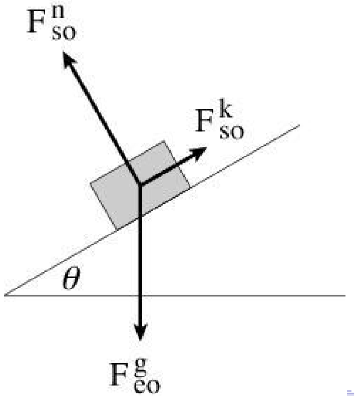
Physics 8 — Wednesday, October 9, 2019

- Pick up HW6 handout, due on Friday 10/18.
- This week you read Ch11 (motion in a circle). For next week, you'll read Ch12 (torque). We're still doing Ch10 in class.

An object "O" of mass m slides down an inclined surface "S" at constant velocity. What is the magnitude of the **normal force** F^n_{so} exerted by the surface on the object?

(A) $F^n_{so} = mg$
(B) $F^n_{so} = mg \sin\theta$
(C) $F^n_{so} = mg \cos\theta$
(D) $F^n_{so} = mg \tan\theta$
(E) $F^n_{so} = \mu_k mg$
(F) $F^n_{so} = \mu_k mg \sin\theta$
(G) $F^n_{so} = \mu_k mg \cos\theta$
(H) $F^n_{so} = \mu_k mg \tan\theta$



An object "O" of mass m slides down an inclined surface "S" at constant velocity. What is the magnitude of the (kinetic) **frictional force** F_{so}^{k} exerted by the surface on the object?

(A) $F_{so}^{k} = mg$

(B) $F_{so}^{k} = mg \sin\theta$

(C) $F_{so}^{k} = mg \cos\theta$

(D) $F_{so}^{k} = mg \tan\theta$

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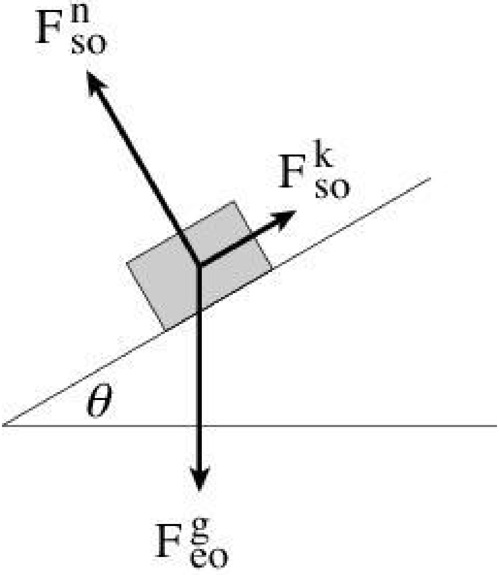

An object "O" of mass m slides down an inclined surface "S" at constant velocity. What is the magnitude of the **gravitational force** F_{eo}^{g} exerted by Earth on the object?

(A) $F_{eo}^{g} = mg$

(B) $F_{eo}^{g} = mg \sin\theta$

(C) $F_{eo}^{g} = mg \cos\theta$

(D) $F_{eo}^{g} = mg \tan\theta$

(E) $F_{eo}^{g} = \mu_k mg$

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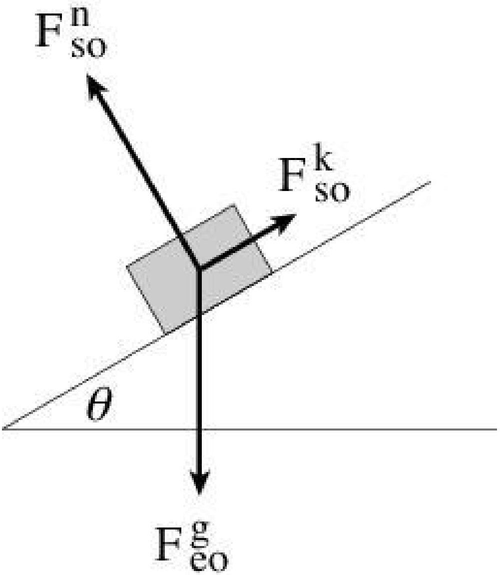

An object "O" of mass m slides down an inclined surface "S" at constant velocity. Let the x-axis point downhill. What is the magnitude of the **downhill (tangential) component** $F^g_{eo,x}$ of the gravitational force exerted by Earth on the object?

(A) $F^g_{eo,x} = mg$

(B) $F^g_{eo,x} = mg\sin\theta$

(C) $F^g_{eo,x} = mg\cos\theta$

(D) $F^g_{eo,x} = mg\tan\theta$

(E) $F^g_{eo,x} = \mu_k mg$

(F) $F^g_{eo,x} = \mu_k mg\sin\theta$

(G) $F^g_{eo,x} = \mu_k mg\cos\theta$

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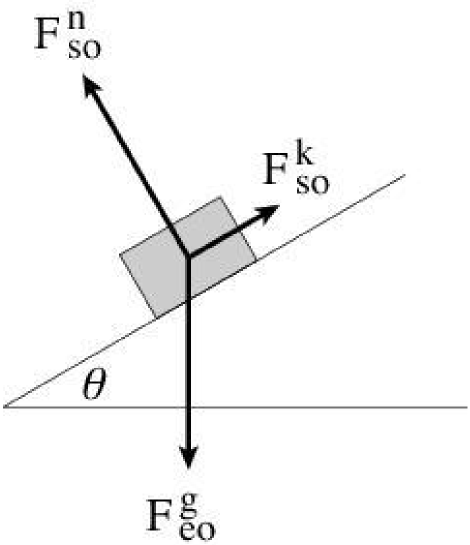


Since object "O" slides down surface "S" at constant velocity, the forces on O must sum vectorially to zero. How do I express this fact for the forces acting along the downhill (tangential) axis?

(A) $\mu_k mg = mg\cos\theta$

(B) $\mu_k mg = mg\sin\theta$

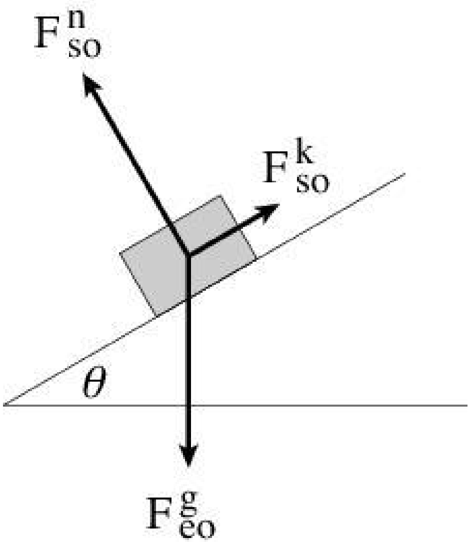
(C) $\mu_k mg\cos\theta = mg$

(D) $\mu_k mg\sin\theta = mg$

(E) $\mu_k mg\cos\theta = mg\sin\theta$

(F) $\mu_k mg\sin\theta = mg\cos\theta$

(G) $mg\sin\theta = mg\cos\theta$



Suppose friction holds object "O" at rest on surface "S." Which statement is true?

(A) $mg\sin\theta = F^s_{so} = \mu_k mg\cos\theta$

(B) $mg\sin\theta = F^s_{so} = \mu_s mg\cos\theta$

(C) $mg\sin\theta = F^s_{so} \leq \mu_k mg\cos\theta$

(D) $mg\sin\theta = F^s_{so} \leq \mu_s mg\cos\theta$

(E) $mg\cos\theta = F^s_{so} = \mu_k mg\sin\theta$

(F) $mg\cos\theta = F^s_{so} = \mu_s mg\sin\theta$

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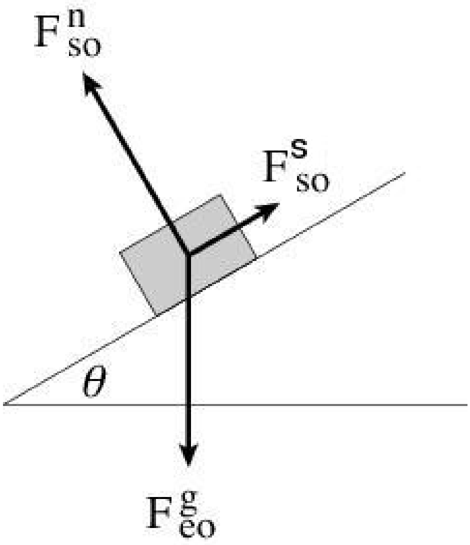

Suppose friction holds object "O" at rest on surface "S." Then I gradually increase θ until the block just begins to slip. Which statement is true at the instant when the block starts slipping?

(A) $mg\sin\theta = F^s_{so} = \mu_k mg\cos\theta$

(B) $mg\sin\theta = F^s_{so} = \mu_s mg\cos\theta$

(C) $mg\sin\theta = F^s_{so} \leq \mu_k mg\cos\theta$

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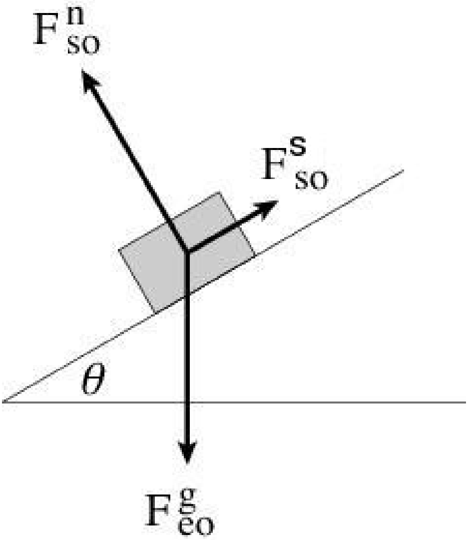
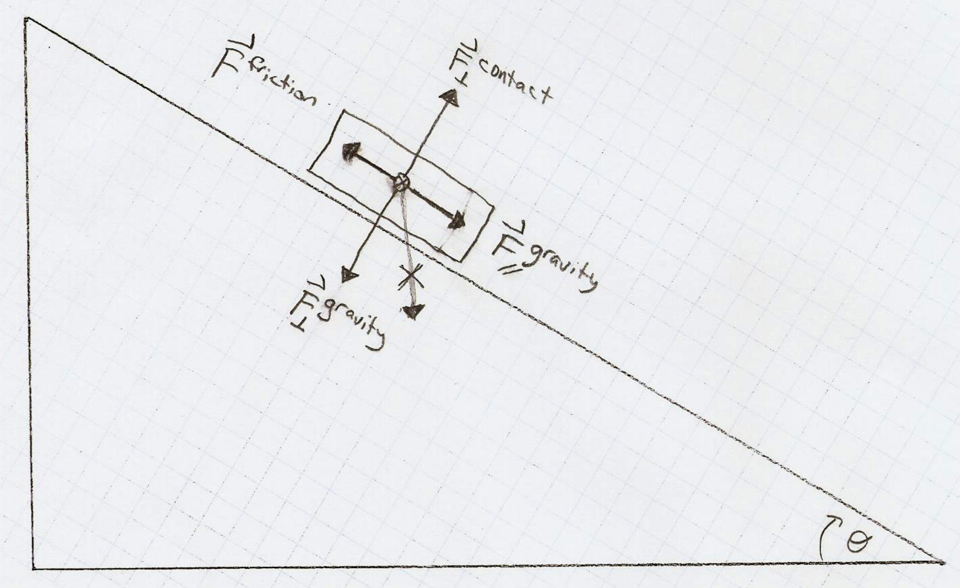
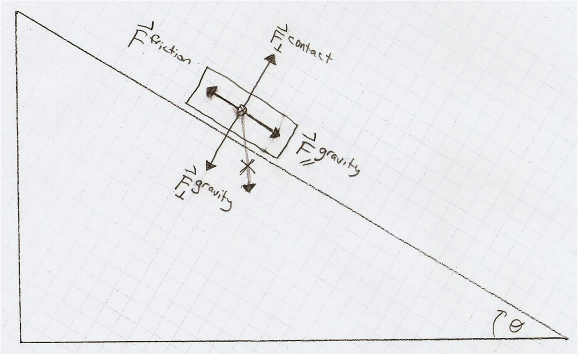


Friction on inclined plane



Why do I "cross off" the downward gravity arrow?



Take x-axis to be downhill, y-axis to be upward $\perp$ from surface.

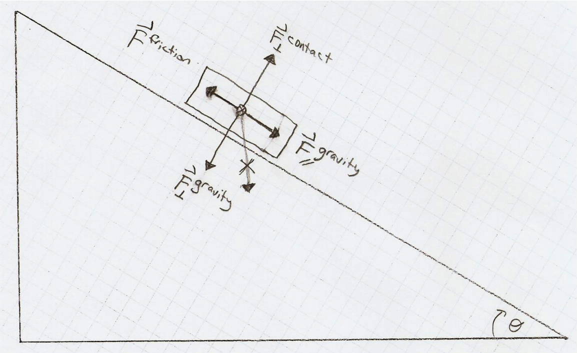
$$\vec{F}^G_\perp = -mg\cos\theta\ \hat{j}, \qquad \vec{F}^N = +mg\cos\theta\ \hat{j}$$

$$\vec{F}^G_\parallel = +mg\sin\theta\ \hat{i}$$

If block is not sliding then friction balances downhill gravity:

$$\vec{F}^S = -mg\sin\theta\ \hat{i}$$

(I'll skip this slide, but it's here for reference.)



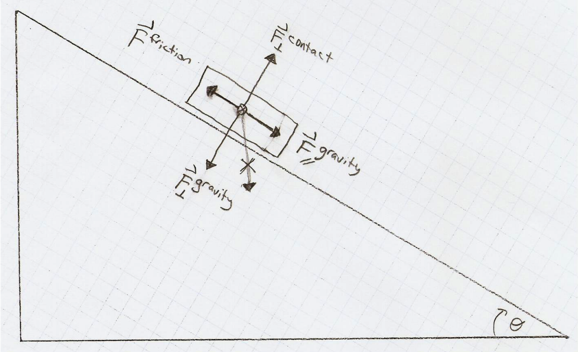
Magnitude of "normal" force ("normal" is a synonym for "perpendicular") between surfaces is

$$F^N = mg\cos\theta$$

Magnitude of static friction must be less than maximum:

$$F^S \;\leq\; \mu_S F^N = \mu_S\; mg\cos\theta$$

Block begins sliding when downhill component of gravity equals maximum magnitude of static friction ...



Block begins sliding when downhill component of gravity equals maximum magnitude of static friction:

$$\mu_S \; mg \cos\theta = mg \sin\theta$$

$$\mu_S = \frac{mg \sin\theta}{mg \cos\theta}$$

$$\mu_S = \tan\theta$$

A Ch10 problem that may not fit into HW6

The coefficient of static friction of tires on ice is about 0.10. (a) What is the steepest driveway on which you could park under those circumstances? (b) Draw a free-body diagram for the car when it is parked (successfully) on an icy driveway that is just a tiny bit less steep than this maximum steepness. [We might want to do (b) before we do (a).]

Answering part (a) starts by expressing (in math) which statement:

(A) (total gravitational force on car) equals (kinetic friction)

(B) (total gravitational force on car) equals (largest possible value of static friction)

(C) (downhill component of gravity) equals (kinetic friction)

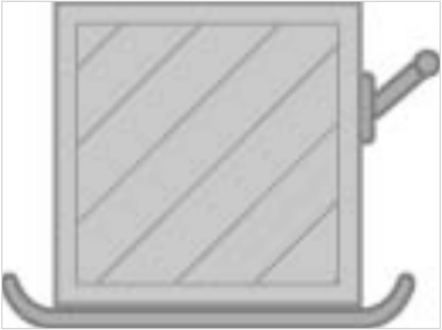
(D) (downhill component of gravity) equals (largest possible value of static friction)

A heavy crate has plastic skid plates beneath it and a tilted handle attached to one side. Which requires a smaller force (directed along the diagonal rod of the handle) to move the box? Why?

(A) Pushing the crate is easier than pulling.

(B) Pulling the crate is easier than pushing.

(C) There is no difference.



Example (tricky!) problem

A woman applies a constant force to pull a 50 kg box across a floor **at constant speed**. She applies this force by pulling on a rope that makes an angle of 37° above the horizontal. The friction coefficient between the box and the floor is $\mu_k = 0.10$.

(a) Find the tension in the rope.

(b) How much work does the woman do in moving the box 10 m?

free-body diagram for box

What are all of the forces acting on the box? Try drawing your own FBD for the box. It's tricky!

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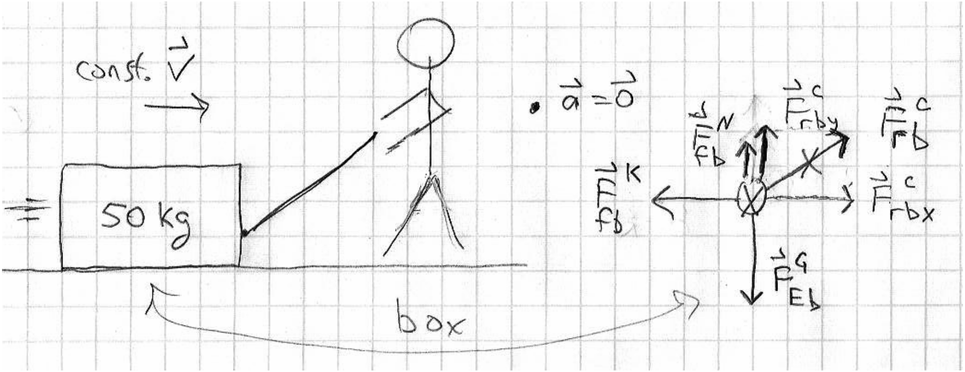


(I should redraw the RHS of this diagram on the board.)

find tension in rope

Step one: If T is the tension in the rope, then what is the normal force (by floor on box)?

(A) $F^N = mg$
(B) $F^N = mg + T\cos\theta$
(C) $F^N = mg + T\sin\theta$
(D) $F^N = mg - T\cos\theta$
(E) $F^N = mg - T\sin\theta$

find tension in rope

Step two: what is the frictional force exerted by the floor on the box (which is sliding across the floor at constant speed)?

(A) $F^K = \mu_K(mg - T\sin\theta)$
(B) $F^K = \mu_K(mg - T\cos\theta)$
(C) $F^K = \mu_S(mg - T\sin\theta)$
(D) $F^K = \mu_S(mg - T\cos\theta)$
(E) $F^K = (mg - T\sin\theta)$
(F) $F^K = (mg - T\cos\theta)$

We stopped here.

find tension in rope

Step three: how do I use the fact that the box is moving at constant velocity (and hence is not accelerating)?

(A) $T = F^K = \mu_K(mg - T\sin\theta)$

(B) $T\cos\theta = F^K = \mu_K(mg - T\sin\theta)$

(C) $T\sin\theta = F^K = \mu_K(mg - T\sin\theta)$

solution (part a): find tension in rope

Force by rope on box has upward vertical component $T \sin\theta$. So the normal force (by floor on box) is $\boxed{F^N = mg - T\sin\theta}$.

Force of friction is $\boxed{F^K = \mu_K(mg - T\sin\theta)}$. To keep box sliding at constant velocity, horizontal force by rope on box must balance F^K.

$$\boxed{T\cos\theta = F^K = \mu_K(mg - T\sin\theta)} \quad \Rightarrow \quad T = \frac{\mu_K mg}{\cos\theta + \mu_K \sin\theta}$$

This reduces to familiar $T = \mu_K mg$ if $\theta = 0°$ (pulling horizontally) and even reduces to a sensible $T = mg$ if $\theta = 90°$ (pulling vertically).

Plugging in $\theta = 37°$, so $\cos\theta = 4/5 = 0.80$, $\sin\theta = 3/5 = 0.60$,

$$T = \frac{(0.10)(50\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{(0.80) + (0.10)(0.60)} = 57\ \mathrm{N}$$

solution (part b): work done by pulling for 10 meters

In part (a) we found tension in rope is $T = 57\ \mathrm{N}$ and is oriented at an angle $\theta = 36.9^\circ$ above the horizontal.

In 2D, work is displacement times **component of force along direction of displacement** (which is horizontal in this case). So the work done by the rope on the box is

$$W = \vec{F}_{rb} \cdot \Delta\vec{r}_b$$

This is the dot product (or "scalar product") of the force $\vec{F}_{rb}$ (by rope on box) with the displacement $\Delta\vec{r}_b$ of the point of application of the force.

In part (a) we found tension in rope is $T = 57\ \mathrm{N}$ and is oriented at an angle $\theta = 36.9^\circ$ above the horizontal.

What is the work done by the rope on the box by pulling the box across the floor for 10 meters? (Assume my arithmetic is correct.)

(In two dimensions, work is the dot product of the force $\vec{F}_{rb}$ with the displacement $\Delta\vec{r}_b$ of the point of application of the force.)

(A) $W = (10\ \mathrm{m})(T) = (10\ \mathrm{m})(57\ \mathrm{N}) = 570\ \mathrm{J}$

(B) $W = (10\ \mathrm{m})(T\cos\theta) = (10\ \mathrm{m})(57\ \mathrm{N})(0.80) = 456\ \mathrm{J}$

(C) $W = (10\ \mathrm{m})(T\sin\theta) = (10\ \mathrm{m})(57\ \mathrm{N})(0.60) = 342\ \mathrm{J}$

(D) $W = (8.0\ \mathrm{m})(T\cos\theta) = (8.0\ \mathrm{m})(57\ \mathrm{N})(0.80) = 365\ \mathrm{J}$

(E) $W = (8.0\ \mathrm{m})(T\sin\theta) = (8.0\ \mathrm{m})(57\ \mathrm{N})(0.60) = 274\ \mathrm{J}$

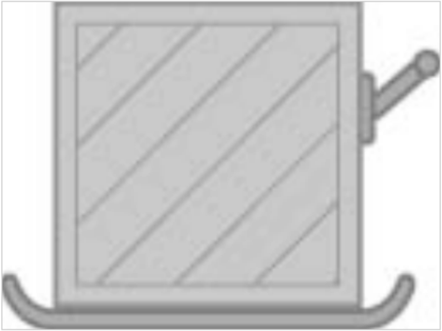
Repeat, now that we've analyzed this quantitatively

A heavy crate has plastic skid plates beneath it and a tilted handle attached to one side. Which requires a smaller force (directed along the diagonal rod of the handle) to move the box? Why?

(A) Pushing the crate is easier than pulling.

(B) Pulling the crate is easier than pushing.

(C) There is no difference.



Easier example

How hard do you have to push a 1000 kg car (with brakes on, all wheels, on level ground) to get it to start to slide? Let's take $\mu_S \approx 1.2$ for rubber on dry pavement.

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$$F^{\mathrm{Normal}} = mg = 9800 \mathrm{~N}$$

$$F^{\mathrm{Static}} \leq \mu_S F^N = (1.2)(9800 \mathrm{~N}) \approx 12000 \mathrm{~N}$$

So the static friction gives out (hence car starts to slide) when your push exceeds 12000 N.

How hard do you then have to push to keep the car sliding at constant speed? Let's take $\mu_K \approx 0.8$ for rubber on dry pavement.

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$$F^{\text{Static}} \leq \mu_S F^N = (1.2)(9800 \text{ N}) \approx 12000 \text{ N}$$

So the static friction gives out (hence car starts to slide) when your push exceeds 12000 N.

How hard do you then have to push to keep the car sliding at constant speed? Let's take $\mu_K \approx 0.8$ for rubber on dry pavement.

$$F^{\text{Kinetic}} = \mu_K F^N = (0.8)(9800 \text{ N}) \approx 8000 \text{ N}$$

How far does your car slide on dry, level pavement if you jam on the brakes, from 60 mph (27 m/s)?

$$F^N = mg, \quad F^K = \mu_K mg$$

$$a = ? \qquad \Delta x = ?$$

(The math is worked out on the next slides, but we won't go through them in detail. It's there for you to look at later.)

How far does your car slide on dry, level pavement if you jam on the brakes, from 60 mph (27 m/s)?

$$F^N = mg, \quad F^K = \mu_K mg$$

$$a = -F^K/m = -\mu_K g = -(0.8)(9.8\ \mathrm{m/s^2}) \approx -8\ \mathrm{m/s^2}$$

Constant force → constant acceleration from 27 m/s down to zero:

$$v_f^2 = v_i^2 + 2ax$$

$$x = \frac{v_i^2}{-2a} = \frac{(27\ \mathrm{m/s})^2}{2 \times (8\ \mathrm{m/s^2})} \approx 45\ \mathrm{m}$$

How much time elapses before you stop?

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{27\ \mathrm{m/s}}{8\ \mathrm{m/s^2}} = 3.4\ \mathrm{s}$$

How does this change if you have anti-lock brakes (or good reflexes) so that the tires never skid?

How does this change if you have anti-lock brakes (or good reflexes) so that the tires never skid? Remember $\mu_S > \mu_K$. For rubber on dry pavement, $\mu_S \approx 1.2$ (though there's a wide range) and $\mu_K \approx 0.8$. The best you can do is *maximum* static friction:

$$F^S \leq \mu_S mg$$

$$a = -F^S/m = -\mu_S g = -(1.2)(9.8\ \mathrm{m/s^2}) \approx -12\ \mathrm{m/s^2}$$

Constant force $\rightarrow$ constant acceleration from 27 m/s down to zero:

$$v_f^2 = v_i^2 + 2ax$$

$$x = \frac{v_i^2}{-2a} = \frac{(27\ \mathrm{m/s})^2}{2 \times (12\ \mathrm{m/s^2})} \approx 30\ \mathrm{m}$$

How much time elapses before you stop?

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{27\ \mathrm{m/s}}{15\ \mathrm{m/s^2}} = 2.2\ \mathrm{s}$$

So you can stop in about 2/3 the time (and 2/3 the distance) if you don't let your tires skid. Or whatever μ_K/μ_S ratio is.

A Ch10 problem that may not fit into HW6

Calculate $\vec{C} \cdot (\vec{B} - \vec{A})$ if $\vec{A} = 3.0\hat{i} + 2.0\hat{j}$, $\vec{B} = 1.0\hat{i} - 1.0\hat{j}$, and $\vec{C} = 2.0\hat{i} + 2.0\hat{j}$. Remember that there are two ways to compute a dot product—choose the easier method in a given situation: one way is $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}| \cos\varphi$, where φ is the angle between vectors $\vec{P}$ and $\vec{Q}$, and the other way is $\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y$.

A Ch10 problem that may not fit into HW6

A child rides her bike 1.0 block east and then $\sqrt{3} \approx 1.73$ blocks north to visit a friend. It takes her 10 minutes, and each block is 60 m long. What are (a) the magnitude of her displacement, (b) her average velocity (magnitude and direction), and (c) her average speed?

A Ch10 problem that may not fit into HW6

A fried egg of inertia m slides (at constant speed) down a Teflon frying pan tipped at an angle θ above the horizontal. [This only works if the angle θ is just right.] (a) Draw the free-body diagram for the egg. Be sure to include friction. (b) What is the "net force" (i.e. the vector sum of forces) acting on the egg? (c) How do these answers change if the egg is instead speeding up as it slides?

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