

## Physics 8 — Monday, September 16, 2019

- ▶ You read Ch7 (interactions) for today and you'll read Ch8 (force) for Wednesday. [Then we can finally start using Newton's three laws, as we will for the rest of the semester!]
- ▶ Work on this problem with your neighbor(s) while we get started today:

A system consists of two 1.00 kg carts attached to each other by a compressed spring. Initially, the system is at rest on a low-friction track. When the spring is released, internal energy that was initially stored in the spring is converted into kinetic energy of the carts. The change in the spring's internal energy during the separation is 4.00 joules. What are the two carts' final velocities?

A system consists of two 1.00 kg carts attached to each other by a compressed spring. Initially, the system is at rest on a low-friction track. When the spring is released, internal energy that was initially stored in the spring is converted into kinetic energy of the carts. The change in the spring's internal energy during the separation is 4.00 joules. What are the two carts' final velocities?

```
ClearAll["Global`*"];
```

```
m1 = 1.00; m2 = 1.00; v1xi = 0.0; v2xi = 0.0;
```

```
Eispring = 4.00; Efspring = 0.00;
```

```
Reduce[ {
```

$$m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,$$

$$\frac{1}{2} m1 v1xi^2 + \frac{1}{2} m2 v2xi^2 + Eispring == \frac{1}{2} m1 v1xf^2 + \frac{1}{2} m2 v2xf^2 + Efspring,$$

$$v2xf > 0$$

```
}]
```

```
v2xf == 2. && v1xf == -2.
```

## HW3 problem 10

A system consists of a 2.00 kg cart and a 1.00 kg cart attached to each other by a compressed spring. Initially, the system is at rest on a low-friction track. When the spring is released, an explosive separation occurs at the expense of the internal energy of the compressed spring. If the decrease in the spring's internal energy during the separation is 10.0 J, what is the speed of each cart right after the separation?

Since the two-cart system is isolated, what equation can we write down?

Since the spring's internal energy is converted into the carts' kinetic energies, we can account for the initial and final energies of the cart + spring + cart system and can see that this system is closed. (No energy goes in or out of the system.) What second equation can we write down?

(This one can be done without writing down much at all.)

Two carts, of inertias  $m_1 = 1.0$  kg and  $m_2 = 1.0$  kg, collide head-on on a low-friction track. Before the collision, cart 1 is moving to the right at 2.0 m/s and cart 2 is moving to the left at 2.0 m/s. After the collision, cart 1 is moving to the left at 1.0 m/s and cart 2 is moving to the right at 1.0 m/s.

Let “the system” be cart 1 + cart 2. With the given values, is the system’s total momentum the same before and after the collision?

What is the coefficient of restitution,  $e$ , for this collision?

(This one can be done without writing down much at all.)

Two carts, of inertias  $m_1 = 1.0$  kg and  $m_2 = 1.0$  kg, collide head-on on a low-friction track. Before the collision, cart 1 is moving to the right at 2.0 m/s and cart 2 is moving to the left at 2.0 m/s. After the collision, cart 1 is moving to the left at 1.0 m/s and cart 2 is moving to the right at 1.0 m/s.

Let “the system” be cart 1 + cart 2. With the given values, is the system’s total momentum the same before and after the collision?

What is the coefficient of restitution,  $e$ , for this collision?

Initial and final momentum are both zero, as you can verify. The relative speed of the two objects is reduced by a factor  $e = 0.5$ .

A battery-powered car, with bald tires, sits on a sheet of ice. Friction between the bald tires and the ice is negligible. The driver steps on the accelerator, but the wheels just spin (frictionlessly) on the ice without moving the car. Is the car an isolated system (considering only the coordinate along the car's axis) — i.e. does nothing outside the system push/pull on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

- (A) Closed but not isolated.
- (B) Isolated but not closed.
- (C) Both closed and isolated.
- (D) Isolated: yes. Closed: very nearly so, yes.
- (E) Neither closed nor isolated.

A battery-powered Aston Martin car, with James-Bond-like spiked tires, sits on a sheet of ice. Agent 007 (or maybe it is really Austin Powers?) steps on the pedal, and the car accelerates forward. Is the car an isolated system (considering only the coordinate along the car's axis), i.e. nothing outside the system pushes/pulls on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

- (A) Closed but not isolated.
- (B) Isolated: no. Closed: very nearly so, yes.
- (C) Isolated but not closed.
- (D) Both closed and isolated.
- (E) Neither closed nor isolated.

A battery-powered Aston Martin car, with James-Bond-like spiked tires, sits on a sheet of ice. Agent 007 steps on the accelerator, and the car accelerates forward. **All the while, a high-tech solar panel on the car's roof rapidly charges the car's battery.** Is the car an isolated system (considering only the coordinate along the car's axis), i.e. nothing outside the system pushes/pulls on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

- (A) Closed but not isolated.
- (B) Isolated but not closed.
- (C) Both closed and isolated.
- (D) Neither closed nor isolated.



A battery-powered Aston Martin, with James-Bond-like spiked tires, sits atop an iceberg that floats in the North Sea. Agent 007 steps on the accelerator, and the car accelerates forward. (What happens to the iceberg?) All the while, a high-tech solar panel on the car's roof rapidly charges the car's battery. Ignore any friction (or viscosity, drag, etc.) between the water and the iceberg. Which statement is true?

- (A) "Car alone" system is isolated but not closed.
- (B) "Car + iceberg" system is isolated but not closed.
- (C) "Iceberg alone" system is isolated but not closed.
- (D) "Car alone" system is isolated and closed.
- (E) "Car + iceberg" system is isolated and closed.
- (F) "Iceberg alone" system is isolated and closed.
- (G) None of the above.

- ▶ An **isolated** system has no mechanism for momentum to get in/out of the system from/to outside of the system. This means nothing outside of the system can push/pull on anything inside of the system. (Later this week, we'll say: "no external forces act on the system.")
- ▶ This will make more sense when we discuss *forces*, next time!
- ▶ A hugely important idea in physics is that if the parts of a system interact only with each other (do not push/pull on anything outside of the system), then the total momentum of that system does not change.
- ▶ A **closed** system has no mechanism for energy to get in/out of the system. Examples so far are contrived, but soon we will learn to calculate energy stored in springs, energy stored in Earth's gravitational field, etc. The concept of a closed system is much more useful once we learn how to account for the many ways energy can be stored.
- ▶ Accounting for movement of energy in/out of a system will make more sense when we discuss *work*, just after forces.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. What system is isolated?

- (A) One cart.
- (B) Cart + spring + other cart.
- (C) One cart plus the spring.
- (D) None of the above.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. Is the cart + spring + other cart system closed?

- (A) Yes, for all practical purposes, because the system's total energy  $K_1 + K_2 + E_{\text{spring}}$  is the same before and after releasing the spring, and other tiny transfers of energy (escaping sound, etc.) are negligible by comparison.
- (B) No.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. Is the spring alone a closed system?

(A) Yes.

(B) No, because it transferred energy to the carts, which are outside of what you're now calling "the system."

I put two carts on a low-friction track, with a lighted firecracker between them. The firecracker explodes, which sets the carts moving apart. Is the cart + firecracker + other cart system closed?

- (A) Yes, by analogy with the cart + spring + cart system.
- (B) Yes, for some other reason.
- (C) No, because realistically, some of the firecracker's energy will escape in the form of heat, flying debris, etc. So really energy conservation only provides an upper limit on  $K_1 + K_2$  after the explosion, because accounting for where the energy goes is more difficult here than for a simple spring.
- (D) No, for some other reason.
- (E) I still don't understand what "closed" means.

## A variation on HW2 #8

A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (Homework asked, “How tall is the building?”) **Which of the following statements is true?** (Let  $x$ -axis point upward.)

- (A) The rock's average velocity  $v_{x,av}$  during the last 1.0 s of its fall is  $-21.5$  m/s.
- (B) The rock's instantaneous velocity  $v_x$  one second before it hits the ground is  $-21.5$  m/s.
- (C) The rock's instantaneous velocity  $v_x$  at the instant just before it hits the ground is  $-21.5$  m/s.
- (D) Statements (A), (B), (C) are all true.
- (E) Statements (A), (B), (C) are all false.

A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (Homework asked, “How tall is the building?”) **At the instant just before hitting the ground, the rock’s speed is**

- (A) 21.5 m/s
- (B)  $-21.5$  m/s
- (C) Somewhat faster than 21.5 m/s
- (D) Somewhat slower than 21.5 m/s
- (E) We don’t have enough information to decide.



A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (Homework asked, “How tall is the building?”) Let the building height be  $h$ . Let the total time the rock falls be  $t$ . Which is a true statement about the problem?

(A)  $h - \frac{1}{2}gt^2 = 0$

(B)  $0 - gt = -21.5 \text{ m/s}$

(C)  $h - \frac{1}{2}g[t - 1.0 \text{ s}]^2 = 21.5 \text{ m}$

(D)  $0 - g[t - 1.0 \text{ s}] = -21.5 \text{ m/s}$

(E) (A) and (B) are both true.

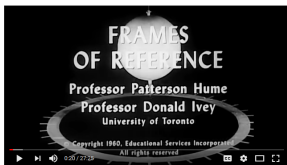
(F) (A) and (C) are both true.

(G) (A), (B), (C), and (D) are all true.

(H) (A), (B), (C), and (D) are all false.

## If “inertial reference frames” baffled you:

- ▶ Imagine yourself trying to pour a cup of coffee while standing up on an airplane that is cruising smoothly at constant velocity. No problem.
- ▶ Now imagine trying to pour coffee while the airplane is taking off, landing, turning sharply, or experiencing turbulence. Your eye and hand are working from the perspective of a non-inertial reference frame — a set of coordinate axes that is accelerating w.r.t. “the fixed stars.” The usual rules of physics don’t work. To use the usual rules of physics, you have to analyze the situation from the perspective of an inertial frame.
- ▶ If you want more detail on frames of reference, watch this 30-minute educational video from 1960. Email me a few sentences detailing what you learned for extra credit.



Physics: Frames of Reference 1960 PSSC Physical Science Study Committee; Reference & Relativity

<https://youtu.be/bJMYoj4hHqU>

Chapter 6 included a few key ideas, some of which were obscured by the notation and equations.

### Law of inertia — this is big deal! (a.k.a. Newton's law #1.)

- ▶ In an inertial reference frame, an isolated object at rest remains at rest, and an isolated object in motion keeps moving at a constant velocity.
- ▶ You can't "feel" the difference between being at rest in Earth's frame vs. being at rest in some other inertial reference frame.
- ▶ Imagine that you're sitting on an airplane, pouring a cup of coffee, juggling, or maybe just tossing a single ball into the air and catching it. If the airplane is cruising at constant velocity, is all of this activity feasible?
- ▶ What if you try the same thing while the airplane is rapidly screeching to a halt on the runway immediately after landing?
- ▶ (Illustrate with "ball popper.")

The **law of inertia** states that in an inertial reference frame, any isolated object that is at rest remains at rest, and any isolated object in motion keeps moving at a constant velocity.

Imagine that you are in a jet airplane that has just landed and is in the midst of screeching to a stop on the runway. (You are wearing your seatbelt!) Is the frame of reference in which **the airplane** is at rest an inertial frame? Will a marble, initially sitting at rest on the floor of the airplane, **as observed from the frame in which the airplane is at rest (i.e. as observed by a passenger, with the window shades down)**, remain at rest as the airplane screeches to a stop?

- (A) Yes (inertial frame) and Yes (marble)
- (B) Yes (inertial frame) and No (marble)
- (C) No (inertial frame) and Yes (marble)
- (D) No (inertial frame) and No (marble)

Suppose I'm a passenger on a train that is speeding toward NYC at  $40 \text{ m/s}$  (heading "north"). In search of coffee, I walk toward the back of the train at  $2 \text{ m/s}$ , just as the train whizzes past Princeton Junction. From the perspective of a passenger watching me from the train platform, my velocity is

- (A)  $2 \text{ m/s}$  northward
- (B)  $2 \text{ m/s}$  southward
- (C)  $42 \text{ m/s}$  northward
- (D)  $40 \text{ m/s}$  northward
- (E)  $38 \text{ m/s}$  northward

Suppose I'm a passenger on a train that is speeding toward NYC at 40 m/s (heading "north"). In search of coffee, I walk toward the back of the train at 2 m/s, just as the train whizzes past Princeton Junction. From the perspective of a passenger watching me from the train platform, my velocity is

- (A) 2 m/s northward
- (B) 2 m/s southward
- (C) 42 m/s northward
- (D) 40 m/s northward
- (E) 38 m/s northward

$$\vec{v}_{\text{Earth,me}} = \vec{v}_{\text{Earth,Train}} + \vec{v}_{\text{Train,me}}$$

“(My velocity w.r.t. Earth) =  
(Train's velocity w.r.t. Earth) + (my velocity w.r.t. Train)”

I'm driving east at 50 kph. A little kid looks out the window of a westbound car that is going 40 kph. From the kid's point of view, what is my velocity?

- (A) 10 kph east
- (B) 40 kph east
- (C) 50 kph east
- (D) 90 kph east
- (E) 10 kph west
- (F) 40 kph west
- (G) 50 kph west
- (H) 90 kph west

I'm driving east at 50 kph. A truck driving east at 60 kph overtakes me. As I look out my window, how fast does the truck appear to be moving?

- (a) 10 kph
- (b) 50 kph
- (c) 60 kph
- (d) 110 kph



We stopped here on Monday. We'll finish up this stuff from chapters 6 & 7 and then we'll start talking about forces some time during Wednesday's class.

## More chapter 6 key ideas

- ▶ Center of mass: basically a weighted-average of positions.

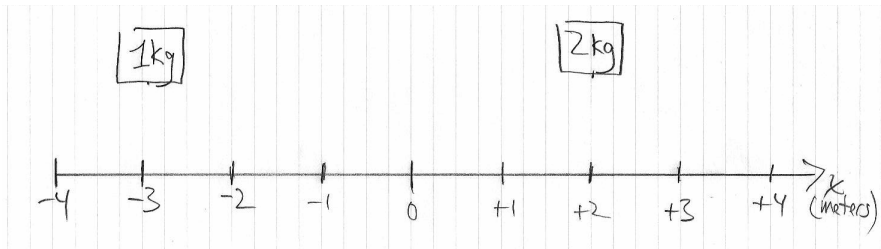
$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$

- ▶ CoM of an object lies along axis of symmetry (if there is one).
- ▶ When analyzing the motion of a complicated object (composed of many pieces), it is often useful to consider separately the motion of its CoM and the motion of the various internal parts w.r.t. the CoM.
- ▶ Illustrate by tossing complicated object in the air.

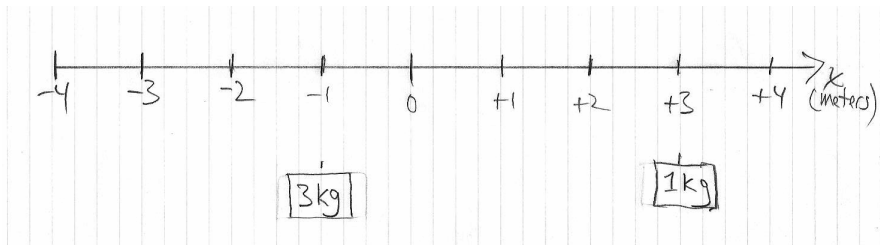
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$

At what value of  $x$  is the CoM of this pair of masses?



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$

At what value of  $x$  is the CoM of this pair of masses?



## More chapter 6 key ideas

- ▶ “Center-of-mass velocity” is the velocity of the CoM of a system of objects:

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots}{m_1 + m_2 + \dots}$$

$$v_{x,cm} = \frac{m_1 v_{x1} + m_2 v_{x2} + \dots}{m_1 + m_2 + \dots}$$

- ▶ An isolated system’s CoM velocity cannot change!
- ▶ You can see this by noticing that the numerator in  $\vec{v}_{cm}$  is the system’s total momentum, which you know is constant for an isolated system.
- ▶ If you observe this system from a camera that is moving at  $\vec{v}_{cm}$ , the system’s CoM will appear to be at rest. This camera’s frame-of-reference is called the “ZM frame,” because the system’s momentum is zero as seen from that frame (i.e. as seen by that moving camera).

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity  $30 \text{ m/s}$ ), with our little track aligned with the train axis.

I push a  $1 \text{ kg}$  cart north toward my friend at  $1 \text{ m/s}$ . She pushes a  $1 \text{ kg}$  cart south toward me at  $1 \text{ m/s}$ .

As seen by a camera mounted on the ceiling of the train, what is the velocity (north) of the center-of-mass of the two-cart system?

- (A)  $+30 \text{ m/s}$    (B)  $-30 \text{ m/s}$    (C)  $+1 \text{ m/s}$    (D)  $-1 \text{ m/s}$    (E)  $0 \text{ m/s}$

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity  $30 \text{ m/s}$ ), with our little track aligned with the train axis.

I push a  $1 \text{ kg}$  cart north toward my friend at  $1 \text{ m/s}$ . She pushes a  $1 \text{ kg}$  cart south toward me at  $1 \text{ m/s}$ .

As seen by a camera mounted on the ceiling of the train, what is the velocity (north) of the center-of-mass of the two-cart system?

- (A)  $+30 \text{ m/s}$    (B)  $-30 \text{ m/s}$    (C)  $+1 \text{ m/s}$    (D)  $-1 \text{ m/s}$    (E)  $0 \text{ m/s}$

As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity  $30 \text{ m/s}$ ), with our little track aligned with the train axis.

I push a  $1 \text{ kg}$  cart north toward my friend at  $1 \text{ m/s}$ . She pushes a  $1 \text{ kg}$  cart south toward me at  $1 \text{ m/s}$ .

As seen by a camera mounted on the ceiling of the train, what is the velocity (north) of the center-of-mass of the two-cart system?

- (A)  $+30 \text{ m/s}$    (B)  $-30 \text{ m/s}$    (C)  $+1 \text{ m/s}$    (D)  $-1 \text{ m/s}$    (E)  $0 \text{ m/s}$

As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

Is one of these two cameras watching from the “zero-momentum” frame of the two-cart system?



A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity  $30 \text{ m/s}$ ), with our little track aligned with the train axis.

I push a  $1 \text{ kg}$  cart north toward my friend at  $3 \text{ m/s}$ . She pushes a  $2 \text{ kg}$  cart south toward me at  $6 \text{ m/s}$ .

As seen by a camera mounted on the ceiling of the train, what is the velocity of the center-of-mass of the two-cart system? (Let the  $+x$  axis point north.)

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity  $30 \text{ m/s}$ ), with our little track aligned with the train axis.

I push a  $1 \text{ kg}$  cart north toward my friend at  $3 \text{ m/s}$ . She pushes a  $2 \text{ kg}$  cart south toward me at  $6 \text{ m/s}$ .

As seen by a camera mounted on the ceiling of the train, what is the velocity of the center-of-mass of the two-cart system? (Let the  $+x$  axis point north.)

As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity  $30 \text{ m/s}$ ), with our little track aligned with the train axis.

I push a  $1 \text{ kg}$  cart north toward my friend at  $3 \text{ m/s}$ . She pushes a  $2 \text{ kg}$  cart south toward me at  $6 \text{ m/s}$ .

As seen by a camera mounted on the ceiling of the train, what is the velocity of the center-of-mass of the two-cart system? (Let the  $+x$  axis point north.)

As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

Is one of these two cameras watching from the “zero-momentum” frame of the two-cart system?

If I observe a system from its zero-momentum reference frame, what can I say about its center-of-mass velocity?

- (A) The center-of-mass velocity (as seen from the ZM frame) is the same as the velocity of the ZM reference frame (as seen from the Earth frame).
- (B) The center-of-mass velocity (as seen from the ZM frame) is zero.
- (C) When observing from **Earth's** frame of reference, a system's center-of-mass velocity will be the same as the velocity (w.r.t. Earth) of the ZM reference frame.
- (D) (A) and (B)
- (E) (A) and (C)
- (F) (B) and (C)
- (G) (A), (B), and (C)

## More chapter 6 key ideas

- ▶ An isolated system's CoM velocity cannot change!
- ▶ A somewhat obscure consequence of this fact is that even in a totally inelastic collision, it is not necessarily possible to convert 100% of the initial kinetic energy into heating up, mangling, etc. the colliding objects.
- ▶ Momentum conservation requires that the CoM velocity cannot change, so if the CoM is moving initially, it has to keep moving after the collision.
- ▶ Textbook: “convertible” vs. “translational” parts of a system's kinetic energy.
- ▶ That idea is worth remembering, **but the math is not.**
- ▶ (Can illustrate using colliding carts.)

You really only need these first two equations from Ch6. The third one is in the “obscure” category. Don’t worry about it.

### (Chapter 6: relative motion)

Center of mass:

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

Center of mass velocity (equals velocity of ZM frame):

$$v_{ZM,x} = \frac{m_1v_{1x} + m_2v_{2x} + m_3v_{3x} + \dots}{m_1 + m_2 + m_3 + \dots}$$

Convertible kinetic energy:  $K_{\text{conv}} = K - \frac{1}{2}mv_{CM}^2$

## Zero-Momentum (ZM) frame for two-object collisions

- ▶ Very useful (but difficult to visualize) tool: ZM frame.
- ▶ Elastic collision analyzed in ZM (“\*”) frame:

$$v_{1i,x}^* = v_{1i,x} - v_{ZM,x}, \quad v_{2i,x}^* = v_{2i,x} - v_{ZM,x}$$

$$v_{1f,x}^* = -v_{1i,x}^*, \quad v_{2f,x}^* = -v_{2i,x}^*$$

$$v_{1f,x} = v_{1f,x}^* + v_{ZM,x}, \quad v_{2f,x} = v_{2f,x}^* + v_{ZM,x}$$

- ▶ Inelastic collision analyzed in ZM frame (restitution coeff.  $e$ ):

$$v_{1f,x}^* = -ev_{1i,x}^*, \quad v_{2f,x}^* = -ev_{2i,x}^*$$

- ▶ Step 1: shift velocities into ZM frame, by subtracting  $v_{ZM,x}$
- ▶ Step 2: write down (very simple!!) answer in ZM frame
- ▶ Step 3: shift velocities back into Earth frame, by adding  $v_{ZM,x}$

You can try this on some XC problems. Otherwise, skip it.

There are actually three pretty neat situations that you can analyze quite easily using the “ZM frame” trick:

- ▶ When a stationary golf ball is hit by a much more massive golf club, the golf ball’s outgoing speed is  $2\times$  the incoming speed of the (end of the) club.
- ▶ It’s easier to hit a home run off of a fastball than a slow pitch.
- ▶ When you drop a basketball with a tennis ball resting atop the basketball, the result is quite remarkable.

I was planning to skip these as “obscure,” and leave them as topics for extra-credit problems. But if there is overwhelming demand, we could work them out one day in class?

(A) Leave it for XC

(B) Do it in class



Here are the only two equations worth knowing from Chapter 7.  
By contrast, Chapter 8 will have quite a few worth knowing!

**(Chapter 7: interactions)**

For two objects that form an isolated system (i.e. interacting only with one another), the ratio of accelerations is

$$\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

When an object near Earth's surface moves a distance  $\Delta x$  in the direction away from Earth's center (i.e. upward), the change in gravitational potential energy of the Earth+object system is

$$\Delta U = mg\Delta x$$

- ▶ **Hugely important:** when two objects interact only with one another:

$$\Delta p_{1x} = -\Delta p_{2x}$$

$$\Delta v_{1x}/\Delta v_{2x} = -m_2/m_1$$

$$\boxed{\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}}$$

- ▶ When the medicine ball and I push apart from one another, we both accelerate: in opposite directions, and in inverse proportion to our masses.
- 
- ▶ Lifting an object up a height  $\Delta x$  in Earth's gravity changes its gravitational **potential energy** by

$$\Delta U^G = mg\Delta x$$

- ▶ I usually remember  $U = mgh$  where  $h$  is height
- ▶ Basketball: back & forth between  $\frac{1}{2}mv^2$  and  $mgh$  until mechanical energy is dissipated into thermal energy

Problem: I release a 1 kg ball from rest, from an initial height  $x_i = +5.0$  m above the ground. (Use  $g \approx 10$  m/s<sup>2</sup>.)

- (A) What is the ball's initial G.P.E. ?  
(Let's define  $x = 0$  to be  $U^G = 0$ .)
- (B) What is the ball's initial K.E. ?
- (C) What is the ball's G.P.E. immediately before it reaches the ground?
- (D) What is the ball's K.E. immediately before it reaches the ground?
- (E) What is the ball's speed immediately before it reaches the ground?
- (F) If the ball bounces elastically off of the floor, what height will it reach after bouncing?
- (G) If instead the ball bounces off of the floor with a restitution coefficient  $e = 0.9$ , what height will it reach after bouncing?

Problem: Suppose your friend's mass is about 50 kg, and she climbs up 30 flights of stairs (that's about 100 m) to check out a great rooftop view of the city's architecture.

- (A) By how many joules did climbing the stairs change her G.P.E.?
- (B) Where did this gravitational potential energy come from? I mean what source energy was converted into this G.P.E.?

Problem: Suppose your friend's mass is about 50 kg, and she climbs up 30 flights of stairs (that's about 100 m) to check out a great rooftop view of the city's architecture.

- (A) By how many joules did climbing the stairs change her G.P.E.?
- (B) Where did this gravitational potential energy come from? I mean what source energy was converted into this G.P.E.?
- (C) How many food Calories did she burn (assuming, unrealistically, that one's muscles are 100% efficient at converting food into mechanical work)?

## Physics 8 — Monday, September 16, 2019

- ▶ You read Ch7 (interactions) for today and you'll read Ch8 (force) for Wednesday. [Then we can finally start using Newton's three laws, as we will for the rest of the semester!]
- ▶ If you find it tedious to do algebra by hand, you could consider learning to use Mathematica, which is free (via site license) for all SAS and Wharton students. I have some excellent self-study Mathematica materials you could go through for extra credit. Email if you're interested.

```
m1 = 1.0; m2 = 9.0; v1xi = +1.0; v2xi = 0.0;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  (v1xf - v2xf) == -(v1xi - v2xi)
}]
```

```
Out[3]= v2xf == 0.2 && v1xf == -0.8
```