Physics 8 — Monday, September 23, 2019

- For today, you read Ch9 (work). For Wednesday (though we won't get to it until next week), you'll read the first half of Ch10 (motion in a plane).
- If you weren't here last Wed/Fri, pick up your "clicker" with your graded HW2 paper. We'll try them today!
- If you have little or no coding experience and you're interested in an XC option to learn Python for quantitative tasks like graphing and modeling data, email me ASAP.
- Wolfram Mathematica is free (site license) for SAS and Wharton students. I have some very helpful self-study Mathematica materials you can do for XC. Email if interested.
- If you're interested in learning to do a bit of Python coding in a drawing/animation system called "Processing" made by and for visual artists, you can look at my Fall 2017 day-before-Thanksgiving lecture here: http://xray.hep.upenn.edu/wja/p008/2017/files/phys8_notes_20171122.pdf

Chapter 8 ("force") reading Q #2

"Explain briefly in your own words what it means for the interaction between two objects to involve 'equal and opposite' forces. Can you illustrate this with an everyday example?"

- For instance, if I push against some object O that moves, deforms, or collapses in response to my push, is the force exerted by O on me still equal in magnitude and opposite in direction to the force exerted by me on O?
- If every force is paired with an equal and opposite force, why is it ever possible for any object to be accelerated? Don't they all just cancel each other out?
- (I think the next example may help.)

Have you ever spotted the Tropicana juice train?!

vocab: powered “locomotive” pulls the unpowered “cars”

Equal and opposite forces?

An engine ("locomotive") (the first vehicle of the train) pulls a series of train cars. Which is the correct analysis of the situation?

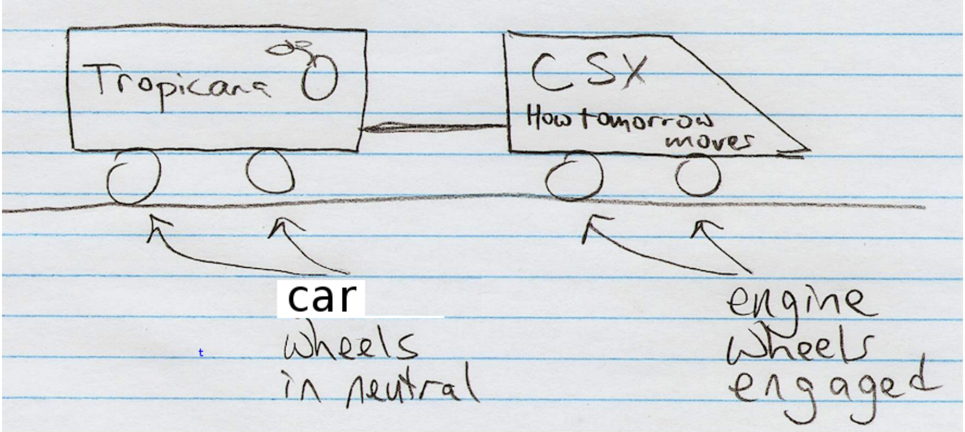
(A) The train moves forward because the locomotive pulls forward slightly harder on the cars than the cars pull backward on the locomotive.

(B) Because action always equals reaction, the locomotive cannot pull the cars — the cars pull backward just as hard as the locomotive pulls forward, so there is no motion.

(C) The locomotive gets the cars to move by giving them a tug during which the force on the cars is momentarily greater than the force exerted by the cars on the locomotive.

(D) The locomotive's force on the cars is as strong as the force of the cars on the locomotive, but the frictional force by the track on the locomotive is forward and large while the backward frictional force by the track on the cars is small.

(E) The locomotive can pull the cars forward only if its inertia (i.e. mass) is larger than that of the cars.



Let's see the effect of including or not including the frictional force of the tracks pushing forward on the wheels of the engine.

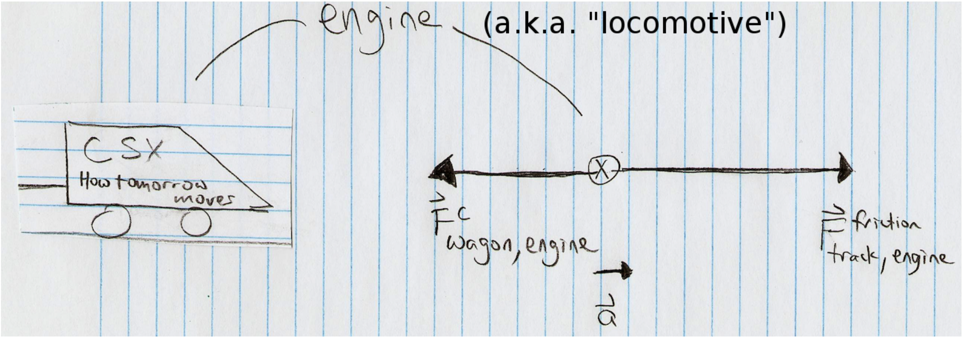
I'll pretend to be the engine!

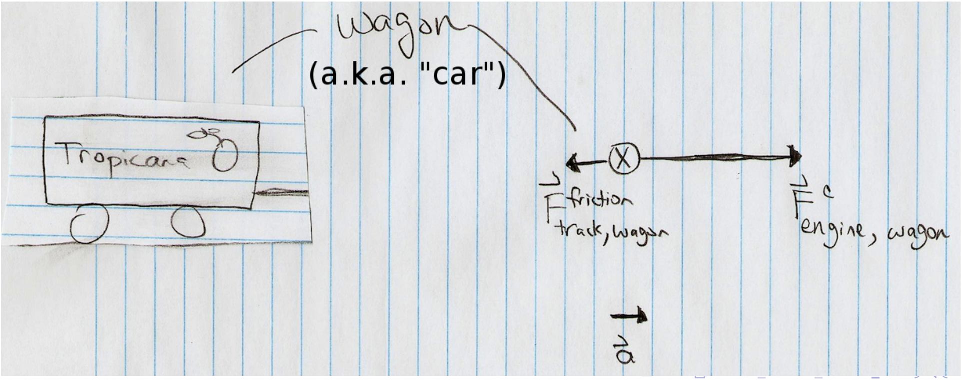
Only *external* forces can accelerate a system's CoM

Let's define "system" to be locomotive+car. Remember that forces internal to system cannot accelerate system's CoM.

To change the velocity of the CoM, we need a force that is *external* to the system.

(By the way, when you look at the two free-body diagrams on the next page, tell me if you see an "interaction pair" of forces somewhere!)


engine
(a.k.a. "locomotive")
CSX
How tomorrow moves
$\vec{F}^{c}_{wagon, engine}$
$\vec{F}^{friction}_{track, engine}$
$\vec{a}$


wagon
(a.k.a. "car")
Tropicana
$\vec{F}^{friction}_{track, wagon}$
$\vec{F}^{c}_{engine, wagon}$
$\vec{a}$

$$\vec{a}_{\mathrm{CoM}} = \frac{\sum \vec{F}^{\mathrm{external}}}{m_{\mathrm{total}}}$$

It's useful to remember that even if the several pieces of a system are behaving in a complicated way, you can find the acceleration of the CoM of the system by considering only the **external** forces that act **on** the system.

Once again, a careful choice of "system" boundary often makes the analysis much easier. We'll see more examples of this soon. (This topic also arises in HW4 #9 and #10, so we'll try to practice it today or Wednesday.)

Hooke's law

- When you pull on a spring, it stretches
- When you stretch a spring, it pulls back on you
- When you compress a spring, it pushes back on you
- For an ideal spring, the pull is proportional to the stretch
- Force **by** spring, **on** load is

$$F_x = -k\,(x - x_0)$$

- The constant of proportionality is the "spring constant" k, which varies from spring to spring. When we talk later about properties of building materials, we'll see where k comes from.
- The minus sign indicates that if I move my end of the spring to the right of its relaxed position, the force exerted by the the spring on my finger points left.

Let's look at some examples of springs.

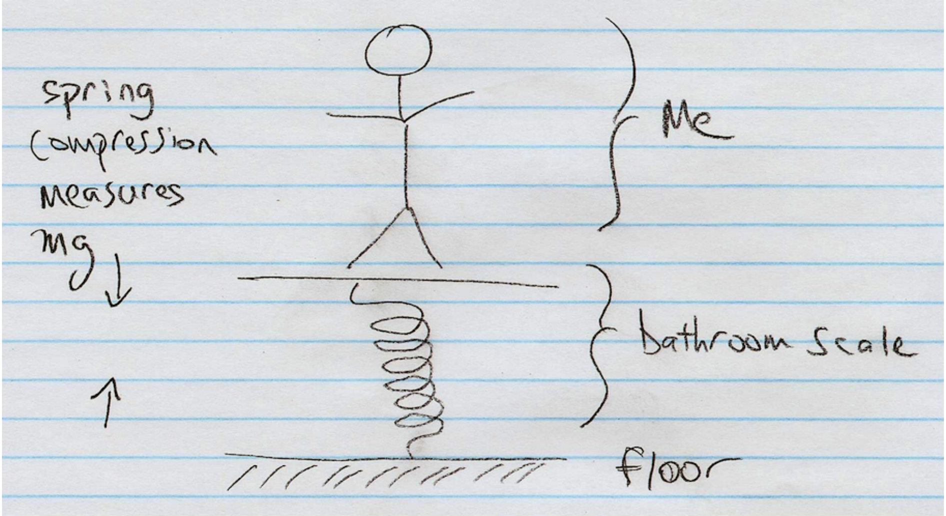
A spring hanging from the ceiling is 1.0 m long when there is no object attached to its free end. When a 4.0 kg brick is attached to the free end, the spring is 2.0 m long. (For easier math, use $g = 10\,\mathrm{m/s^2} = 10\,\mathrm{N/kg}$.) What is the spring constant of the spring?

[Hint: draw a FBD for the brick, to figure out what magnitude force the spring must be exerting on the brick. The magnitude of the force exerted by the spring is the spring constant (k) times how far the spring is stretched w.r.t. its relaxed length.]

(A) 5.0 N/m
(B) 10 N/m
(C) 20 N/m
(D) 30 N/m
(E) 40 N/m

Measuring your weight ($F = mg$) with a spring scale

Most bathroom scales work something like this:



Now suppose I take my bathroom scale on an elevator . . .

Bathroom scale on an accelerating elevator

A bathroom scale typically uses the compression of a spring to infer the gravitational force ($F = mg$) exerted by Earth on you, which we call your *weight*.

Suppose I am standing on such a scale while riding an elevator. With the elevator parked at the bottom floor, the scale reads 700 N. I push the button for the top floor. The door closes. The elevator begins moving upward. At the moment when I can feel (e.g. in my stomach) that the elevator has begun moving upward, the scale reads

(A) a value smaller than 700 N.

(B) the same value: 700 N.

(C) a value larger than 700 N.

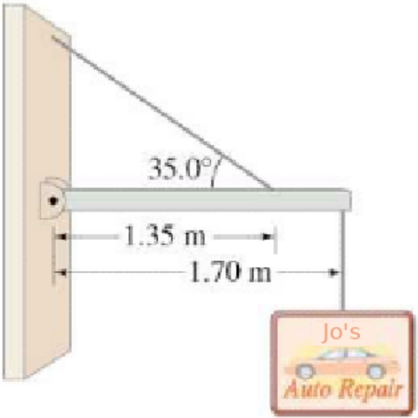
You might want to try drawing a free-body diagram for your body, showing the downward force of gravity, the upward force of the scale pushing on your feet, **and your body's acceleration**.

Tension vs. compression

- When a force tries to squish a spring, that is called *compression*, or a compressive force
- When a force tries to elongate a spring, that is called *tension*, or a tensile force
- We'll spend a lot of time next month talking about compression and tension in columns, beams, etc.
- For now, remember that tension is the force trying to pull apart a spring, rope, etc., and compression is the force trying to squeeze a post, a basketball, a mechanical linkage, etc.

Tension in cables

- A large category of physics problems (and even architectural structures, e.g. a suspension bridge) involves two objects connected by a rope, a cable, a chain, etc.
- These things (cables, chains, ropes) can pull but can't push. There are two cables in this figure:

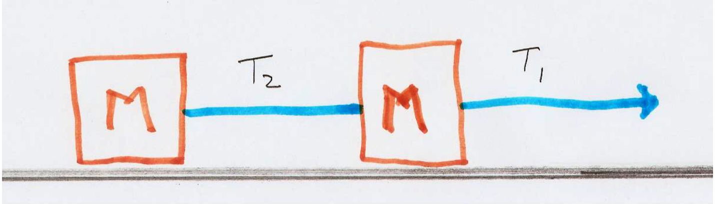


Tension in cables

- Usually the cables in physics problems are considered light enough that you don't worry about their inertia (we pretend $m = 0$), and stiff enough that you don't worry about their stretching when you pull on them (we pretend $k = \infty$).
- The cable's job is just to transmit a force from one end to the other. We call that force the cable's *tension*, T.
- A cable always pulls on both ends with same magnitude (T), though in opposite directions. [Formally: we neglect the cable's mass, and the cable's acceleration must be finite.]
- E.g. hang basketball from ceiling. Cable transmits mg to ceiling. Gravity pulls ball down. Tension pulls ball up. Forces on ball add (vectorially) to zero.

- Let's try an example.

Two blocks of equal mass are pulled to the right by a constant force, which is applied by pulling at the arrow-tip on the right. The blue lines represent two identical sections of rope (which can be considered massless). Both cables are taut, and friction (if any) is the same for both blocks. What is the ratio of T_1 to T_2?



(A) zero: $T_1 = 0$ and $T_2 \neq 0$.
(B) $T_1 = \frac{1}{2} T_2$
(C) $T_1 = T_2$
(D) $T_1 = 2T_2$
(E) infinite: $T_2 = 0$ and $T_1 \neq 0$.

It's worth drawing an FBD first for the two-mass system, then for the left mass, then for the right mass.

Forces recap, before we do more examples

- The **force** concept quantifies interaction between two objects.
- Forces always come in "**interaction pairs**." The force exerted by object "A" on object "B" is equal in magnitude and opposite in direction to the force exerted by B on A:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

- The acceleration of object "A" is given by the vector sum of the forces acting **on** A, divided by the mass of A:

$$\vec{a}_A = \frac{\sum \vec{F}_{(\text{on A})}}{m_A}$$

- The vector sum of the forces acting **on** an object equals the rate of change of the object's momentum:

$$\sum \vec{F}_{(\text{on A})} = \frac{\mathrm{d}\vec{p}_A}{\mathrm{d}t}$$

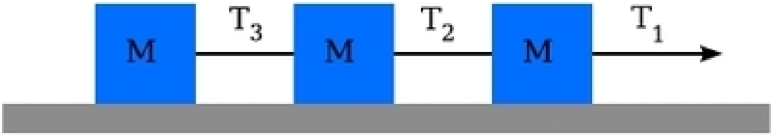
- An object whose momentum is not changing is in translational **equilibrium**. We'll see later that this will be a big deal for the members of a structure! To achieve this, we will want all forces acting **on** each member to sum vectorially to zero.
- The unit of force is the **newton**. $1\ \mathrm{N} = 1\ \mathrm{kg} \cdot \mathrm{m/s^2}$.
- **Free-body diagrams** depict all of the forces acting **on** a given object. They are used all the time in analyzing structures!

- The force exerted by a compressed or stretched spring is proportional to the displacement of the end of the spring w.r.t. its relaxed value x_0. k is "spring constant."

$$F_x^{\mathrm{spring}} = -k(x - x_0)$$

- When a rope is held taut, it exerts a force called the **tension** on each of its ends. Same magnitude T on each end.

Three blocks of equal mass are pulled to the right by a constant force. The blocks are connected by identical sections of rope (which can be considered massless). All cables are taut, and friction (if any) is the same for all blocks. What is the ratio of T_1 to T_2?



(A) $T_1 = \frac{1}{3} T_2$

(B) $T_1 = \frac{2}{3} T_2$

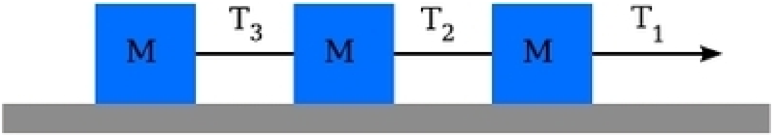
(C) $T_1 = T_2$

(D) $T_1 = \frac{3}{2} T_2$

(E) $T_1 = 2 T_2$

(F) $T_1 = 3 T_2$

Three blocks of equal mass are pulled to the right by a constant force. The blocks are connected by identical sections of rope (which can be considered massless). All cables are taut, and friction (if any) is the same for all blocks. What is the ratio of T_1 to T_3?



(A) $T_1 = \frac{1}{3} T_3$

(B) $T_1 = \frac{2}{3} T_3$

(C) $T_1 = T_3$

(D) $T_1 = \frac{3}{2} T_3$

(E) $T_1 = 2T_3$

(F) $T_1 = 3T_3$

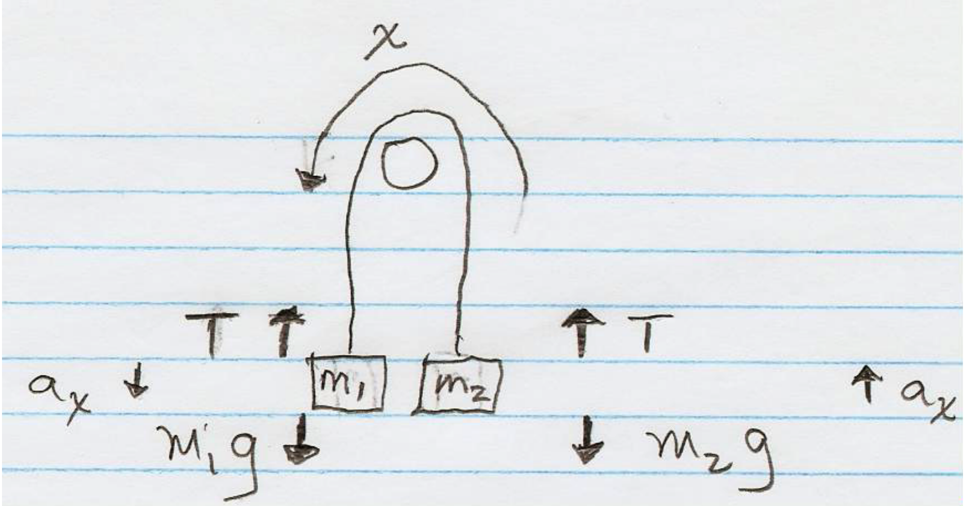
Atwood machine — discuss with your neighbors

A contraption something like this appears in HW4 (but with a spring added, to keep things interesting).

- Why aren't the two masses accelerating?
- What is the tension in the cable when the two masses are equal (both 5.0 kg) and stationary, as they are now?
- If I make one mass equal 5.0 kg and the other mass equal 5.1 kg, what will happen? Can you predict what the acceleration will be?
- If I make one mass equal 5.0 kg and the other mass equal 6.0 kg, will the acceleration be larger or smaller than in the previous case?
- Try drawing a free-body diagram for each of the two masses
- By how much do I change the gravitational potential energy of the machine+Earth system when I raise the 6 kg mass 1 m?

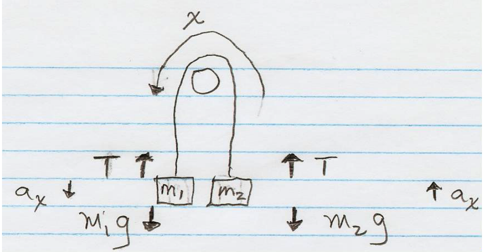
- Two more comments:
- This machine was originally invented as a mechanism for measuring g and for studying motion with constant acceleration.
- The same concept is used by the "counterweight" in an elevator for a building.

Atwood machine: take $m_1 > m_2$



Pause here: how can we solve for a_x? Try it before we go on.

Atwood machine: write masses' equations of motion



$$m_1 g - T = m_1 a_x$$

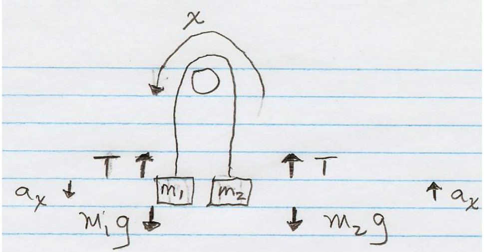
$$T - m_2 g = m_2 a_x$$

Solve second equation for T; plug T into first equation; solve for a_x:

$$T = m_2 a_x + m_2 g \quad \Rightarrow \quad m_1 g - (m_2 a_x + m_2 g) = m_1 a_x \quad \Rightarrow$$

$$(m_1 - m_2) g = (m_1 + m_2) a_x \quad \Rightarrow \quad \boxed{a_x = \frac{m_1 - m_2}{m_1 + m_2}\, g}$$

Atwood machine: write masses' equations of motion



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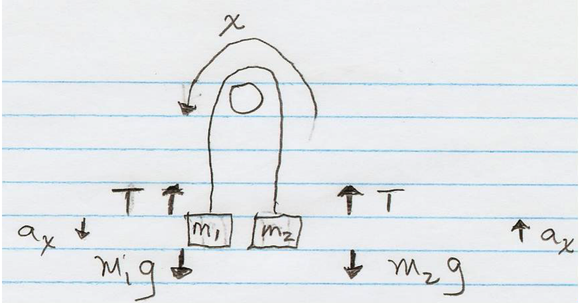
$$(m_1 - m_2) g = (m_1 + m_2) a_x \quad \Rightarrow \quad \boxed{a_x = \frac{m_1 - m_2}{m_1 + m_2}\, g}$$

For $m_2 = 0$, $\quad a_x = g \quad$ (just like picking up m_1 and dropping it)

For $m_1 \approx m_2$, $a_x \ll g$: small difference divided by large sum.

$$\boxed{a_x = \frac{m_1 - m_2}{m_1 + m_2}\ g}$$

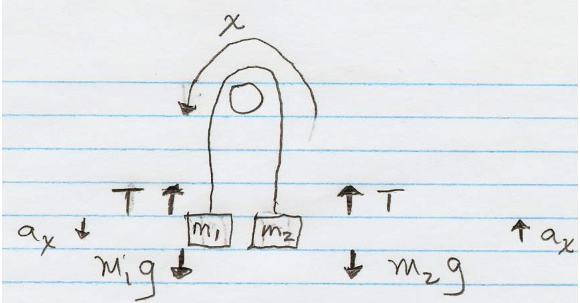


For example, $m_1 = 4.03\ \mathrm{kg}$, $m_2 = 3.73\ \mathrm{kg}$:

$$a_x = \frac{m_1 - m_2}{m_1 + m_2}\ g = \left(\frac{0.30\ \mathrm{kg}}{7.76\ \mathrm{kg}}\right)\left(9.8\ \mathrm{m/s^2}\right) = 0.38\ \mathrm{m/s^2}$$

How long does it take m_1 to fall 2 meters?

$$a_x = \frac{m_1 - m_2}{m_1 + m_2}\, g$$



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How long does it take m_1 to fall 2 meters?

$$x = \frac{a_x t^2}{2} \quad \Rightarrow \quad t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{(2)(2\ \mathrm{m})}{(0.38\ \mathrm{m/s^2})}} \approx 3.2\ \mathrm{s}$$

You can also solve for T if you like (eliminate a_x), to find the tension while the two masses are free to accelerate (no interaction with my hand or the floor).

Start from masses' equations of motion:

$$m_1 g - T = m_1 a_x, \qquad T - m_2 g = m_2 a_x$$

Eliminate a_x:

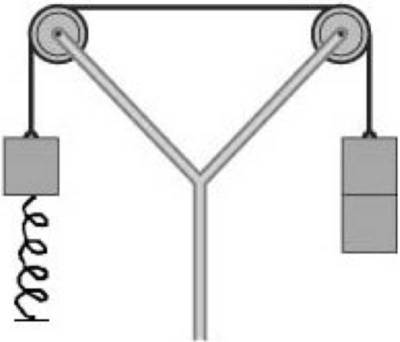
$$\frac{m_1 g - T}{m_1} = \frac{T - m_2 g}{m_2} \quad \Rightarrow \quad m_1 m_2 g - m_2 T = m_1 T - m_1 m_2 g$$

$$\Rightarrow 2 m_1 m_2 g = (m_1 + m_2) T \quad \Rightarrow \quad T = \frac{2 m_1 m_2}{m_1 + m_2}\, g$$

consider extreme cases: $m_2 = m_1$ vs. $m_2 \ll m_1$.

HW4 / problem 7: tricky!

7*. A modified Atwood machine is shown below. Each of the three blocks has the same inertia m. One end of the vertical spring, which has spring constant k, is attached to the single block, and the other end of the spring is fixed to the floor. The positions of the blocks are adjusted until the spring is at its **relaxed** length. The blocks are then released from rest. What is the acceleration of the two blocks on the right after they have fallen a distance D?



In the 17th century, Otto von Güricke, a physicist in Magdeburg, fitted two hollow bronze hemispheres together and removed the air from the resulting sphere with a pump. Two eight-horse teams could not pull the halves apart even though the hemispheres fell apart when air was readmitted. Suppose von Güricke had tied both teams of horses to one side and bolted the other side to a giant tree trunk. In this case, the tension on the hemispheres would be

(A) twice

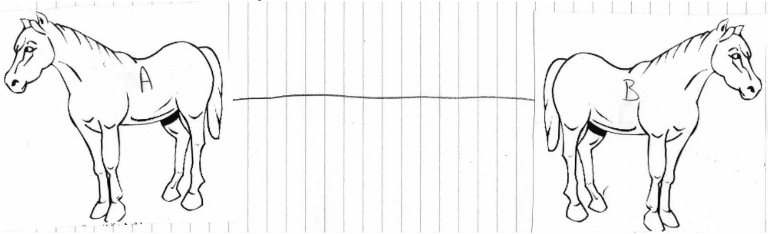
(B) exactly the same as

(C) half

what it was before.

(To avoid confusion, you can replace the phrase "the hemispheres" with the phrase "the cable" if you like. The original experiment was a demonstraton of air pressure, but we are interested in tension.)

Suppose a horse can pull 1000 N



$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

$$|\vec{F}_{A \text{ on } B}| = |\vec{F}_{B \text{ on } A}| = 1000 \text{ N}$$

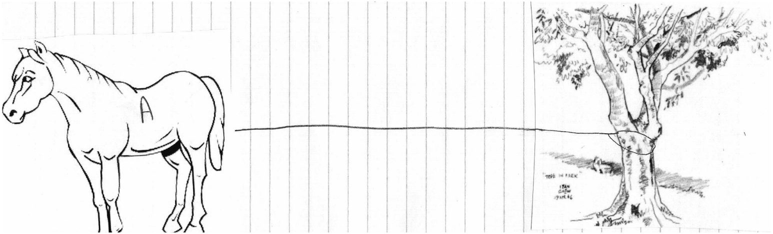
$$T = 1000 \text{ N}$$

$$\vec{a}_A = \vec{0}$$

$$\vec{a}_B = \vec{0}$$

The acceleration of each horse is zero. What are the two horizontal forces acting on horse A? What are the two horizontal forces acting on horse B?

Suppose tree stays put, no matter how hard horse pulls



$$\vec{F}_{A \text{ on tree}} = -\vec{F}_{\text{tree on } A}$$

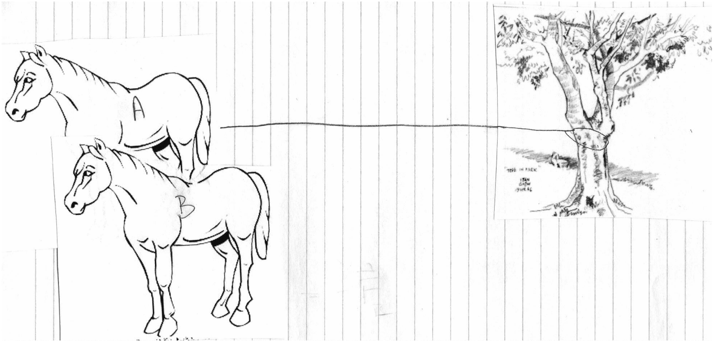
$$|\vec{F}_{A \text{ on tree}}| = |\vec{F}_{\text{tree on } A}| = 1000 \text{ N}$$

$$T = 1000 \text{ N}$$

$$\vec{a_A} = \vec{0}$$

What are the two horizontal forces acting on horse A?

Suppose tree stays put, no matter how hard horses pull. Somehow we attach both horses to the left end of the same cable.



$$\vec{F}_{A+B \text{ on tree}} = -\vec{F}_{\text{tree on } A+B}$$

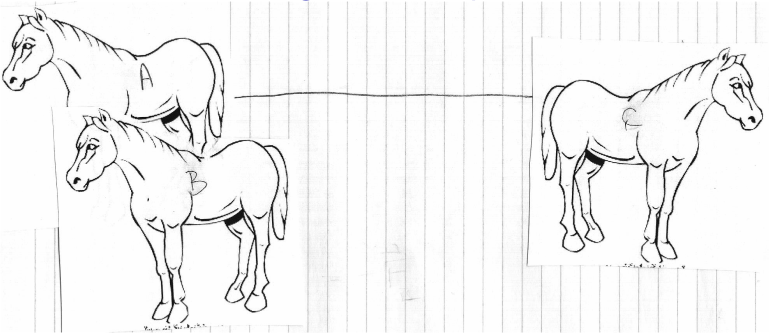
$$|\vec{F}_{A+B \text{ on tree}}| = |\vec{F}_{\text{tree on } A+B}| = 2000 \text{ N}$$

$$T = 2000 \text{ N}$$

$$\vec{a}_{\text{horses}A+B} = \vec{0}$$

What are the external forces acting on the two-horse system (system = horse A + horse B)?

Horse C loses his footing when he pulls > 1000 N



$$|\vec{F}_{A+B \text{ on C}}| = |\vec{F}_{\text{C on } A+B}| = 2000 \text{ N}$$

$$T = 2000 \text{ N}$$

Force of ground on C is 1000 N to the right. Tension pulls on C 2000 N to the left. C accelerates to the left.

$$|\vec{a_C}| = (2000 \text{ N} - 1000 \text{ N})/m_C$$

Estimate the spring constant of your car springs. (Experiment: sit on one fender.)

(What do you think?)

When a 5.0 kg box is suspended from a spring, the spring stretches to 1.0 m beyond its equilibrium length. In an elevator accelerating upward at 0.98 $\mathrm{m/s^2}$ (that's "0.1 g"), how far will the spring stretch with the same box attached? (Assume that the spring adjusts such that the box and the elevator have the same acceleration.)

(A) 0.50 m

(B) 0.90 m

(C) 1.0 m

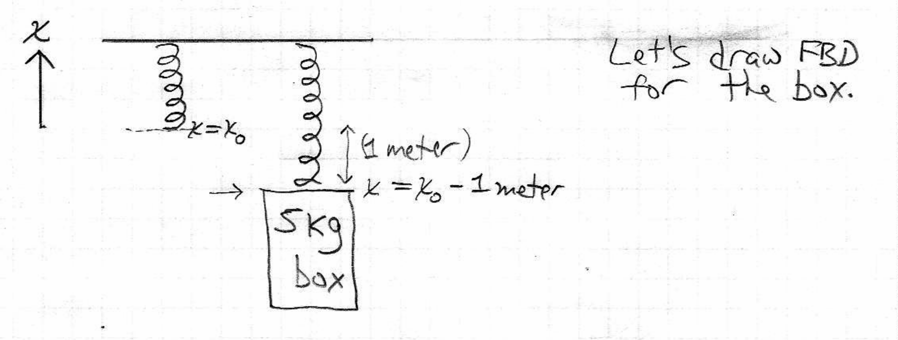
(D) 1.1 m

(E) 1.2 m

(F) 1.9 m

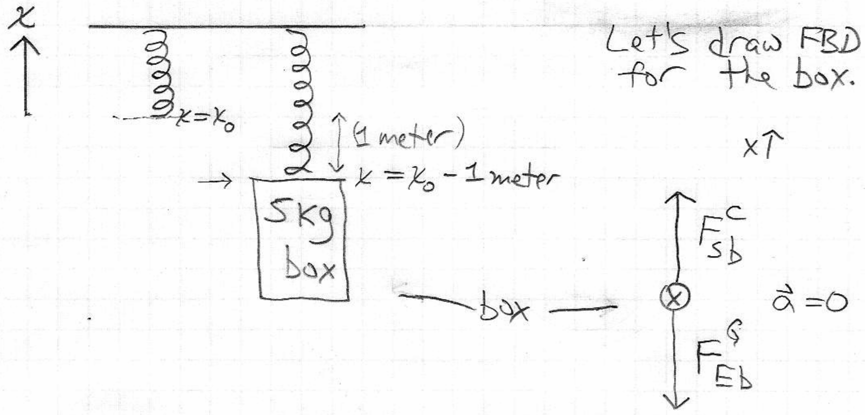
(G) 2.0 m

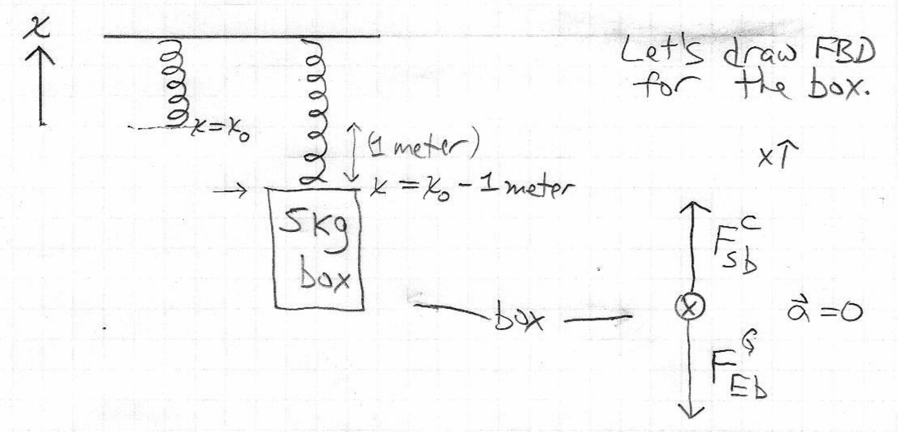
(By the way: When a tall building sways back and forth in the wind, the uncomfortable acceleration experienced by the occupants is often measured as a fraction of "g.")



Let's start by drawing a FBD for the box when the elevator is **not** accelerating.

Let's draw FBD for the box.



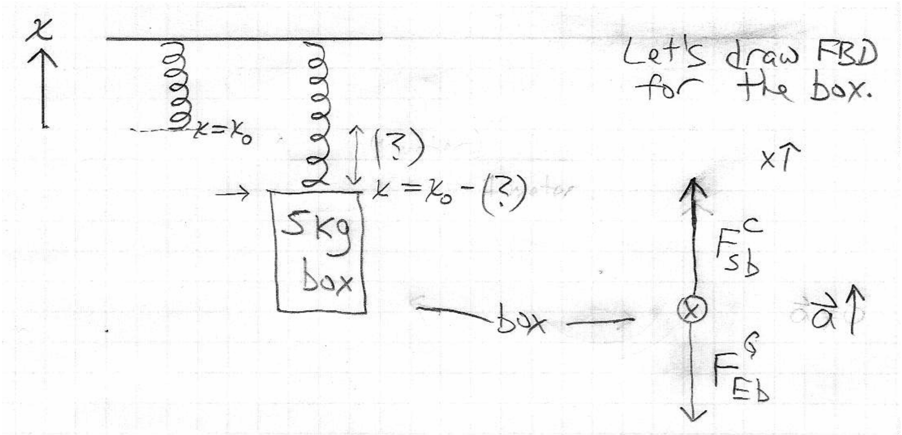


$$F^c_{sb,x} + F^G_{Eb,x} = ma_x = 0$$

$$F^c_{sb,x} = -k\,(x - x_0) = -k(-1\ \text{meter}) \qquad F^G_{Eb,x} = -mg$$

$$+k(1\ \text{meter}) - mg = ma_x = 0$$

Next, what happens if elevator is accelerating upward at $1\ \text{m/s}^2$?



$$F^c_{sb,x} + F^G_{Eb,x} = ma_x = +1\ \mathrm{m/s^2}$$

$$F^c_{sb,x} = -k\,(x - x_0) \qquad F^G_{Eb,x} = -mg$$

$$-k(x - x_0) - mg = ma_x = +0.1mg$$

combine with $+k(1\ \mathrm{meter}) - mg = 0$ from last page

$$-k(x - x_0) - mg = ma_x = +0.1g \quad \Rightarrow \quad \boxed{-k(x - x_0) = +1.1mg}$$

combine with $+k(1\ \text{meter}) - mg = 0 \quad \Rightarrow \quad \boxed{+k(1\ \text{meter}) = mg}$

Divide two boxed equations: get $\quad x - x0 = -1.1\ \text{meters}$

So the spring is now stretching 1.1 meters beyond its relaxed length (vs. 1.0 meters when $a_x = 0$).

The upward force exerted by the spring on the box is $\quad m(g + a_x)$.

When a 5.0 kg box is suspended from a spring, the spring stretches to 1.0 m beyond its equilibrium length. In an elevator accelerating upward at 0.98 $\mathrm{m/s^2}$, how far will the spring stretch with the same box attached? (Assume that the spring adjusts such that the box and the elevator have the same acceleration.)

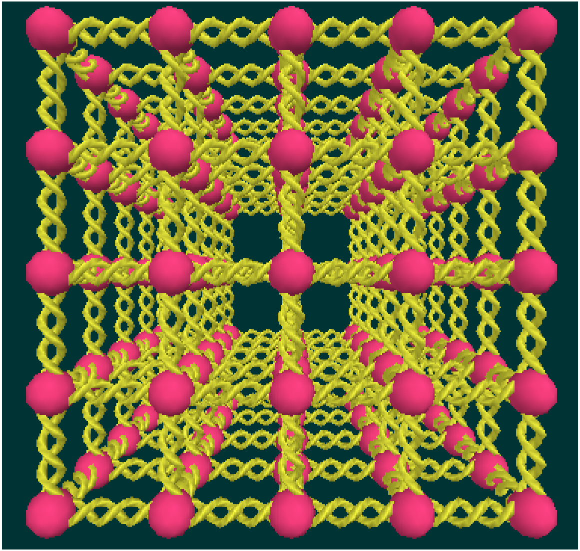
(A) 0.50 m
(B) 0.90 m
(C) 1.0 m
(D) 1.1 m
(E) 1.2 m
(F) 1.9 m
(G) 2.0 m

(Begin digression.)

Dissipative / incoherent / irreversible

A simple ball / spring model of the atoms in a solid.

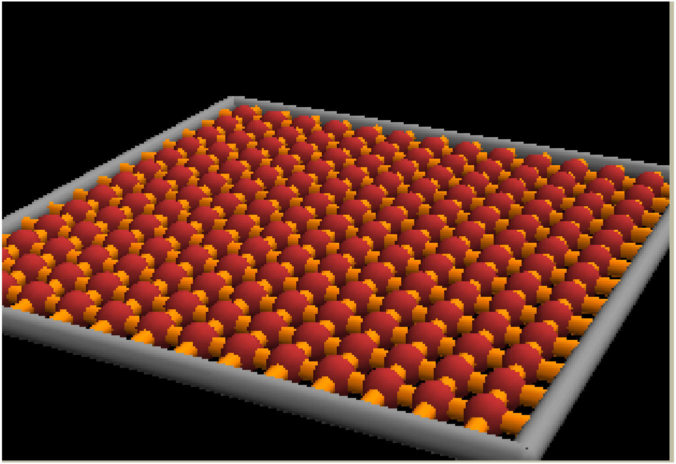
This is sometimes a useful picture to keep in your head.



Dissipative / incoherent / irreversible

2D version for simplicity

illustrate "reversible" and "irreversible" deformation with e.g. marbles and egg crate



Dissipative / incoherent / irreversible

I showed you once before my low-tech animation of two objects in a totally inelastic collision. Collision dissipates coherent motion (kinetic energy) into incoherent vibration of atoms (thermal energy)

`https://youtu.be/SJIKCmg2Uzg`

Here's a high-speed movie of a (mostly) reversible process a golf ball bouncing off of a wall at 150 mph.

https://www.youtube.com/watch?v=AkB81u5IM3I

(End digression.)

Physics 8 — Monday, September 23, 2019

- For today, you read Ch9 (work). For Wednesday (though we won't get to it until next week), you'll read the first half of Ch10 (motion in a plane).
- If you weren't here last Wed/Fri, pick up your "clicker" with your graded HW2 paper. We'll try them today!
- If you have little or no coding experience and you're interested in an XC option to learn Python for quantitative tasks like graphing and modeling data, email me ASAP.
- Wolfram Mathematica is free (site license) for SAS and Wharton students. I have some very helpful self-study Mathematica materials you can do for XC. Email if interested.
- If you're interested in learning to do a bit of Python coding in a drawing/animation system called "Processing" made by and for visual artists, you can look at my Fall 2017 day-before-Thanksgiving lecture here: http://xray.hep.upenn.edu/wja/p008/2017/files/phys8_notes_20171122.pdf