# Physics 8 — Wednesday, September 25, 2019

- Remember homework #4 due this Friday, at the start of class. It covers Chapters 6, 7, 8.
- Homework study/help sessions: Greg is in DRL 3C4 Wednesdays 4–6pm. Bill is in DRL 2C4 Thursdays 6–8pm.
- For today (though we won't get to it until next week), you read the first half of Ch10 (motion in a plane).
- Thanks to Jerod B, here's a neat video showing that the CoM of a dropped slinky falls at acceleration $g$, even though the top and bottom of the slinky do not move in unison:
  `https://www.youtube.com/watch?v=eCMmmEEyOO0&t=43`
  super-sized version (harder to see than original version):
  `https://www.youtube.com/watch?v=JsytnJ_pSf8&t=88`

# Since Monday, we are finally talking about forces

- The **force** concept quantifies interaction between two objects.
- Forces always come in "**interaction pairs**." The force exerted by object "A" on object "B" is equal in magnitude and opposite in direction to the force exerted by B on A:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

- The acceleration of object "A" is given by the vector sum of the forces acting **on** A, divided by the mass of A:

$$\vec{a}_A = \frac{\sum \vec{F}_{(\mathrm{on\ A})}}{m_A}$$

- The vector sum of the forces acting **on** an object equals the rate of change of the object's momentum:

$$\sum \vec{F}_{(\mathrm{on\ A})} = \frac{\mathrm{d}\vec{p}_A}{\mathrm{d}t}$$

- An object whose momentum is not changing is in translational **equilibrium**. We'll see later that this will be a big deal for the members of a structure! To achieve this, we will want all forces acting **on** each member to sum vectorially to zero.
- The unit of force is the **newton**. $1\ \mathrm{N} = 1\ \mathrm{kg} \cdot \mathrm{m/s^2}$.
- **Free-body diagrams** depict all of the forces acting **on** a given object. They are used all the time in analyzing structures!

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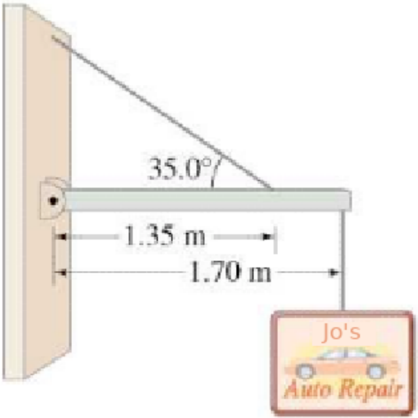
- The force exerted by a compressed or stretched spring is proportional to the displacement of the end of the spring w.r.t. its relaxed value $x_0$. $k$ is "spring constant."

$$F_x^{\mathrm{spring}} = -k(x - x_0)$$

- When a rope is held taut, it exerts a force called the **tension** on each of its ends. Same magnitude $T$ on each end.

# Tension in cables (repeated from Monday)

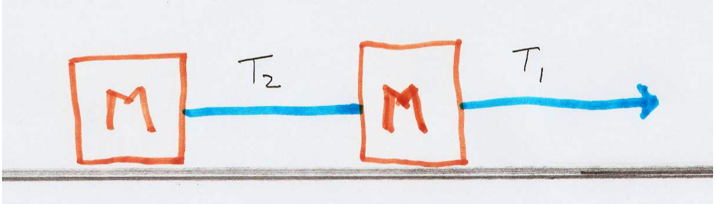
- A large category of physics problems (and even architectural structures, e.g. a suspension bridge) involves two objects connected by a rope, a cable, a chain, etc.
- These things (cables, chains, ropes) can pull but can't push. There are two cables in this figure:



# Tension in cables

- Usually the cables in physics problems are considered light enough that you don't worry about their inertia (we pretend $m = 0$), and stiff enough that you don't worry about their stretching when you pull on them (we pretend $k = \infty$).
- The cable's job is just to transmit a force from one end to the other. We call that force the cable's *tension*, $T$.
- A cable always pulls on both ends with same magnitude ($T$), though in opposite directions. [Formally: we neglect the cable's mass, and the cable's acceleration must be finite.]
- (We stopped here on Monday.)
- E.g. hang basketball from ceiling. Cable transmits $mg$ to ceiling. Gravity pulls ball down. Tension pulls ball up. Forces on ball add (vectorially) to zero.
- Let's try an example.

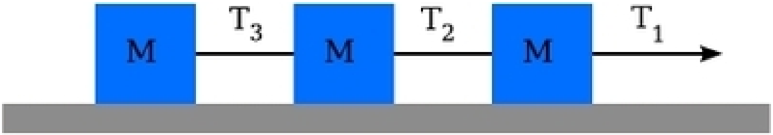
Two blocks of equal mass are pulled to the right by a constant force, which is applied by pulling at the arrow-tip on the right. The blue lines represent two identical sections of rope (which can be considered massless). Both cables are taut, and friction (if any) is the same for both blocks. What is the ratio of $T_1$ to $T_2$?




(A) zero: $T_1 = 0$ and $T_2 \neq 0$.
(B) $T_1 = \frac{1}{2} T_2$
(C) $T_1 = T_2$
(D) $T_1 = 2 T_2$
(E) infinite: $T_2 = 0$ and $T_1 \neq 0$.

It's worth drawing an FBD first for the two-mass system, then for the left mass, then for the right mass.

Three blocks of equal mass are pulled to the right by a constant force. The blocks are connected by identical sections of rope (which can be considered massless). All cables are taut, and friction (if any) is the same for all blocks. What is the ratio of $T_1$ to $T_2$?




(A) $T_1 = \frac{1}{3} T_2$

(B) $T_1 = \frac{2}{3} T_2$

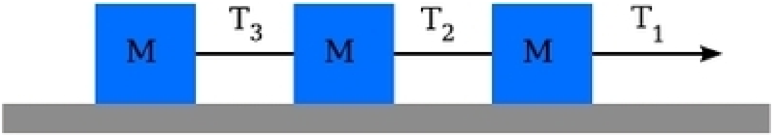
(C) $T_1 = T_2$

(D) $T_1 = \frac{3}{2} T_2$

(E) $T_1 = 2 T_2$

(F) $T_1 = 3 T_2$

Three blocks of equal mass are pulled to the right by a constant force. The blocks are connected by identical sections of rope (which can be considered massless). All cables are taut, and friction (if any) is the same for all blocks. What is the ratio of $T_1$ to $T_3$?

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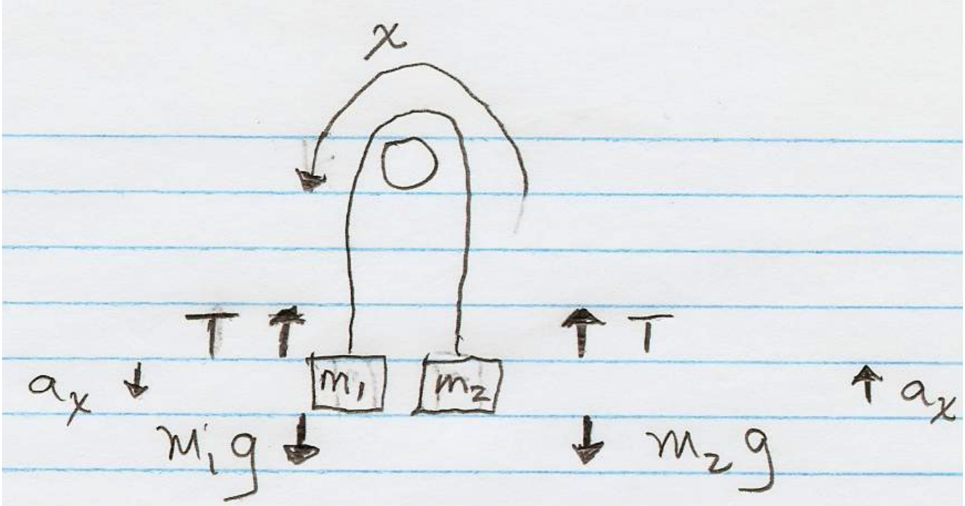
# Atwood machine — discuss with your neighbors

A contraption something like this appears in HW4 (but with a spring added, to keep things interesting).

- Why aren't the two masses accelerating?
- What is the tension in the cable when the two masses are equal (both 5.0 kg) and stationary, as they are now?
- If I make one mass equal 5.0 kg and the other mass equal 5.1 kg, what will happen? Can you predict what the acceleration will be?
- If I make one mass equal 5.0 kg and the other mass equal 6.0 kg, will the acceleration be larger or smaller than in the previous case?
- Try drawing a free-body diagram for each of the two masses
- By how much do I change the gravitational potential energy of the machine+Earth system when I raise the 6 kg mass 1 m?

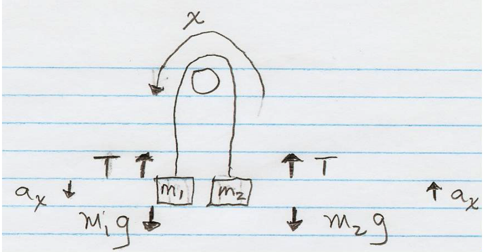
- Two more comments:
- This machine was originally invented as a mechanism for measuring $g$ and for studying motion with constant acceleration.
- The same concept is used by the “counterweight” in an elevator for a building.

# Atwood machine: take $m_1 > m_2$




Pause here: how can we solve for $a_x$? Try it before we go on.

# Atwood machine: write masses' equations of motion




$$m_1 g - T = m_1 a_x$$

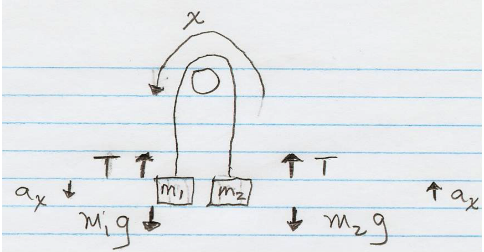
$$T - m_2 g = m_2 a_x$$

Solve second equation for $T$; plug $T$ into first equation; solve for $a_x$:

$$T = m_2 a_x + m_2 g \quad \Rightarrow \quad m_1 g - (m_2 a_x + m_2 g) = m_1 a_x \quad \Rightarrow$$

$$(m_1 - m_2) g = (m_1 + m_2) a_x \quad \Rightarrow \quad \boxed{a_x = \frac{m_1 - m_2}{m_1 + m_2}\, g}$$

# Atwood machine: write masses' equations of motion

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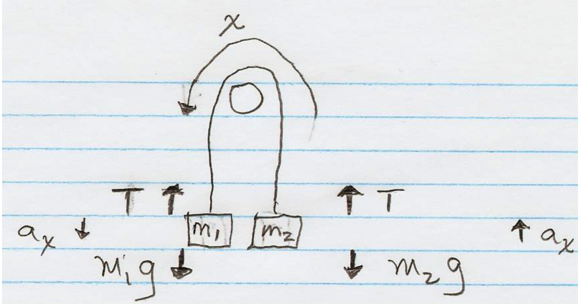
For $m_2 = 0$, $\quad a_x = g \quad$ (just like picking up $m_1$ and dropping it)

For $m_1 \approx m_2$, $a_x \ll g$: small difference divided by large sum.

$$\boxed{a_x = \frac{m_1 - m_2}{m_1 + m_2}\ g}$$

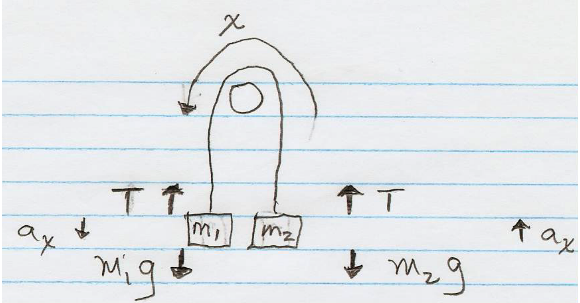



For example, $m_1 = 4.03\ \mathrm{kg}$, $m_2 = 3.73\ \mathrm{kg}$:

$$a_x = \frac{m_1 - m_2}{m_1 + m_2}\ g = \left(\frac{0.30\ \mathrm{kg}}{7.76\ \mathrm{kg}}\right)\left(9.8\ \mathrm{m/s^2}\right) = 0.38\ \mathrm{m/s^2}$$

How long does it take $m_1$ to fall 2 meters?

$$\boxed{a_x = \frac{m_1 - m_2}{m_1 + m_2}\, g}$$

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How long does it take $m_1$ to fall 2 meters?

$$x = \frac{a_x t^2}{2} \quad \Rightarrow \quad t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{(2)(2\ \mathrm{m})}{(0.38\ \mathrm{m/s^2})}} \approx 3.2\ \mathrm{s}$$

You can also solve for $T$ if you like (eliminate $a_x$), to find the tension while the two masses are free to accelerate (no interaction with my hand or the floor).

Start from masses' equations of motion:

$$m_1 g - T = m_1 a_x, \qquad T - m_2 g = m_2 a_x$$

Eliminate $a_x$:

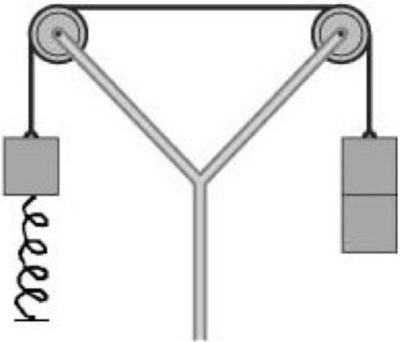
$$\frac{m_1 g - T}{m_1} = \frac{T - m_2 g}{m_2} \quad \Rightarrow \quad m_1 m_2 g - m_2 T = m_1 T - m_1 m_2 g$$

$$\Rightarrow 2 m_1 m_2 g = (m_1 + m_2) T \quad \Rightarrow \quad T = \frac{2 m_1 m_2}{m_1 + m_2}\, g$$

consider extreme cases: $m_2 = m_1$ vs. $m_2 \ll m_1$.

# HW4 / problem 7: tricky!

7*. A modified Atwood machine is shown below. Each of the three blocks has the same inertia $m$. One end of the vertical spring, which has spring constant $k$, is attached to the single block, and the other end of the spring is fixed to the floor. The positions of the blocks are adjusted until the spring is at its **relaxed** length. The blocks are then released from rest. What is the acceleration of the two blocks on the right after they have fallen a distance $D$?



(we stopped before this.)

In the 17th century, Otto von Güricke, a physicist in Magdeburg, fitted two hollow bronze hemispheres together and removed the air from the resulting sphere with a pump. Two eight-horse teams could not pull the halves apart even though the hemispheres fell apart when air was readmitted. Suppose von Güricke had tied both teams of horses to one side and bolted the other side to a giant tree trunk. In this case, the tension on the hemispheres would be

(A) twice

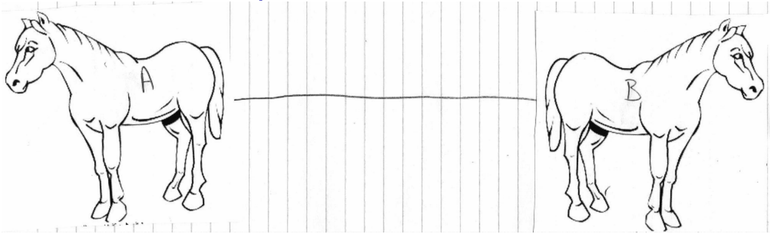
(B) exactly the same as

(C) half

what it was before.

(To avoid confusion, you can replace the phrase "the hemispheres" with the phrase "the cable" if you like. The original experiment was a demonstraton of air pressure, but we are interested in tension.)

# Suppose a horse can pull 1000 N




$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

$$|\vec{F}_{A \text{ on } B}| = |\vec{F}_{B \text{ on } A}| = 1000 \text{ N}$$

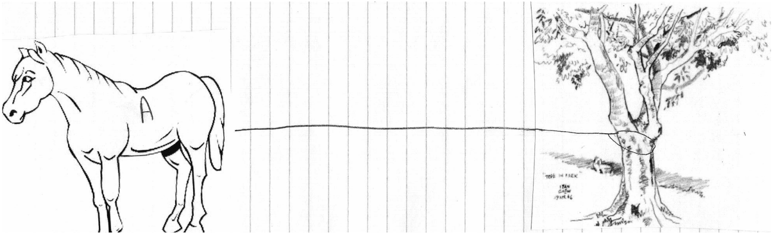
$$T = 1000 \text{ N}$$

$$\vec{a}_A = \vec{0}$$

$$\vec{a}_B = \vec{0}$$

The acceleration of each horse is zero. What are the two horizontal forces acting on horse A? What are the two horizontal forces acting on horse B?

# Suppose tree stays put, no matter how hard horse pulls




$$\vec{F}_{A \text{ on tree}} = -\vec{F}_{\text{tree on } A}$$

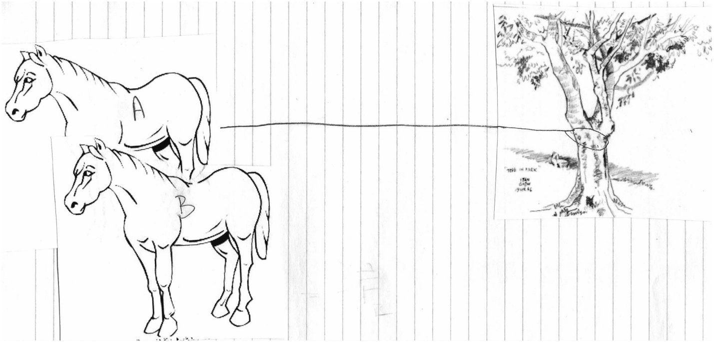
$$|\vec{F}_{A \text{ on tree}}| = |\vec{F}_{\text{tree on } A}| = 1000 \text{ N}$$

$$T = 1000 \text{ N}$$

$$\vec{a_A} = \vec{0}$$

What are the two horizontal forces acting on horse A?

Suppose tree stays put, no matter how hard horses pull. Somehow we attach both horses to the left end of the same cable.




$$\vec{F}_{A+B \text{ on tree}} = -\vec{F}_{\text{tree on } A+B}$$

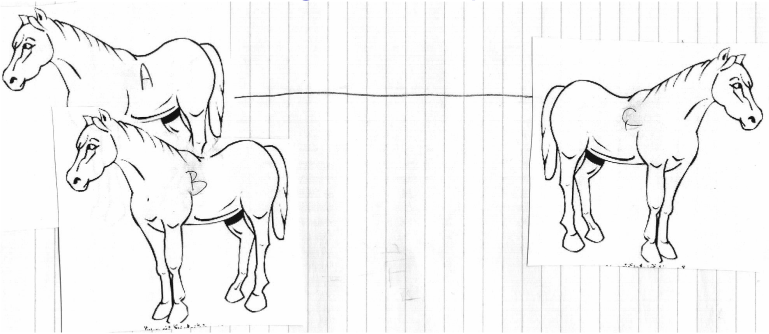
$$|\vec{F}_{A+B \text{ on tree}}| = |\vec{F}_{\text{tree on } A+B}| = 2000 \text{ N}$$

$$T = 2000 \text{ N}$$

$$\vec{a}_{\text{horses} A+B} = \vec{0}$$

What are the external forces acting on the two-horse system (system = horse A + horse B)?

# Horse $C$ loses his footing when he pulls $> 1000$ N




$$|\vec{F}_{A+B \text{ on C}}| = |\vec{F}_{\text{C on } A+B}| = 2000 \text{ N}$$

$$T = 2000 \text{ N}$$

Force of ground on $C$ is 1000 N to the right. Tension pulls on $C$ 2000 N to the left. C accelerates to the left.

$$|\vec{a_C}| = (2000 \text{ N} - 1000 \text{ N})/m_C$$

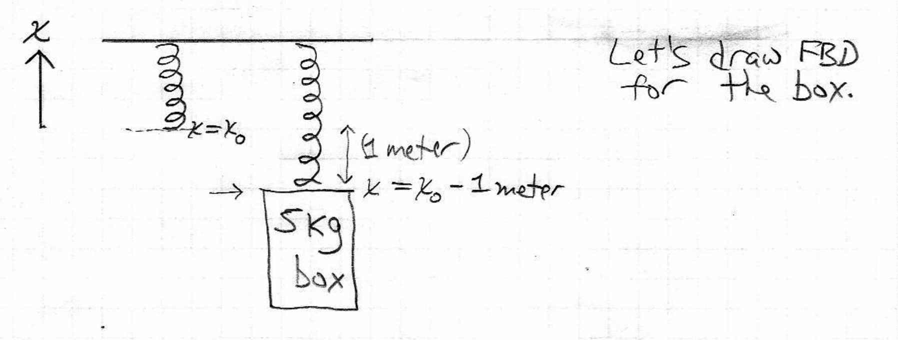
Estimate the spring constant of your car springs. (Experiment: sit on one fender.)

(What do you think?)

When a 5.0 kg box is suspended from a spring, the spring stretches to 1.0 m beyond its equilibrium length. In an elevator accelerating upward at 0.98 $\mathrm{m/s^2}$ (that's "0.1 $g$"), how far will the spring stretch with the same box attached? (Assume that the spring adjusts such that the box and the elevator have the same acceleration.)

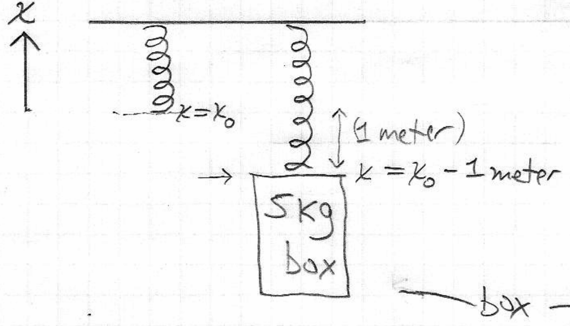
(A) 0.50 m
(B) 0.90 m
(C) 1.0 m
(D) 1.1 m
(E) 1.2 m
(F) 1.9 m
(G) 2.0 m

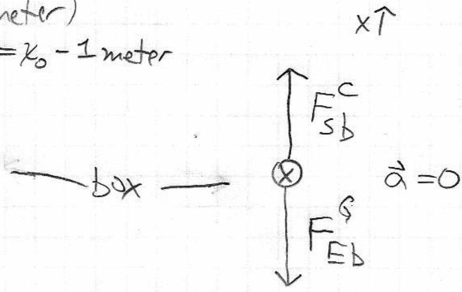
(By the way: When a tall building sways back and forth in the wind, the uncomfortable acceleration experienced by the occupants is often measured as a fraction of "$g$.")

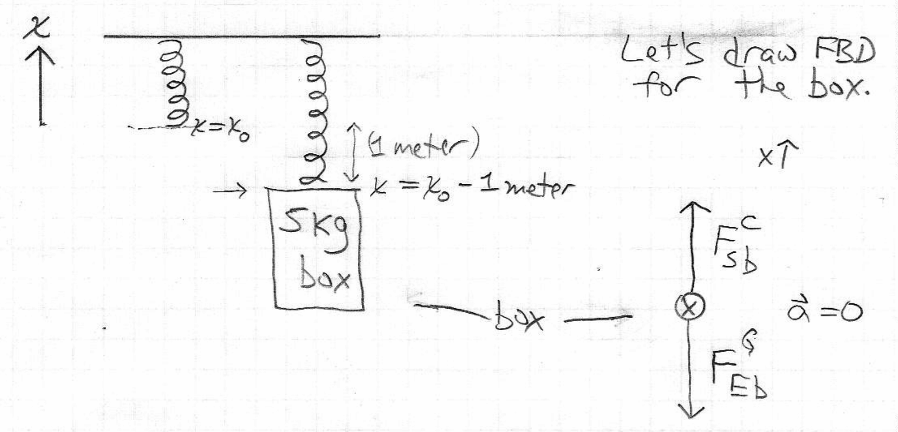



Let’s start by drawing a FBD for the box when the elevator is **not** accelerating.

Let's draw FBD for the box.



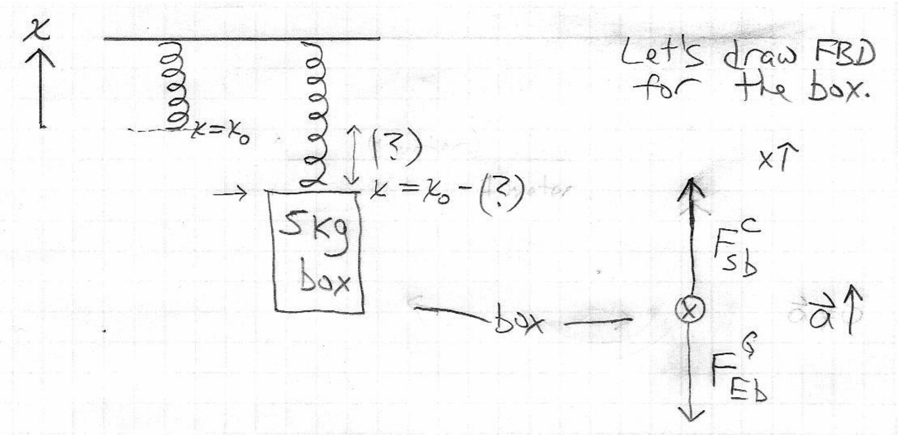



$$F^c_{sb,x} + F^G_{Eb,x} = ma_x = 0$$

$$F^c_{sb,x} = -k\,(x - x_0) = -k(-1 \text{ meter}) \qquad F^G_{Eb,x} = -mg$$

$$+k(1 \text{ meter}) - mg = ma_x = 0$$

Next, what happens if elevator is accelerating upward at $1\ \mathrm{m/s^2}$?




$$F^c_{sb,x} + F^G_{Eb,x} = ma_x = +1\ \mathrm{m/s^2}$$

$$F^c_{sb,x} = -k\,(x - x_0) \qquad F^G_{Eb,x} = -mg$$

$$-k(x - x_0) - mg = ma_x = +0.1mg$$

combine with $+k(1\ \mathrm{meter}) - mg = 0$ from last page

$$-k(x - x_0) - mg = ma_x = +0.1g \quad \Rightarrow \quad \boxed{-k(x - x_0) = +1.1mg}$$

combine with $+k(1 \text{ meter}) - mg = 0 \quad \Rightarrow \quad \boxed{+k(1 \text{ meter}) = mg}$

Divide two boxed equations: get $\quad x - x0 = -1.1 \text{ meters}$

So the spring is now stretching 1.1 meters beyond its relaxed length (vs. 1.0 meters when $a_x = 0$).

The upward force exerted by the spring on the box is $\quad m(g + a_x)$.

When a 5.0 kg box is suspended from a spring, the spring stretches to 1.0 m beyond its equilibrium length. In an elevator accelerating upward at 0.98 $\mathrm{m/s^2}$, how far will the spring stretch with the same box attached? (Assume that the spring adjusts such that the box and the elevator have the same acceleration.)

(A) 0.50 m
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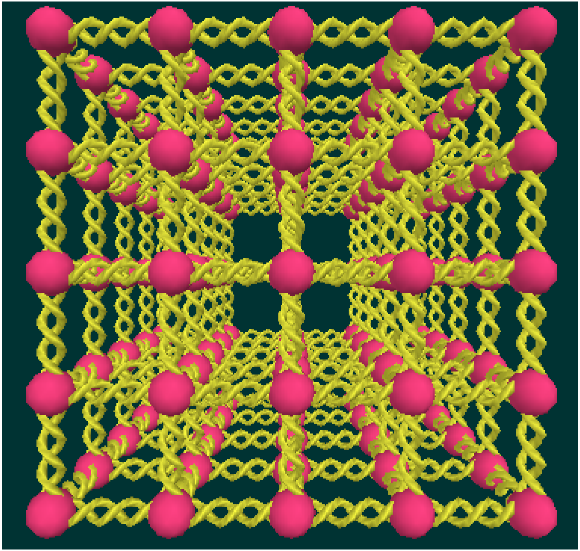
(F) 1.9 m
(G) 2.0 m

(Begin digression.)

# Dissipative / incoherent / irreversible

A simple ball / spring model of the atoms in a solid.

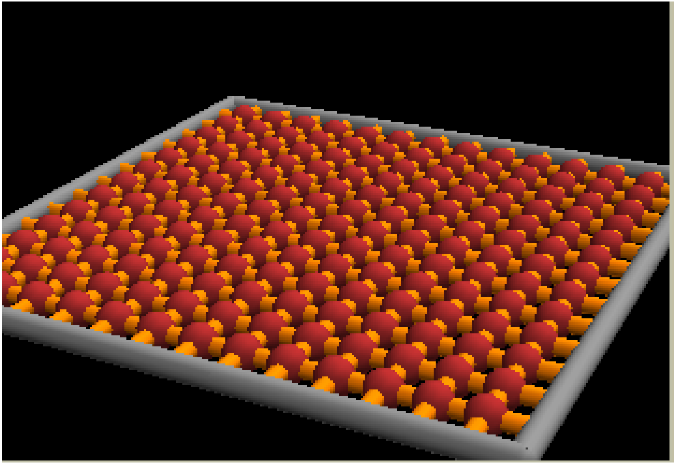
This is sometimes a useful picture to keep in your head.



# Dissipative / incoherent / irreversible

2D version for simplicity

*illustrate "reversible" and "irreversible" deformation with e.g. marbles and egg crate*



# Dissipative / incoherent / irreversible

I showed you once before my low-tech animation of two objects in a totally inelastic collision. Collision dissipates coherent motion (kinetic energy) into incoherent vibration of atoms (thermal energy)

`https://youtu.be/SJIKCmg2Uzg`

Here's a high-speed movie of a (mostly) reversible process a golf ball bouncing off of a wall at 150 mph.


https://www.youtube.com/watch?v=AkB81u5IM3I

(End digression.)

# Physics 8 — Wednesday, September 25, 2019

- Remember homework #4 due this Friday, at the start of class. It covers Chapters 6, 7, 8.
- Homework study/help sessions: Greg is in DRL 3C4 Wednesdays 4–6pm. Bill is in DRL 2C4 Thursdays 6–8pm.
- For today (though we won't get to it until next week), you read the first half of Ch10 (motion in a plane).
- If you have little or no coding experience and you're interested in an XC option to learn Python for quantitative tasks like graphing and modeling data, email me ASAP.
- Wolfram Mathematica is free (site license) for SAS and Wharton students. I have some very helpful self-study Mathematica materials you can do for XC. Email if interested.
- If you're interested in learning to do a bit of Python coding in a drawing/animation system called "Processing" made by and for visual artists, you can look at my Fall 2017 day-before-Thanksgiving lecture here: `http://xray.hep.upenn.edu/wja/p008/2017/files/phys8_notes_20171122.pdf`