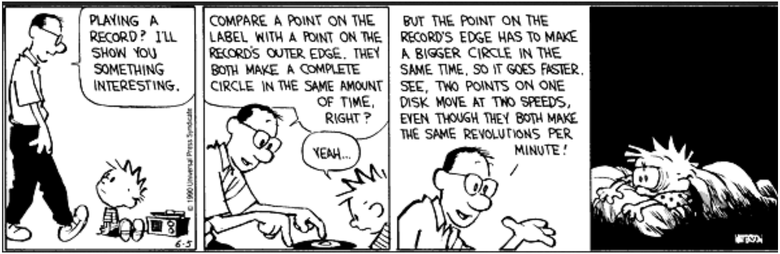
Physics 8 — Monday, October 14, 2019

- Pick up HW6 handout, due this Friday, if you haven't already done so.
- This week you're reading Ch12 (torque). Today in class we'll finish Ch10 and start Ch11.



A woman applies a constant force to pull a 50 kg box across a floor **at constant speed**. She applies this force by pulling on a rope that makes an angle of 37° above the horizontal. The friction coefficient between the box and the floor is $\mu_k = 0.10$.

While you wait for us to get started today, draw a free-body diagram for the box. It's tricky! (We started this on Wednesday.)

Example (tricky!) problem

A woman applies a constant force to pull a 50 kg box across a floor **at constant speed**. She applies this force by pulling on a rope that makes an angle of 37° above the horizontal. The friction coefficient between the box and the floor is $\mu_k = 0.10$.

(a) Find the tension in the rope.

(b) How much work does the woman do in moving the box 10 m?

free-body diagram for box

What are all of the forces acting on the box? Try drawing your own FBD for the box. It's tricky!

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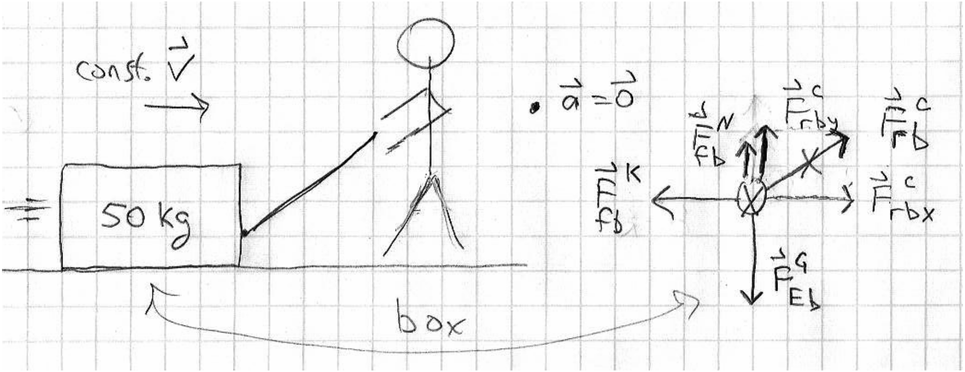


(I should redraw the RHS of this diagram on the board.)

find tension in rope

Step one: If T is the tension in the rope, then what is the normal force (by floor on box)?

(A) $F^N = mg$
(B) $F^N = mg + T\cos\theta$
(C) $F^N = mg + T\sin\theta$
(D) $F^N = mg - T\cos\theta$
(E) $F^N = mg - T\sin\theta$

find tension in rope

Step two: what is the frictional force exerted by the floor on the box (which is sliding across the floor at constant speed)?

(A) $F^K = \mu_K(mg - T\sin\theta)$
(B) $F^K = \mu_K(mg - T\cos\theta)$
(C) $F^K = \mu_S(mg - T\sin\theta)$
(D) $F^K = \mu_S(mg - T\cos\theta)$
(E) $F^K = (mg - T\sin\theta)$
(F) $F^K = (mg - T\cos\theta)$

find tension in rope

Step three: how do I use the fact that the box is moving at constant velocity (and hence is not accelerating)?

(A) $T = F^K = \mu_K(mg - T\sin\theta)$

(B) $T\cos\theta = F^K = \mu_K(mg - T\sin\theta)$

(C) $T\sin\theta = F^K = \mu_K(mg - T\sin\theta)$

solution (part a): find tension in rope

Force by rope on box has upward vertical component $T \sin\theta$. So the normal force (by floor on box) is $\boxed{F^N = mg - T\sin\theta}$.

Force of friction is $\boxed{F^K = \mu_K(mg - T\sin\theta)}$. To keep box sliding at constant velocity, horizontal force by rope on box must balance F^K.

$$\boxed{T\cos\theta = F^K = \mu_K(mg - T\sin\theta)} \quad \Rightarrow \quad T = \frac{\mu_K mg}{\cos\theta + \mu_K \sin\theta}$$

This reduces to familiar $T = \mu_K mg$ if $\theta = 0^\circ$ (pulling horizontally) and even reduces to a sensible $T = mg$ if $\theta = 90^\circ$ (pulling vertically).

Plugging in $\theta = 37^\circ$, so $\cos\theta = 4/5 = 0.80$, $\sin\theta = 3/5 = 0.60$,

$$T = \frac{(0.10)(50\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{(0.80) + (0.10)(0.60)} = 57\ \mathrm{N}$$

solution (part b): work done by pulling for 10 meters

In part (a) we found tension in rope is $T = 57\ \mathrm{N}$ and is oriented at an angle $\theta = 36.9^\circ$ above the horizontal.

In 2D, work is displacement times **component of force along direction of displacement** (which is horizontal in this case). So the work done by the rope on the box is

$$W = \vec{F}_{rb} \cdot \Delta\vec{r}_b$$

This is the dot product (or "scalar product") of the force $\vec{F}_{rb}$ (by rope on box) with the displacement $\Delta\vec{r}_b$ of the point of application of the force.

In part (a) we found tension in rope is $T = 57\ \mathrm{N}$ and is oriented at an angle $\theta = 36.9^\circ$ above the horizontal.

What is the work done by the rope on the box by pulling the box across the floor for 10 meters? (Assume my arithmetic is correct.)

(In two dimensions, work is the dot product of the force $\vec{F}_{rb}$ with the displacement $\Delta\vec{r}_b$ of the point of application of the force.)

(A) $W = (10\ \mathrm{m})(T) = (10\ \mathrm{m})(57\ \mathrm{N}) = 570\ \mathrm{J}$

(B) $W = (10\ \mathrm{m})(T\cos\theta) = (10\ \mathrm{m})(57\ \mathrm{N})(0.80) = 456\ \mathrm{J}$

(C) $W = (10\ \mathrm{m})(T\sin\theta) = (10\ \mathrm{m})(57\ \mathrm{N})(0.60) = 342\ \mathrm{J}$

(D) $W = (8.0\ \mathrm{m})(T\cos\theta) = (8.0\ \mathrm{m})(57\ \mathrm{N})(0.80) = 365\ \mathrm{J}$

(E) $W = (8.0\ \mathrm{m})(T\sin\theta) = (8.0\ \mathrm{m})(57\ \mathrm{N})(0.60) = 274\ \mathrm{J}$

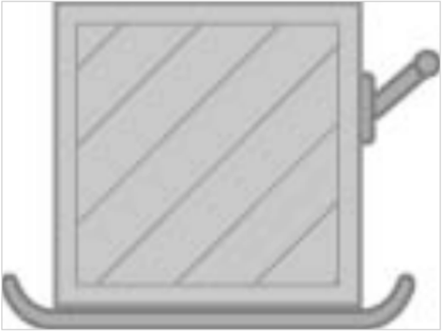
Repeat, now that we've analyzed this quantitatively

A heavy crate has plastic skid plates beneath it and a tilted handle attached to one side. Which requires a smaller force (directed along the diagonal rod of the handle) to move the box? Why?

(A) Pushing the crate is easier than pulling.

(B) Pulling the crate is easier than pushing.

(C) There is no difference.



Easier example (quickly, or skip)

How hard do you have to push a 1000 kg car (with brakes on, all wheels, on level ground) to get it to start to slide? Let's take $\mu_S \approx 1.2$ for rubber on dry pavement.

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How hard do you have to push a 1000 kg car (with brakes on, all wheels, on level ground) to get it to start to slide? Let's take $\mu_S \approx 1.2$ for rubber on dry pavement.

$$F^{\mathrm{Normal}} = mg = 9800 \ \mathrm{N}$$

$$F^{\mathrm{Static}} \le \mu_S F^N = (1.2)(9800 \ \mathrm{N}) \approx 12000 \ \mathrm{N}$$

So the static friction gives out (hence car starts to slide) when your push exceeds 12000 N.

How hard do you then have to push to keep the car sliding at constant speed? Let's take $\mu_K \approx 0.8$ for rubber on dry pavement.

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How hard do you then have to push to keep the car sliding at constant speed? Let's take $\mu_K \approx 0.8$ for rubber on dry pavement.

$$F^{\text{Kinetic}} = \mu_K F^N = (0.8)(9800 \text{ N}) \approx 8000 \text{ N}$$

How far does your car slide on dry, level pavement if you jam on the brakes, from 60 mph (27 m/s)?

$$F^N = mg, \quad F^K = \mu_K mg$$

$$a = ? \qquad \Delta x = ?$$

(The math is worked out on the next slides, but we won't go through them in detail. It's there for you to look at later.)

How far does your car slide on dry, level pavement if you jam on the brakes, from 60 mph (27 m/s)?

$$F^N = mg, \quad F^K = \mu_K mg$$

$$a = -F^K/m = -\mu_K g = -(0.8)(9.8\ \mathrm{m/s^2}) \approx -8\ \mathrm{m/s^2}$$

Constant force → constant acceleration from 27 m/s down to zero:

$$v_f^2 = v_i^2 + 2ax$$

$$x = \frac{v_i^2}{-2a} = \frac{(27\ \mathrm{m/s})^2}{2 \times (8\ \mathrm{m/s^2})} \approx 45\ \mathrm{m}$$

How much time elapses before you stop?

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{27\ \mathrm{m/s}}{8\ \mathrm{m/s^2}} = 3.4\ \mathrm{s}$$

How does this change if you have anti-lock brakes (or good reflexes) so that the tires never skid?

How does this change if you have anti-lock brakes (or good reflexes) so that the tires never skid? Remember $\mu_S > \mu_K$. For rubber on dry pavement, $\mu_S \approx 1.2$ (though there's a wide range) and $\mu_K \approx 0.8$. The best you can do is *maximum* static friction:

$$F^S \leq \mu_S mg$$

$$a = -F^S/m = -\mu_S g = -(1.2)(9.8\ \mathrm{m/s^2}) \approx -12\ \mathrm{m/s^2}$$

Constant force $\rightarrow$ constant acceleration from 27 m/s down to zero:

$$v_f^2 = v_i^2 + 2ax$$

$$x = \frac{v_i^2}{-2a} = \frac{(27\ \mathrm{m/s})^2}{2 \times (12\ \mathrm{m/s^2})} \approx 30\ \mathrm{m}$$

How much time elapses before you stop?

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{27\ \mathrm{m/s}}{15\ \mathrm{m/s^2}} = 2.2\ \mathrm{s}$$

So you can stop in about 2/3 the time (and 2/3 the distance) if you don't let your tires skid. Or whatever μ_K/μ_S ratio is.

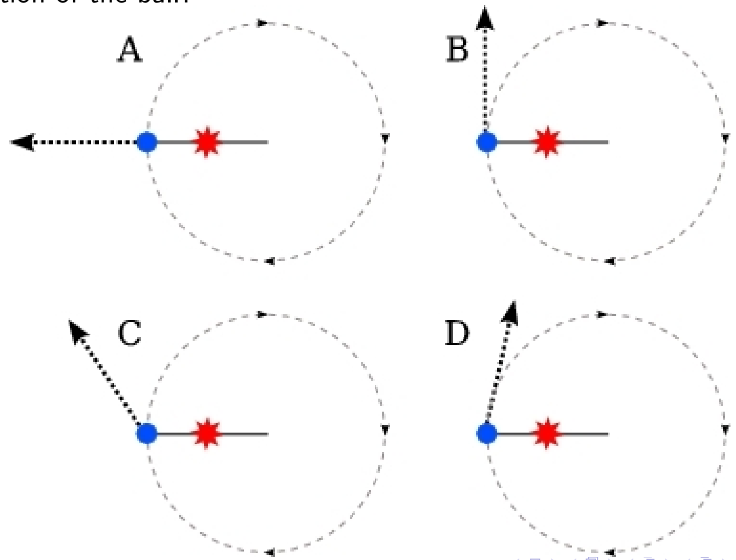
Chapter 11: motion in a circle

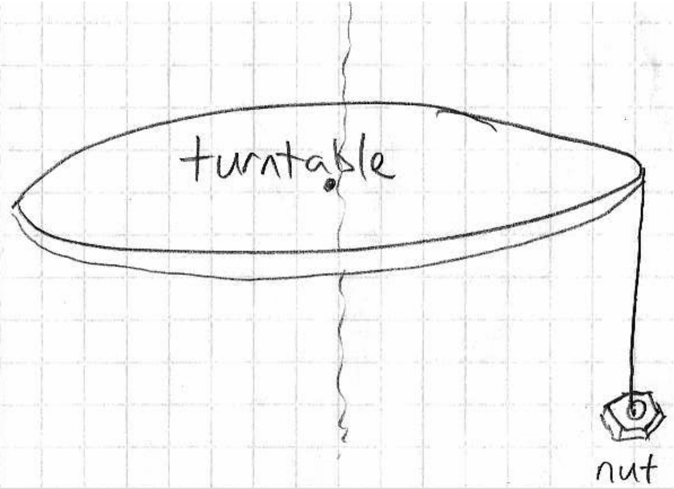
- If you go around in a circle at constant speed, your velocity vector is always changing direction.
- A change in velocity (whether magnitude, direction, or both) requires acceleration.
- For motion in a circle of radius R at constant speed v

$$a = \frac{v^2}{R}$$

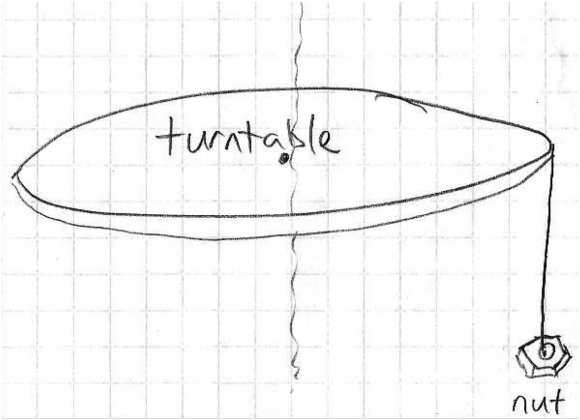
- This is called **centripetal acceleration**, and points toward the center of the circle.
- In the absence of a force (i.e. if vector sum of forces (if any) is zero), there is no acceleration, hence no change in velocity.

You are looking down (plan view) as I spin a (blue) ball on a string above my head in a circle at constant speed. The string breaks at the instant shown below. Which picture depicts the subsequent motion of the ball?





If the nut has mass m and the turntable is sitting idle, what is the tension in the string?

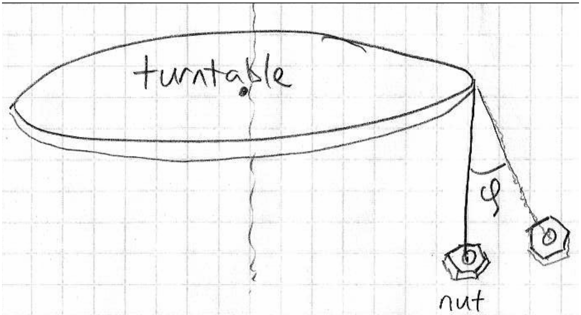


What will happen when I start the turntable spinning?

(A) The nut will continue to hang down vertically.

(B) The nut will move inward somewhat, making some angle φ w.r.t. the vertical axis.

(C) The nut will move outward somewhat, making some angle φ w.r.t. the vertical axis.



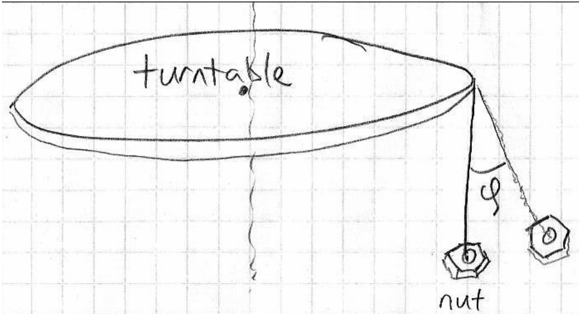
What will happen if I spin the turntable faster? Let T be the tension in the string.

(A) The nut will move farther outward. $T \sin\varphi$ provides the centripetal force mv/R^2, while $T \cos\varphi$ balances gravity mg.

(B) The nut will move farther outward. $T \cos\varphi$ provides the centripetal force mv/R^2, while $T \sin\varphi$ balances gravity mg.

(C) The nut will move farther outward. $T \sin\varphi$ provides the centripetal force mv^2/R, while $T \cos\varphi$ balances gravity mg.

(D) The nut will move farther outward. $T \cos\varphi$ provides the centripetal force mv^2/R, while $T \sin\varphi$ balances gravity mg.



$$m\vec{a}_{\text{nut}} = \vec{F}^{\text{tension}}_{s,\text{nut}} + \vec{F}^{G}_{E,\text{nut}}$$

$$0 = ma_y = T\cos\varphi - mg$$

$$T\cos\varphi = mg$$

We'll resume here next time.

$$-\frac{mv^2}{R} = ma_x = -T\sin\varphi$$

$$\frac{T\sin\varphi}{T\cos\varphi} = \tan\varphi = \frac{mv^2/R}{mg} = \frac{v^2}{gR}$$

Now suppose that friction provides the centripetal force

Suppose that a highway offramp that I often use bends with a radius of 20 meters. I notice that my car tires allow me (in good weather) to take this offramp at 15 m/s without slipping. How large does the offramp's bending radius need to be for me to be able to make the turn at 30 m/s instead?

(Assume that the frictional force between the road and my tires is the same in both cases and that the offramp is level (horizontal), i.e. not "banked.")

(A) 5 meters
(B) 10 meters
(C) 20 meters
(D) 30 meters
(E) 40 meters
(F) 80 meters

Now suppose that friction provides the centripetal force

- If velocity gets too large, penny flies off of turntable, as friction is no longer large enough to hold it in place.
- What if there are several pennies placed on the turntable at several different radii?
- As I slowly increase the speed at which the turntable rotates, do all of the pennies fly off at the same time?! Discuss!

If I put several pennies on the turntable at several different radii and turn the turntable slowly enough that all pennies stay put, which of the following statements is true?

(A) All pennies have the same velocity.

(B) All pennies make the same number of revolutions per second.

(C) All pennies have the same "angular velocity" $\omega = v/r$, but r will vary from penny to penny, so v will also vary.

(D) B and C are both true.

(E) A, B, and C are all true.

How can I best express the centripetal acceleration for each penny on the turntable?

(A) $a = v^2/r$

(B) $a = v^2/r = (r\omega)^2/r = \omega^2 r$

(C) a is the same for all pennies on the turntable

(D) (A) and (B) are both true, but (B) is a more useful way to describe what is happening on the turntable, because v varies from penny to penny, while ω is the same for all pennies.

(E) (A), (B), and (C) are all true

(F) (A), (B), and (C) are all false

Now we know that the centripetal acceleration can be written $a = \omega^2 r$ and varies with the radius of each penny. We also know that static friction must provide the force $m\omega^2 r$ to keep each penny going in a circle. Can we predict which pennies will slide off of the turntable first as I gradually increase the rotational velocity of the turntable?

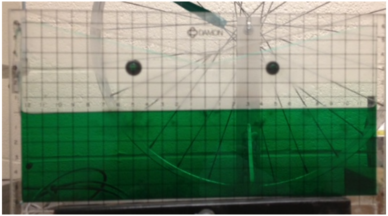
(A) The inside pennies will fly off first. This makes sense, because (for a given speed v) your tires screech more when you go around a turn with small radius than when you go around a turn with large radius.

(B) The outside pennies will fly off first. This makes sense because ω is the same for all pennies (v is not the same for all pennies), but $m\omega^2 r$ is largest for the outermost pennies.

(C) They all slide off at the same time.

(D) No way to predict.

What happens to the surface of this liquid if I center the tank atop the turntable and spin the turntable? (You'll have to make a leap of intuition, by analogy with the spinning nut-on-string, as we haven't studied fluids in this course.)



(A) The water surface will stay horizontal.

(B) The water surface is (when not spinning) perpendicular to the vector $(0, -g)$, i.e. it is horizontal. When spinning, the water surface will be perpendicular to the vector $(\omega^2 r, -g)$, with slope $= \omega^2 r/g$. So the surface will be triangular.

(C) The water surface is (when not spinning) perpendicular to the vector $(0, -mg)$, i.e. it is horizontal. When spinning, the water surface will be perpendicular to the vector $(\omega^2 r, -g)$, with slope $= \omega^2 r/g$. So the surface will be a parabola.

(D) All of the water will be stuck against the outer walls, as if trying to escape from a salad spinner.

Why does a salad spinner work?

(A) The outer wall of the spinner provides the centripetal force that pushes the lettuce toward the center of rotation, but the water feels no such force, because it can flow through the holes in the outer wall, thus separating water from lettuce.

(B) I really want to say "centrifugal force," even though my high-school teachers told me that there is really no such thing as "centrifugal force" — it's just a pseudo-force that one perceives when observing from the confusing perspective of a non-inertial reference frame.

(C) I guess you could say (A) or (B), but (A) is the way we've learned to analyze the situation methodically from Earth's reference frame. We haven't learned how to do calculations in non-inertial reference frames.

(D) While the obvious answer is (A), I am so fascinated by the pseudo-forces that appear in non-inertial reference frames that I went and read the Wikipedia article on the Coriolis effect!

Here is a good answer to the salad-spinner question: "The explanation for the physics going on as the spinner does its job is centripetal acceleration. The centripetal acceleration of an object in circular motion at constant speed tells us that the vector sum of the forces exerted on the object must be directed toward the center of the circle, continuously adjusting the objects direction. Without this inward pointing vector sum of forces, the object would move in a straight line. Centripetal force between the lettuce and the inside of the spinner pushes the lettuce around in a circle. On the other hand, the water can slip through the drain holes, so there's nothing to give it the same kind of push (and consequently there's no centripetal force to make it go in a circle). Thus, the lettuce experiences centripetal force while the water doesn't. In this way, the spinner manages to separate the two as the lettuce goes round in a circle and the water in a straight line through the holes."

Several people pointed out that we expect the water to shoot out *tangentially* from the spinner, since the water, once it loses contact with the lettuce, should travel in a straight line in the absence of a centripetal force. Need transparent salad spinner to verify!

Suppose I try to spin a pail of water in a vertical circle at constant rotational speed ω, with the water a distance R from the pivot point at my shoulder. So the water is moving at speed $v = \omega R$. (I'll demonstrate first with an empty pail.) Will the water fall out of the pail?

(A) The water will fall out while the pail is upside down, no matter how fast you spin it around.

(B) The water will stay in the pail, no matter how slowly you spin it around.

(C) The water will stay in the pail as long as you spin it fast enough. "Fast enough" means $v/R^2 > g$ (or equivalently $\omega^2/R > g$) when the bucket is upside-down.

(D) The water will stay in the pail as long as you spin it fast enough. "Fast enough" means $v^2/R > g$ (or equivalently $\omega^2 R > g$) when the bucket is upside-down.

The way to think about the water-in-bucket problem is

(A) The bottom surface of the bucket can both push and pull on the water, as if the water and bucket were glued together.

(B) The bottom surface of the bucket can push on the water (compressive force) but cannot pull on the water (no tensile force). If the required centripetal acceleration is large enough that the bucket must push on the water to keep it moving in a circle (even when Earth's gravity is pulling down on the water), then the water will stay in the bucket.

(C) When the bucket is upside down, the bottom surface of the bucket must "pull up" on the water to keep it inside the bucket, or else the water will spill out.

(D) The water stays in the upside-down bucket if the outward "centrifugal pseudo-force" (magnitude mv^2/R or $m\omega^2 R$) is at least as large as the downward force of gravity.

(E) I think you could say (B) or (D), but we haven't learned in this course how to analyze the "pseudo-forces" that one perceives when working in a non-inertial reference frame. So I prefer (B), which uses the Earth reference frame.

How does this thing work? (Discuss!)

`http://www.youtube.com/watch?v=oh9sn5gn2fk`

Can you tell me what movie this is from?
(Hints: directed by Stanley Kubrick, story by A.C. Clarke.)

An ice cube and a rubber ball are both placed at one end of a warm cookie sheet, and the sheet is then tipped up. The ice cube slides down with virtually no friction, and the ball rolls down without slipping. Which one makes it to the bottom first?

(A) They reach the bottom at the same time.

(B) The ball gets there slightly faster, because the ice cube's friction (while very small) is kinetic and dissipates some energy, while the rolling ball's friction is static and does not dissipate energy.

(C) The ice cube gets there substantially faster, because the ball's initial potential energy mgh gets shared between $\frac{1}{2}mv^2$ (translational) and $\frac{1}{2}I\omega^2$ (rotational), while essentially all of the ice cube's initial mgh goes into $\frac{1}{2}mv^2$ (translational).

(D) The ice cube gets there faster because the ice cube's friction is negligible, while the frictional force between the ball and the cookie sheet dissipates the ball's kinetic energy into heat.

A hollow cylinder and a solid cylinder both roll down an inclined plane without slipping. Does friction play an important role in the cylinders' motion?

(A) No, friction plays a negligible role.

(B) Yes, (kinetic) friction dissipates a substantial amount of energy as the objects roll down the ramp.

(C) Yes, (static) friction is what causes the objects to roll rather than to slide. Without static friction, they would just slide down, so there would be no rotational motion (if you just let go of each cylinder from rest at the top of the ramp).

Why are people who write physics problems (e.g. about cylinders rolling down inclined planes) so fond of the phrase "rolls without slipping?"

(A) Because Nature abhors the frictional dissipation of energy.

(B) Because "rolls without slipping" implies that $v = \omega R$, where v is the cylinder's (translational) speed down the ramp. This lets you directly relate the rotational and translational parts of the motion.

(C) No good reason. You could analyze the problem just as easily if the cylinders were slipping somewhat while they roll.

How do I write the total kinetic energy of an object that has both translational motion at speed v and rotational motion at speed ω?

(Note that the symbol I is a capital I (for rotational "inertia") in the sans-serif font that I use to make my slides. Sorry!)

(A) $K = \frac{1}{2}mv^2$
(B) $K = \frac{1}{2}I\omega^2$
(C) $K = \frac{1}{2}I^2\omega$
(D) $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
(E) $K = \frac{1}{2}mv^2 + \frac{1}{2}I^2\omega$
(F) $K = \frac{1}{2}m\omega^2 + \frac{1}{2}Iv^2$

While you discuss, I'll throw a familiar object across the room, for you to look at now from the perspective of Chapters 11 and 12.

Sliding vs. rolling downhill:

For translational motion with no friction, $v_f = \sqrt{2gh}$ because

$$mgh_i = \frac{1}{2}mv_f^2$$

For rolling without slipping, we can write $\omega_f = v_f/R$:

$$mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\left(\frac{v_f}{R}\right)^2$$

$$mgh_i = \frac{1}{2}mv_f^2\left(1 + \frac{I}{mR^2}\right)$$

So the final velocity is slower (as are all intermediate velocities):

$$v_f = \sqrt{\frac{2gh}{1 + \frac{I}{mR^2}}}$$

A hollow cylinder and a solid cylinder both roll down an inclined plane without slipping. Assuming that the two cylinders have the same mass and same outer radius, which one has the larger rotational inertia?

(A) The hollow cylinder has the larger rotational inertia, because the material is concentrated at larger radius.

(B) The solid cylinder has the larger rotational inertia, because the material is distributed over more area.

(C) The rotational inertias are the same, because the masses and radii are the same.

The rolling object's downhill acceleration is smaller by a factor

$$\left(\frac{1}{1+\frac{I}{mR^2}}\right)$$

$I = mR^2$ for hollow cylinder. $\frac{1}{1+1} = 0.5$

$I = \frac{2}{3}mR^2$ for hollow sphere. $\frac{1}{1+(2/3)} = 0.60$

$I = \frac{1}{2}mR^2$ for solid cylinder. $\frac{1}{1+(1/2)} = 0.67$

$I = \frac{2}{5}mR^2$ for solid sphere. $\frac{1}{1+(2/5)} = 0.71$

Using Chapter 11 ideas, we know how to analyze the rolling objects' motion using energy arguments. With Chapter 12 ideas, we can look again at the same problem using torque arguments, and directly find each object's downhill acceleration.

Rotational inertia

For an extended object composed of several particles, with particle j having mass m_j and distance r_j from the rotation axis,

$$I = \sum_{j \text{ in particles}} m_j r_j^2$$

For a continuous object like a sphere or a solid cylinder, you have to integrate (or more often just look up the answer):

$$I = \int r^2 \, \mathrm{d}m$$

If you rearrange the same total mass to put it at larger distance from the axis of rotation, you get a larger rotational inertia.

(Which of these two adjustable cylinder-like objects has the larger rotational inertia?)

inertia

$$m$$

translational velocity

$$v$$

translational K.E.

$$K = \frac{1}{2}mv^2$$

momentum

$$p = mv$$

rotational inertia

$$I = \sum mr^2$$

rotational velocity

$$\omega$$

rotational K.E.

$$K = \frac{1}{2}I\omega^2$$

angular momentum

$$L = I\omega$$

We learned earlier that momentum can be transferred from one object to another, but cannot be created or destroyed.

Consequently, a system on which no external forces are exerted (an "isolated system") has a constant momentum ($\vec{p} = m\vec{v}$):

$$\Delta \vec{p} = 0$$

We now also know that angular momentum can be transferred from one object to another, but cannot be created or destroyed.

So a system on which no external torques are exerted has a constant angular momentum ($L = I\omega$):

$$\Delta \vec{L} = 0$$

If I spin around while sitting on a turntable (so that I am rotationally "isolated") and suddenly decrease my own rotational inertia, what happens to my rotational velocity?

position

$$\vec{r} = (x, y)$$

velocity

$$\vec{v} = (v_x, v_y) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}$$

acceleration

$$\vec{a} = (a_x, a_y) = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

if a_x is constant then:

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$v_{x,f}^2 = v_{x,i}^2 + 2 a_x \Delta x$$

rotational coordinate

$$\vartheta = s/r$$

rotational velocity

$$\omega = \frac{\mathrm{d}\vartheta}{\mathrm{d}t}$$

rotational acceleration

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$$

if α is constant then:

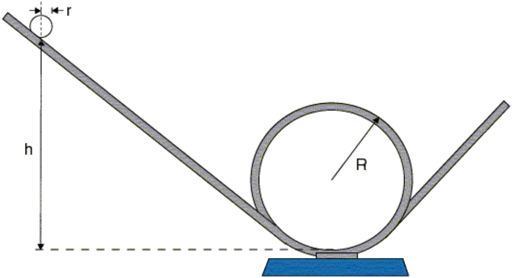
$$\omega_f = \omega_i + \alpha t$$

$$\vartheta_f = \vartheta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\vartheta$$

4. An automobile accelerates from rest starting at t = 0 such that its tires undergo a constant rotational acceleration $\alpha = 5.9\ \mathrm{s}^{-2}$. The radius of each tire is 0.29 m. At t = 11 s after the acceleration begins, find (a) the instantaneous rotational speed ω of the tires, (b) the total rotational displacement $\Delta\vartheta$ of each tire, (c) the linear speed v of the automobile (assuming the tires stay perfectly round) and (d) the total distance the car travels in the 11 s.

Let R be the **radius** of the circle in this loop-the-loop demo. I want the ball to make it all the way around the loop without falling off. What is the lowest height h at which I can start the ball (from rest)?



(A) The ball will make it all the way around if $h \geq R$.

(B) The ball will make it all the way around if $h \geq 2R$.

(C) If $h = 2R$, the ball will just make it to the top and will then fall down. When the ball is at the top of the circle, its velocity must still be large enough to require a downward normal force exerted by the track on the ball. So the minimum h is even larger than $2R$. My neighbor and I are discussing now just how much higher that should be.

Suppose the ball makes it all the way around the loop-the-loop with much more than sufficient speed to stay on the circular track. Let the y-axis point upward, and let $v_{\rm top}$ be the ball's speed when it reaches the top of the loop. What is the y component, a_y, of the ball's acceleration when it is at the very top of the loop?

(A) $a_y = -g$

(B) $a_y = +g$

(C) $a_y = +v_{\rm top}^2/R$

(D) $a_y = -v_{\rm top}^2/R$

(E) $a_y = +g + v_{\rm top}^2/R$

(F) $a_y = -g - v_{\rm top}^2/R$

(G) $a_y = +g + v_{\rm top}/R^2$

(H) $a_y = -g - v_{\rm top}/R^2$

The track can push on the ball, but it can't pull on the ball! How do I express the fact that the track is still pushing on the ball even at the very top of the loop?

(A) Write the equation of motion for the ball: $m\vec{a} = \sum \vec{F}_{\text{on ball}}$, and require the normal force exerted by the track on the ball to point inward, even at the very top.

(B) Use conservation of angular momentum.

(C) Draw a free-body diagram for the ball, and require that gravity and the normal force point in opposite directions.

(D) Draw a free-body diagram for the ball, and require that the magnitude of the normal force be at least as large as the force of Earth's gravity on the ball.

For the ball to stay in contact with the track when it is at the top of the loop, there must still be an inward-pointing normal force exerted by the track on the ball, even at the very top. How can I express this fact using $\boxed{ma_y = \sum F_y}$? Let $v_{\rm top}$ be the ball's speed at the top of the loop.

(A) $+mv_{\rm top}^2/R = +mg + F_{tb}^N$ with $F_{tb}^N > 0$

(B) $+mv_{\rm top}^2/R = +mg - F_{tb}^N$ with $F_{tb}^N > 0$

(C) $+mv_{\rm top}^2/R = -mg + F_{tb}^N$ with $F_{tb}^N > 0$

(D) $+mv_{\rm top}^2/R = -mg - F_{tb}^N$ with $F_{tb}^N > 0$

(E) $-mv_{\rm top}^2/R = +mg + F_{tb}^N$ with $F_{tb}^N > 0$

(F) $-mv_{\rm top}^2/R = +mg - F_{tb}^N$ with $F_{tb}^N > 0$

(G) $-mv_{\rm top}^2/R = -mg + F_{tb}^N$ with $F_{tb}^N > 0$

(H) $-mv_{\rm top}^2/R = -mg - F_{tb}^N$ with $F_{tb}^N > 0$

How do I decide the minimum height h from which the ball will make it all the way around the loop without losing contact with the track? For simplicity, assume that the track is very slippery, so that you can neglect the ball's rotational kinetic energy.

(A) $2mgR = \frac{1}{2}mv_{\rm top}^2 + mgh$ with $v_{\rm top} = \sqrt{gR}$

(B) $mgR = \frac{1}{2}mv_{\rm top}^2 + mgh$ with $v_{\rm top} = \sqrt{gR}$

(C) $mgh = \frac{1}{2}mv_{\rm top}^2 + 2mgR$ with $v_{\rm top} = \sqrt{gR}$

(D) $mgh = \frac{1}{2}mv_{\rm top}^2 + mgR$ with $v_{\rm top} = \sqrt{gR}$

(By the way, how would the answer change if I said instead that the (solid) ball rolls without slipping on the track?)

Physics 8 — Monday, October 14, 2019

- Pick up HW6 handout, due this Friday, if you haven't already done so.
- This week you're reading Ch12 (torque). Today in class we'll finish Ch10 and start Ch11.