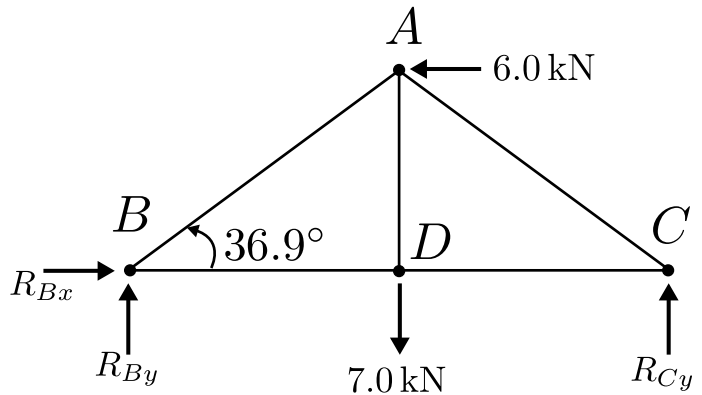
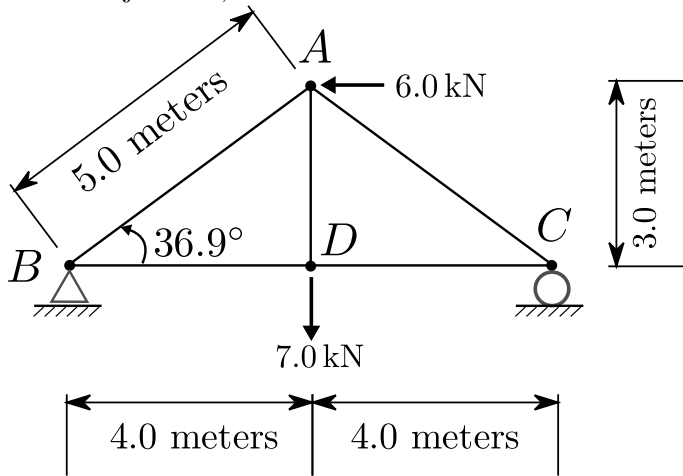
6. (10%) A race car is negotiating a curve, of radius R , on a track which is banked at an angle θ with respect to horizontal. There is a certain speed v_{critical} at which friction is not needed to keep the car on the track in the curve. (This is the speed the car should travel if there is an oil slick on the track, for example.) Draw a Mazur-style free-body diagram showing the forces exerted on the car when the car is moving at speed

- (a) $v = v_{\text{critical}}$,
- (b) $v > v_{\text{critical}}$, and
- (c) $v < v_{\text{critical}}$.



(d) Using your drawing for (a) to guide your reasoning, write v_{critical} in terms of the radius of curvature R , the bank angle θ , and the gravitational acceleration constant g .

7. (20%) The truss shown below is simply supported at joints B and C, carries a 6.0 kN horizontal load at joint A, and carries a 7.0 kN vertical load at joint D.

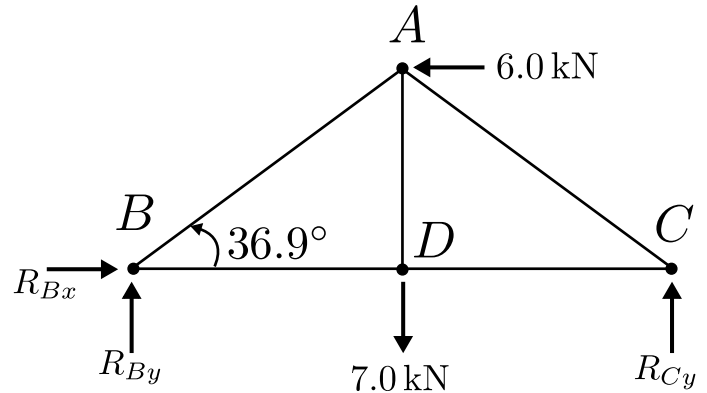


(a) I've drawn, above-right, an Extended Free Body Diagram for the truss as a whole. Using joint B as a pivot, write the moment equation about joint B to solve for the vertical "reaction" force R_{Cy} exerted on the truss by the roller support at joint C. As a check against careless mistakes, you should expect your answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

(b) Now use $\sum F_x$ and $\sum F_y$ for the truss as a whole to find the two reaction forces, R_{Bx} and R_{By} exerted on the truss by the hinge support at joint B. As a check against careless mistakes, you should expect each answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

(Problem continues on next page.)

(c) Using the method of joints, write $\sum F_y$ for joint D to solve for the bar tension T_{AD} . Indicate whether bar AD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

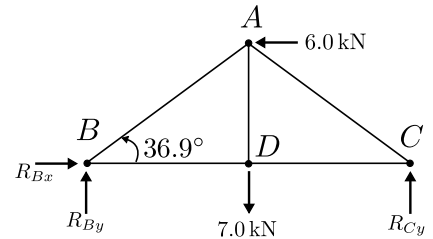


(d) Using the method of joints, write $\sum F_y$ for joint C to solve for the bar tension T_{AC} . Indicate whether bar AC is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

(e) Using the method of joints, write $\sum F_y$ for joint B to solve for the bar tension T_{AB} . Indicate whether bar AB is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

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(f) Using the method of joints, write $\sum F_y$ for joint A to check that your results for T_{AB} , T_{AD} , and T_{AC} are consistent.



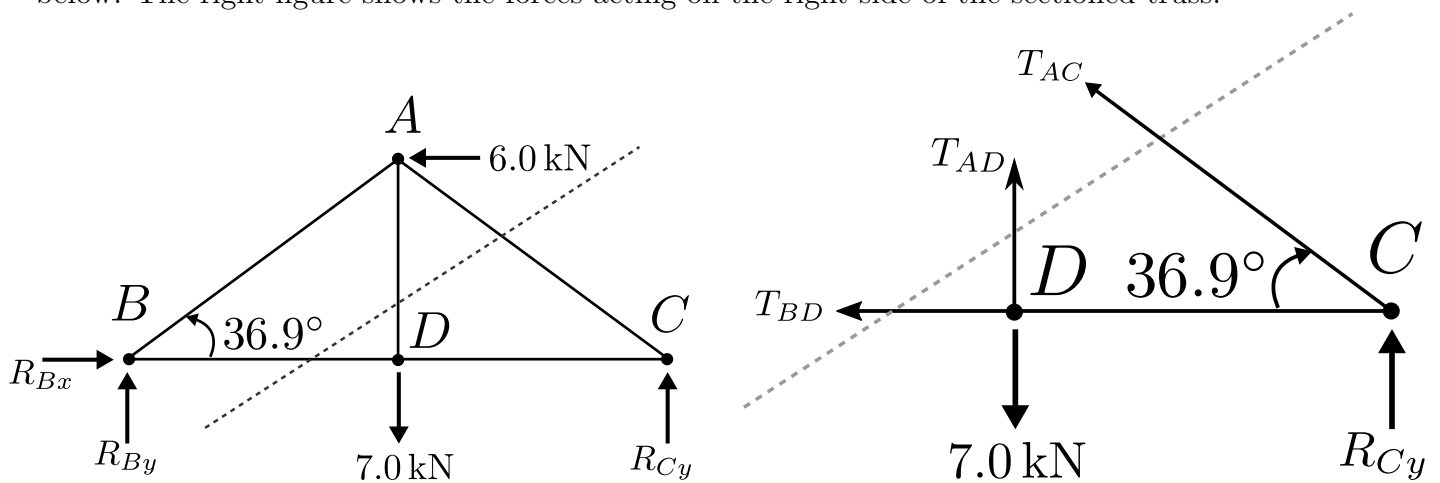
(g) Using the method of joints, write $\sum F_x$ for joint C to solve for the bar tension T_{CD} . Indicate whether bar CD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

(h) Using the method of joints, write $\sum F_x$ for joint D to solve for the bar tension T_{BD} . Indicate whether bar BD is in tension or in compression. As a check against careless mistakes, you should expect your answer to be one of the following: ± 1.250 kN, ± 1.667 kN, ± 2.083 kN, ± 5.750 kN, ± 6.000 kN, ± 7.000 kN, ± 9.583 kN.

(i) Using the method of joints, write $\sum F_x$ for joint B to check that your results for T_{AB} , T_{BD} , and R_{Bx} are consistent.

(Problem continues on next page.)

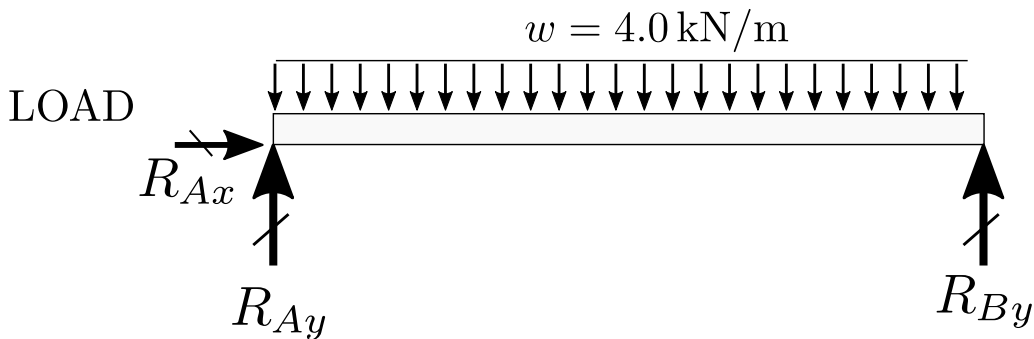
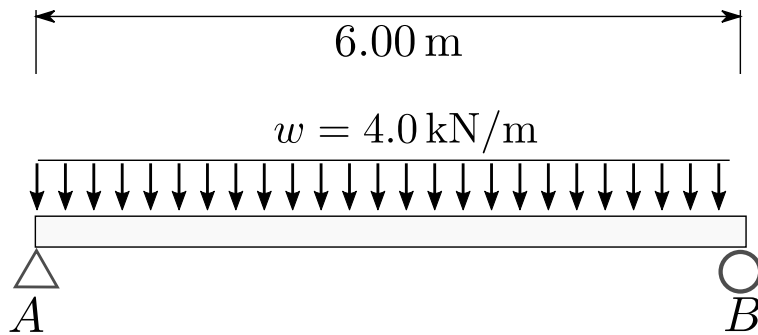
Now let's use the method of sections to analyze the right-hand side of the section shown in the figure below. The right figure shows the forces acting on the right side of the sectioned truss.



(j) Using moments about joint D, write an equation that lets you check the consistency of your results for T_{AC} and R_{Cy} . (Alas, this equation may feel a bit redundant to you.)

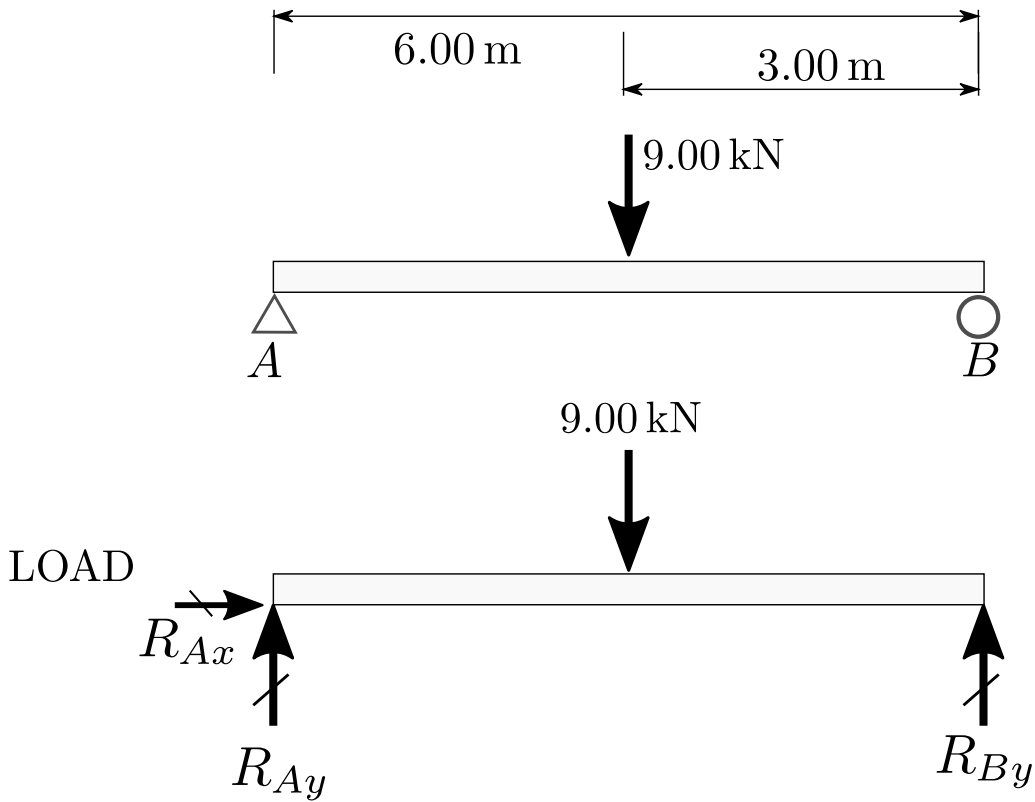
(k) Using moments about joint A (which is invisible here, but you know where it is), write an equation that lets you check the consistency of your results for T_{BD} and R_{Cy} .

8. (20%) (a) A simply-supported beam of length $L = 6.00$ m carries a uniform distributed load $w = 4.00$ kN/m along its entire length. Solve for the three “reaction” forces exerted by the supports: the upward force R_{Ay} exerted by the support at A , the horizontal force R_{Ax} exerted by the support at A , and the upward force R_{By} exerted by the support at B .



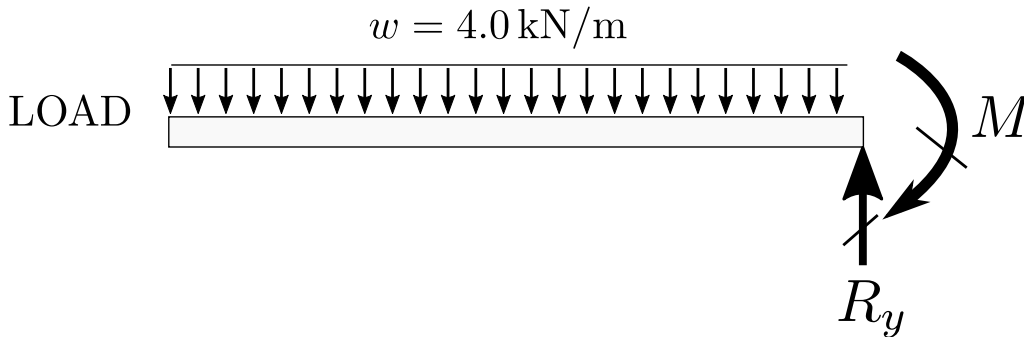
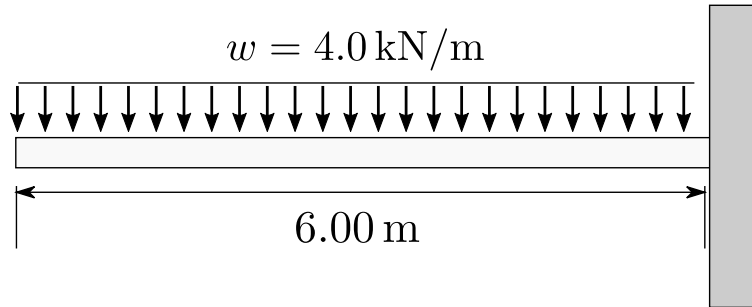
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(b) A simply-supported beam of length $L = 6.00$ m carries a single concentrated load 9.00 kN at mid-span. Solve for the three “reaction” forces exerted by the supports: the upward force R_{Ay} exerted by the support at A , the horizontal force R_{Ax} exerted by the support at A , and the upward force R_{By} exerted by the support at B .



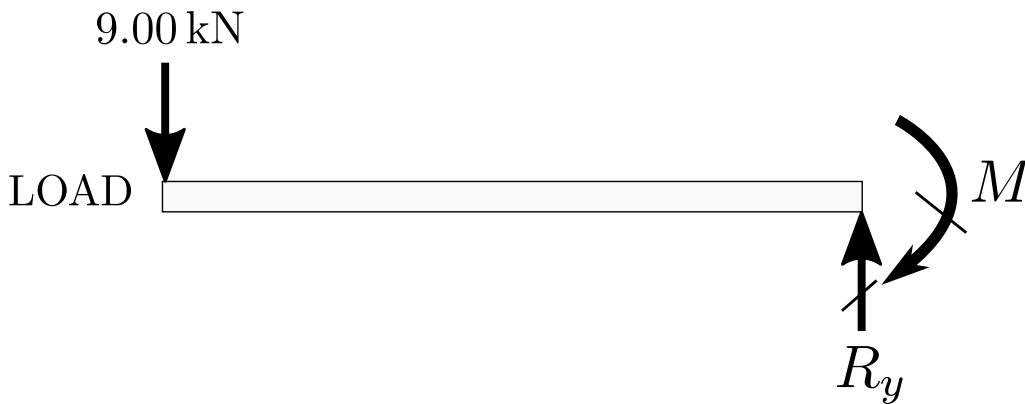
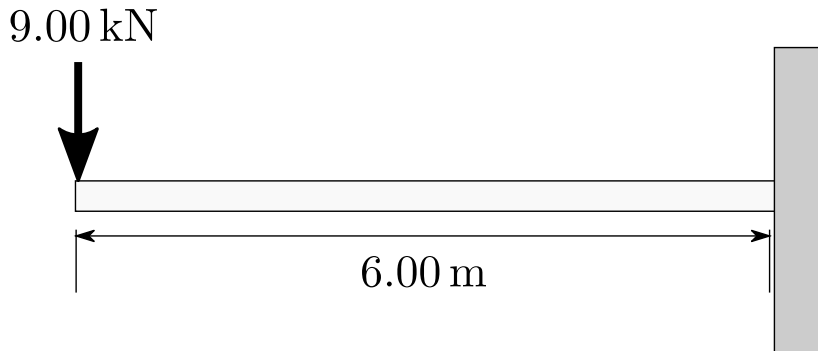
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(c) A cantilever beam of length $L = 6.00$ m carries a uniform distributed load $w = 4.00$ kN/m along its entire length. Solve for the upward “reaction” force R_y exerted by the wall on the right end of the beam. Also solve for the moment (torque) M exerted by the wall on the beam: to do this, use the right end of the beam (at the wall) as a pivot, and let M be whatever torque is needed to make all torques (moments) add up to zero for equilibrium.



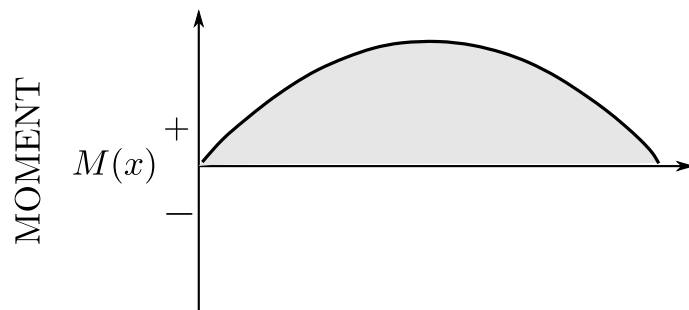
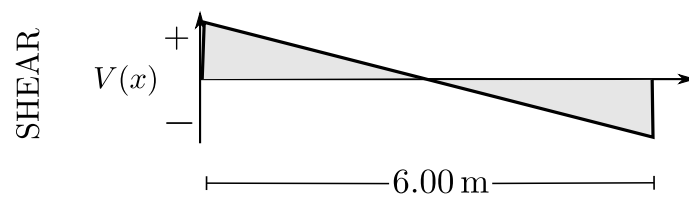
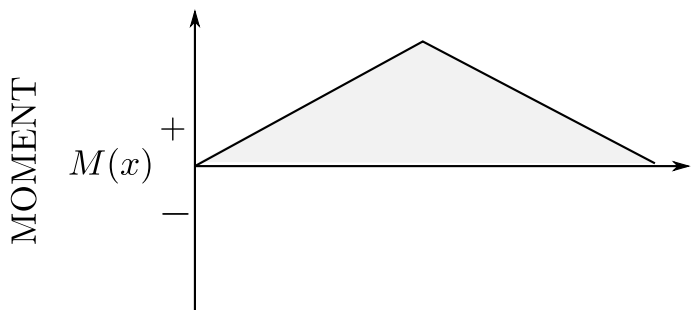
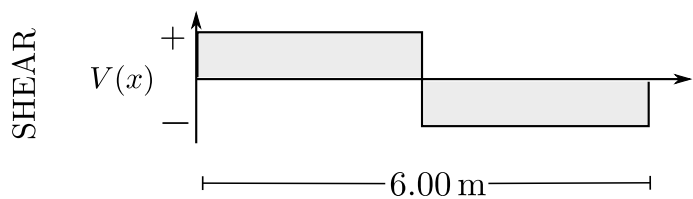
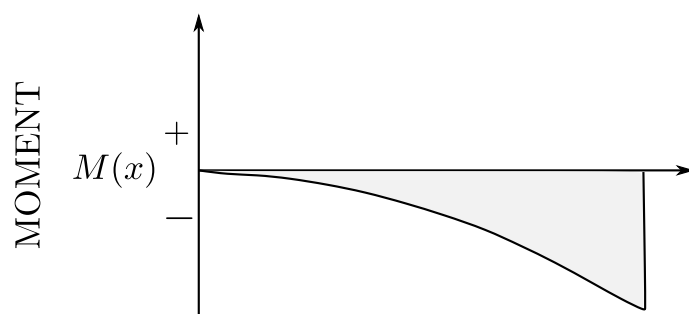
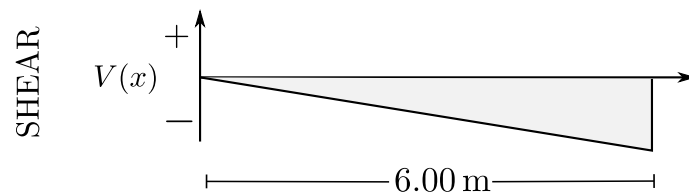
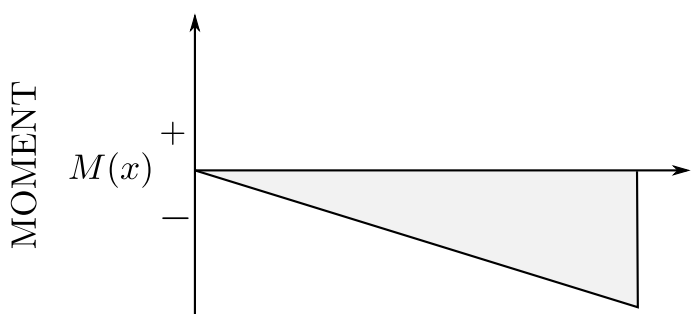
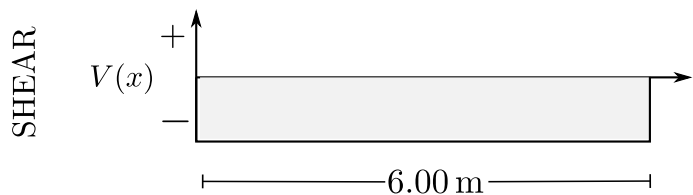
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(d) A cantilever beam of length $L = 6.00$ m carries a single concentrated load of $w = 9.00$ kN at its left end. Solve for the upward “reaction” force R_y exerted by the wall on the right end of the beam. Also solve for the moment (torque) M exerted by the wall on the beam: to do this, use the right end of the beam (at the wall) as a pivot, and let M be whatever torque is needed to make all torques (moments) add up to zero for equilibrium.



(Problem continues on next page.)

(e) For each of the following four shear/moment diagram pairs, write in the letter **A**, **B**, **C**, or **D** to indicate whether the graphs correspond to the beam shown in part (a), part (b), part (c), or part (d) of this problem (above).



Possibly useful equations

$$\cos(30^\circ) = \frac{\sqrt{3}}{2} \approx 0.866 \quad \sin(30^\circ) = \frac{1}{2} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}} \approx 0.577$$

$$\cos(60^\circ) = \frac{1}{2} \quad \sin(60^\circ) = \frac{\sqrt{3}}{2} \approx 0.866 \quad \tan(60^\circ) = \sqrt{3} \approx 1.732$$

$$\cos(36.9^\circ) = \frac{4}{5} \quad \sin(36.9^\circ) = \frac{3}{5} \quad \tan(36.9^\circ) = \frac{3}{4}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$f_{\text{spring}} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad f_{\text{pendulum}} = \frac{1}{2\pi} \sqrt{\frac{g}{\ell}}$$

$$\sum \vec{F} = m\vec{a} \quad \sum \vec{F} = \frac{d\vec{p}}{dt} \quad \vec{p} = m\vec{v}$$

$$F_c = \frac{mv^2}{r} \quad F_c = m\omega^2 r \quad v = \omega r$$

$$F^K = \mu^K F^N \quad F^s \leq \mu_s F^N$$

$$F_x^{\text{spring}} = -k(x - x_0)$$

$$F_y^{\text{grav}} = -mg \quad g = 9.8 \text{ m/s}^2$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \tau = rF \sin \theta = r_\perp F = rF_\perp$$

$$\frac{F}{A} = (\text{stress}) = (E)(\text{strain}) = E \frac{\Delta L}{L_0}$$

$$V = \frac{dM}{dx} \quad (M_2 - M_1) = (x_2 - x_1) \bar{V}_{1 \rightarrow 2} \quad w = -\frac{dV}{dx} \quad V(x) = \sum_{0 \rightarrow x} F_y (\text{up minus down})$$