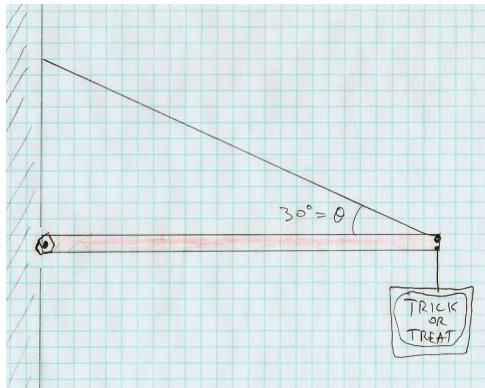


## Physics 8 — Friday, November 1, 2019

- ▶ Turn in HW8. Pick up HW9 handout in back corner of room.
- ▶ This week, you read Ch2 (statics) and Ch3 (determinate systems: equilibrium, trusses, arches) of Onouye/Kane. Next week, you'll skim Ch4 (load tracing) and read Ch5 (strength of materials). Feel free to buy one of my \$10 used copies if you wish. At the end of the term, you can keep it, or sell it back to me for \$10.

A beam of mass  $M = 20$  kg and length  $L = 2$  m is attached to a wall by a hinge. A sign of mass  $m = 10$  kg hangs from the end of the beam. The end of the beam is supported by a cable (at  $\theta = 30^\circ$  angle w.r.t. horizontal beam), which is anchored to the wall above the hinge.

Let's start by drawing an EFBF for the beam, showing each force acting on the beam and its line of action.



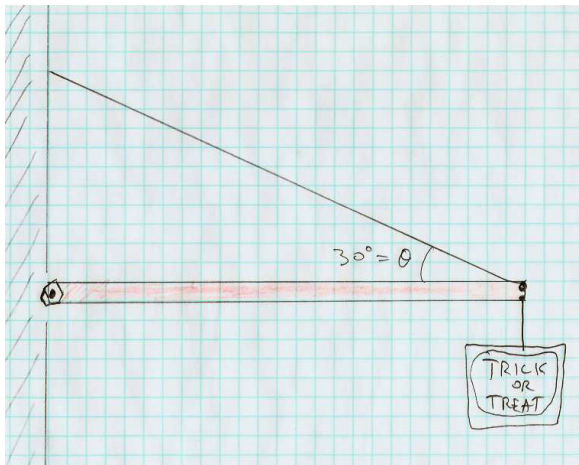
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What forces act on the beam? (Draw EFBD.)

Find the cable tension  $T$ .

Find the “reaction” forces  $F_x$  and  $F_y$  exerted by the hinge on the beam.

What 3 equations can we write for the beam? (Next few slides.)



(Redraw this on the board.)

Beam mass  $M$ , length  $L$ . Sign mass  $m$ . Cable angle  $\theta$  from horizontal. Hinge exerts forces  $F_x$ ,  $F_y$  on beam.

How do we write “sum of horizontal forces (on beam) = 0” ?

(A)  $F_x + T \cos \theta = 0$

(B)  $F_x + T \sin \theta = 0$

(C)  $F_x - T \cos \theta = 0$

(D)  $F_x - T \sin \theta = 0$

Beam mass  $M$ , length  $L$ . Sign mass  $m$ . Cable angle  $\theta$  from horizontal. Hinge exerts forces  $F_x$ ,  $F_y$  on beam.

How do we write “sum of vertical forces (on beam) = 0” ?

(A)  $F_y + T \cos \theta + (M + m)g = 0$

(B)  $F_y + T \cos \theta - (M + m)g = 0$

(C)  $F_y + T \cos \theta - (M + m)g = 0$

(D)  $F_y + T \sin \theta + (M + m)g = 0$

(E)  $F_y + T \cos \theta - (M + m)g = 0$

(F)  $F_y + T \sin \theta - (M + m)g = 0$

(G)  $F_y - T \cos \theta - (M + m)g = 0$

(H)  $F_y - T \sin \theta - (M + m)g = 0$

Beam mass  $M$ , length  $L$ . Sign mass  $m$ . Cable angle  $\theta$  from horizontal. Hinge exerts forces  $F_x$ ,  $F_y$  on beam.

How do we write “sum of torques (about hinge) = 0” ?

(A)  $+\frac{L}{2}Mg + Lmg + LT \cos \theta = 0$

(B)  $+\frac{L}{2}Mg + Lmg + LT \sin \theta = 0$

(C)  $-\frac{L}{2}Mg + Lmg + LT \cos \theta = 0$

(D)  $-\frac{L}{2}Mg + Lmg + LT \sin \theta = 0$

(E)  $-\frac{L}{2}Mg - Lmg + LT \cos \theta = 0$

(F)  $-\frac{L}{2}Mg - Lmg + LT \sin \theta = 0$

## The 3 equations for static equilibrium in the $xy$ plane

sum of horizontal forces = 0:

$$F_x - T \cos \theta = 0$$

sum of vertical forces = 0:

$$F_y + T \sin \theta - (M + m)g = 0$$

sum of torques (a.k.a. moments) about hinge = 0:

$$-\frac{L}{2}Mg - Lmg + LT \sin \theta = 0$$

Here's my solution

$$F_x - T \cos \theta = 0$$

$$F_y - Mg - mg + T \sin \theta = 0$$
$$-\left(\frac{L}{2}\right)(Mg) - (L)(mg) + (L)(T \sin \theta) = 0$$

$$LT \sin \theta = Lmg + \frac{1}{2}LMg = L\left(m + \frac{M}{2}\right)g$$

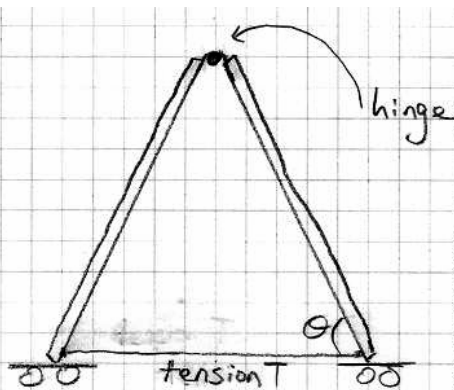
$$T = \frac{\left(m + \frac{M}{2}\right)g}{\sin \theta}$$

$$F_x = T \cos \theta = \frac{\left(m + \frac{M}{2}\right)g}{\tan \theta}$$

$$F_y = (M+m)g - T \sin \theta = (M+m)g - \left(m + \frac{M}{2}\right)g = \frac{Mg}{2}$$

## Let's build & measure a simplified arch

Often the essence of physics is to reduce a complicated problem to a similar problem that is easier to analyze.

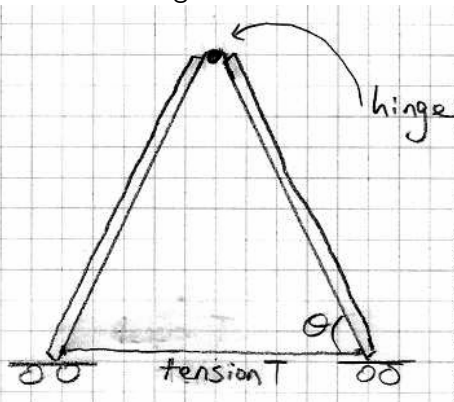


(Does this make the function of a "roller support" more obvious?!)



## (We'll emphasize function over form here ...)

Often the essence of physics is to reduce a complicated problem to a similar problem that is easier to analyze. Use a cable to hold bottom together so that we can use scale to measure tension.

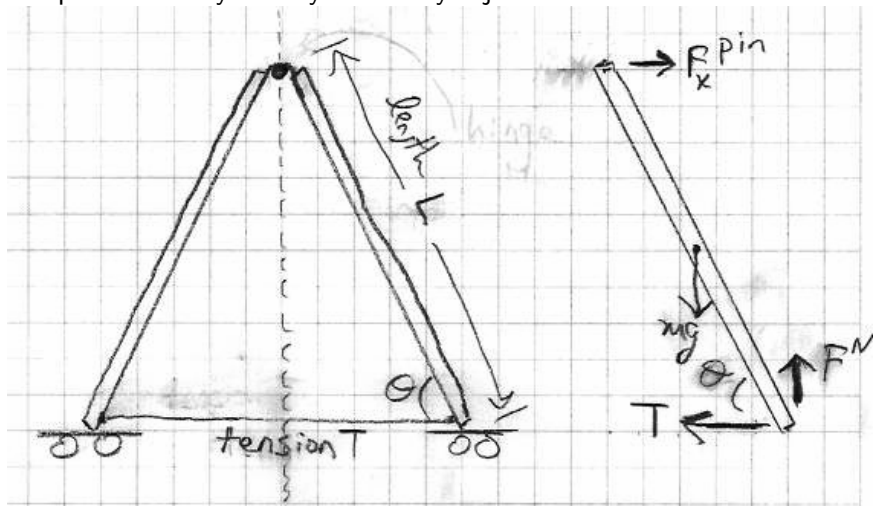


Weight ( $mg$ ) of each side is 20 N.

We'll exploit mirror symmetry and analyze just one side of arch.

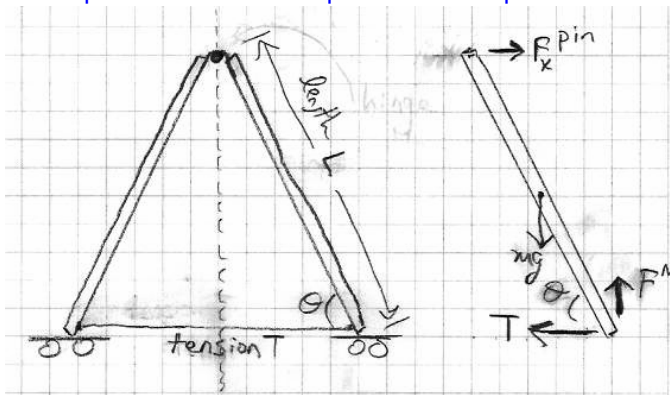
What forces act (and where) on the r.h.s. of the arch? **(Draw EFBF for the right-hand board.)**

Use a cable to hold bottom of "arch" together so that we can use scale to measure tension. Weight ( $mg$ ) of each side is 20 N. We'll exploit mirror symmetry and analyze just one side of arch.



Right side shows EFB for right-hand board.

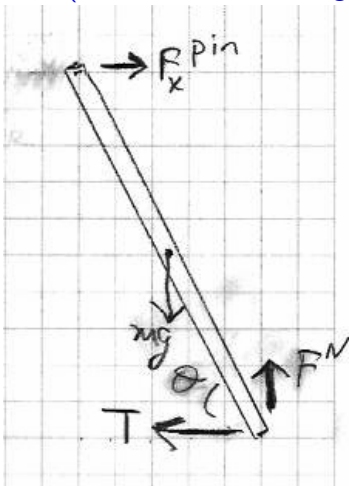
How many unknown variables is it possible to determine using the equations for static equilibrium in a plane?



- (A) one
- (B) two
- (C) three
- (D) four
- (E) five

Static equilibrium lets us write down three equations for a given object:  $\sum F_x = 0$ ,  $\sum F_y = 0$ ,  $\sum M_z = 0$ . Let's first sum up the "moments" (a.k.a. torques) **about the top hinge**.

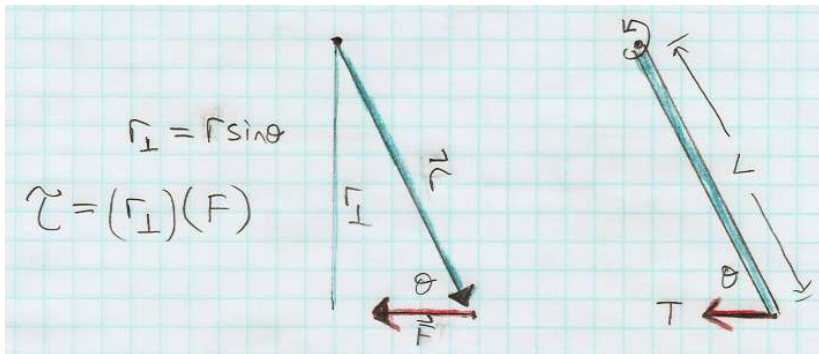
Which statement correctly expresses  $\sum M_z = 0$  (a.k.a.  $\sum \tau = 0$ )?  
(Let the mass and length of each wooden board be  $L$  and  $m$ .)



- (A)  $-mg \left(\frac{L}{2}\right) \cos \theta + F^N L \cos \theta - TL \sin \theta = 0$
- (B)  $-mg \left(\frac{L}{2}\right) \cos \theta + F^N L \cos \theta + TL \sin \theta = 0$
- (C)  $-mg \left(\frac{L}{2}\right) \sin \theta + F^N L \sin \theta - TL \cos \theta = 0$
- (D)  $-mg \left(\frac{L}{2}\right) \sin \theta + F^N L \sin \theta + TL \cos \theta = 0$
- (E)  $+mg \left(\frac{L}{2}\right) \cos \theta + F^N L \cos \theta - TL \sin \theta = 0$
- (F)  $+mg \left(\frac{L}{2}\right) \cos \theta + F^N L \cos \theta + TL \sin \theta = 0$
- (G)  $+mg \left(\frac{L}{2}\right) \sin \theta + F^N L \sin \theta - TL \cos \theta = 0$
- (H)  $+mg \left(\frac{L}{2}\right) \sin \theta + F^N L \sin \theta + TL \cos \theta = 0$

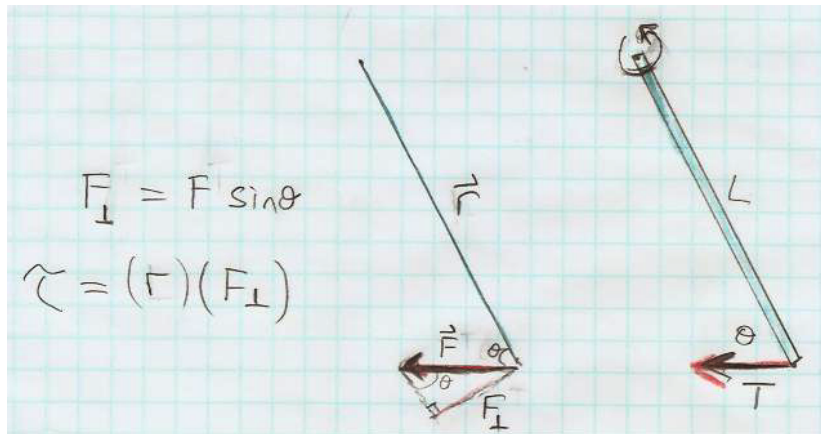
Let's start with torque (about top hinge) due to tension  $T$ .

- ▶ Usual convention: clockwise = negative, ccw = positive.
- ▶ Draw vector  $\vec{r}$  from pivot to point where force is applied.
- ▶ Draw force vector  $\vec{F}$ , with its line-of-action passing through the point where the force is applied.
- ▶ Decompose  $\vec{r}$  to find component  $r_{\perp}$  that is perpendicular to  $\vec{F}$ . The component  $r_{\perp}$  is called the "lever arm."
- ▶ Magnitude of torque is  $|\tau| = (r_{\perp})(F)$ .



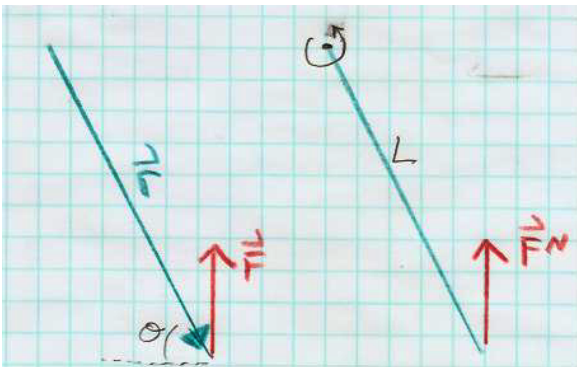
Alternative method: use  $(r)(F_{\perp})$  instead of  $(r_{\perp})(F)$ .

- ▶ Draw vector  $\vec{r}$  from pivot to point where force is applied.
- ▶ Draw force vector  $\vec{F}$ , with its line-of-action passing through the point where the force is applied.
- ▶ Decompose  $\vec{F}$  to find component  $F_{\perp}$  perpendicular to  $\vec{r}$ .
- ▶ Magnitude of torque is  $|\tau| = (r)(F_{\perp})$ .



Now you try it for the normal force  $\vec{F}^N$ .

- ▶ Draw vector  $\vec{r}$  from pivot to point where force is applied.
- ▶ Draw force vector  $\vec{F}$ , with its line-of-action passing through the point where the force is applied.
- ▶ Decompose  $\vec{r}$  to find component  $r_{\perp}$  that is perpendicular to  $\vec{F}$ . The component  $r_{\perp}$  is called the “lever arm.”
- ▶ Magnitude of torque is  $|\tau| = r_{\perp} F$ .

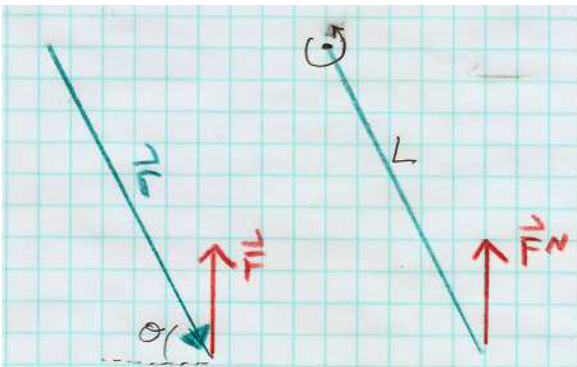


Which component of  $\vec{r}$  is perpendicular to the normal force  $\vec{F}^N$  ?

- (A) horizontal component
- (B) vertical component

Now you try it for the normal force  $\vec{F}^N$ .

- ▶ Draw vector  $\vec{r}$  from pivot to point where force is applied.
- ▶ Draw force vector  $\vec{F}$ , with its line-of-action passing through the point where the force is applied.
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- ▶ Magnitude of torque is  $|\tau| = r_{\perp} F$ .



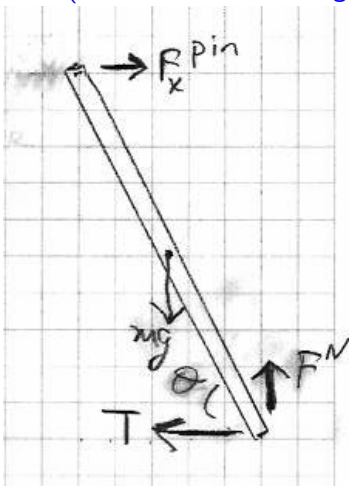
How long is the horizontal component of  $\vec{r}$  (i.e. the  $\vec{r}$  component which is perpendicular to  $\vec{F}$ ) ?

- (A)  $L \cos \theta$
- (B)  $L \sin \theta$
- (C)  $L \tan \theta$



OK, now back to the original question: Let's sum up the "moments" (a.k.a. torques) **about the top hinge**.

Which statement correctly expresses  $\sum M_z = 0$  (a.k.a.  $\sum \tau = 0$ )?  
(Let the mass and length of each wooden board be  $L$  and  $m$ .)



- (A)  $-mg \left(\frac{L}{2}\right) \cos \theta + F^N L \cos \theta - TL \sin \theta = 0$
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- (C)  $-mg \left(\frac{L}{2}\right) \sin \theta + F^N L \sin \theta - TL \cos \theta = 0$
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Next: what about  $\sum F_x = 0$  and  $\sum F_y = 0$  ?

Next: what about  $\sum F_x = 0$  and  $\sum F_y = 0$  ?

$$-mg \left(\frac{L}{2}\right) \cos\theta + F^N L \cos\theta - TL \sin\theta = 0$$

$$\Sigma F_y = 0 \Rightarrow -mg + F^N = 0$$

$$-mg \frac{L}{2} \cos\theta + mg L \cos\theta - TL \sin\theta = 0$$

$$\Rightarrow mg \frac{L}{2} \cos\theta = TL \sin\theta$$

$$T = \frac{mg \cos\theta}{2 \sin\theta}$$

$$= \boxed{\frac{mg}{2 \tan\theta} = T}$$

We said  $mg = 20 \text{ N}$ , so we expect the string tension to be

$$T = \frac{10 \text{ N}}{\tan\theta}$$

How would this change if we suspended a weight  $Mg$  from the hinge? (By symmetry, each side of arch carries **half** of this  $Mg$ .)

Let's use forces and torques to analyze the big red wheel that we first saw on Monday. The wheel has rotational inertia  $I$ .

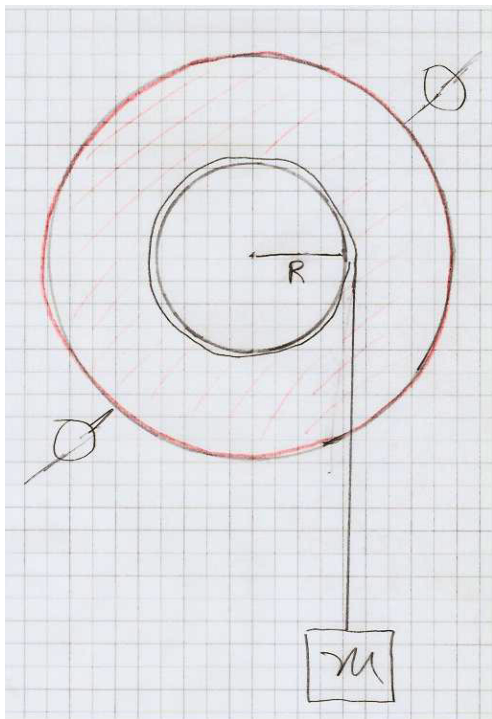
The string is wrapped at radius  $R$ , with an object of mass  $m$  dangling on the string.

For the dangling object, write

$$ma_y = \sum F_y$$

For the cylinder, write

$$I\alpha = \sum \tau$$



After some math, I get

$$\alpha = \frac{mgR}{I_{\text{wheel}} + mR^2} \approx \frac{mgR}{I_{\text{wheel}}}$$

(The approximation is for the limit where the object falls at  $a \ll g$ , so the string tension is  $T = (mg - ma) \approx mg$ .)

$$I\alpha = \tau = RT \Rightarrow T = \frac{I\alpha}{R}$$

$$ma = mg - T \Rightarrow m(\alpha R) = mg - \left(\frac{I}{R}\alpha\right)$$

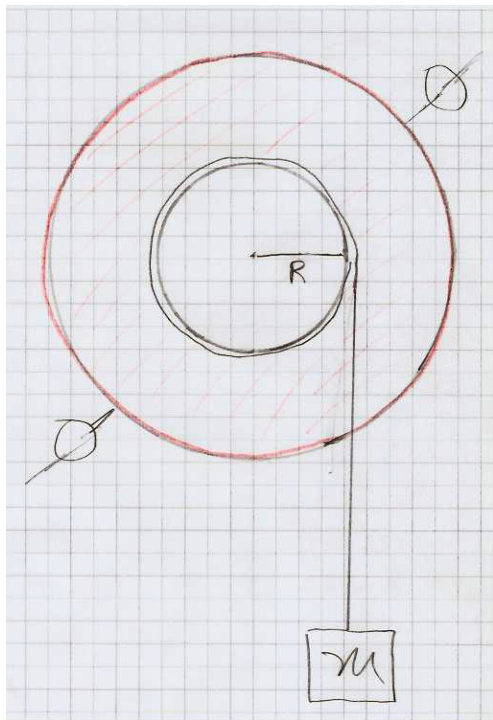
$$\alpha = \frac{g}{R} - \left(\frac{I}{mR^2}\right)\alpha \Rightarrow \alpha\left(1 + \frac{I}{mR^2}\right) = \frac{g}{R}$$

$$\alpha = \frac{g}{R\left(1 + \frac{I}{mR^2}\right)} = \frac{mgR}{I + mR^2}$$

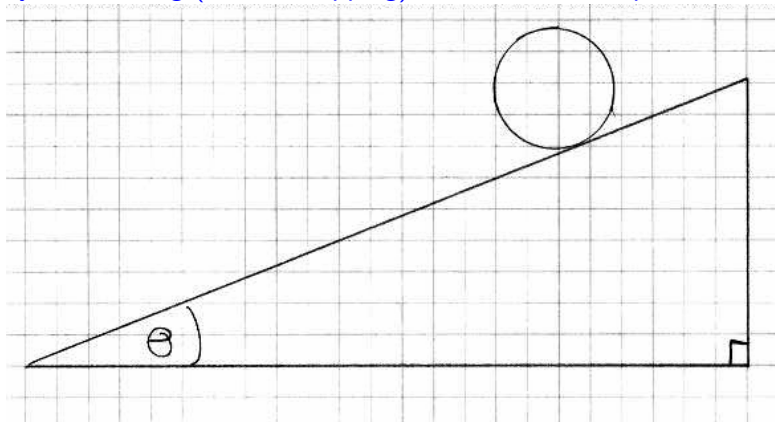
Why did increasing the dangling mass  $m$  **increase** the wheel's rotational acceleration  $\alpha$  ?

Why did increasing the radius  $R$  from which the dangling mass was suspended **increase** the wheel's rotational acceleration?

Why did sliding the big rotating masses farther out on the extended "arms" **decrease** the wheel's rotational acceleration?



Let's go back and use torque to analyze another problem that last week we were only able to analyze using energy conservation:  
a cylinder rolling (without slipping) down an inclined plane.



What 3 forces act on the cylinder? What is the rotation axis?  
Draw FBD and extended FBD. What are the torque(s) about this axis? How are  $\alpha$  and  $a$  related? Write  $\vec{F} = m\vec{a}$  and  $\tau = I\alpha$ .

$$I\alpha = \sum \tau$$
$$I\left(\frac{a_x}{R}\right) = RF^s$$
$$F^s = \left(\frac{I}{R^2}\right) a_x$$

$$ma_x = mg \sin \theta - F^s$$
$$ma_x = mg \sin \theta - \left(\frac{I}{R^2}\right) a_x$$
$$\left(m + \frac{I}{R^2}\right) a_x = mg \sin \theta$$

$$a_x = \frac{mg \sin \theta}{m + \frac{I}{R^2}} = \frac{g \sin \theta}{1 + \left(\frac{I}{mR^2}\right)}$$

Remember that the object with the larger “shape factor”  $I/(mR^2)$  rolls downhill more slowly.



While we're here, let's revisit the "center-of-mass chalkline" demonstration from a few weeks ago.

Now that we know about torque, we can see why the CoM always winds up directly beneath the pivot, once we understand that the line-of-action for gravity passes through the CoM.

## Another equilibrium problem!

The top end of a ladder of inertia  $m$  rests against a smooth (i.e. slippery) wall, and the bottom end rests on the ground. The coefficient of static friction between the ground and the ladder is  $\mu_s$ . What is the minimum angle between the ground and the ladder such that the ladder does not slip?

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Let's start by drawing an EFBD for the ladder.

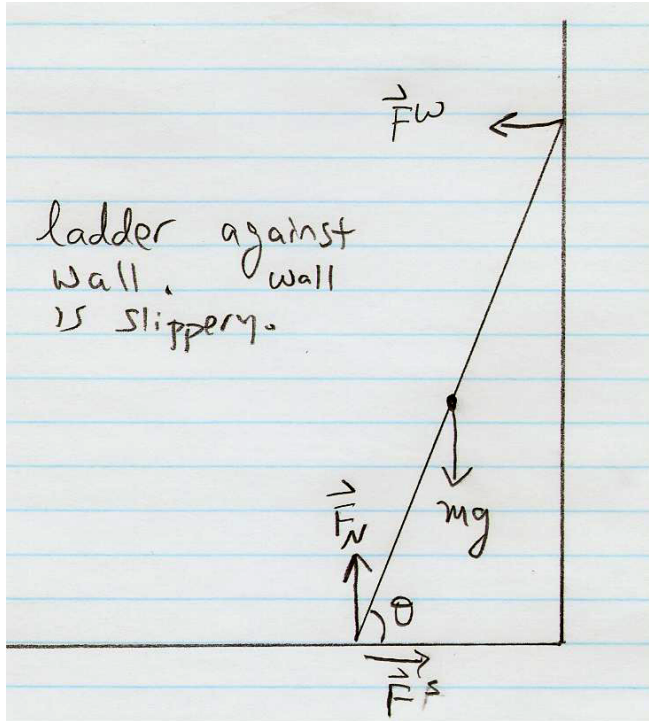
Why must we say the wall is slippery?

Is the slippery wall more like a pin or a roller support?

What plays the role here that string tension played in the previous problem?

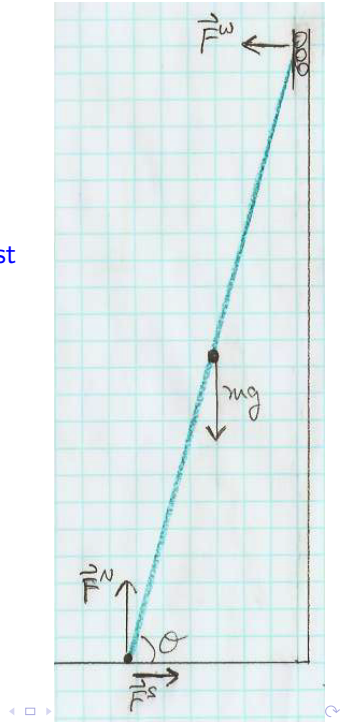
Does the combination of two forces at the bottom act more like a pin or a roller support?

Which forces would an engineer call "reaction" forces?



Which choice of pivot axis will give us the simplest equation for  $\sum M_z = 0$ ? (We'll get an equation involving only two forces if we choose this axis.)

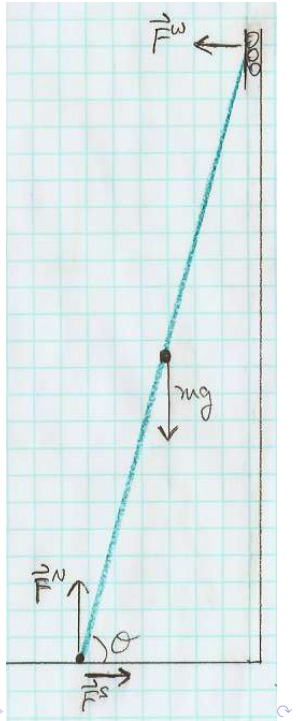
- (A) Use bottom of ladder as pivot axis.
- (B) Use center of ladder as pivot axis.
- (C) Use top of ladder as pivot axis.



How would I write  $\sum M_z = 0$  about the bottom end of the ladder? (Take length of ladder to be  $L$ .)

- (A)  $F^W L \cos \theta + mgL \sin \theta = 0$
- (B)  $F^W L \cos \theta + mg \frac{L}{2} \sin \theta = 0$
- (C)  $F^W L \cos \theta - mgL \sin \theta = 0$
- (D)  $F^W L \cos \theta - mg \frac{L}{2} \sin \theta = 0$
- (E)  $F^W L \sin \theta + mgL \cos \theta = 0$
- (F)  $F^W L \sin \theta + mg \frac{L}{2} \cos \theta = 0$
- (G)  $F^W L \sin \theta - mgL \cos \theta = 0$
- (H)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta = 0$

What do we learn from  $\sum F_x = 0$  and  $\sum F_y = 0$  ?



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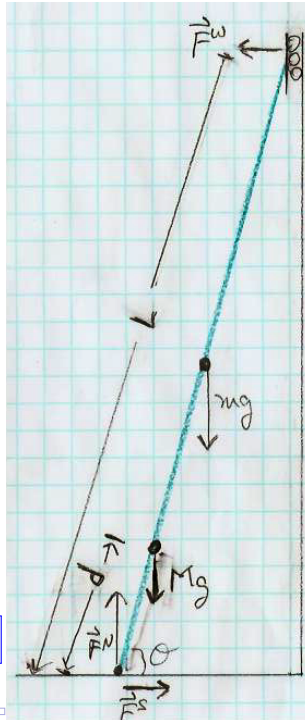
Let's answer the original question:

What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Suppose we add to this picture a woman of mass  $M$  who has climbed up a distance  $d$  along the length of the ladder. Now how do we write the moment equation  $\sum M_z = 0$  ?

- (A)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta + Mg \frac{d}{2} \cos \theta = 0$
- (B)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta + Mg \frac{d}{2} \sin \theta = 0$
- (C)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta + Mgd \cos \theta = 0$
- (D)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta + Mgd \sin \theta = 0$
- (E)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta - Mg \frac{d}{2} \cos \theta = 0$
- (F)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta - Mg \frac{d}{2} \sin \theta = 0$
- (G)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta - Mgd \cos \theta = 0$
- (H)  $F^W L \sin \theta - mg \frac{L}{2} \cos \theta - Mgd \sin \theta = 0$

What do we learn from  $\sum F_x = 0$  and  $\sum F_y = 0$  ?



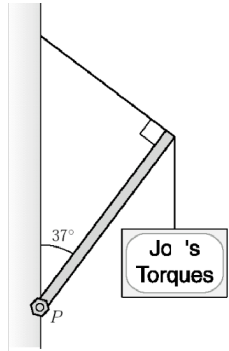
What do we learn from  $\sum F_x = 0$  and  $\sum F_y = 0$  ?

For a given  $\theta$ , how far up can she climb before the ladder slips?



## Here's a trickier equilibrium problem:

4\*. You want to hang a 22 kg sign (shown at right) that advertises your new business. To do this, you attach a 7.0 kg beam of length 1.0 m to a wall at its base by a pivot  $P$ . You then attach a thin cable to the beam and to the wall in such a way that the cable and beam are perpendicular to each other. The beam makes an angle of  $37^\circ$  with the vertical. You hang the sign from the end of the beam to which the cable is attached. (a) What must be the minimum tensile strength of the cable (the amount of tension it can sustain) if it is not to snap? (b) Determine the horizontal and vertical components of the force the pivot exerts on the beam.



What forces act on the beam?

What 3 equations can we write for the beam?

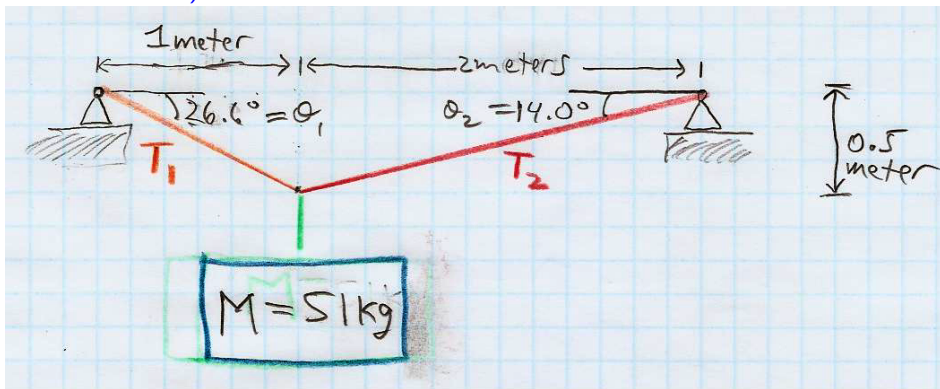
A tightly stretched “high wire” has length  $L = 50$  m. It sags by  $d = 1.0$  m when a tightrope walker of mass  $M = 51$  kg stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that  $d = 0$ )?

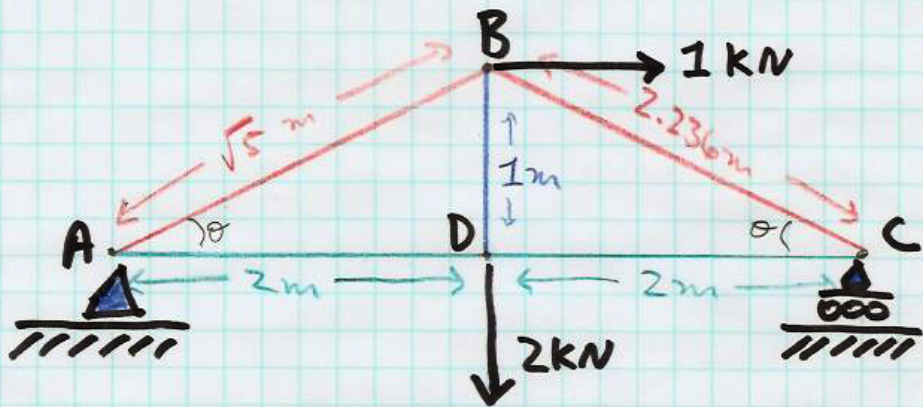
What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



How would you find the tensions  $T_1$  and  $T_2$ ?

Once you know  $T_1$  and  $T_2$ , what are the horizontal and vertical forces exerted by the two supports on the cable?



How many equations does the “method of joints” allow us to write down for this truss? (Consider how many joints the truss has.)

- (A) 4                      (B) 8                      (C) 12                      (D) 15

## Physics 8 — Friday, November 1, 2019

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- ▶ This week, you read Ch2 (statics) and Ch3 (determinate systems: equilibrium, trusses, arches) of Onouye/Kane. Next week, you'll skim Ch4 (load tracing) and read Ch5 (strength of materials). Feel free to buy one of my \$10 used copies if you wish. At the end of the term, you can keep it, or sell it back to me for \$10.