Physics 8 — Monday, September 30, 2019

- For today, you finished reading Ch10 (motion in a plane).
- The next few chapters (10,11,12) are the most difficult material in the course. We will slow down for them. After that, the fun begins: we can apply our knowledge of forces (ch8), vectors (ch10), and torque (ch12) to structures.
- You probably noticed by now that I try my best to motivate you to spend time each week reading, working checkpoints, and solving problems. I give you a significant amount of work to do. By **doing** all of the work, you will learn a lot, and you will do very well in the course. That's the bargain we offer.
- I'm not allowed to have any required work due today, so if you "click" today, I'll use it to fill in one missed class from elsewhere in the term. And there is no penalty for turning in today's reading by Wednesday — though your having first read chapter 10 will be a big help to your understanding what we do in class today.

Fall break is just over a week away. Your **other** courses are subjecting you to midterm exams and studio critiques. Last weekend, you did your hardest physics homework set so far.

So let's do something fun for most of the hour, to get started on Chapter 10: projectile motion in 2 dimensions!

If there's time left over at the end, we can go back to talking about work and the rest of Chapter 9.

Let's start with the familiar "ball-popper" cart

New (ch10): use two coordinate axes. In most cases, make y-axis point upward (vertical), and x-axis point to the right (horizontal).

Vertical equation of motion ($a_y = -g$ is constant):

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Vertical equation of motion ($a_y = -g$ is constant):

$$y = y_i + v_{i,y}t - \frac{1}{2}gt^2$$

$$v_y = v_{i,y} - gt$$

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Horizontal equation of motion ($v_x = v_{i,x}$ is constant):

$$x = x_i + v_{i,x} t$$

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Vertical equation of motion ($a_y = -g$ is constant):

$$y = y_i + v_{i,y}t - \frac{1}{2}gt^2$$

$$v_y = v_{i,y} - gt$$

Horizontal equation of motion ($v_x = v_{i,x}$ is constant):

$$x = x_i + v_{i,x}t$$

If you let $x_i = 0$ (simpler) and solve horizontal eqn. for t, you get

$$t = \frac{x}{v_{i,x}}$$

Now plug this into the equation for y ...

$$y = y_i + v_{i,y} t - \frac{1}{2} g t^2$$

Now plug $t = \frac{x}{v_{i,x}}$ into the equation for y:

$$y = y_i + v_{i,y} \left(\frac{x}{v_{i,x}} \right) - \frac{1}{2} g \left(\frac{x}{v_{i,x}} \right)^2$$

$$y = y_i + v_{i,y} t - \frac{1}{2} g t^2$$

Now plug $t = \frac{x}{v_{i,x}}$ into the equation for y:

$$y = y_i + v_{i,y} \left(\frac{x}{v_{i,x}} \right) - \frac{1}{2} g \left(\frac{x}{v_{i,x}} \right)^2$$

Separate out the constants to see that $y(x)$ is a parabola:

$$y = y_i + \left(\frac{v_{i,y}}{v_{i,x}} \right) x - \left(\frac{g}{2 v_{i,x}^2} \right) x^2$$

(You can "see" this either by drawing a graph or by happening to remember from math that $y = Ax^2 + Bx + C$ is a parabola.)

$$y = y_i + v_{i,y}t - \frac{1}{2}gt^2$$

Now plug $t = \frac{x}{v_{i,x}}$ into the equation for y:

$$y = y_i + v_{i,y}\left(\frac{x}{v_{i,x}}\right) - \frac{1}{2}g\left(\frac{x}{v_{i,x}}\right)^2$$

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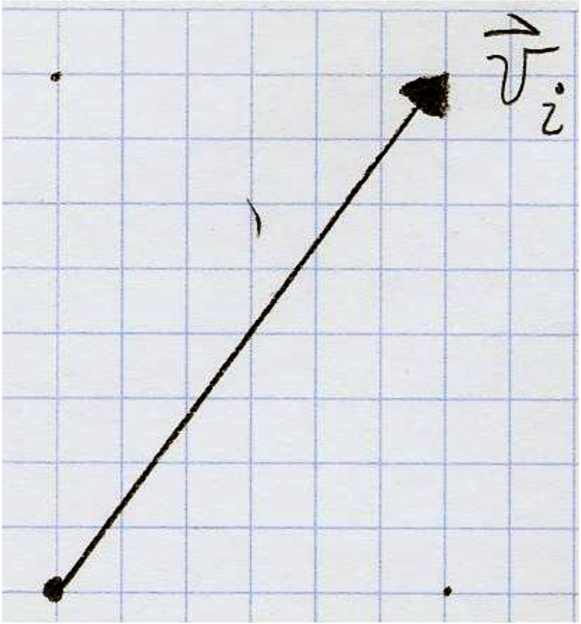
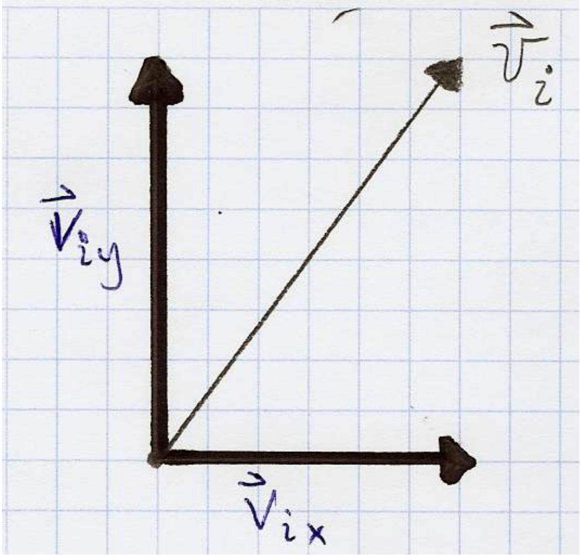
(You can "see" this either by drawing a graph or by happening to remember from math that $y = Ax^2 + Bx + C$ is a parabola.)

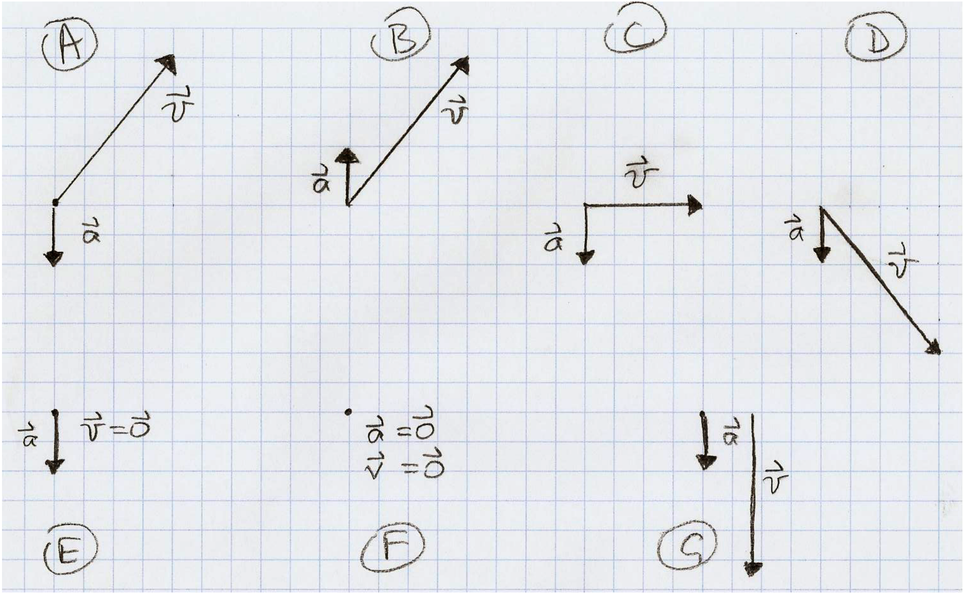
Let's draw and "decompose" the velocity vector at the moment the ball is launched from the cart.



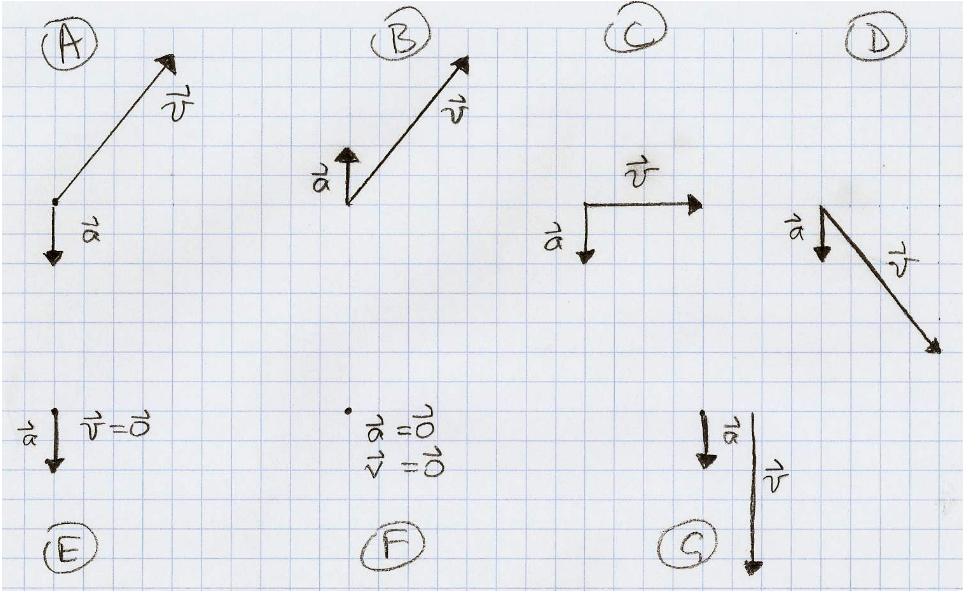
Now decompose into x and y components . . .



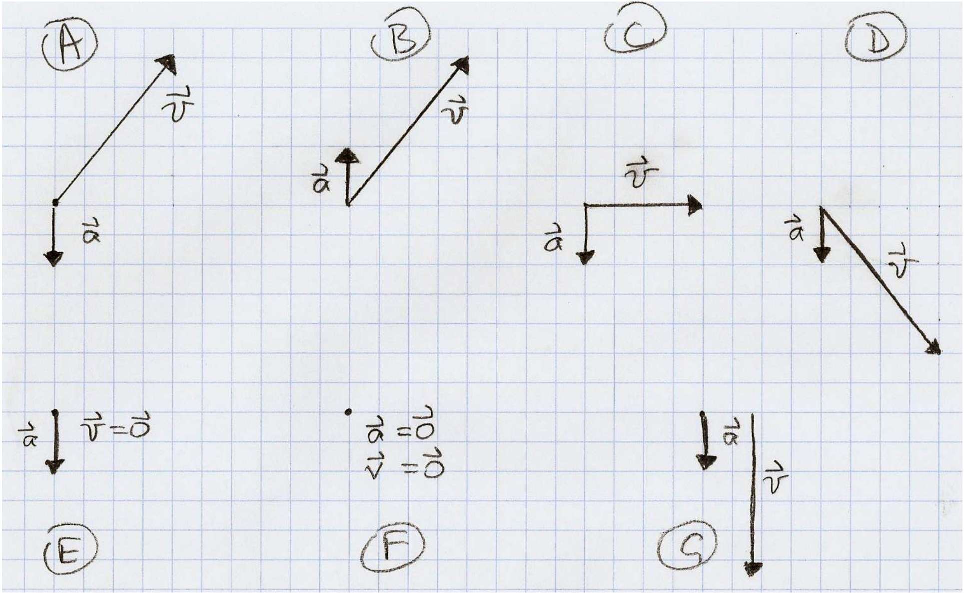
Notice (blackboard) that adding the two components together gives back the original vector.



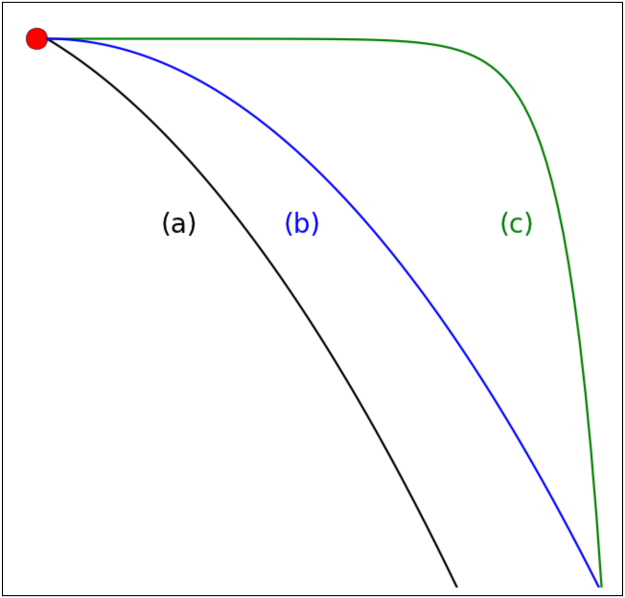
Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ the instant **after** the ball is launched from the cart?



Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ at the top of the ball's trajectory?

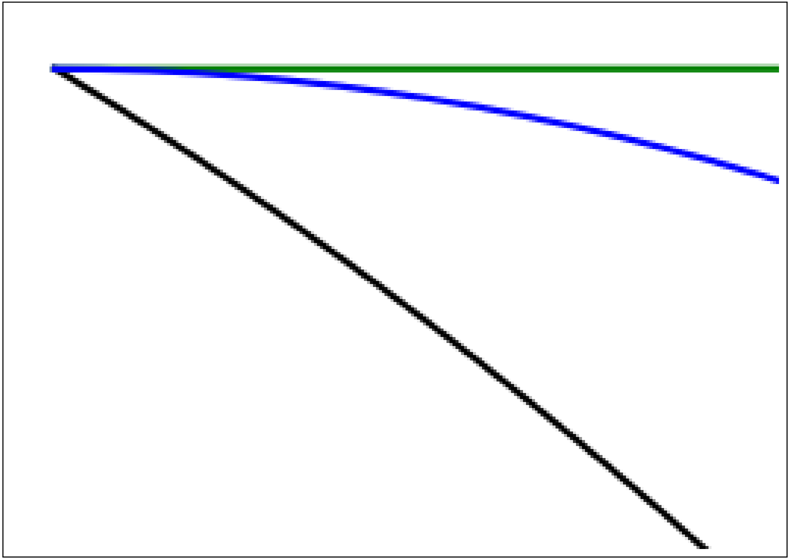


Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ the instant before the ball lands in the cart?



Which path best represents the trajectory of a cantaloupe *thrown horizontally* off a bridge? (What's wrong with the other two?)
(Next slide zooms in on corner.)

zoom in on top-left corner (launch position)



Which path best represents the trajectory of a cantaloupe *thrown horizontally* off a bridge? (What's wrong with the other two?)

Two steel balls are released simultaneously from the same height above the ground.

One ball is simply dropped (zero initial velocity).

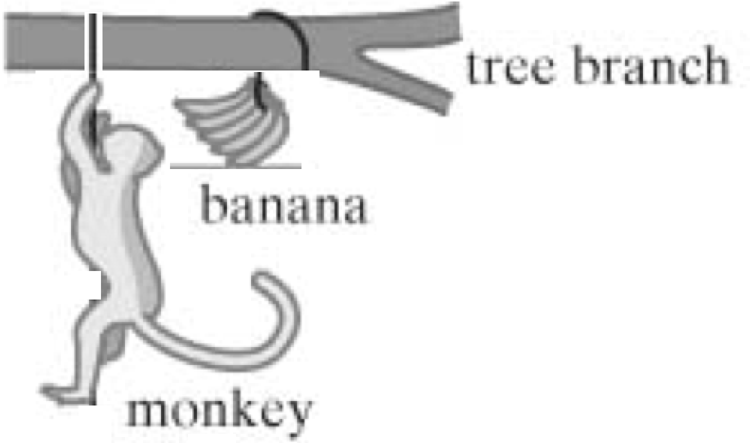
The other ball is thrown horizontally (initial velocity is nonzero, but is purely horizontal).

Which ball will hit the ground first?

(A) The ball thrown horizontally will hit the ground first.
(B) The ball released from rest will hit the ground first.
(C) Both balls will hit the ground at the same time.

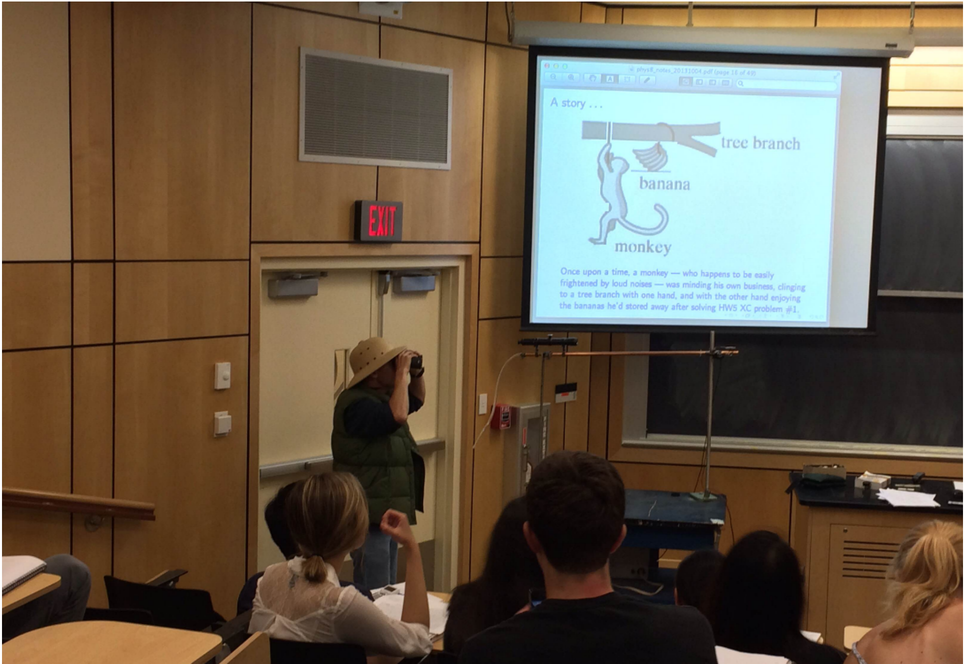
(I should draw a picture of both trajectories on the board.)

A story . . .



Once upon a time, a monkey — who happened to be easily frightened by loud noises — was minding his own business, clinging to a tree branch with one hand, and with the other hand enjoying the bananas he'd stored away after solving HW4 XC problem #7.

Look out . . .

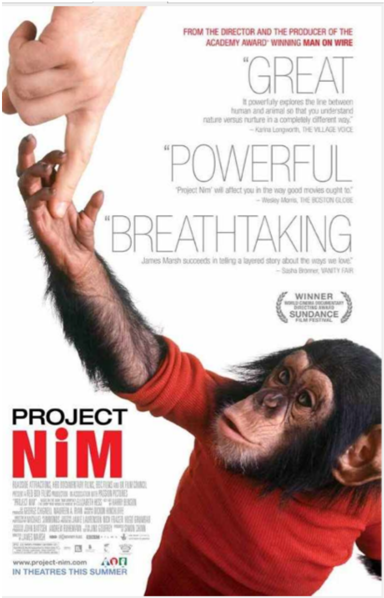

EXIT
A story . . .
tree branch
banana
monkey
Once upon a time, a monkey — who happens to be easily frightened by loud noises — was minding his own business, clinging to a tree branch with one hand, and with the other hand enjoying the bananas he'd stored away after solving HW5 XC problem #1.

Now let's move on to two questions of much more practical significance:

1. Should the "ecologist" shoot the "tranquilizer dart" at Nim Chimpsky, or at Mr. Bill? (She needs to collect a harmless DNA sample from one of these two characters for the Primate Genome Project.)

(A) Tranquilize Nim Chimpsky! (His DNA sample may explain why he was smart enough to learn all those words of American Sign Language.)

(B) Tranquilize Mr. Bill! (If you manage to find any real DNA in his sample, the result will definitely be a publishable paper, if not a Nobel Prize.)



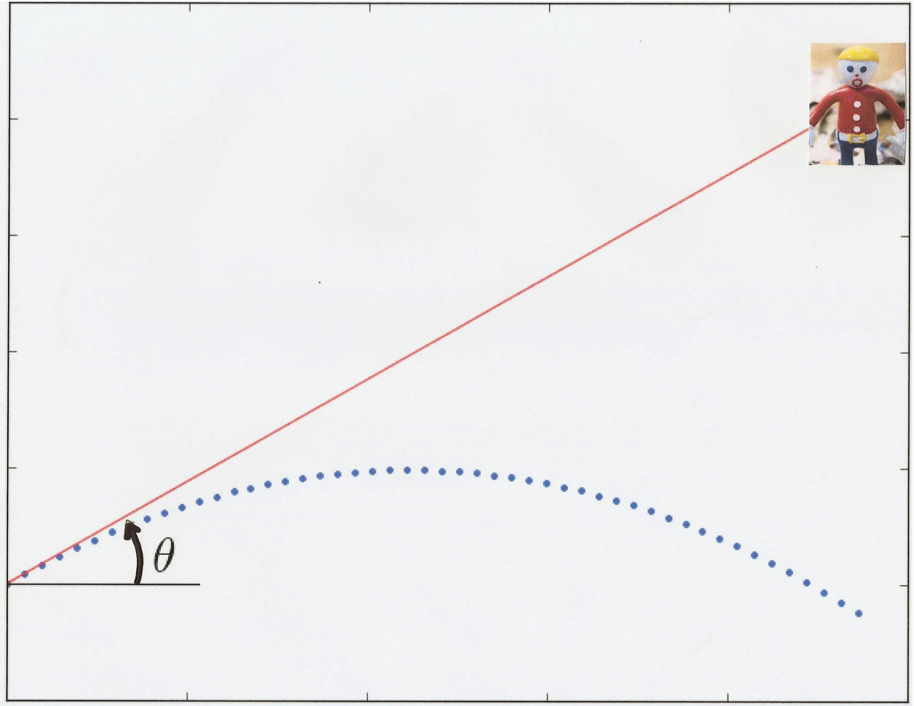
(A) study Nim Chimpsky.
(B) study Mr. Bill.

Now let's move on to two questions of much more practical significance:

1. Should the "ecologist" shoot the "tranquilizer pellet" at Nim Chimpsky, or at Mr. Bill?

2. It takes the pellet some time to travel across the width of the room.
   - In that time interval, gravity will cause Nim/Bill to fall.
   - So where should I aim the pea-shooter so that the pellet hits Nim/Bill as he drops?

Before you answer, let's explain in detail how this game works, why Nim/Bill lets go of the tree, what each trajectory will look like, etc.


θ

What shall I aim for?

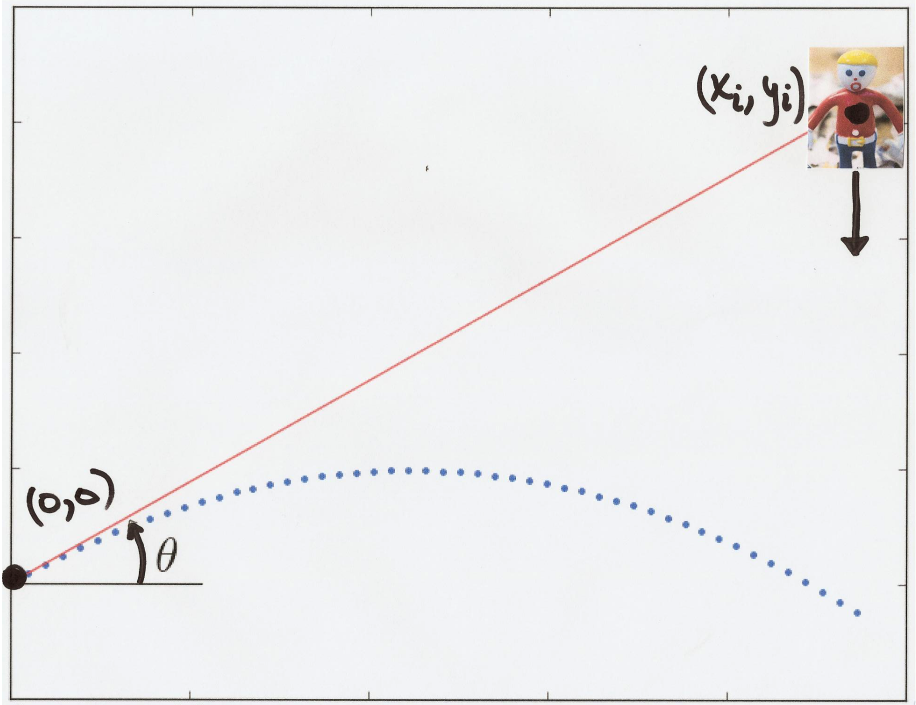
(A) Aim high, because the steel pellet is so much heavier than Mr. Bill, and will be pulled down more by gravity.

(B) Aim low, because Mr. Bill will be falling while the pellet travels.

(C) Aim directly for Mr. Bill. This is clearly what you would do if gravity were absent. The presence of gravity will affect Mr. Bill and the pellet in the same way (they experience the same downward gravitational acceleration), so aiming directly for Mr. Bill will result in a direct hit.

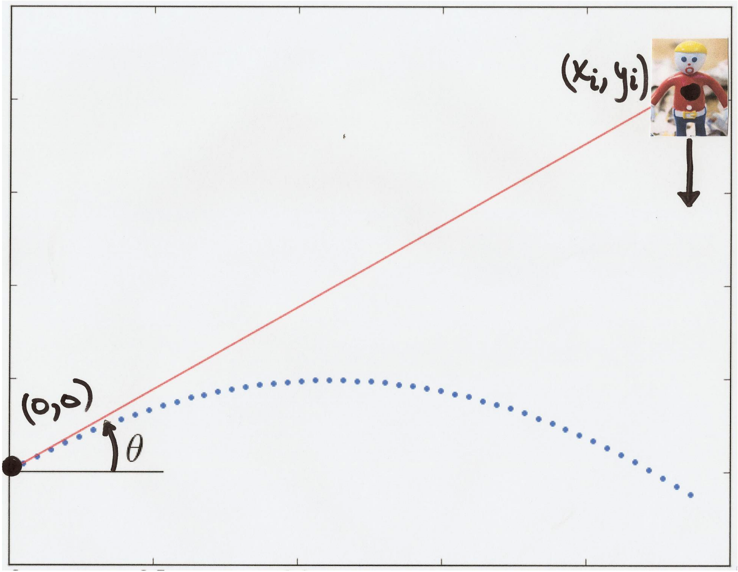
(D) How much below Mr. Bill you need to aim depends on the speed with which you fire the pellet, because the time that it takes the pellet to reach Mr. Bill will depend on how fast the pellet is shot.

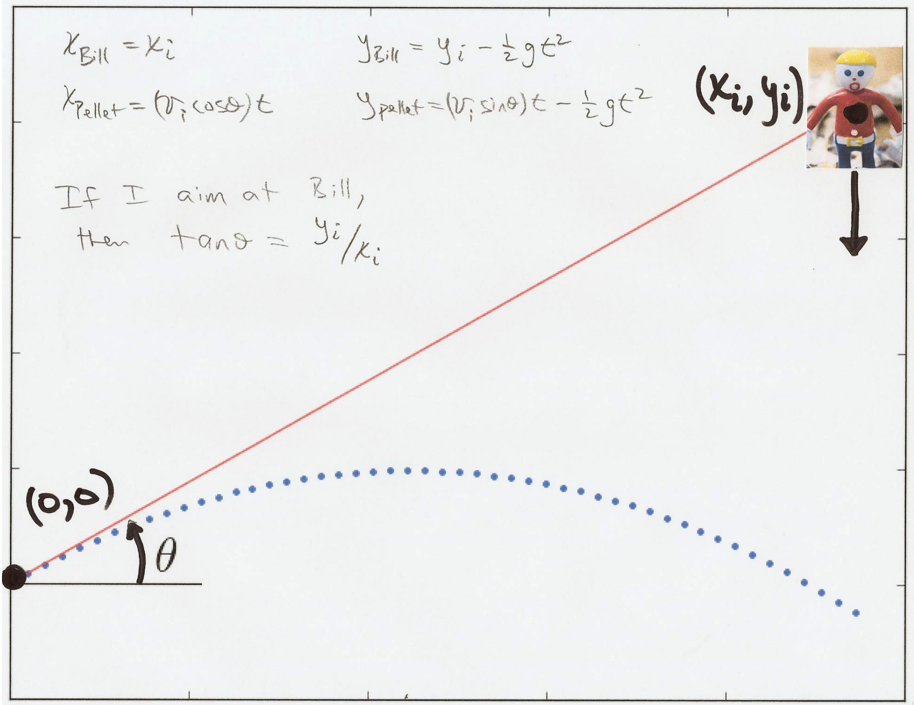
(I'm not going to give away my own answer yet!)



(xi, yi)
(0,0)
θ

Try writing equations for $\boxed{x_{\text{Bill}}(t),\ y_{\text{Bill}}(t),\ x_{\text{pellet}}(t),\ y_{\text{pellet}}(t)}$, in terms of x_i, y_i, θ (shown on diagram) and initial pellet speed v_i.





$x_{Bill} = x_i$
$y_{Bill} = y_i - \frac{1}{2}gt^2$
$x_{Pellet} = (v_i \cos\theta)t$
$y_{pellet} = (v_i \sin\theta)t - \frac{1}{2}gt^2$
(x_i, y_i)
If I aim at Bill,
then $\tan\theta = y_i/x_i$
$(0,0)$
θ

Anybody want to change his/her vote?

(A) Aim high, because the steel pellet is so much heavier than Mr. Bill, and will be pulled down more by gravity.

(B) Aim low, because Mr. Bill will be falling while the pellet travels.

(C) Aim directly for Mr. Bill. **This is clearly what you would do if gravity were absent.** The presence of gravity will affect Mr. Bill and the pellet in the same way, so aiming directly for Mr. Bill will result in a direct hit.

(D) How much below Mr. Bill you need to aim depends on the speed with which you fire the pellet, because the time that it takes the pellet to reach Mr. Bill will depend on how fast the pellet is shot.

Mr. Bill starts from rest at (x_i, y_i). Pellet starts at $(0, 0)$ with initial velocity $(v_i \cos\theta, v_i \sin\theta)$. Equations of motion:

$$x_{\text{bill}} = x_i$$

$$y_{\text{bill}} = y_i - \frac{1}{2}gt^2$$

$$x_{\text{pellet}} = v_i \cos\theta \; t$$

$$y_{\text{pellet}} = v_i \sin\theta \; t - \frac{1}{2}gt^2$$

When does pellet cross Mr. Bill's downward path?

$$x_{\text{pellet}} = x_{\text{bill}} \quad \Rightarrow \quad v_i \cos\theta \; t = x_i$$

$$t = \frac{x_i}{v_i \cos\theta}$$

Plugging in $t = \left(\frac{x_i}{v_i \cos\theta}\right)$:

$$x_{\text{bill}} = x_i$$

$$x_{\text{pellet}} = v_i \cos\theta \left(\frac{x_i}{v_i \cos\theta}\right) = x_i$$

$$y_{\text{bill}} = y_i - \frac{1}{2} g \left(\frac{x_i}{v_i \cos\theta}\right)^2$$

$$y_{\text{pellet}} = v_i \sin\theta \left(\frac{x_i}{v_i \cos\theta}\right) - \frac{1}{2} g \left(\frac{x_i}{v_i \cos\theta}\right)^2$$

What is vertical separation between Mr. Bill and the pellet at the instant when $x_{\text{pellet}} = x_{\text{bill}} = x_i$?

$$y_{\text{bill}} - y_{\text{pellet}} = y_i - v_i \sin\theta \left(\frac{x_i}{v_i \cos\theta}\right)$$

$$y_{\text{bill}} - y_{\text{pellet}} = y_i - x_i \tan\theta = y_i - y_i = 0$$

Anybody want to change his/her vote?

(A) Aim high, because the steel pellet is so much heavier than Mr. Bill, and will be pulled down more by gravity.

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(C) Aim directly for Mr. Bill. **This is clearly what you would do if gravity were absent.** The presence of gravity will affect Mr. Bill and the pellet in the same way, so aiming directly for Mr. Bill will result in a direct hit.

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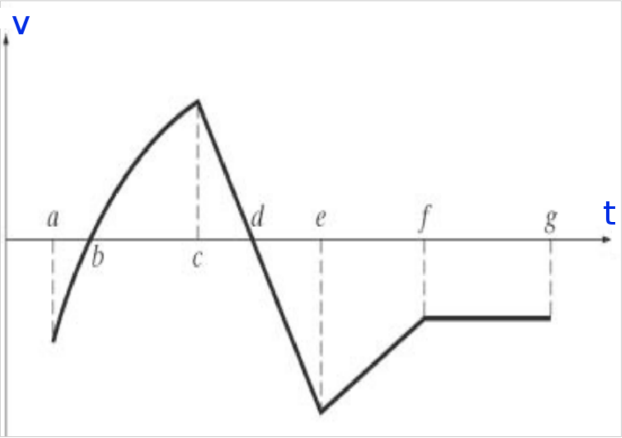
Oh noooo . . .

https://en.wikipedia.org/wiki/Mr._Bill

I had more material just in case, but we ran out of time once we got here!

2. The velocity of an object as a function of time is shown in the figure below. Over what intervals is the work done on the object (a) positive, (b) negative, (c) zero? (Hint: make a table showing sign of acceleration (hence sign of net force), sign of displacement, and sign of their product, for each segment.)



(Consider work done by whatever external force is causing the object's velocity to change.)

Stretching a certain spring 0.10 m from its equilibrium length requires 10 J of work. How much more work does it take to stretch this spring an additional 0.10 m from its equilibrium length?

(A) No additional work

(B) An additional 10 J

(C) An additional 20 J

(D) An additional 30 J

(E) An additional 40 J

A block initially at rest is allowed to slide down a frictionless ramp and attains a speed v at the bottom. To achieve a speed $2v$ at the bottom, how many times as high must a new ramp be?

(A) 1

(B) 1.414

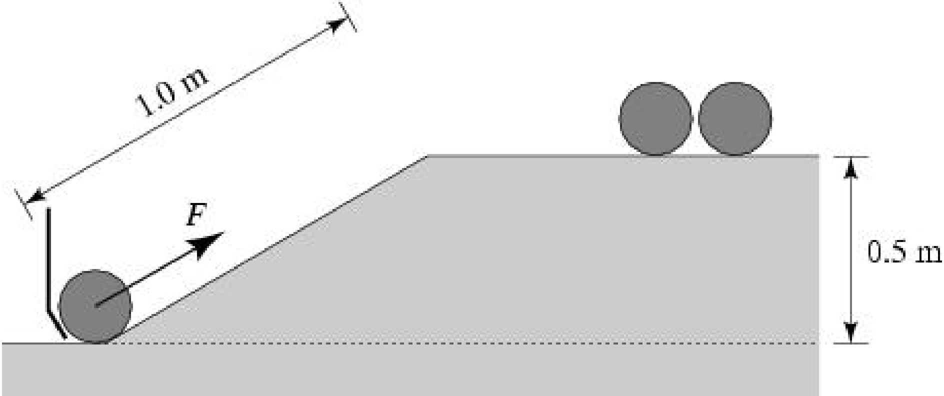
(C) 2

(D) 3

(E) 4

(F) 5

(G) 6



At the bowling alley, the ball-feeder mechanism must exert a force to push the bowling balls up a 1.0 m long ramp. The ramp leads the balls to a chute 0.5 m above the base of the ramp. About how much force must be exerted on a 5.0 kg bowling ball?

(A) 200 N
(B) 100 N
(C) 50 N
(D) 25 N
(E) 5.0 N
(F) impossible to determine.

Suppose you drop a 1 kg rock from a height of 5 m above the ground. When it hits, how much force does the rock exert on the ground? (Take $g \approx 10\ \mathrm{m/s^2}$.)

(A) 0.2 N

(B) 5 N

(C) 50 N

(D) 100 N

(E) impossible to determine without knowing over what distance the rock slows when it impacts the ground.

Chapter 9 reading question

2. When you stand up from a seated position, you push down with your legs. So then do you do negative work when you stand up?

"In this situation, we have 2 systems. Firstly, in the system of just the person, the action of standing up will result in a loss of internal or chemical energy, thereby resulting in a loss of system energy and hence positive work (BY the system) [which implies negative work done ON the system, by Earth's gravitational force]. For the system of the person and Earth, the action of standing up increases the person's potential energy at the expense of internal energy. In this situation, there is no change in system energy and therefore no work is done."

Reading question 2 had no really simple answer

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose "system" = me + Earth + floor + chair

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = -mg\ (\Delta x)_{\text{my c.o.m.}} < 0$

- $W = 0$

There are no external forces. Everything of interest is inside the system boundary.

Let's try choosing a different "system."

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose "system" = me + floor + chair

- $\Delta K = 0$
- $\Delta U = 0$ (U^G undefined if Earth not in system)
- $\Delta E_{\rm thermal} = 0$ (debatable but irrelevant)
- $\Delta E_{\rm source} = -mg\ (\Delta x)_{\rm my\ c.o.m.} < 0$
- $W = -mg\ (\Delta x)_{\rm my\ c.o.m.} < 0$

External gravitational force, exerted by Earth on me, does negative work on me. Point of application of this external force is my body's center of mass. Force points downward, but displacement is upward. $W < 0$. System's total energy decreases.

Let's try answering a slightly different question.

When **a friend** *stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?*

Suppose "system" = me + Earth + floor + chair

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = 0$

- $W = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$

My friend applies an upward force beneath my arms. The point of application of force is displaced upward.

Let's include my friend as part of "the system."

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

Suppose "system" = me + Earth + floor + chair + friend

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = -mg\ (\Delta x)_{\text{my c.o.m.}} < 0$

- $W = 0$

There is no external force. Everything is within the system.

Back to the original reading question

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

I think the work done **ON the system BY my legs** is either positive (if my legs are considered "external" to the me+Earth+floor system and are supplying the energy to lift me) or zero (if my legs are part of the system).

Remember the one way we got a negative answer: In the case in which Earth was not part of the system, we found that the external force of Earth's gravity did negative work on me. But I was pushing Earth downward, away from me. I lost energy. So even in this case (where the work done **on me** was negative), the work done **by me** was positive.

Key point: what you call "work" depends on how you define "the system."

A few key ideas from Chapters 8 (force) and 9 (work)

Impulse (i.e. momentum change) delivered by external force:

$$\text{force} = \frac{\mathrm{d(momentum)}}{\mathrm{d}t} \quad \Leftrightarrow \quad \vec{J} = \int \vec{F}_{\text{external}}\, \mathrm{d}t$$

External force exerted ON system:

$$\text{force} = \frac{\mathrm{d(work)}}{\mathrm{d}x} \quad \Leftrightarrow \quad W = \int F_x\, \mathrm{d}x$$

Force exerted BY spring, gravity, etc.:

$$\text{force} = -\frac{\mathrm{d}(\textit{potential energy})}{\mathrm{d}x}$$

ΔE_{system} = flow of energy into system = work done ON system:

$$\text{work} = \Delta(\text{energy}) = \Delta K + \Delta U + \Delta E_{\text{source}} + \Delta E_{\text{thermal}}$$

Notice that work : energy :: impulse : momentum

Some equation sheet entries for Chapters 8+9

http://positron.hep.upenn.edu/physics8/files/equations.pdf

Work (external, nondissipative, 1D):

$$W = \int F_x(x)\ dx$$

which for a constant force is

$$W = F_x\ \Delta x$$

Power is rate of change of energy:

$$P = \frac{\mathrm{d}E}{\mathrm{d}t}$$

Constant external force, 1D:

$$P = F_x v_x$$

G.P.E. near earth's surface:

$$U_{\mathrm{gravity}} = mgh$$

Force of gravity near earth's surface (force is $-\frac{\mathrm{d}U_{\mathrm{gravity}}}{\mathrm{d}x}$):

$$F_x = -mg$$

Potential energy of a spring:

$$U_{\mathrm{spring}} = \frac{1}{2}k(x - x_0)^2$$

Hooke's Law (force is $-\frac{\mathrm{d}U_{\mathrm{spring}}}{\mathrm{d}x}$):

$$F_{\mathrm{by\ spring\ ON\ load}} = -k(x - x_0)$$

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- You probably noticed by now that I try my best to motivate you to spend time each week reading, working checkpoints, and solving problems. I give you a significant amount of work to do. By **doing** all of the work, you will learn a lot, and you will do very well in the course. That's the bargain we offer.
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