Physics 8 — Wednesday, September 4, 2019

- Course www: `http://positron.hep.upenn.edu/physics8`
- After first spending a moment to finish up unit vectors (left over from Friday, Ch 2), we'll spend today's class on Ch 3 ("acceleration"), which you read during the weekend. I got reading responses from 40/51 of you. Good work! Let's get 51/51 next time!
- We will spend Friday's class on Ch 4 ("momentum"), which you read for today. If you haven't turned in the reading yet, it's always better late than never.
- Remember homework #1 due this Friday, at the start of class. It covers Chapters 1 and 2.
- Homework study/help sessions (optional):
  Greg will be in DRL 3C4 Wednesdays from 4–6pm (today!).
  Bill will be in DRL 2C4 on Thursdays from 6–8pm.
- Why are we talking about velocity and acceleration, when architectural structures generally do not move? Answer: to understand force and torque, we need first to discuss motion.

Unit vectors (yuck)

- We can define **unit vectors** in the x, y, and z directions: $\hat{i} = (1, 0, 0)$, $\hat{j} = (0, 1, 0)$, and $\hat{k} = (0, 0, 1)$.
- Then we can write $\vec{r} = (x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$.
- It's often convenient to define a coordinate system where the x-axis points east, the y-axis points north, and the z-axis points up, with the origin at some specified location (e.g. the center of the ground floor).
- Then if I'm standing 5 meters east of the origin, my position vector is $+5\,\mathrm{m}\ \hat{i}$, which we could also write as $(+5\,\mathrm{m}, 0, 0)$.
- If I'm 3 m west of the origin, then $\vec{r} = -3\,\mathrm{m}\ \hat{i} = (-3\,\mathrm{m}, 0, 0)$.
- If I'm 2 m north of the origin, then my position is $\vec{r} = +2\,\mathrm{m}\ \hat{j} = (0, +2\,\mathrm{m}, 0)$.
- Most students dislike Mazur's unit-vector notation, so I try to avoid using it. I will instead write, "The displacement is +5 meters eastward." I will usually use a word like "east" or "north" or "up" to avoid writing $\hat{i}$ or other unit vectors.

Vectors

- Vectors are very useful on a 2D map ((x, y) or geocode) or in a 3D CAD model (x, y, z).
- For the first 10 chapters of our textbook, all problems will be one-dimensional (we will use the x-axis only), which makes the use of vectors seem contrived at this stage.
- The reason for doing this is so that we can focus on the physics first before reviewing too much math.
- In one dimension, position is $\vec{r} = (x, 0, 0) = x\ \hat{i}$.
- The x component of vector $\vec{v}$ is v_x, and in one dimension $\vec{v} = (v_x, 0, 0) = v_x\ \hat{i}$.
- The x component of vector $\vec{r}$ is x. (Special case notation.)
- In 1D, magnitude of $\vec{r}$ is $|x|$, and magnitude of $\vec{v}$ is $|v_x|$.
- Vectors will seem more natural starting in Chapter 10, when we study motion in a two-dimensional plane.

- **position:** where is it located in space? $\vec{r} = (x, y, z)$
- **displacement:** where is it w.r.t. some earlier position?
- $\Delta\vec{r} \;=\; (\Delta x, \Delta y, \Delta z) \;=\; \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$
- position and displacement are both **vectors**: they have both a direction in space and a magnitude
- **distance** is a scalar (magnitude only, never negative)
- **unit vectors** $\hat{\imath} = (1, 0, 0)$, $\hat{\jmath} = (0, 1, 0)$, $\hat{k} = (0, 0, 1)$ are vectors pointing along x,y,z axes, with "unit" magnitude (length $= 1$). Until Chapter 10, we use only the x-axis. So $\hat{\imath}$ is the only unit vector introduced in Chapter 2.
- **average velocity** $\vec{v}_{\rm av} = \frac{\Delta\vec{r}}{\Delta t}$: (displacement) / (time interval)
  x-component of $\vec{v}_{\rm av}$ is $v_{x,\rm av} = \frac{\Delta x}{\Delta t}$
- **(instantaneous) velocity** $\vec{v} = \frac{{\rm d}\vec{r}}{{\rm d}t} = (\frac{{\rm d}x}{{\rm d}t}, \frac{{\rm d}y}{{\rm d}t}, \frac{{\rm d}z}{{\rm d}t})$
  x-component of $\vec{v}$ is $v_x = \frac{{\rm d}x}{{\rm d}t}$
- velocity is a vector (it has a direction in space),
  speed is a scalar (it has only a magnitude)
- For many people, the hardest part of this reading was getting used to the author's notation.

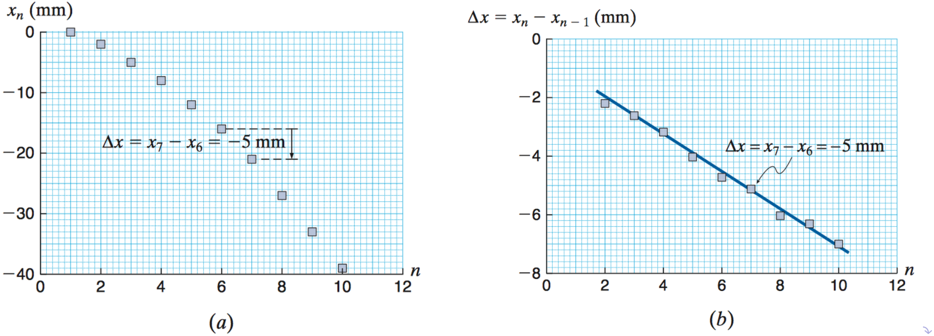
Potential sources of confusion from Chapter 2

- It takes a while to get used to the textbook's vector notation. Some people positively hate the book's notation!
  - But the book's notation is extremely self-consistent, even if the many subscripts and superscripts can be annoying.
  - And this book is excellent on the concepts.
- Also, it might take some practice to re-acclimate your brain to reading lots of equations, if it has been several years since your last math course. No worries.
- What is a unit vector? Yuck!
- Using only a single spatial dimension (until Chapter 10) makes the discussion of vectors seem contrived.
- Distinction between displacement & position vectors.
- Difference between average and instantaneous velocity.
- Anything to add to this list?

Now — onward to chapter 3 . . .

Potential sources of confusion from **today's** reading (Chapter 3)

- Inclined planes are new to many people.
- How do you draw a motion diagram?
- Don't follow Eric's reasoning about what is happening (v_x, a_x) at the very top of the motion for a ball tossed upward.
- Some of the mathy parts at the end are hard to follow.
- For checkpoint 3.6, you have to stare at Figure 3.6b for a while before you see that, since the points are all equal steps in time, the quantity being graphed is proportional to v_x, the x component of velocity.



Defining acceleration

- Last week, we defined velocity as the rate of change of position with respect to time

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$$

(considering only the x component for now), and we learned to identify v_x visually as the slope on a graph of $x(t)$

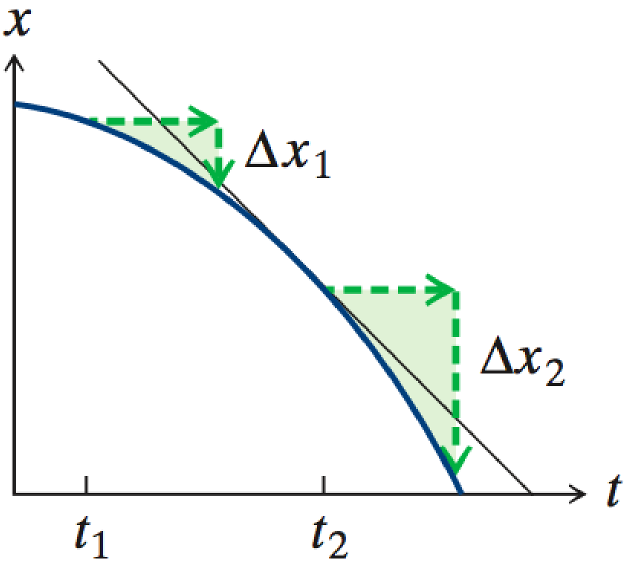
- Moving at constant velocity is not very interesting! So we need to be able to talk about changes in velocity.
- The rate of change of velocity with respect to time is called acceleration:

$$a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t}$$

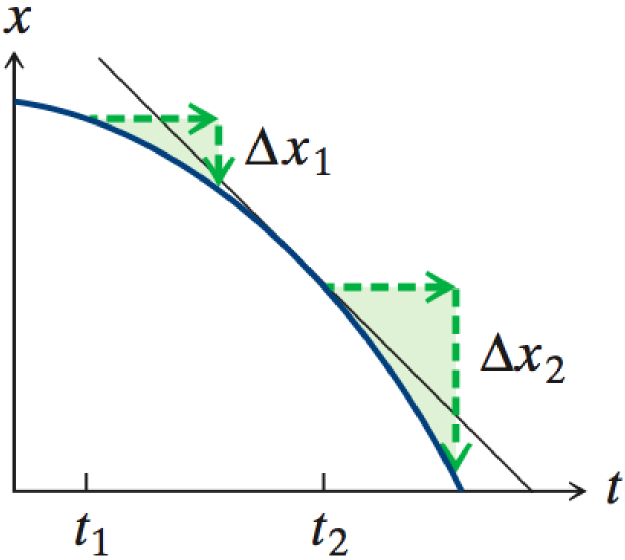
- While acceleration can also vary with time (!), there are many situations in which constant acceleration ($a_x = \mathrm{constant}$) gives a good description of the motion. We'll see soon what math lets us conclude, if we start with $a_x = \mathrm{constant}$.

At time t_2 in the position-vs-time graph below, the object is

(A) not moving
(B) moving at constant speed
(C) speeding up
(D) slowing down



At time t_2 in the position-vs-time graph below, is v_x (the x component of velocity) is



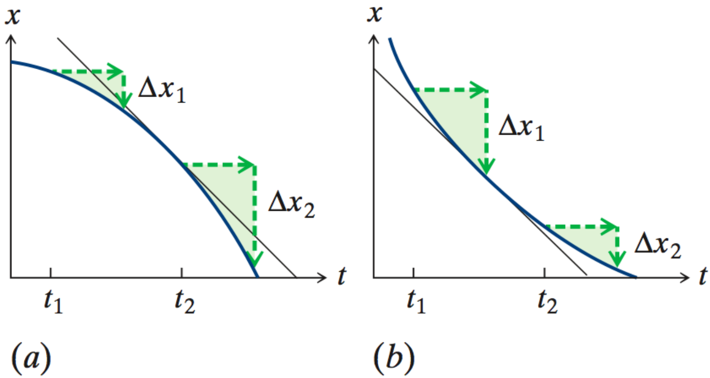
(A) zero

(B) not changing

(C) increasing

(D) decreasing

The x component of acceleration in these two graphs is



(A) positive in (a), negative in (b)

(B) negative in (a), positive in (b)

(C) negative in both (a) and (b)

(D) positive in both (a) and (b)

(E) zero in both (a) and (b)

Accelerating under gravity's influence

- One important situation in which constant acceleration ($a_x = \mathrm{constant}$) gives a good description of the motion is "free fall" near Earth's surface.
- (Until Chapter 10, we will use only one axis in any given problem, and we will call that axis x. So for free-fall problems, for now, the x axis will be vertical, pointing upward.)
- *Free fall* is the motion of an object subject only to the influence of gravity.
  - Not being pushed or held by your hand or by the ground
  - When air resistance is small enough to neglect
- Close to Earth's surface, an object in free fall experiences a constant acceleration, of magnitude $|\vec{a}| = 9.8\ \mathrm{m/s^2}$ and pointing in the *downward* direction.
- If we define the x axis to point upward (as we often will, for free-fall problems before Ch10), then $a_x = -9.8\ \mathrm{m/s^2}$.
- Since we see the quantity $9.8\ \mathrm{m/s^2}$ so often, we give it a name: $g = 9.8\ \mathrm{m/s^2}$. Then $a_x = -g$.

(Checkpoint 3.7)

Let's pause here to go through Checkpoint 3.7 together.

- Does the speed of a falling object (A) increase or (B) decrease?
- If the positive x axis points up, does v_x (A) increase or (B) decrease as the object falls?
- is the x component of the acceleration (A) positive or (B) negative?

Discuss with your neighbor for a moment, and then we'll compare answers.

You and your neighbor might even want to graph $v_x(t)$, for a falling object, while you discuss.

Let's do what Galileo could only imagine doing!

- Let's see if different objects really do fall with the same acceleration

$$a_x = -g$$

if we are able to remove the effects of air resistance.

Equations we can derive from $a_x = \text{constant}$

- **You don't need to know how to do these derivations,** but if you like calculus, you might enjoy seeing where these often-used results come from.
- We defined $a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t}$ and $v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$, without worrying so far about whether or not a_x is changing with time.
- Integrating the first equation $\left(\frac{\mathrm{d}v_x}{\mathrm{d}t} = a_x\right)$ over time,

$$v_x(t) = v_{x,i} + \int_0^t a_x \,\mathrm{d}t$$

- If $a_x = \text{constant}$, then this integral becomes easy to do:

$$v_x(t) = v_{x,i} + a_x t$$

- We can also integrate the equation $\left(\frac{\mathrm{d}x}{\mathrm{d}t} = v_x\right)$ over time:

$$x(t) = x_i + \int_0^t v_x \,\mathrm{d}t$$

keeping in mind that v_x (unlike a_x) is changing with time

Equations we can derive from $a_x = \text{constant}$

$$v_x(t) = v_{x,i} + a_x t$$

$$x(t) = x_i + \int_0^t v_x \, \mathrm{d}t$$

- Plugging our $v_x(t)$ result into the second integal:

$$x(t) = x_i + \int_0^t (v_{xi} + a_x t) \, \mathrm{d}t$$

$$x(t) = x_i + v_{x,i}\, t + \frac{1}{2} a_x\, t^2$$

Equations we can derive from $a_x = \text{constant}$

- That's all there is to it. Just writing down the assumption that a_x is constant allows us to integrate twice to get two results that you will use many times:

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

- If you plug one of these equations into the other, you can eliminate t to get one more very useful result

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x \left(x_f - x_i\right)$$

- This last one is helpful e.g. to know how fast the dropped steel ball is traveling at the instant before it hits the ground.
- My point is that these equations are just the result of taking $a_x = \text{constant}$ and doing some math.

Inclined planes

- Falling to the ground at $a_x = -g$ happens so quickly that it can be difficult to see exactly what is happening.
- Maybe there is a way to "fall" in slow motion?
- Yes! We can slide down a hill.

$$|g| \rightarrow |g \sin \theta|$$

(We'll see in Chapter 10 why it's $\sin \theta$ here. Don't worry.)

- To get the $\pm$ sign right, you have to choose which direction to draw the x axis. Eric chooses the x axis to point *downhill*

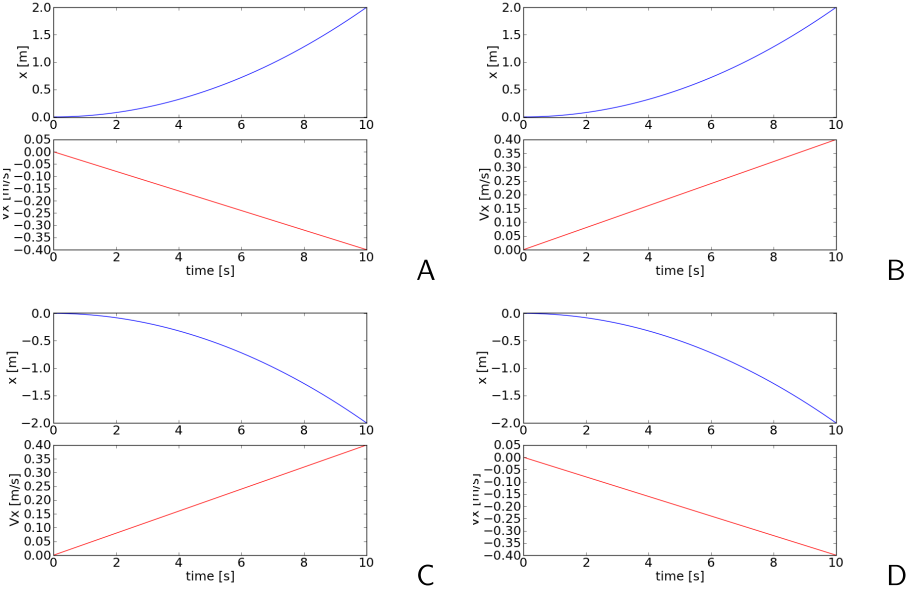
$$a_x = +g \sin \theta$$

- Let's look at **this contraption** and figure out which way it defines the x axis to point

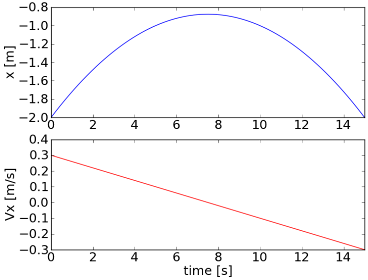
Inclined air track

- It looks as if the x axis points *downhill*, and the point on the top of the ramp is called $x = 0$.

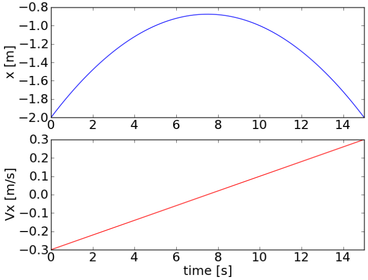
Which of the following shows the expected shapes of $x(t)$ [blue] and $v_x(t)$ [red] if I release the cart (at rest) from $x = 0$?



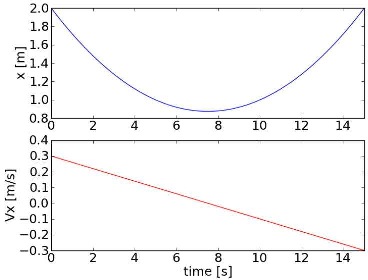
Which of the following shows the expected shapes of $x(t)$ [blue] and $v_x(t)$ [red] if I shove the cart upward starting from $x = +2$ m?



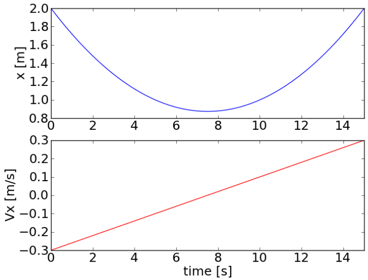
A



B



C



D

I shove the cart uphill and watch it travel up and back. We define the x axis to point *downhill*. At the top of its trajectory (where it turns around), v_x is

(A) positive

(B) negative

(C) zero

(D) infinite

(E) undefined

I shove the cart uphill and watch it travel up and back. We define the x axis to point *downhill*. At the top of its trajectory (where it turns around), a_x is

(A) positive

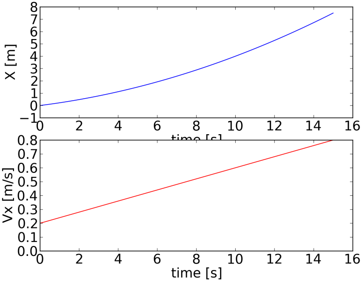
(B) negative

(C) zero

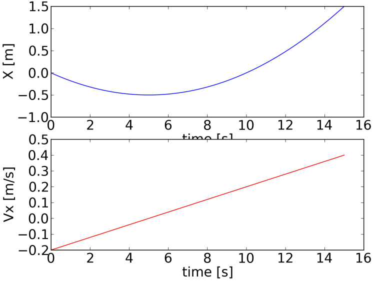
(D) infinite

(E) undefined

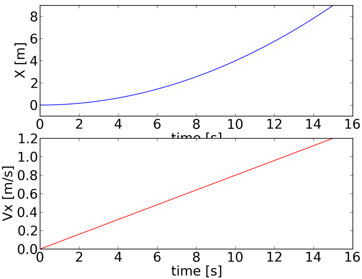
Which of the following shows the expected shapes of $x(t)$ [blue] and $v_x(t)$ [red] if I shove the cart gently downward from $x = 0\ \mathrm{m}$?



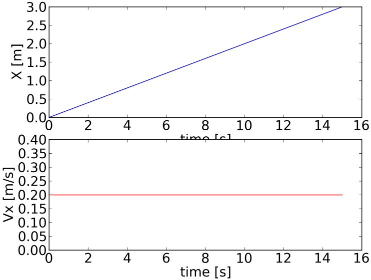
A



B



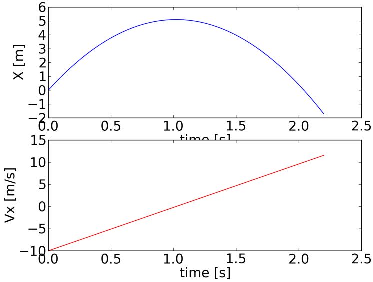
C



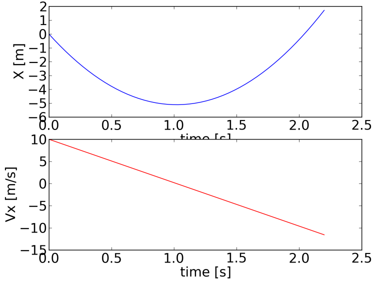
D

We stopped here. But someone asked an excellent question after class about the difference, in the previous slide, between scenario (A) and scenario (C). Let's discuss that first.

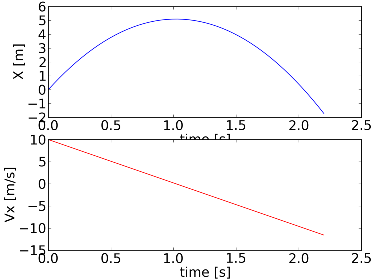
What are the expected shapes of $x(t)$ [blue] and $v_x(t)$ [red] for a **basketball tossed upward**, when the x axis points *upward*?



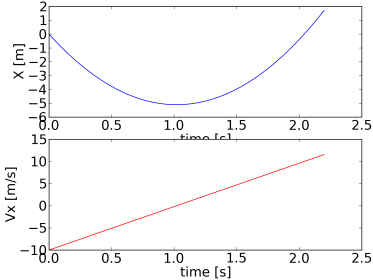
A



B



C



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