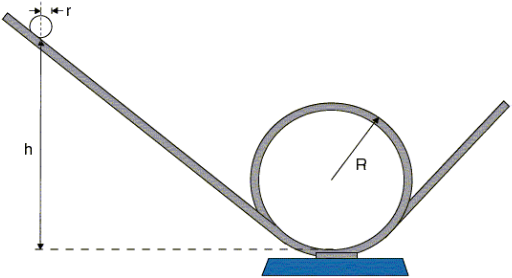
Physics 8 — Monday, October 21, 2019

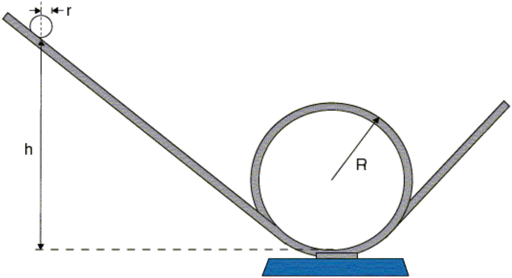
- This week you're reading Giancoli Ch9 (static equilibrium, etc.), and O/K Ch1. In class, we're still finishing up Mazur Ch11 (motion in a circle). You may want to buy one of my $10 used copies of Onouye/Kane, which you can either keep or sell back to me for $10 in December.

Let R be the **radius** of the circle in this loop-the-loop demo. I want the ball to make it all the way around the loop without falling off. What is the lowest height h at which I can start the ball (from rest)?



(Think now — we'll do multiple-choice on the next page.)

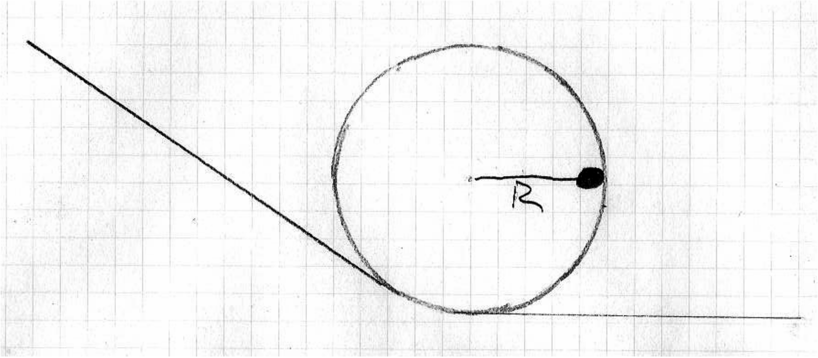
Let R be the **radius** of the circle in this loop-the-loop demo. I want the ball to make it all the way around the loop without falling off. What is the lowest height h at which I can start the ball (from rest)?



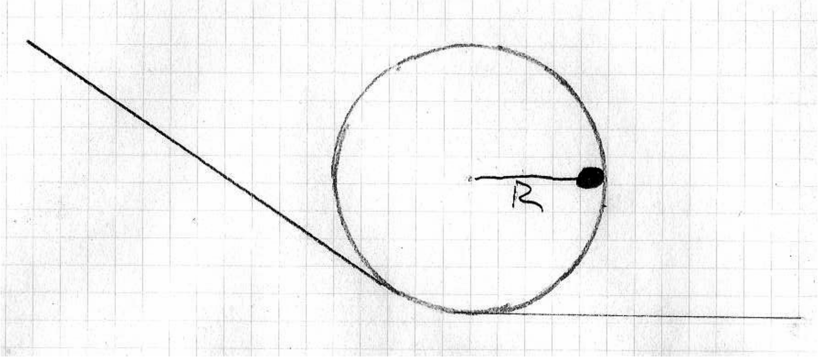
(A) The ball will make it all the way around if $h \geq R$.

(B) The ball will make it all the way around if $h \geq 2R$.

(C) If $h = 2R$, the ball will just make it to the top and will then fall down (assuming, for the moment, that it slides frictionlessly along the track). When the ball is at the top of the circle, its velocity must still be large enough to require a downward normal force exerted by the track on the ball. So the minimum h is even larger than $2R$. My neighbor and I are discussing now just how much higher that should be.

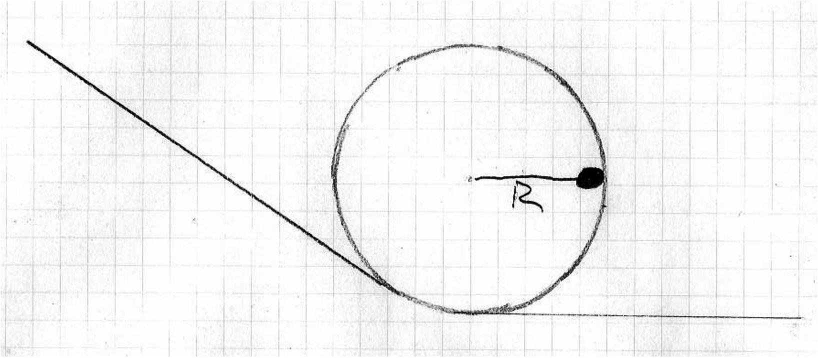


The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), in what direction does its velocity vector point?



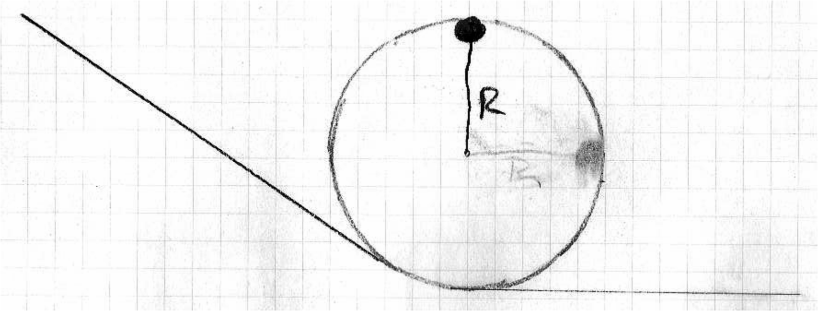
The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), what do we know about the vertical (y axis points up) component, a_y, of the ball's acceleration vector?

If $a_y \neq 0$, what vertical force(s) F_y is/are responsible?



The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), what do we know about the horizontal (x axis points right) component, a_x, of the ball's acceleration vector?

If $a_x \neq 0$, what horizontal force(s) F_x is/are responsible?



Suppose the ball makes it all the way around the circle without falling off. At the instant when the ball is at the position shown (at top of circle), what do we know about the vertical component, a_y, of the ball's acceleration vector?

If $a_y \neq 0$, what vertical force(s) F_y is/are responsible?

Suppose the ball makes it all the way around the loop-the-loop with much more than sufficient speed to stay on the circular track. Let the y-axis point upward, and let v_{top} be the ball's speed when it reaches the top of the loop. What is the y component, a_y, of the ball's acceleration when it is at the very top of the loop?

(A) $a_y = -g$

(B) $a_y = +g$

(C) $a_y = +v_{\mathrm{top}}^2/R$

(D) $a_y = -v_{\mathrm{top}}^2/R$

(E) $a_y = +g + v_{\mathrm{top}}^2/R$

(F) $a_y = -g - v_{\mathrm{top}}^2/R$

(G) $a_y = +g + v_{\mathrm{top}}/R^2$

(H) $a_y = -g - v_{\mathrm{top}}/R^2$

The track can push on the ball, but it can't pull on the ball! How do I express the fact that the track is still pushing on the ball even at the very top of the loop?

(A) Write the equation of motion for the ball: $m\vec{a} = \sum \vec{F}_{\text{on ball}}$, and require the normal force exerted by the track on the ball to point inward, even at the very top. (At the very top, "inward" is "downward.") If the equation $m\vec{a} = \sum \vec{F}_{\text{on ball}}$ gave us an outward-pointing normal force (exerted by track on ball), that would be inconsistent with the ball's staying in contact with the track.

(B) Use conservation of angular momentum.

(C) Draw a free-body diagram for the ball, and require that gravity and the normal force point in opposite directions.

(D) Draw a free-body diagram for the ball, and require that the magnitude of the normal force be at least as large as the force of Earth's gravity on the ball.

For the ball to stay in contact with the track when it is at the top of the loop, there must still be an inward-pointing normal force exerted by the track on the ball, even at the very top. How can I express this fact using $\boxed{ma_y = \sum F_y}$? Let $v_{\rm top}$ be the ball's speed at the top of the loop.

(A) $+mv_{\rm top}^2/R = +mg + F_{tb}^N$ with $F_{tb}^N > 0$

(B) $+mv_{\rm top}^2/R = +mg - F_{tb}^N$ with $F_{tb}^N > 0$

(C) $+mv_{\rm top}^2/R = -mg + F_{tb}^N$ with $F_{tb}^N > 0$

(D) $+mv_{\rm top}^2/R = -mg - F_{tb}^N$ with $F_{tb}^N > 0$

(E) $-mv_{\rm top}^2/R = +mg + F_{tb}^N$ with $F_{tb}^N > 0$

(F) $-mv_{\rm top}^2/R = +mg - F_{tb}^N$ with $F_{tb}^N > 0$

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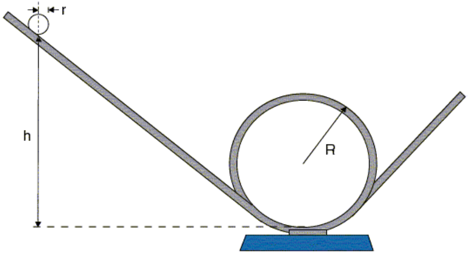


How do I decide the minimum height h from which the ball will make it all the way around the loop without losing contact with the track? For simplicity, assume that the track is very slippery, so that you can neglect the ball's rotational kinetic energy.

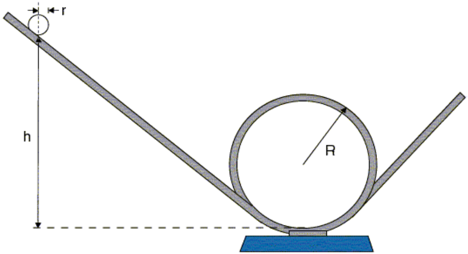
(A) $2mgR = \frac{1}{2}mv_{\rm top}^2 + mgh$ with $v_{\rm top} = \sqrt{gR}$

(B) $mgR = \frac{1}{2}mv_{\rm top}^2 + mgh$ with $v_{\rm top} = \sqrt{gR}$

(C) $mgh = \frac{1}{2}mv_{\rm top}^2 + 2mgR$ with $v_{\rm top} = \sqrt{gR}$

(D) $mgh = \frac{1}{2}mv_{\rm top}^2 + mgR$ with $v_{\rm top} = \sqrt{gR}$

(By the way, how would the answer change if I said instead that the (solid) ball rolls without slipping on the track?)



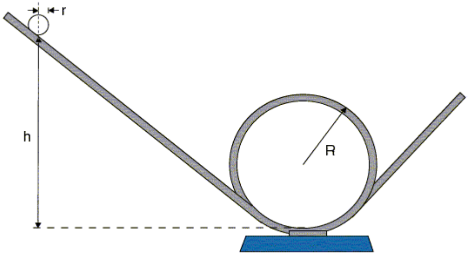
How do I decide the minimum height h from which the ball will make it all the way around the loop without losing contact with the track? Let's now be realistic: the ball is a solid sphere that rolls without slipping on the track.

(A) $mgh = \frac{1}{2}mv_{\text{top}}^2 + 2mgR$

(B) $mgh = \frac{1}{2}mv_{\text{top}}^2 + \frac{1}{2}I\omega_{\text{top}}^2 + 2mgR$

with $v_{\text{top}} = \sqrt{gR}$ and $\omega_{\text{top}} = v_{\text{top}}/r_{\text{ball}}$

(Little "r_{ball}" is the radius of the ball. Big "R" is the radius of the loop-the-loop.)



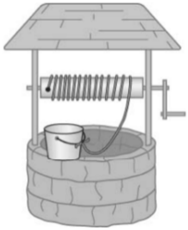
$$mgh = \frac{1}{2} m v_{\text{top}}^2 + \frac{1}{2} I \omega_{\text{top}}^2 + 2mgR$$

with $v_{\text{top}} = \sqrt{gR}$ and $\omega_{\text{top}} = v_{\text{top}}/r_{\text{ball}}$ and $I = \frac{2}{5} m r_{\text{ball}}^2$.

$$mgh = \frac{1}{2} m(gR) + \frac{\frac{2}{5} m r_{\text{ball}}^2}{2 r_{\text{ball}}^2}(gR) + 2mgR = 2.7mgR$$

We stopped after this.

6*. You accidentally knock a full bucket of water off the side of the well shown in the figure at right. The bucket plunges 18 m to the bottom of the well. Attached to the bucket is a light rope that is wrapped around the crank cylinder. How fast is the handle turning (rotational speed) when the bucket hits bottom? The inertia of the bucket plus water is 13 kg. The crank cylinder is a solid cylinder of radius 0.65 m and inertia 5.0 kg. (Assume the small handle's inertia is negligible in comparison with the crank cylinder.)



How would you approach this problem? Discuss with your neighbor while I set up a demonstration along the same lines . . .

(A) initial angular momentum of bucket equals final angular momentum of cylinder + bucket
(B) initial G.P.E. equals final K.E. (translational for bucket + rotational for cylinder)
(C) initial G.P.E. equals final K.E. of bucket
(D) initial G.P.E. equals final K.E. of cylinder
(E) initial K.E. of bucket equals final G.P.E.
(F) use torque = mgR to find constant angular acceleration

- What is the rotational inertia for a solid cylinder?
- How do you relate v of the bucket with ω of the cylinder? Why is this true?
- What is the expression for the total kinetic energy?
- Why is angular momentum not the same for the initial and final states?
- What are the two expressions for angular momentum used in Chapter 11?
- Does anyone know (though this is in Chapter 12 and is tricky) why using $\tau = mgR$ would not give the correct angular acceleration? What if you used $\tau = TR$, where T is the tension in the rope?

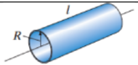
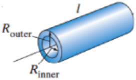
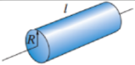
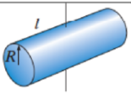
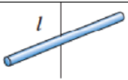
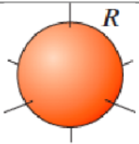
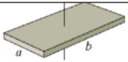
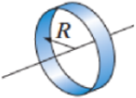
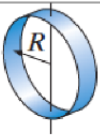
configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder $\perp$ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod $\perp$ to axis		$(1/12)m\ell^2$

Table 11.3. Also in "equation sheet"
http://positron.hep.upenn.edu/p8/files/equations.pdf

hollow sphere		$(2/3)mR^2$
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2+b^2)$
thin hoop about its axis		mR^2
thin hoop $\perp$ to axis		$(1/2)mR^2$

5. A long, thin rod is pivoted from a hinge such that it hangs vertically from one end. (The hinge is at the top.) The length of the rod is 1.23 m. You want to hit the lower end of the rod just hard enough to get the rod to swing all the way up and over the pivot (i.e. to swing more than 180°). How fast do you have to make the end go?

How would you approach this problem? Discuss with neighbors!
Which (if any) of these statements is **false** ?

(A) I know the change in G.P.E from the initial to the desired final states. So the initial K.E. (which is rotational) of the rod needs to be at least this large.

(B) The book (or equation sheet) gives rotational inertia I for a long, thin rod about its center. So I can use the parallel-axis theorem to get I for the rod about one end.

(C) The angular momentum, $L = I\omega$, is the same for the initial and final states.

(D) Because the rod pivots about one end, the speed of the other end is $v = \omega\ell$ (where ℓ is length of rod)

(E) None. (All of the above statements are true.)

The rotational inertia for a long, thin rod of length ℓ about a perpendicular axis through its center is

$$I = \frac{1}{12} m\ell^2$$

What is its rotational inertia about one end?

(A) $\frac{1}{12} m\ell^2$
(B) $\frac{1}{24} m\ell^2$
(C) $\frac{1}{2} m\ell^2$
(D) $\frac{1}{3} m\ell^2$
(E) $\frac{1}{4} m\ell^2$
(F) $\frac{1}{6} m\ell^2$

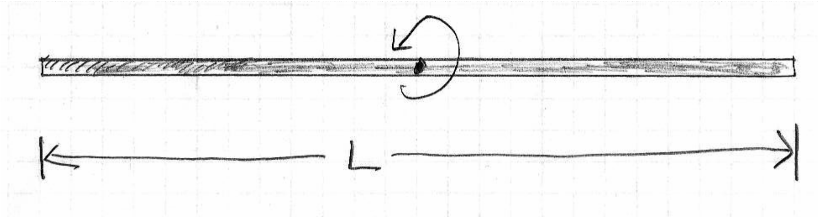
If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance $d_\perp$ from CoM), the rotational inertia is larger, because the object's CoM revolves around a circle of radius $d_\perp$ and in addition the object rotates about its own CoM.

This larger rotational inertia is given by the *parallel axis theorem*:

$$I = I_{\rm cm} + Md_\perp^2$$

where $I_{\rm cm}$ is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's CoM.

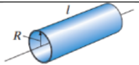
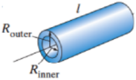
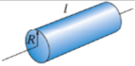
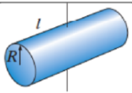
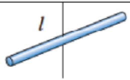
(We'll go over the parallel-axis theorem again next time. First I want to make sure you know what you need for this week's HW.)



The rotational inertia of a long, thin rod (whose thickness is negligible compared with its length) of mass M and length L, for rotation about its CoM, is

$$I = \frac{1}{12} M L^2$$

Using the parallel axis theorem, what is the rod's rotational inertia for rotation about one end?

configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder $\perp$ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod $\perp$ to axis		$(1/12)m\ell^2$

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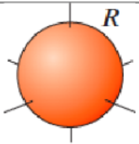
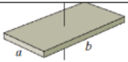
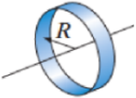
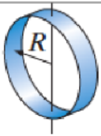
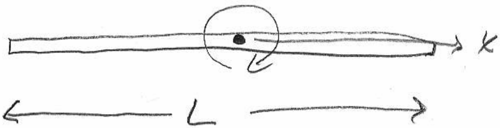
http://positron.hep.upenn.edu/p8/files/equations.pdf

hollow sphere		$(2/3)mR^2$
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2+b^2)$
thin hoop about its axis		mR^2
thin hoop $\perp$ to axis		$(1/2)mR^2$

(In case you're curious where that $I = ML^2/12$ comes from.)



$$I = \sum m r^2 \rightarrow \int r^2\, dm$$

$$dm = \frac{M}{L}\, dx \qquad r = |x|$$

$$I = \int_{-L/2}^{L/2} x^2 \left(\frac{M}{L}\right) dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{x=-\frac{L}{2}}^{x=+\frac{L}{2}}$$

$$I = \frac{M}{L}\left[\frac{(L/2)^3}{3} - \frac{(-L/2)^3}{3}\right] = \frac{ML^2}{12}$$

3*. You have a weekend job selecting speed-limit signs to put at road curves. The speed limit is determined by the radius of the curve and the bank angle of the road w.r.t. horizontal. Your first assignment today is a turn of radius 250 m at a bank angle of 4.8°. (a) What speed limit sign should you choose for that curve such that a car traveling at the speed limit negotiates the turn successfully even when the road is wet and slick? (So at this speed, it stays on the road even when friction is negligible.) (b) Draw a free-body diagram showing all of the forces acting on the car when it is moving at this maximum speed. (Your diagram should also indicate the direction of the car's acceleration vector.)

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