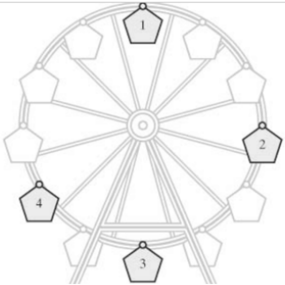
Physics 8 — Friday, October 25, 2019

- **Hold on to your hw7** paper until the end of class. We're going to talk through some of the more challenging problems together, so you may want to check or update your results. Then you can turn in your paper at the end of class. Or if you email me a scanned PDF (one file, PDF, readable, easily printed) by Sunday afternoon, it will count as on-time.
- Pick up HW8 handout, due next Friday, Nov 1.
- For next week, you'll read Ch2 (statics) and Ch3 (determinate systems: equilibrium, trusses, arches) of Onouye/Kane. Feel free to buy one of my $10 used copies if you wish.
- After spending next week's class time on torque, we'll spend 4 weeks applying the ideas of forces, vectors, and torque to the analysis of architectural structures. Fun reward for your work!
- The Ferris wheel problem was my favorite problem on hw7: let's discuss that one first.



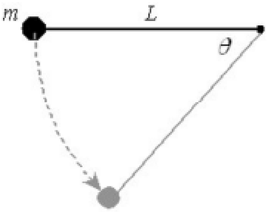
A Ferris wheel rotates at constant speed. Draw an FBD for each numbered carriage. For scale, draw $m\vec{a}$ on each FBD instead of the usual $\vec{a}$. Then the two force vectors must sum to $m\vec{a}$. For a realistic Ferris wheel, ma would be much smaller than mg, but to make drawings that bring out the physics, let's make ma be half the size of mg. In other words, for the sake of illustration, we'll spin the Ferris wheel fast enough that $v^2/R = g/2$.

I'll try to use these values for some ad-hoc questions:

(A) E (B) NE (C) N (D) NW (E) W (F) SW (G) S (H) SE

(A) 0.5 (B) 1.0 (C) $\sqrt{\frac{5}{4}} = 1.118$ (D) 1.32 (E) 1.5 (F) 2.0

2. You attach one end of a string of length L to a small ball of inertia m. You attach the string's other end to a pivot that allows free revolution. You hold the ball out to the side with the string taut along a horizontal line, as the in figure (below, left). (a) If you release the ball from rest, what is the tension in the string as a function of angle ϑ swept through? (b) What should the tensile strength of the string be (the maximum tension it can sustain without breaking) if you want it not to break through the ball's entire motion (from $\vartheta = 0$ to $\vartheta = \pi$)?



- Since the string's length L stays constant, what shape does the ball's path trace out as it moves?
- Does the ball's acceleration have a component that points along the axis of the string? If so, does its magnitude depend on the ball's speed?
- What two forces are acting on the ball?
- Assuming that no energy is dissipated, how can we relate the ball's speed v to its height y ?
- Can you write $m\vec{a} = \sum \vec{F}$ for the component of $\vec{a}$ and $\sum \vec{F}$ that points along the string?

("small" ball $\Rightarrow$ neglect the ball's rotation about its own CoM)

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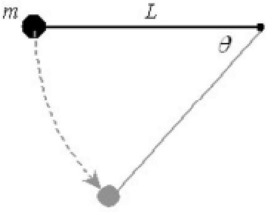
How do I relate angle θ to speed v? $E_i = mgL \rightarrow E = \frac{1}{2}mv^2 + mgy$



(A) $\frac{1}{2}mv^2 = mg(L - y) = mgL(1 - \cos\theta)$

(B) $\frac{1}{2}mv^2 = mg(L - y) = mgL(1 - \sin\theta)$

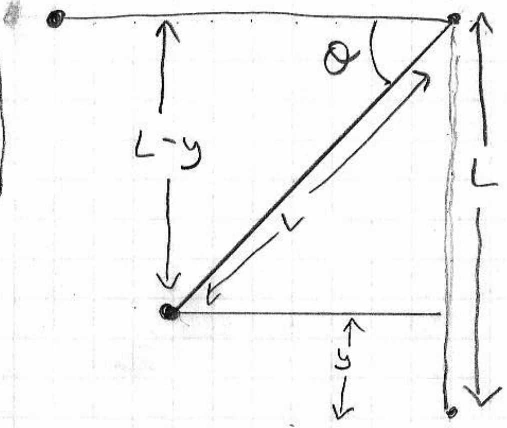
(C) $\frac{1}{2}mv^2 = mg(L - y) = mgL\cos\theta$

(D) $\frac{1}{2}mv^2 = mg(L - y) = mgL\sin\theta$

Hint: draw on the figure a vertical line of length $L - y = y_i - y$

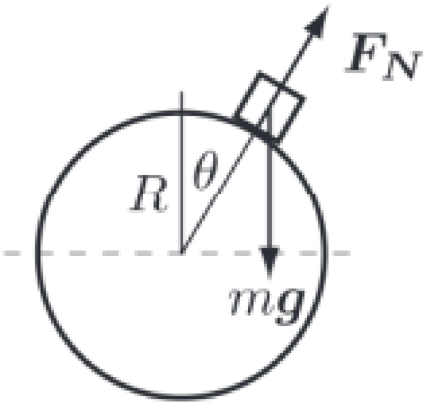
Next: write "radial" component of $m\vec{a} = \sum \vec{F}$ to find T

$$\sin\theta = \frac{L-y}{L}$$

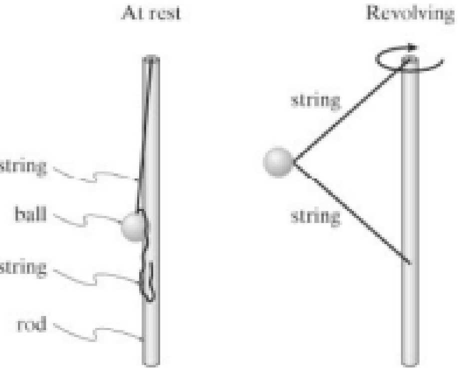


When does the block lose contact with the sphere?

A small block of mass m slides down a sphere of radius R, starting from rest at the top. The sphere is immobile, and friction between the block and the sphere is negligible. In terms of m, g, R, and θ, determine:
(a) the K.E. of the block;
(b) the centripetal acceleration of the block;
(c) the normal force exerted by the sphere on the block.
(d) At what value of θ does the block lose contact with the sphere?

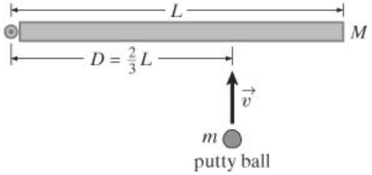


8*. A ball is attached to a vertical rod by two strings of equal strength and equal length. (See figure, below left.) The strings are very light and do not stretch. The rod begins to rotate under the influence of a constant rotational acceleration. (a) Which string breaks first? (b) Draw a free-body diagram for the ball, indicating all forces and their relative magnitudes, to justify your answer to (a).



- Are the angles of the two strings w.r.t. horizontal equal?
- Are the tensions in the two strings equal? How do you know?
- What three forces act on the ball?
- Is the ball accelerating vertically? Horizontally?
- Draw a FBD for the ball, showing both horizontal (radial) and vertical component of each force.

Notice that the ball's speed v increases with time, until finally one string breaks. Which one? (Which string's tension is larger?)



(plan view — from above)

9*. An open door of inertia M and width L is at rest when it is struck by a thrown putty ball of inertia m that is moving at linear speed v at the instant it strikes the door. (See figure, above right.) The impact point is a distance $D = \frac{2}{3}L$ from the rotation axis through the hinges. The putty ball strikes at a right angle to the door face and sticks after it hits. What is the rotational speed of the door and putty? Do not ignore the inertia m.

How would you approach this problem? Discuss with neighbors!

(A) The final K.E. (rotational+translational) equals the initial K.E. of the ball.

(B) The initial momentum $m\vec{v}$ of the ball equals the final momentum $(m+M)\vec{v}$ of the door+ball.

(C) The initial angular momentum $L = r_\perp mv$ of the ball w.r.t. the hinge axis equals the final angular momentum $L = I\omega$ of the door+ball.

9*. An open door of inertia M and width L is at rest when it is struck by a thrown putty ball of inertia m that is moving at linear speed v at the instant it strikes the door. (See figure, above right.) The impact point is a distance $D = \frac{2}{3}L$ from the rotation axis through the hinges. The putty ball strikes at a right angle to the door face and sticks after it hits. What is the rotational speed of the door and putty? Do not ignore the inertia m.

I know that the rotational inertia of a thin rod of length L about a perpendicular axis through its center is $I = \frac{1}{12}mL^2$. The rotational inertia I to use for the final state here is

(A) $I = ML^2 + mL^2$

(B) $I = \frac{1}{12}ML^2 + M(\frac{L}{2})^2 + m(\frac{2}{3}L)^2$

(C) $I = \frac{1}{12}ML^2 + \frac{2}{3}mL^2$

(D) $I = \frac{1}{12}ML^2 + m(\frac{2}{3}L)^2$

(E) $I = \frac{1}{12}ML^2 + mL^2$

(Challenge: Also think how the answer would change if the radius of the putty ball were non-negligible. What if the thickness of the door were non-negligible? Does the height of the door matter?)

we stopped just before this point

Three different expressions for angular momentum:

$$L = I\omega$$

$$L = r_{\perp}\ mv$$

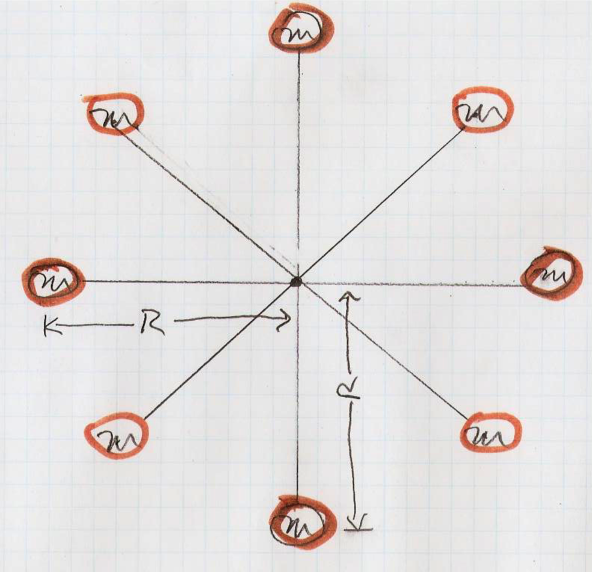
$$L = r\ mv_{\perp}$$

The second expression is telling you that momentum times lever arm (w.r.t. the relevant pivot axis) equals angular momentum.

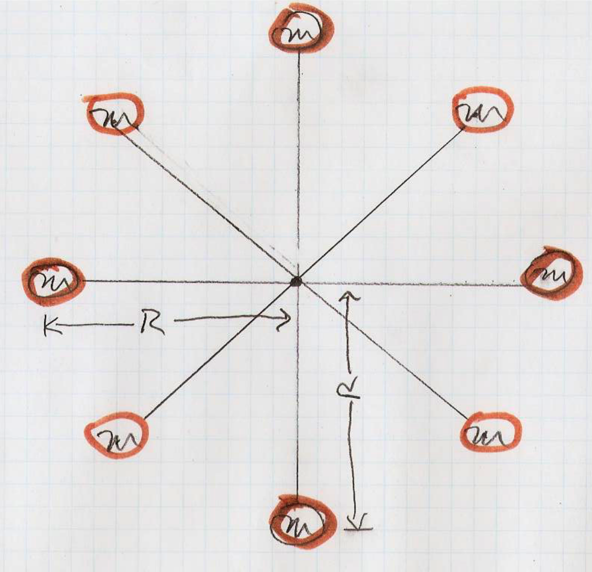
The second and third expressions are both simplified ways of writing the more general (but more difficult) expression

$$\vec{L} = \vec{r} \times \vec{p}$$

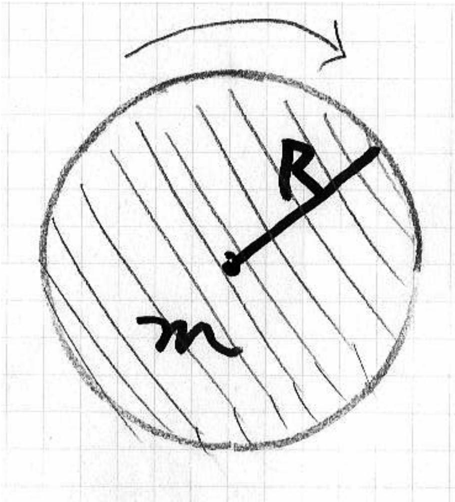
10*. Two skaters skate toward each other, each moving at 3.1 m/s. Their lines of motion are separated by a perpendicular distance of 1.8 m. Just as they pass each other (still 1.8 m apart), they link hands and spin about their common center of mass. What is the rotational speed of the couple about the center of mass? Treat each skater as a point particle, each with an inertia of 52 kg.



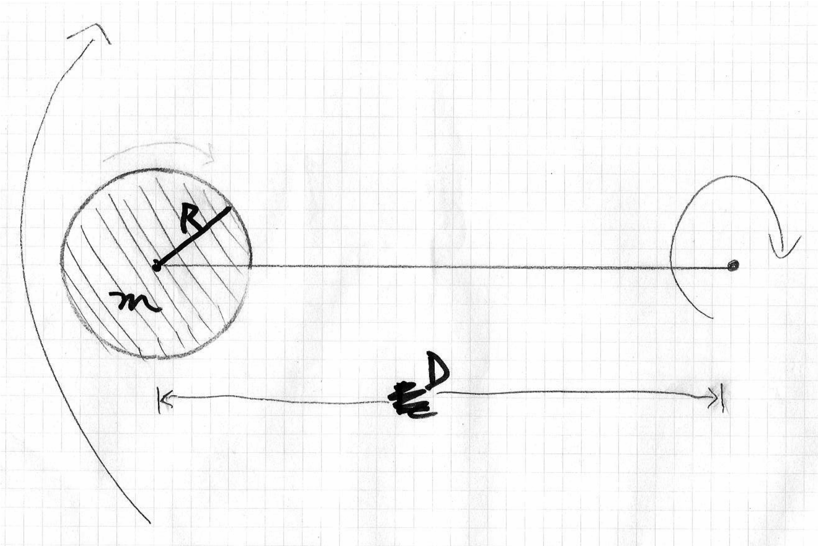
Where is the center of mass of this pinwheel-like object?



What is this object's rotational inertia, for rotation about its center of mass? Assume that all of the mass is concentrated in the orange blobs, and assume that the orange blobs are "point masses," i.e. that their size is much smaller than R.



Suppose I have a solid disk of radius R and mass m. I rotate it about its CoM, about an axis $\perp$ to the plane of the page. What is its rotational inertia? (If you don't happen to remember — is it bigger than, smaller than, or equal to mR^2 ?)



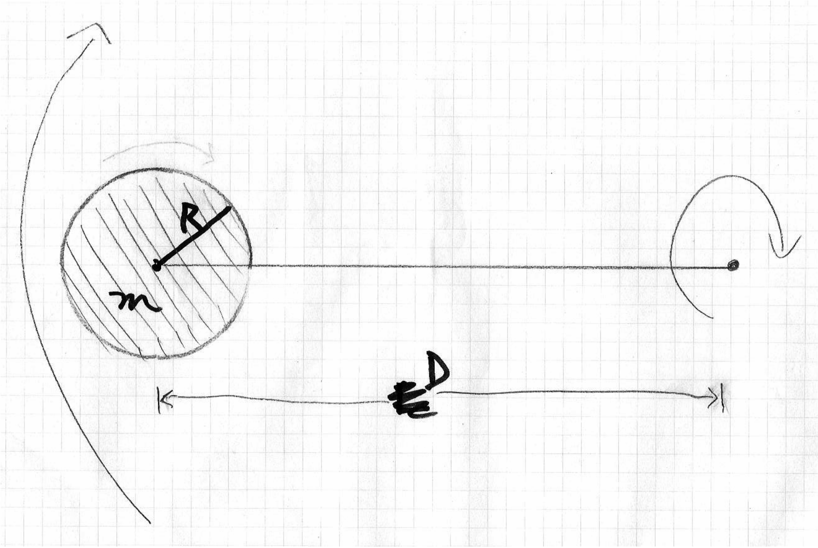
Now I take the same disk, attach it to a string or a lightweight stick of length D, and make the disk's CoM go around in circles of radius D. Is the mass now farther than or closer to the rotation axis than in the original rotation (about CoM)? What happens to I ?

If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance D from CoM), the rotational inertia is larger, because the object's CoM revolves around a circle of radius D and in addition the object rotates about its own CoM.

This larger rotational inertia is given by the *parallel axis theorem*:

$$I = I_{\rm cm} + MD^2$$

where $I_{\rm cm}$ is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's CoM.



Using the parallel axis theorem, what is the disk's rotational inertia about the displaced axis (the axis that is distance D away from the CoM)?

Torque: the rotational analogue of force

Just as an unbalanced force causes linear acceleration

$$\vec{F} = m\vec{a}$$

an unbalanced torque causes rotational acceleration

$$\tau = I\alpha$$

Torque is **(lever arm)** $\times$ **(force)**

$$\tau = r_{\perp} F$$

where $r_{\perp}$ is the "perpendicular distance" from the rotation axis to the line-of-action of the force.

position

$$\vec{r} = (x, y)$$

velocity

$$\vec{v} = (v_x, v_y) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}$$

acceleration

$$\vec{a} = (a_x, a_y) = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

momentum

$$\vec{p} = m\vec{v}$$

force

$$\vec{F} = m\vec{a}$$

rotational coordinate

$$\vartheta = s/r$$

rotational velocity

$$\omega = \mathrm{d}\vartheta/\mathrm{d}t$$

rotational acceleration

$$\alpha = \mathrm{d}\omega/\mathrm{d}t$$

angular momentum

$$L = I\omega$$

$$L = r_\perp \, mv$$

torque

$$\tau = I\alpha$$

$$\tau = r_\perp \, F$$

I wind a string around a coffee can of radius $R = 0.05\ \mathrm{m}$. (That's 5 cm.) Friction prevents the string from slipping. I apply a tension $T = 20\ \mathrm{N}$ to the free end of the string. The free end of the string is tangent to the coffee can, so that the radial direction is perpendicular to the force direction. What is the magnitude of the torque exerted by the string on the coffee can?

(A) $1\ \mathrm{N \cdot m}$

(B) $2\ \mathrm{N \cdot m}$

(C) $5\ \mathrm{N \cdot m}$

(D) $10\ \mathrm{N \cdot m}$

(E) $20\ \mathrm{N \cdot m}$

Suppose that the angular acceleration of the can is $\alpha = 2\ \mathrm{s}^{-2}$ when the string exerts a torque of $1\ \mathrm{N \cdot m}$ on the can. What would the angular acceleration of the can be if the string exerted a torque of $2\ \mathrm{N \cdot m}$ instead?

(A) $\alpha = 0.5\ \mathrm{s}^{-2}$

(B) $\alpha = 1\ \mathrm{s}^{-2}$

(C) $\alpha = 2\ \mathrm{s}^{-2}$

(D) $\alpha = 4\ \mathrm{s}^{-2}$

(E) $\alpha = 5\ \mathrm{s}^{-2}$

(F) $\alpha = 10\ \mathrm{s}^{-2}$

I apply a force of $5.0 \mathrm{~N}$ at a perpendicular distance of 5 cm ($r_{\perp} = 0.05 \mathrm{~m}$) from this rotating wheel, and I observe some angular acceleration α. What force would I need to apply to this same wheel at $r_{\perp} = 0.10 \mathrm{~m}$ (that's 10 cm) to get the same angular acceleration α?

(A) $F = 1.0 \mathrm{~N}$

(B) $F = 2.5 \mathrm{~N}$

(C) $F = 5.0 \mathrm{~N}$

(D) $F = 10 \mathrm{~N}$

(E) $F = 20 \mathrm{~N}$

Suppose that I use the tension T in the string to apply a given torque $\tau = r_\perp T$ to this wheel, and it experiences a given angular acceleration α. Now I **increase** the rotational inertia I of the wheel and then apply the same torque. The new angular acceleration $\alpha_{\rm new}$ will be

(A) larger: $\alpha_{\rm new} > \alpha$

(B) the same: $\alpha_{\rm new} = \alpha$

(C) smaller: $\alpha_{\rm new} < \alpha$

I want to tighten a bolt to a torque of 1.0 newton-meter, but I don't have a torque wrench. I do have an ordinary wrench, a ruler, and a 1.0 kg mass tied to a string. How can I apply the correct torque to the bolt?

(A) Orient the wrench horizontally and hang the mass at a distance 0.1 m from the axis of the bolt

(B) Orient the wrench horizontally and hang the mass at a distance 1.0 m from the axis of the bolt

If the wrench is at 45° w.r.t. horizontal, will the 1.0 kg mass suspended at a distance 0.1 m along the wrench still exert a torque of 1.0 newton-meter on the bolt?

(A) Yes. The force of gravity has not changed, and the distance has not changed.

(B) No. The torque is now smaller — about 0.71 newton-meter — because the "perpendicular distance" is now smaller by a factor of $1/\sqrt{2}$.

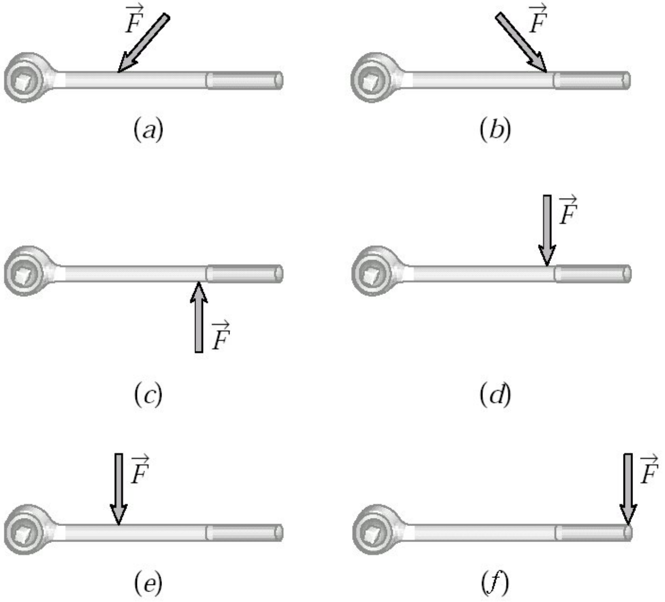
(C) No. The torque is now larger — about 1.4 newton-meter.

$$\tau \; = \; r_\perp F \; = \; rF_\perp \; = \; rF\sin\theta_{rF} \; = \; |\vec{r} \times \vec{F}|$$

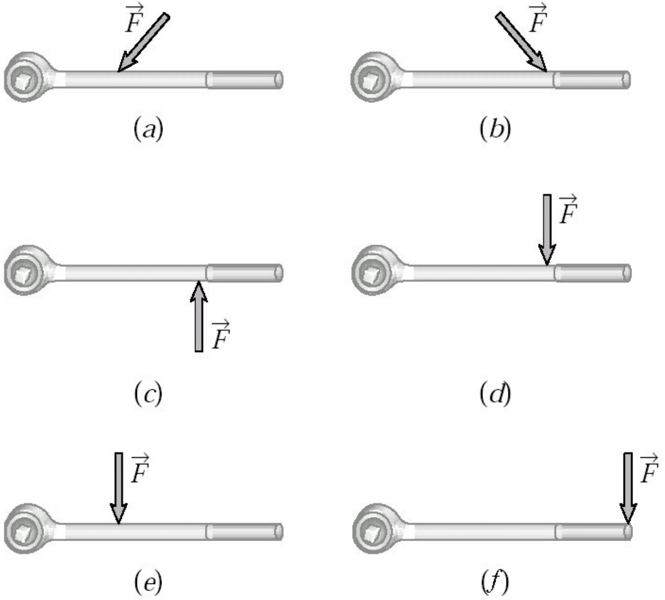
Four ways to get the magnitude of the torque

- (perpendicular component of distance) $\times$ (force)
- (distance) $\times$ (perpendicular component of force)
- (distance) (force) ($\sin\theta$ between $\vec{r}$ and $\vec{F}$)
- use magnitude of "vector product" $\vec{r} \times \vec{F}$ (a.k.a. "cross product")

To tighten a bolt, I apply a force of the same magnitude F at different positions and angles. Which torque is *largest*?



To tighten a bolt, I apply a force of magnitude F at different positions and angles. Which torque is *smallest*?



I want to apply to this meter stick two torques of the same magnitude and opposite sense, so that the stick has zero rotational acceleration. I apply one force of 5 N at a lever arm of 0.5 m. I want to apply an opposing force at a lever arm of 0.2 m, so that the second torque balances the first torque. How large must this second force be?

(A) 1.0 N

(B) 2.0 N

(C) 12.5 N

(D) 25 N

I want to apply to this meter stick two torques of the same magnitude and opposite sense, so that the stick has zero rotational acceleration. I apply one force of 10 N at a lever arm of 0.5 m. I tie a second string on the opposite end, 0.5 m from the pivot point. **The second force is applied at a 45° angle w.r.t. the vertical.** How large must this second force be?

(A) 5 N

(B) 7 N

(C) 10 N

(D) 14 N

(E) 20 N

If the rod doesn't accelerate (rotationally, about the pivot), what force does the scale read?

(A) 1.0 N

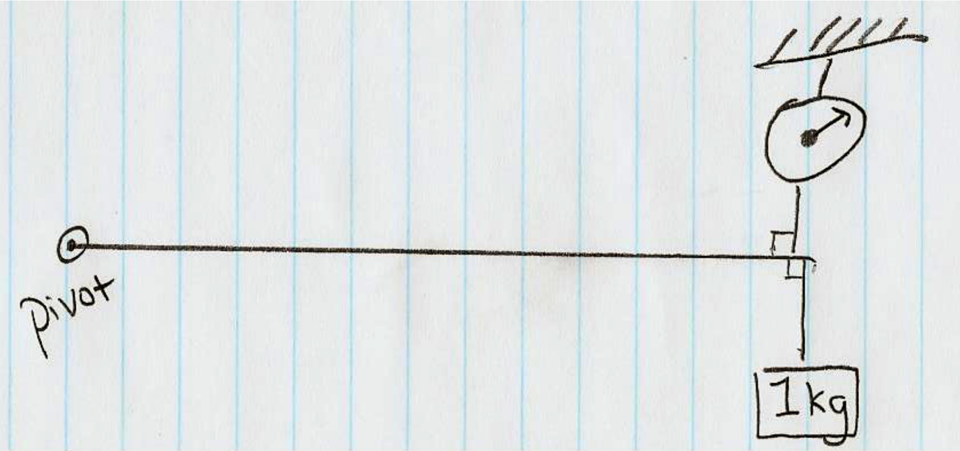
(B) 5.0 N

(C) 7.1 N

(D) 10 N

(E) 14 N

(F) 20 N



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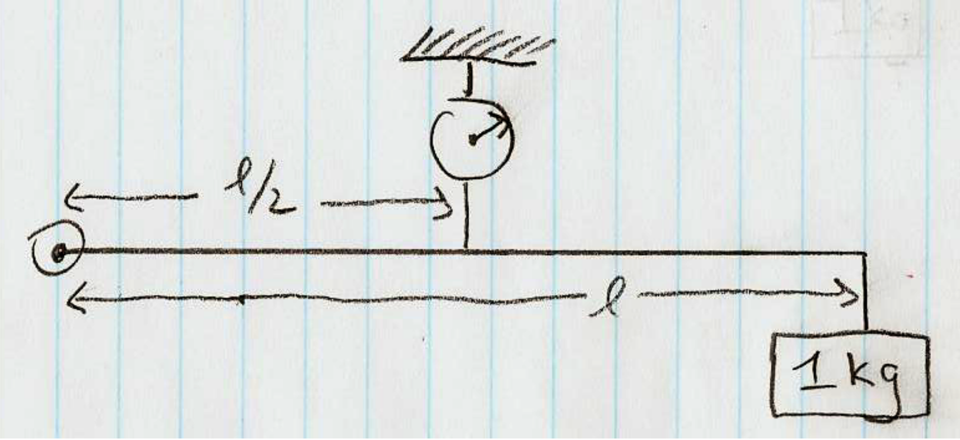

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