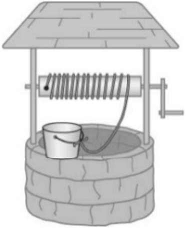
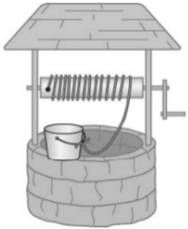
Physics 8 — Wednesday, October 23, 2019

- This week you're reading Giancoli Ch9 (static equilibrium, etc.), and O/K Ch1. In class, we're still finishing up Mazur Ch11 (motion in a circle), and today or Friday we'll start Ch12 (torque). You may want to buy one of my $10 used copies of Onouye/Kane, which you can either keep or sell back to me for $10 in December.
- HW7 due Friday. HW help W4–6pm 3C4, R6–8pm 2C4.
- **How would you approach this problem?**

6*. You accidentally knock a full bucket of water off the side of the well shown in the figure at right. The bucket plunges 18 m to the bottom of the well. Attached to the bucket is a light rope that is wrapped around the crank cylinder. How fast is the handle turning (rotational speed) when the bucket hits bottom? The inertia of the bucket plus water is 13 kg. The crank cylinder is a solid cylinder of radius 0.65 m and inertia 5.0 kg. (Assume the small handle's inertia is negligible in comparison with the crank cylinder.)



6*. You accidentally knock a full bucket of water off the side of the well shown in the figure at right. The bucket plunges 18 m to the bottom of the well. Attached to the bucket is a light rope that is wrapped around the crank cylinder. How fast is the handle turning (rotational speed) when the bucket hits bottom? The inertia of the bucket plus water is 13 kg. The crank cylinder is a solid cylinder of radius 0.65 m and inertia 5.0 kg. (Assume the small handle's inertia is negligible in comparison with the crank cylinder.)



How would you approach this problem? Discuss with your neighbor while I set up a demonstration along the same lines . . .

(A) initial angular momentum of bucket equals final angular momentum of cylinder + bucket

(B) initial G.P.E. equals final K.E. (translational for bucket + rotational for cylinder)

(C) initial G.P.E. equals final K.E. of bucket

(D) initial G.P.E. equals final K.E. of cylinder

(E) initial K.E. of bucket equals final G.P.E.

(F) use torque = mgR to find constant angular acceleration

- What is the rotational inertia for a solid cylinder?
- How do you relate v of the bucket with ω of the cylinder? Why is this true?
- What is the expression for the total kinetic energy?
- Why is angular momentum not the same for the initial and final states?
- What are the two expressions for angular momentum used in Chapter 11?
- Does anyone know (though this is in Chapter 12 and is tricky) why using $\tau = mgR$ would not give the correct angular acceleration? What if you used $\tau = TR$, where T is the tension in the rope?

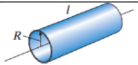
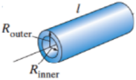
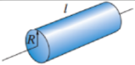
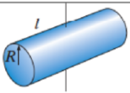
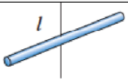
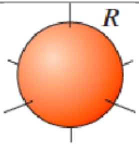
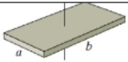
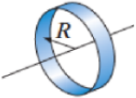
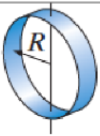
configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder ⊥ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod ⊥ to axis		$(1/12)m\ell^2$

Table 11.3. Also in "equation sheet"

http://positron.hep.upenn.edu/p8/files/equations.pdf

hollow sphere		$(2/3)mR^2$
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2+b^2)$
thin hoop about its axis		mR^2
thin hoop $\perp$ to axis		$(1/2)mR^2$

5. A long, thin rod is pivoted from a hinge such that it hangs vertically from one end. (The hinge is at the top.) The length of the rod is 1.23 m. You want to hit the lower end of the rod just hard enough to get the rod to swing all the way up and over the pivot (i.e. to swing more than 180°). How fast do you have to make the end go?

How would you approach this problem? Discuss with neighbors!
Which (if any) of these statements is **false** ?

(A) I know the change in G.P.E from the initial to the desired final states. So the initial K.E. (which is rotational) of the rod needs to be at least this large.

(B) The book (or equation sheet) gives rotational inertia I for a long, thin rod about its center. So I can use the parallel-axis theorem to get I for the rod about one end.

(C) The angular momentum, $L = I\omega$, is the same for the initial and final states.

(D) Because the rod pivots about one end, the speed of the other end is $v = \omega\ell$ (where ℓ is length of rod)

(E) None. (All of the above statements are true.)

The rotational inertia for a long, thin rod of length ℓ about a perpendicular axis through its center is

$$I = \frac{1}{12} m\ell^2$$

What is its rotational inertia about one end?

(A) $\frac{1}{12} m\ell^2$

(B) $\frac{1}{24} m\ell^2$

(C) $\frac{1}{2} m\ell^2$

(D) $\frac{1}{3} m\ell^2$

(E) $\frac{1}{4} m\ell^2$

(F) $\frac{1}{6} m\ell^2$

(We'll repeat this question after some explanation.)

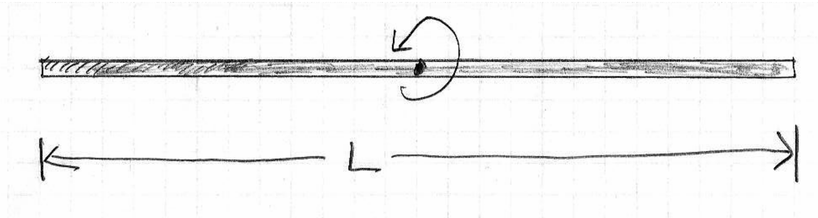
If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance $d_\perp$ from CoM), the rotational inertia is larger, because the object's CoM revolves around a circle of radius $d_\perp$ and in addition the object rotates about its own CoM.

This larger rotational inertia is given by the *parallel axis theorem*:

$$I = I_{\rm cm} + M d_\perp^2$$

where $I_{\rm cm}$ is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's CoM.

(We'll go over the parallel-axis theorem again next time. First I want to make sure you know what you need for this week's HW.)



The rotational inertia of a long, thin rod (whose thickness is negligible compared with its length) of mass M and length L, for rotation about its CoM, is

$$I = \frac{1}{12} ML^2$$

Using the parallel axis theorem, what is the rod's rotational inertia for rotation about one end? (Click next page.)

The rotational inertia for a long, thin rod of length ℓ about a perpendicular axis through its center is

$$I = \frac{1}{12} m\ell^2$$

What is its rotational inertia about one end?

(A) $\frac{1}{12} m\ell^2$

(B) $\frac{1}{24} m\ell^2$

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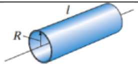
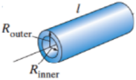
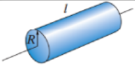
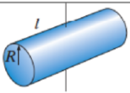
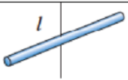
configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder $\perp$ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod $\perp$ to axis		$(1/12)m\ell^2$

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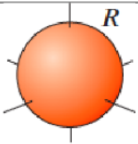
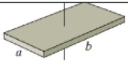
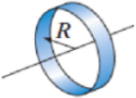
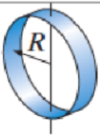
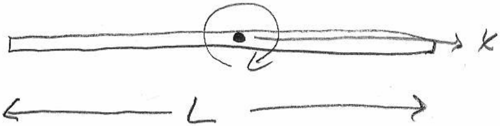
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2+b^2)$
thin hoop about its axis		mR^2
thin hoop $\perp$ to axis		$(1/2)mR^2$

(In case you're curious where that $I = ML^2/12$ comes from.)



$$I = \sum m r^2 \rightarrow \int r^2 \, dm$$

$$dm = \frac{M}{L} dx \qquad r = |x|$$

$$I = \int_{-L/2}^{L/2} x^2 \left(\frac{M}{L}\right) dx = \frac{M}{L} \left[\frac{x^3}{3}\right]_{x=-\frac{L}{2}}^{x=+\frac{L}{2}}$$

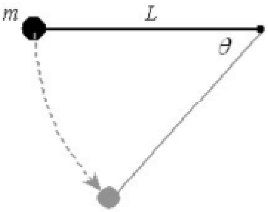
$$I = \frac{M}{L} \left[\frac{(L/2)^3}{3} - \frac{(-L/2)^3}{3}\right] = \frac{ML^2}{12}$$

3*. You have a weekend job selecting speed-limit signs to put at road curves. The speed limit is determined by the radius of the curve and the bank angle of the road w.r.t. horizontal. Your first assignment today is a turn of radius 250 m at a bank angle of 4.8°. (a) What speed limit sign should you choose for that curve such that a car traveling at the speed limit negotiates the turn successfully even when the road is wet and slick? (So at this speed, it stays on the road even when friction is negligible.) (b) Draw a free-body diagram showing all of the forces acting on the car when it is moving at this maximum speed. (Your diagram should also indicate the direction of the car's acceleration vector.)

Let's start by drawing an FBD (for the car) for the case where the car's speed is at exactly the value for which no friction at all is needed to keep the car moving in its circular path. In that case, what are the forces acting on the car?

Alongside the FBD, let's draw (elevation view) the car on the banked road. Let's assume that the road curves to the left.

2. You attach one end of a string of length L to a small ball of inertia m. You attach the string's other end to a pivot that allows free revolution. You hold the ball out to the side with the string taut along a horizontal line, as the in figure (below, left). (a) If you release the ball from rest, what is the tension in the string as a function of angle ϑ swept through? (b) What should the tensile strength of the string be (the maximum tension it can sustain without breaking) if you want it not to break through the ball's entire motion (from $\vartheta = 0$ to $\vartheta = \pi$)?

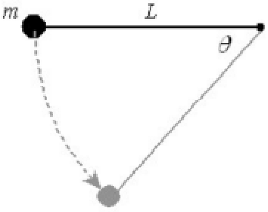


- Since the string's length L stays constant, what shape does the ball's path trace out as it moves?
- Does the ball's acceleration have a component that points along the axis of the string? If so, does its magnitude depend on the ball's speed?
- What two forces are acting on the ball?
- Assuming that no energy is dissipated, how can we relate the ball's speed v to its height y ?
- Can you write $m\vec{a} = \sum \vec{F}$ for the component of $\vec{a}$ and $\sum \vec{F}$ that points along the string?

("small" ball $\Rightarrow$ neglect the ball's rotation about its own CoM)

2. You attach one end of a string of length L to a small ball of inertia m. You attach the string's other end to a pivot that allows free revolution. You hold the ball out to the side with the string taut along a horizontal line, as the in figure (below, left). (a) If you release the ball from rest, what is the tension in the string as a function of angle ϑ swept through? (b) What should the tensile strength of the string be (the maximum tension it can sustain without breaking) if you want it not to break through the ball's entire motion (from $\vartheta = 0$ to $\vartheta = \pi$)?

How do I relate angle θ to speed v? $E_i = mgL \quad \rightarrow \quad E = \frac{1}{2}mv^2 + mgy$



(A) $\frac{1}{2}mv^2 = mg(L-y) = mgL(1-\cos\theta)$

(B) $\frac{1}{2}mv^2 = mg(L-y) = mgL(1-\sin\theta)$

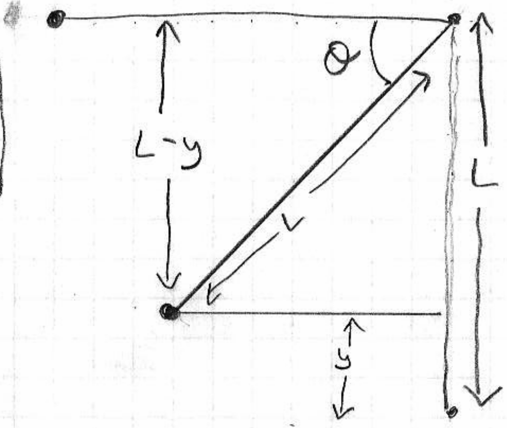
(C) $\frac{1}{2}mv^2 = mg(L-y) = mgL\cos\theta$

(D) $\frac{1}{2}mv^2 = mg(L-y) = mgL\sin\theta$

Hint: draw on the figure a vertical line of length $L - y = y_i - y$

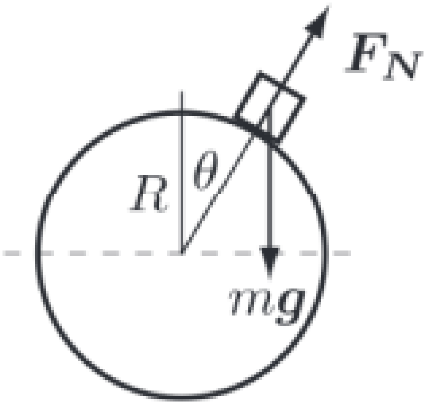
Next: write "radial" component of $m\vec{a} = \sum \vec{F}$ to find T

$$\sin\theta = \frac{L-y}{L}$$

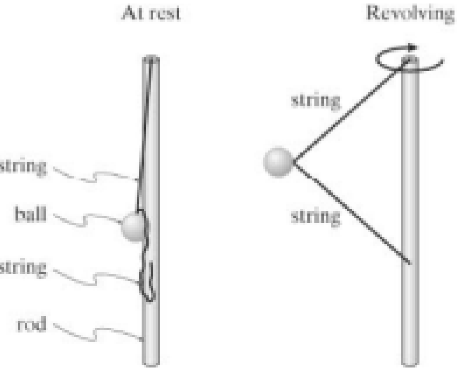


When does the block lose contact with the sphere?

A small block of mass m slides down a sphere of radius R, starting from rest at the top. The sphere is immobile, and friction between the block and the sphere is negligible. In terms of m, g, R, and θ, determine:
(a) the K.E. of the block;
(b) the centripetal acceleration of the block;
(c) the normal force exerted by the sphere on the block.
(d) At what value of θ does the block lose contact with the sphere?

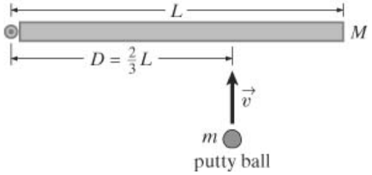


8*. A ball is attached to a vertical rod by two strings of equal strength and equal length. (See figure, below left.) The strings are very light and do not stretch. The rod begins to rotate under the influence of a constant rotational acceleration. (a) Which string breaks first? (b) Draw a free-body diagram for the ball, indicating all forces and their relative magnitudes, to justify your answer to (a).



- Are the angles of the two strings w.r.t. horizontal equal?
- Are the tensions in the two strings equal? How do you know?
- What three forces act on the ball?
- Is the ball accelerating vertically? Horizontally?
- Draw a FBD for the ball, showing both horizontal (radial) and vertical component of each force.

Notice that the ball's speed v increases with time, until finally one string breaks. Which one? (Which string's tension is larger?)



(plan view — from above)

9*. An open door of inertia M and width L is at rest when it is struck by a thrown putty ball of inertia m that is moving at linear speed v at the instant it strikes the door. (See figure, above right.) The impact point is a distance $D = \frac{2}{3}L$ from the rotation axis through the hinges. The putty ball strikes at a right angle to the door face and sticks after it hits. What is the rotational speed of the door and putty? Do not ignore the inertia m.

How would you approach this problem? Discuss with neighbors!

(A) The final K.E. (rotational+translational) equals the initial K.E. of the ball.

(B) The initial momentum $m\vec{v}$ of the ball equals the final momentum $(m+M)\vec{v}$ of the door+ball.

(C) The initial angular momentum $L = r_{\perp}mv$ of the ball w.r.t. the hinge axis equals the final angular momentum $L = I\omega$ of the door+ball.

9*. An open door of inertia M and width L is at rest when it is struck by a thrown putty ball of inertia m that is moving at linear speed v at the instant it strikes the door. (See figure, above right.) The impact point is a distance $D = \frac{2}{3}L$ from the rotation axis through the hinges. The putty ball strikes at a right angle to the door face and sticks after it hits. What is the rotational speed of the door and putty? Do not ignore the inertia m.

I know that the rotational inertia of a thin rod of length L about a perpendicular axis through its center is $I = \frac{1}{12}mL^2$. The rotational inertia I to use for the final state here is

(A) $I = ML^2 + mL^2$

(B) $I = \frac{1}{12}ML^2 + M(\frac{L}{2})^2 + m(\frac{2}{3}L)^2$

(C) $I = \frac{1}{12}ML^2 + \frac{2}{3}mL^2$

(D) $I = \frac{1}{12}ML^2 + m(\frac{2}{3}L)^2$

(E) $I = \frac{1}{12}ML^2 + mL^2$

(Challenge: Also think how the answer would change if the radius of the putty ball were non-negligible. What if the thickness of the door were non-negligible? Does the height of the door matter?)

Three different expressions for angular momentum:

$$L = I\omega$$

$$L = r_{\perp}\ mv$$

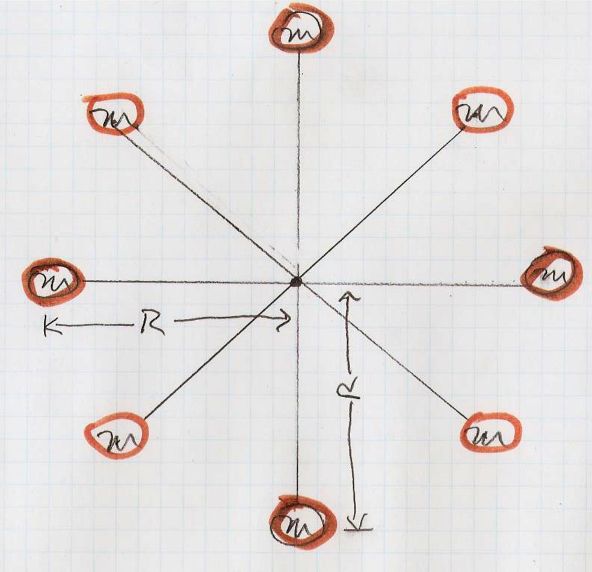
$$L = r\ mv_{\perp}$$

The second expression is telling you that momentum times lever arm (w.r.t. the relevant pivot axis) equals angular momentum.

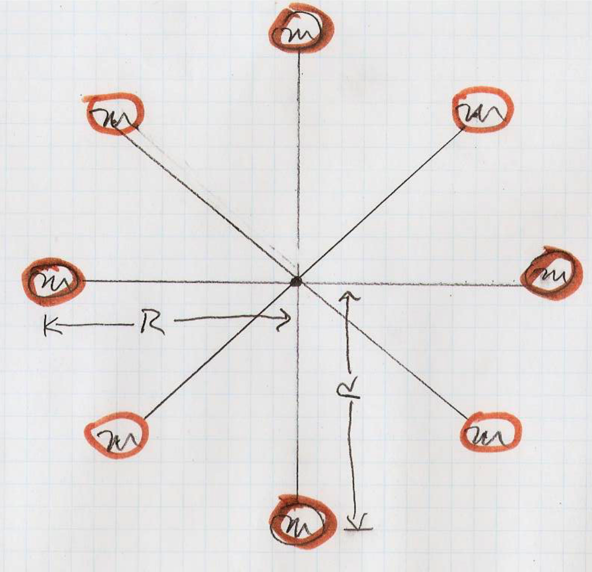
The second and third expressions are both simplified ways of writing the more general (but more difficult) expression

$$\vec{L} = \vec{r} \times \vec{p}$$

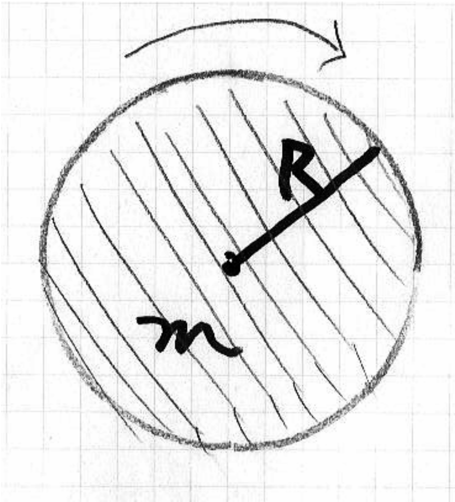
10*. Two skaters skate toward each other, each moving at 3.1 m/s. Their lines of motion are separated by a perpendicular distance of 1.8 m. Just as they pass each other (still 1.8 m apart), they link hands and spin about their common center of mass. What is the rotational speed of the couple about the center of mass? Treat each skater as a point particle, each with an inertia of 52 kg.



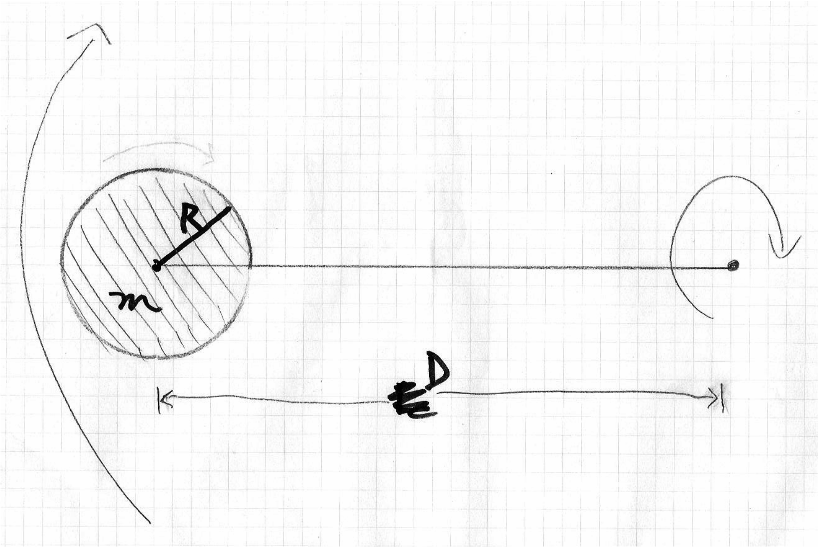
Where is the center of mass of this pinwheel-like object?



What is this object's rotational inertia, for rotation about its center of mass? Assume that all of the mass is concentrated in the orange blobs, and assume that the orange blobs are "point masses," i.e. that their size is much smaller than R.



Suppose I have a solid disk of radius R and mass m. I rotate it about its CoM, about an axis $\perp$ to the plane of the page. What is its rotational inertia? (If you don't happen to remember — is it bigger than, smaller than, or equal to mR^2 ?)



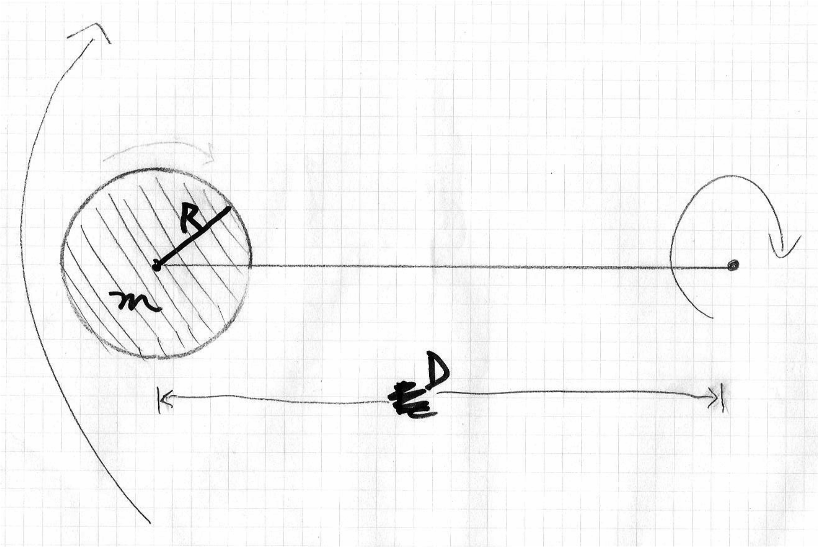
Now I take the same disk, attach it to a string or a lightweight stick of length D, and make the disk's CoM go around in circles of radius D. Is the mass now farther than or closer to the rotation axis than in the original rotation (about CoM)? What happens to I ?

If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance D from CoM), the rotational inertia is larger, because the object's CoM revolves around a circle of radius D and in addition the object rotates about its own CoM.

This larger rotational inertia is given by the *parallel axis theorem*:

$$I = I_{\rm cm} + MD^2$$

where $I_{\rm cm}$ is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's CoM.



Using the parallel axis theorem, what is the disk's rotational inertia about the displaced axis (the axis that is distance D away from the CoM)?

Torque: the rotational analogue of force

Just as an unbalanced force causes linear acceleration

$$\vec{F} = m\vec{a}$$

an unbalanced torque causes rotational acceleration

$$\tau = I\alpha$$

Torque is **(lever arm)** $\times$ **(force)**

$$\tau = r_{\perp}\, F$$

where $r_{\perp}$ is the "perpendicular distance" from the rotation axis to the line-of-action of the force.

Physics 8 — Wednesday, October 23, 2019

- This week you're reading Giancoli Ch9 (static equilibrium, etc.), and O/K Ch1. In class, we're still finishing up Mazur Ch11 (motion in a circle), and today or Friday we'll start Ch12 (torque). You may want to buy one of my $10 used copies of Onouye/Kane, which you can either keep or sell back to me for $10 in December.
- HW7 due Friday. HW help W4–6pm 3C4, R6–8pm 2C4.