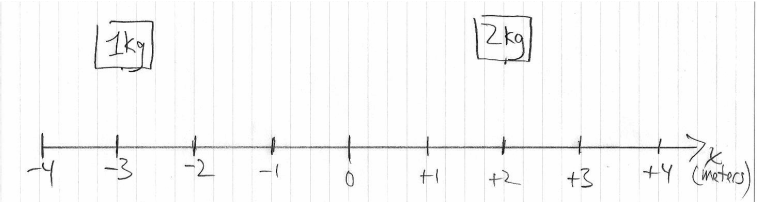
# Physics 8 — Wednesday, September 18, 2019

- Remember homework #3 due this Friday, at the start of class. It covers Chapters 4 and 5.
- Homework study/help sessions: Greg is in DRL 3C4 Wednesdays 4–6pm. Bill is in DRL 2C4 Thursdays 6–8pm.
- You read Ch8 (force) for today. So we can finally start using Newton's three laws, as we will for the rest of the semester!
- If you have little or no coding experience and you're interested in an XC option to learn Python for quantitative tasks like graphing and modeling data, email me ASAP.
- Ponder this with your neighbor(s) while we get started today:

$$x_{cm} = (m_1 x_1 + m_2 x_2 + \cdots) \,/\, (m_1 + m_2 + \cdots)$$

At what value of $x$ is the CoM of this pair of masses?



# More chapter 6 key ideas

- Center of mass: basically a weighted-average of positons.

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

- CoM of an object lies along axis of symmetry (if there is one).
- When analyzing the motion of a complicated object (composed of many pieces), it is often useful to consider separately the motion of its CoM and the motion of the various internal parts w.r.t. the CoM.
- Illustrate by tossing complicated object in the air.

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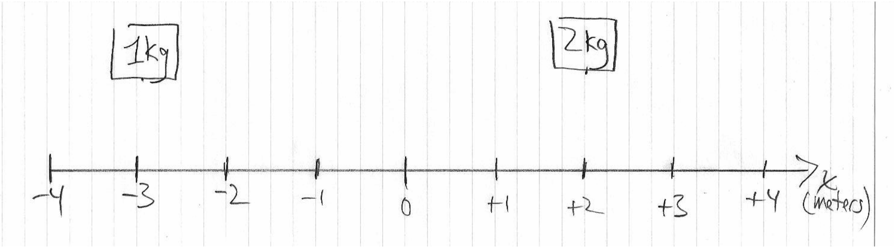

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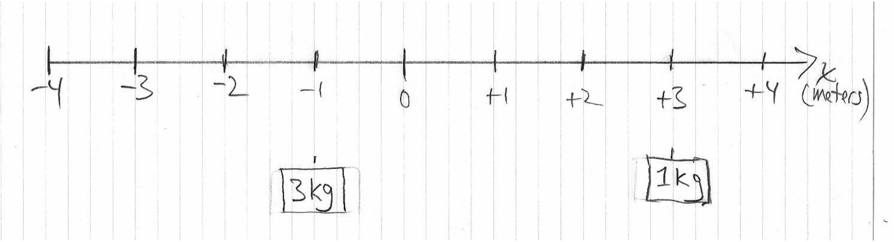

# More chapter 6 key ideas

- "Center-of-mass velocity" is the velocity of the CoM of a system of objects:

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$v_{x,cm} = \frac{m_1 v_{x1} + m_2 v_{x2} + \cdots}{m_1 + m_2 + \cdots}$$

- An isolated system's CoM velocity cannot change!
- You can see this by noticing that the numerator in $\vec{v}_{cm}$ is the system's total momentum, which you know is constant for an isolated system.
- If you observe this system from a camera that is moving at $\vec{v}_{cm}$, the system's CoM will appear to be at rest. This camera's frame-of-reference is called the "ZM frame," because the system's momentum is zero as seen from that frame (i.e. as seen by that moving camera).

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity 30 $\mathrm{m/s}$), with our little track aligned with the train axis.

I push a 1 kg cart north toward my friend at 1 m/s. She pushes a 1 kg cart south toward me at 1 m/s.

As seen by a camera mounted on the ceiling of the train, what is the velocity (north) of the center-of-mass of the two-cart system?

(A) $+30\,\mathrm{m/s}$ (B) $-30\,\mathrm{m/s}$ (C) $+1\,\mathrm{m/s}$ (D) $-1\,\mathrm{m/s}$ (E) $0\,\mathrm{m/s}$

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As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

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As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

Is one of these two cameras watching from the "zero-momentum" frame of the two-cart system?

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I push a 1 kg cart north toward my friend at 3 m/s. She pushes a 2 kg cart south toward me at 6 m/s.

As seen by a camera mounted on the ceiling of the train, what is the velocity of the center-of-mass of the two-cart system? (Let the $+x$ axis point north.)

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity 30 $\mathrm{m/s}$), with our little track aligned with the train axis.

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Is one of these two cameras watching from the "zero-momentum" frame of the two-cart system?

If I observe a system from its zero-momentum reference frame, what can I say about its center-of-mass velocity?

(A) The center-of-mass velocity (as seen from the ZM frame) is the same as the velocity of the ZM reference frame (as seen from the Earth frame).
(B) The center-of-mass velocity (as seen from the ZM frame) is zero.
(C) When observing from **Earth's** frame of reference, a system's center-of-mass velocity will be the same as the velocity (w.r.t. Earth) of the ZM reference frame.
(D) (A) and (B)
(E) (A) and (C)
(F) (B) and (C)
(G) (A), (B), and (C)

# More chapter 6 key ideas

- An isolated system's CoM velocity cannot change!
- A somewhat obscure consequence of this fact is that even in a totally inelastic collision, it is not necessarily possible to convert 100% of the initial kinetic energy into heating up, mangling, etc. the colliding objects.
- Momentum conservation requires that the CoM velocity cannot change, so if the CoM is moving initially, it has to keep moving after the collision.
- Textbook: "convertible" vs. "translational" parts of a system's kinetic energy.
- That idea is worth remembering, **but the math is not.**
- (Can illustrate using colliding carts.)

You really only need these first two equations from Ch6. The third one is in the "obscure" category. Don't worry about it.

**(Chapter 6: relative motion)**

Center of mass:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Center of mass velocity (equals velocity of ZM frame):

$$v_{ZM,x} = \frac{m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Convertible kinetic energy: $K_{\text{conv}} = K - \frac{1}{2} m v_{CM}^2$

# Zero-Momentum (ZM) frame for two-object collisions

- Very useful (but difficult to visualize) tool: ZM frame.
- Elastic collision analyzed in ZM ("$*$") frame:

$$v^*_{1i,x} = v_{1i,x} - v_{ZM,x}, \quad v^*_{2i,x} = v_{2i,x} - v_{ZM,x}$$

$$\boxed{v^*_{1f,x} = -v^*_{1i,x}, \quad v^*_{2f,x} = -v^*_{2i,x}}$$

$$v_{1f,x} = v^*_{1f,x} + v_{ZM,x}, \quad v_{2f,x} = v^*_{2f,x} + v_{ZM,x}$$

- Inelastic collision analyzed in ZM frame (restitution coeff. $e$):

$$\boxed{v^*_{1f,x} = -e v^*_{1i,x}, \quad v^*_{2f,x} = -e v^*_{2i,x}}$$

- Step 1: shift velocities into ZM frame, by subtracting $v_{ZM,x}$
- Step 2: write down (very simple!!) answer in ZM frame
- Step 3: shift velocities back into Earth frame, by adding $v_{ZM,x}$

You can try this on some XC problems. Otherwise, skip it.

There are actually three pretty neat situations that you can analyze quite easily using the "ZM frame" trick:

- When a stationary golf ball is hit by a much more massive golf club, the golf ball's outgoing speed is $2\times$ the incoming speed of the (end of the) club.
- It's easier to hit a home run off of a fastball than a slow pitch.
- When you drop a basketball with a tennis ball resting atop the basketball, the result is quite remarkable.

I was planning to skip these as "obscure," and leave them as topics for extra-credit problems. But if there is overwhelming demand, we could work them out one day in class?

(A) Leave it for XC (B) Do it in class

Here are the only two equations worth knowing from Chapter 7.
By contrast, Chapter 8 will have quite a few worth knowing!

**(Chapter 7: interactions)**

For two objects that form an isolated system (i.e. interacting only with one another), the ratio of accelerations is

$$\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

When an object near Earth's surface moves a distance $\Delta x$ in the direction away from Earth's center (i.e. upward), the change in gravitational potential energy of the Earth+object system is

$$\Delta U = mg\Delta x$$

- **Hugely important:** when two objects interact only with one another:

$$\Delta p_{1x} = -\Delta p_{2x}$$

$$\Delta v_{1x} / \Delta v_{2x} = -m_2 / m_1$$

$$\boxed{\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}}$$

- When the medicine ball and I push apart from one another, we both accelerate: in opposite directions, and in inverse proportion to our masses.

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- Lifting an object up a height $\Delta x$ in Earth's gravity changes its gravitational **potential energy** by

$$\Delta U^G = mg\Delta x$$

- I usually remember $U = mgh$ where $h$ is height
- Basketball: back & forth between $\frac{1}{2}mv^2$ and $mgh$ until mechanical energy is dissipated into thermal energy

Problem: I release a 1 kg ball from rest, from an initial height $x_i = +5.0\ \mathrm{m}$ above the ground. (Use $g \approx 10\ \mathrm{m/s^2}$.)

(A) What is the ball's initial G.P.E. ?
(Let's define $x = 0$ to be $U^G = 0$.)

(B) What is the ball's initial K.E. ?

(C) What is the ball's G.P.E. immediately before it reaches the ground?

(D) What is the ball's K.E. immediately before it reaches the ground?

(E) What is the ball's speed immediately before it reaches the ground?

(F) If the ball bounces elastically off of the floor, what height will it reach after bouncing?

(G) If instead the ball bounces off of the floor with a restitution coefficient $e = 0.9$, what height will it reach after bouncing?

We'll re-visit (G) and then go on from here on Friday.

Problem: Suppose your friend's mass is about 50 $\mathrm{kg}$, and she climbs up 30 flights of stairs (that's about 100 m) to check out a great rooftop view of the city's architecture.

(A) By how many joules did climbing the stairs change her G.P.E.?

(B) Where did this gravitational potential energy come from? I mean what source energy was converted into this G.P.E.?

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(A) By how many joules did climbing the stairs change her G.P.E.?

(B) Where did this gravitational potential energy come from? I mean what source energy was converted into this G.P.E.?

(C) How many food Calories did she burn (assuming, unrealistically, that one's muscles are 100% efficient at converting food into mechanical work)?

# Chapter 8: Force

- Forces **always** come in pairs: when A and B interact,

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

- "Interaction pairs" have equal magnitude, opposite direction. **Always.** That's called Newton's third law. Difficult idea!

- The acceleration of object A is given by vector sum of all of the forces acting **ON** object A, divided by $m_A$. (Law #2.)

$$\vec{a}_A = \frac{1}{m_A} \sum \vec{F}_{\text{on A}}$$

- In an inertial frame of reference, object A moves at constant velocity (or stays at rest) if and only if the vector sum ($\sum \vec{F}_{\text{on A}}$) equals zero. (Law #1.)

You push with a steady force of 25 N on a 50 kg desk fitted with (ultra-low-friction) casters on its four feet. How long does it take you (starting from rest) to get the desk across a room that is 25 m wide?

(A) 0.71 s

(B) 1.0 s

(C) 1.4 s

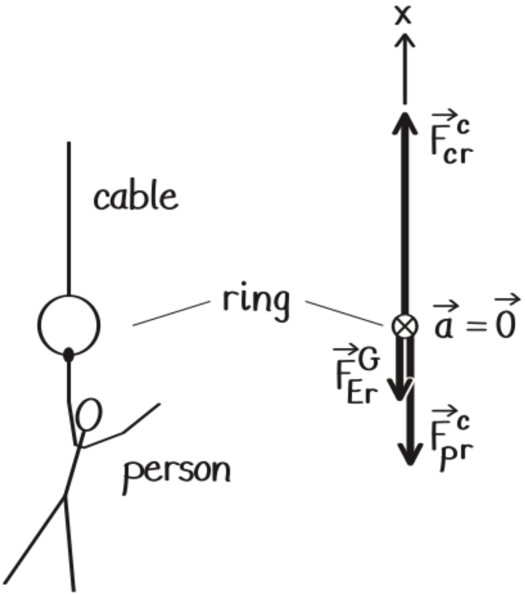
(D) 5.0 s

(E) 7.1 s

(F) 10 s

(G) 14 s

**Free-body diagram:** A sort of visual accounting procedure for adding up the forces acting **ON** a given object. FBD for ring:



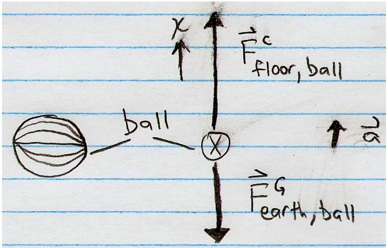
# Chapter 8 (“force”) reading Q #1

“Think about the familiar example of a basketball dropped from eye level and allowed to bounce a few times. Describe the forces acting on the basketball at its lowest point, as it is in contact with the floor and is changing direction from downward to upward motion.”

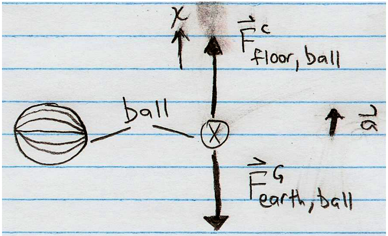
- Working with 1-2 nearby people, draw a free-body diagram of the ball at its lowest point (while it is most squished). Include all forces acting ON the ball. Indicate the direction of each force with its vector arrow. Indicate the relative magnitudes of the forces by the lengths of the arrows. Indicate the direction of the ball’s acceleration with an arrow (or a dot).
- When you finish that, draw a second free-body diagram for the ball — this time while the ball is in the air. Will the diagram be different while the ball is rising vs. falling?
- Discuss! I may call on people to describe their diagrams.

**(My diagram appears on the next slide.)**

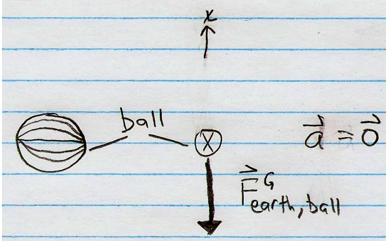
Which free-body diagram best represents the forces acting on the basketball at the *bottom* of its motion?



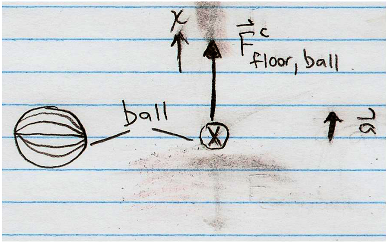

A




B

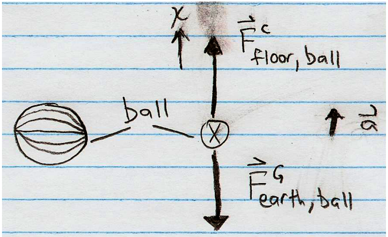



C

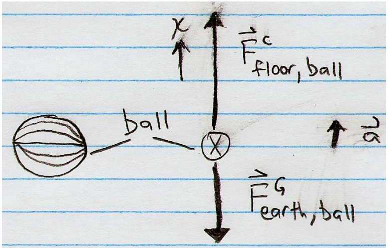



D

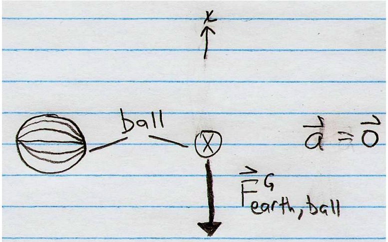
Which free-body diagram best represents the forces acting on the basketball at the *top* of its motion?



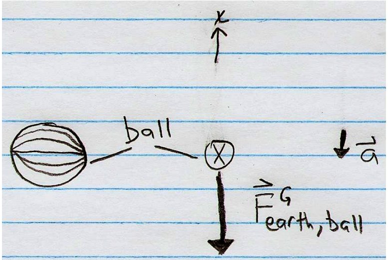

A




B




C




D

If I were to draw a free-body diagram for the basketball when it is halfway back down to the ground, that new diagram would be

(A) the same as

(B) slightly different from

(C) very different from

the drawing for the basketball when it is at the top of its motion? (Neglect air resistance.)

# Equal and opposite forces?

Consider a car at rest on a road. We can conclude that the downward gravitational pull of Earth on the car and the upward contact force of the road on the car are equal and opposite because

(A) the two forces form an interaction pair.

(B) the net force on the car is zero.

(C) neither: the two forces are not equal and opposite

(D) both (A) and (B)

# Chapter 8 (“force”) reading Q #2

“Explain briefly in your own words what it means for the interaction between two objects to involve ‘equal and opposite’ forces. Can you illustrate this with an everyday example?”

- For instance, if I push against some object O that moves, deforms, or collapses in response to my push, is the force exerted by O on me still equal in magnitude and opposite in direction to the force exerted by me on O?
- If every force is paired with an equal and opposite force, why is it ever possible for any object to be accelerated? Don’t they all just cancel each other out?
- (I think the next example may help.)

# Have you ever spotted the Tropicana juice train?!

vocab: powered "locomotive" pulls the unpowered "cars"

# Equal and opposite forces?

An engine ("locomotive") (the first vehicle of the train) pulls a series of train cars. Which is the correct analysis of the situation?

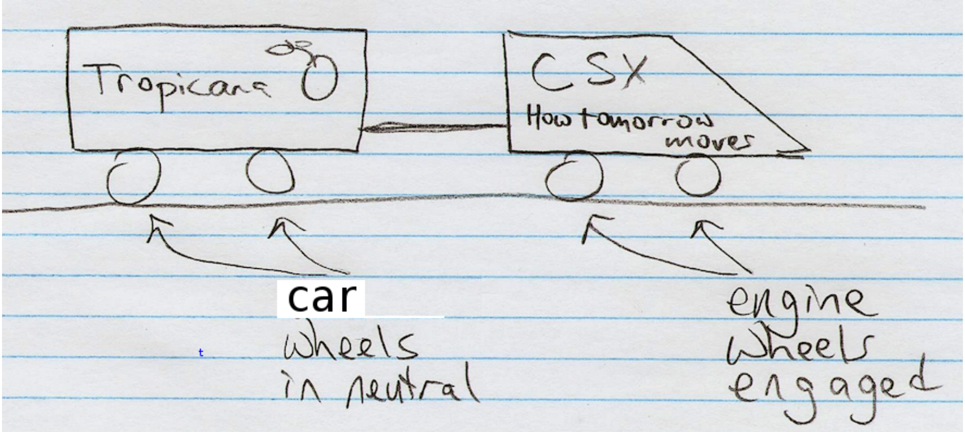
(A) The train moves forward because the locomotive pulls forward slightly harder on the cars than the cars pull backward on the locomotive.

(B) Because action always equals reaction, the locomotive cannot pull the cars — the cars pull backward just as hard as the locomotive pulls forward, so there is no motion.

(C) The locomotive gets the cars to move by giving them a tug during which the force on the cars is momentarily greater than the force exerted by the cars on the locomotive.

(D) The locomotive's force on the cars is as strong as the force of the cars on the locomotive, but the frictional force by the track on the locomotive is forward and large while the backward frictional force by the track on the cars is small.

(E) The locomotive can pull the cars forward only if its inertia is larger than that of the cars.




Let's see the effect of including or not including the frictional force of the tracks pushing forward on the wheels of the engine.

I'll pretend to be the engine!

# Only *external* forces can accelerate a system's CoM

Let's define "system" to be locomotive+car.
Remember that forces internal to system cannot accelerate system's CoM.

To change the velocity of the CoM, we need a force that is *external* to the system.

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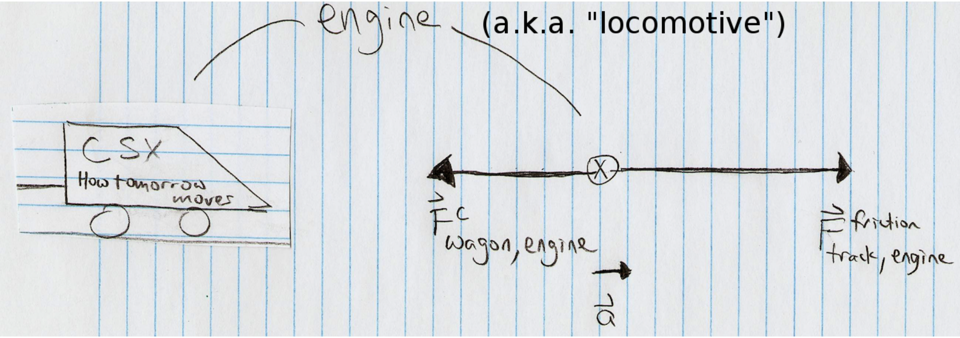
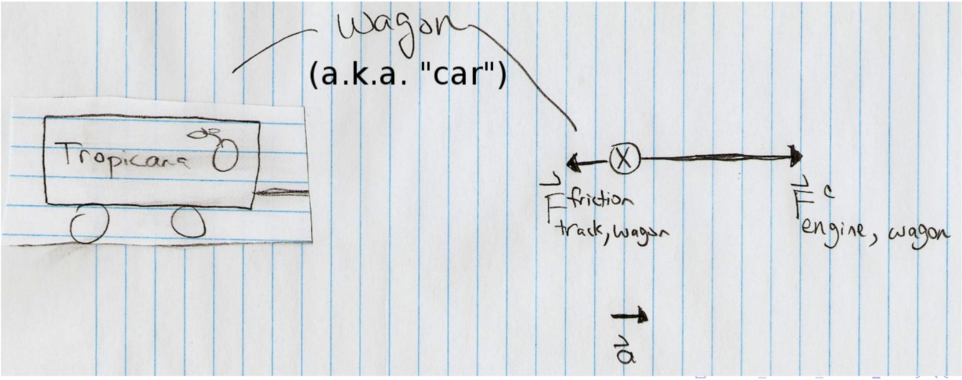
(By the way, when you look at the two free-body diagrams on the next page, tell me if you see an "interaction pair" of forces somewhere!)


engine
(a.k.a. "locomotive")
CSX
How tomorrow moves
$\vec{F}^{c}_{wagon, engine}$
$\vec{F}^{friction}_{track, engine}$
$\vec{a}$


wagon
(a.k.a. "car")
Tropicana
$\vec{F}^{friction}_{track, wagon}$
$\vec{F}^{c}_{engine, wagon}$
$\vec{a}$

$$\vec{a}_{\mathrm{CoM}} = \frac{\sum \vec{F}^{\mathrm{external}}}{m_{\mathrm{total}}}$$

It's useful to remember that even if the several pieces of a system are behaving in a complicated way, you can find the acceleration of the CoM of the system by considering only the **external** forces that act **on** the system.

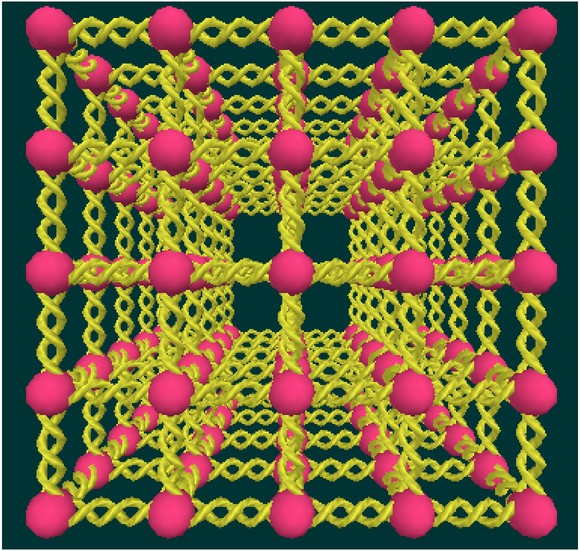
Once again, a careful choice of "system" boundary often makes the analysis much easier. We'll see more examples of this next time.

(Begin digression.)

# Dissipative / incoherent / irreversible

A simple ball / spring model of the atoms in a solid.

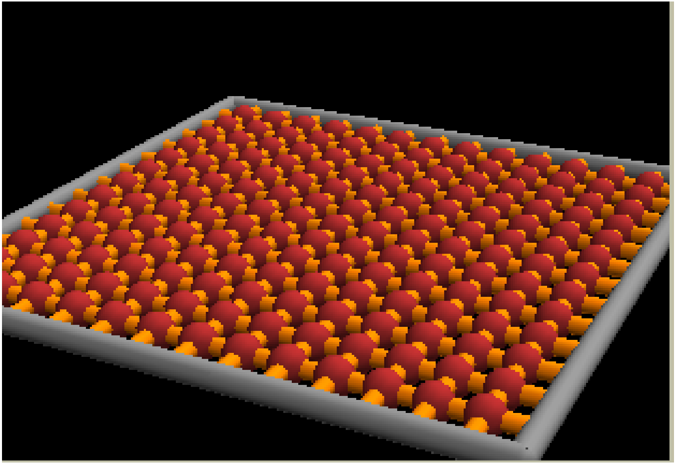
This is sometimes a useful picture to keep in your head.



# Dissipative / incoherent / irreversible

2D version for simplicity

*illustrate "reversible" and "irreversible" deformation with e.g. marbles and egg crate*



# Dissipative / incoherent / irreversible

I showed you once before my low-tech animation of two objects in a totally inelastic collision. Collision dissipates coherent motion (kinetic energy) into incoherent vibration of atoms (thermal energy)

`https://youtu.be/SJIKCmg2Uzg`

Here's a high-speed movie of a (mostly) reversible process a golf ball bouncing off of a wall at 150 mph.


https://www.youtube.com/watch?v=AkB81u5IM3I

(End digression.)

# Hooke's law

- When you pull on a spring, it stretches
- When you stretch a spring, it pulls back on you
- When you compress a spring, it pushes back on you
- For an ideal spring, the pull is proportional to the stretch
- Force **by** spring, **on** load is

$$F_x = -k\,(x - x_0)$$

- The constant of proportionality is the "spring constant" $k$, which varies from spring to spring. When we talk later about properties of building materials, we'll see where $k$ comes from.
- The minus sign indicates that if I move my end of the spring to the right of its relaxed position, the force exerted by the the spring on my finger points left.

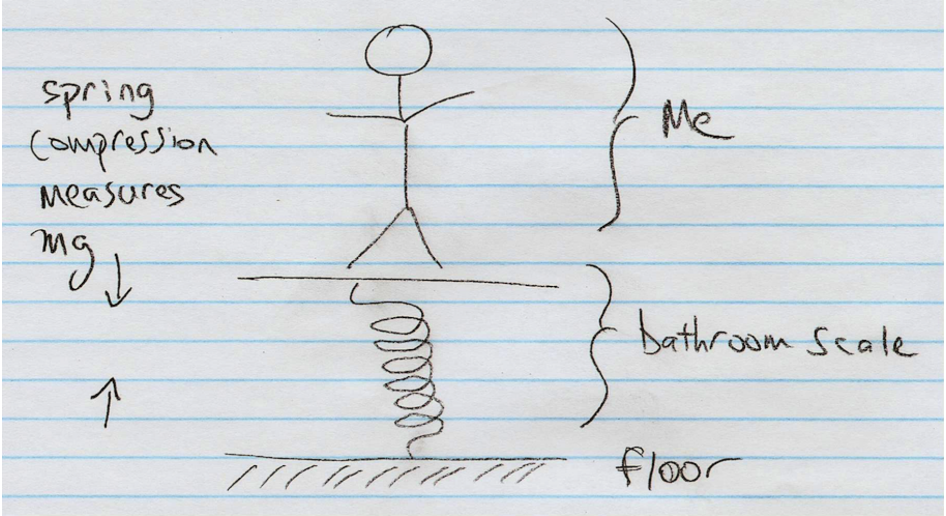
Let's look at some examples of springs.

A spring hanging from the ceiling is 1.00 m long when there is no object attached to its free end. When a 4.0 kg brick is attached to the free end, the spring is 1.98 m long. What is the spring constant of the spring?

(A) 5.0 N/m
(B) 10 N/m
(C) 20 N/m
(D) 30 N/m
(E) 40 N/m

# Measuring your weight ($F = mg$) with a spring scale

Most bathroom scales work something like this:




Now suppose I take my bathroom scale on an elevator . . .

# Bathroom scale on moving elevator

A bathroom scale typically uses the compression of a spring to measure the gravitational force ($F = mg$) exerted by Earth on you, which we call your *weight*.

Suppose I am standing on such a scale while riding an elevator. With the elevator parked at the bottom floor, the scale reads 700 N. I push the button for the top floor. The door closes. The elevator begins moving upward. At the moment when I can feel that the elevator has begun moving upward, the scale reads

(A) a value smaller than 700 N.

(B) the same value: 700 N.

(C) a value larger than 700 N.

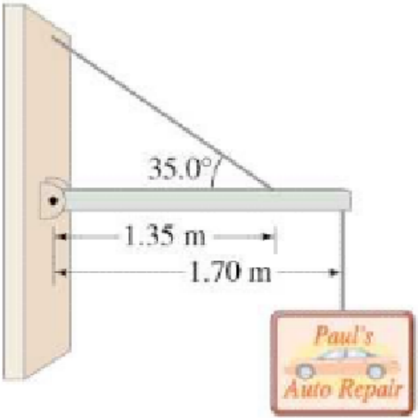
You might want to try drawing a free-body diagram for your body, showing the downward force of gravity, the upward force of the scale pushing on your feet, and your body's acceleration.

# Tension vs. compression

- When a force tries to squish a spring, that is called *compression*, or a compressive force
- When a force tries to elongate a spring, that is called *tension*, or a tensile force
- We'll spend a lot of time next month talking about compression and tension in columns, beams, etc.
- For now, remember that tension is the force trying to pull apart a spring, rope, etc., and compression is the force trying to squeeze a post, a basketball, a mechanical linkage, etc.

# Tension in cables

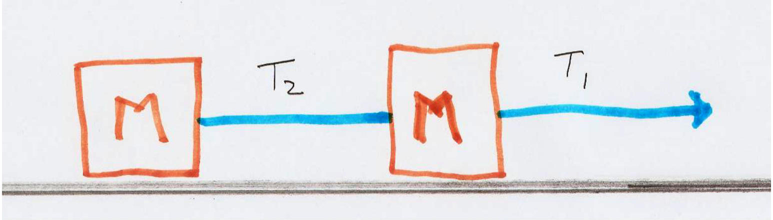
- A large category of physics problems (and even architectural structures, e.g. a suspension bridge) involves two objects connected by a rope, a cable, a chain, etc.
- These things (cables, chains, ropes) can pull but can't push. Two cables in this figure:



# Tension in cables

- Usually the cables in physics problems are considered light enough that you don't worry about their inertia (we pretend $m = 0$), and stiff enough that you don't worry about their stretching when you pull on them (we pretend $k = \infty$).
- The cable's job is just to transmit a force from one end to the other. We call that force the cable's *tension*, $T$.
- Cable always pulls on both ends with same magnitude ($T$), though in opposite directions. [Formally: we neglect the cable's mass, and the cable's acceleration must be finite.]
- E.g. hang basketball from ceiling. Cable transmits $mg$ to ceiling. Gravity pulls ball down. Tension pulls ball up. Forces on ball add to zero.
- Let's try an example.

Two blocks of equal mass are pulled to the right by a constant force, which is applied by pulling at the arrow-tip on the right. The blue lines represent two identical sections of rope (which can be considered massless). Both cables are taut, and friction (if any) is the same for both blocks. What is the ratio of $T_1$ to $T_2$?




(A) zero: $T_1 = 0$ and $T_2 \neq 0$.

(B) $T_1 = \frac{1}{2} T_2$

(C) $T_1 = T_2$

(D) $T_1 = 2T_2$

(E) infinite: $T_2 = 0$ and $T_1 \neq 0$.

# Physics 8 — Wednesday, September 18, 2019

- Remember homework #3 due this Friday, at the start of class. It covers Chapters 4 and 5.
- Homework study/help sessions: Greg is in DRL 3C4 Wednesdays 4–6pm. Bill is in DRL 2C4 Thursdays 6–8pm.
- If you have little or no coding experience and you're interested in an XC option to learn Python for quantitative tasks like graphing and modeling data, email me ASAP.