Physics 8 — Monday, November 4, 2019

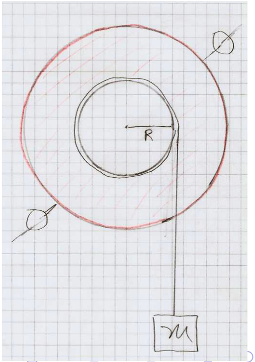
- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you'll skim Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. Feel free to buy one of my \$10 used copies if you wish. At the end of the term, you can keep it, or sell it back to me for \$10.

Let's use forces and torques to analyze the big red wheel that we first saw last Monday. The wheel has rotational inertia I. The string is wrapped at radius R, with an object of mass m dangling on the string. For the dangling object, write

$$ma_y = \sum F_y$$

For the cylinder, write

$$I\alpha = \sum \tau$$



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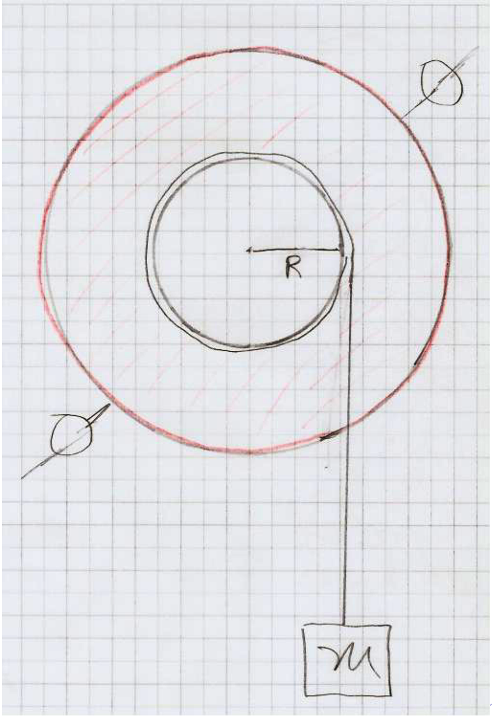


After some math, I get

$$\alpha = \frac{mgR}{I_{\text{wheel}} + mR^2} \approx \frac{mgR}{I_{\text{wheel}}}$$

(The approximation is for the limit where the object falls at $a \ll g$, so the string tension is $T = (mg - ma) \approx mg$.)

$$I\alpha = \tau = RT \Rightarrow T = \frac{I\alpha}{R}$$

$$ma = mg - T \Rightarrow m(\alpha R) = mg - \left(\frac{I\alpha}{R}\right)$$

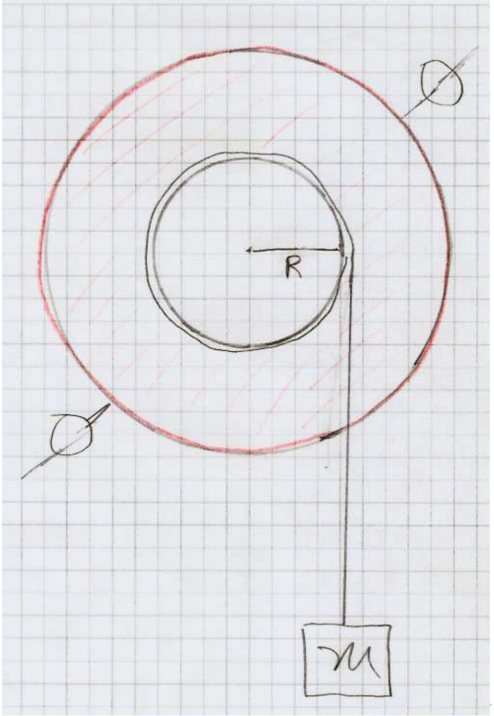
$$\alpha = \frac{g}{R} - \left(\frac{I}{mR^2}\right)\alpha \Rightarrow \alpha\left(1 + \frac{I}{mR^2}\right) = \frac{g}{R}$$

$$\alpha = \frac{g}{R\left(1 + \frac{I}{mR^2}\right)} = \frac{mgR}{I + mR^2}$$

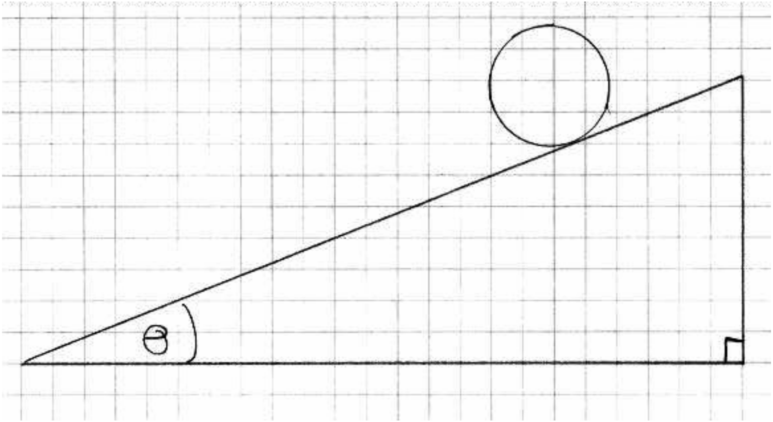
Why did increasing the dangling mass m **increase** the wheel's rotational acceleration α ?

Why did increasing the radius R from which the dangling mass was suspended **increase** the wheel's rotational acceleration?

Why did sliding the big rotating masses farther out on the extended "arms" **decrease** the wheel's rotational acceleration?



Let's go back and use torque to analyze another problem that last week we were only able to analyze using energy conservation:
a cylinder rolling (without slipping) down an inclined plane.



What 3 forces act on the cylinder? What is the rotation axis? Draw FBD and extended FBD. What are the torque(s) about this axis? How are α and a related? Write $\vec{F} = m\vec{a}$ and $\tau = I\alpha$.

$$I\alpha = \sum \tau$$

$$I\left(\frac{a_x}{R}\right) = RF^s$$

$$F^s = \left(\frac{I}{R^2}\right) a_x$$

$$ma_x = mg\sin\theta - F^s$$

$$ma_x = mg\sin\theta - \left(\frac{I}{R^2}\right) a_x$$

$$\left(m + \frac{I}{R^2}\right) a_x = mg\sin\theta$$

$$a_x = \frac{mg\sin\theta}{m + \frac{I}{R^2}} = \frac{g\sin\theta}{1 + \left(\frac{I}{mR^2}\right)}$$

Remember that the object with the larger “shape factor” $I/(mR^2)$ rolls downhill more slowly.

While we're here, let's revisit the "center-of-mass chalkline" demonstration from a few weeks ago.

Now that we know about torque, we can see why the CoM always winds up directly beneath the pivot, once we understand that the line-of-action for gravity passes through the CoM.

We stopped after this.

Another equilibrium problem!

The top end of a ladder of inertia m rests against a smooth (i.e. slippery) wall, and the bottom end rests on the ground. The coefficient of static friction between the ground and the ladder is μ_s. What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Let's start by drawing an EFBD for the ladder.

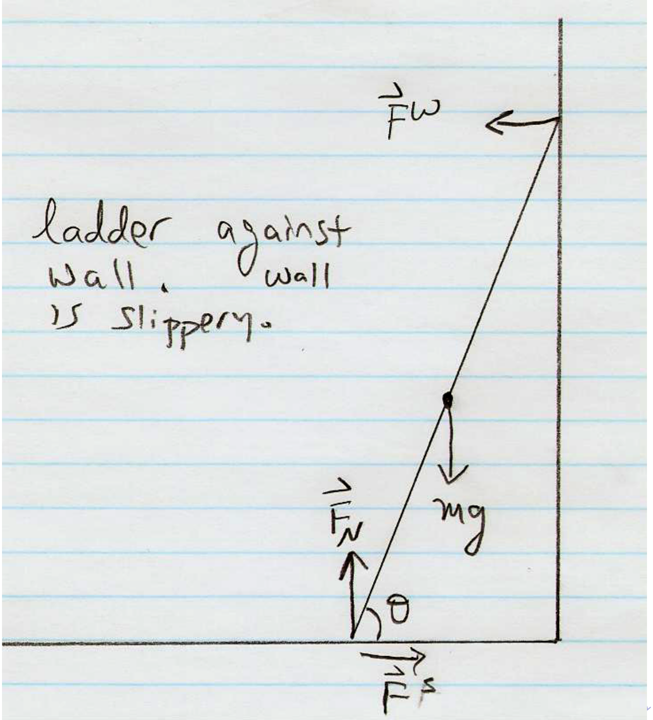
Why must we say the wall is slippery?

Is the slippery wall more like a pin or a roller support?

What plays the role here that string tension played in the previous problem?

Does the combination of two forces at the bottom act more like a pin or a roller support?

Which forces would an engineer call “reaction” forces?

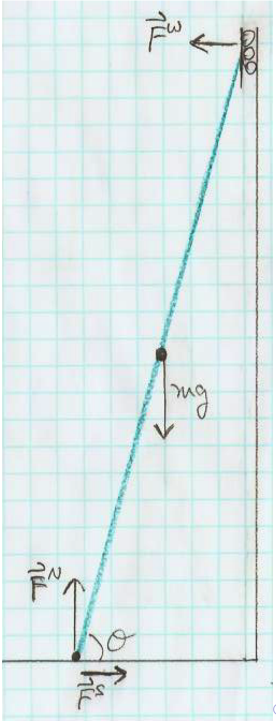


Which choice of pivot axis will give us the simplest equation for $\sum M_z = 0$? (We'll get an equation involving only two forces if we choose this axis.)

(A) Use bottom of ladder as pivot axis.

(B) Use center of ladder as pivot axis.

(C) Use top of ladder as pivot axis.



How would I write $\sum M_z = 0$ about the bottom end of the ladder? (Take length of ladder to be L.)

(A) $F^W L\cos\theta + mgL\sin\theta = 0$

(B) $F^W L\cos\theta + mg\frac{L}{2}\sin\theta = 0$

(C) $F^W L\cos\theta - mgL\sin\theta = 0$

(D) $F^W L\cos\theta - mg\frac{L}{2}\sin\theta = 0$

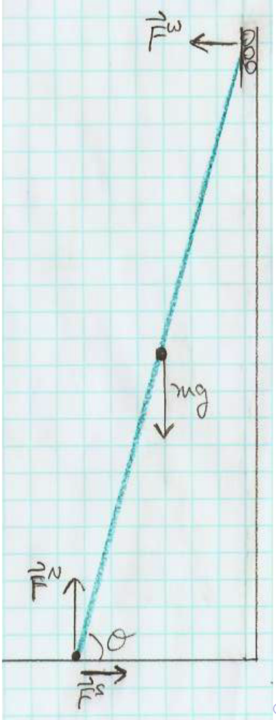
(E) $F^W L\sin\theta + mgL\cos\theta = 0$

(F) $F^W L\sin\theta + mg\frac{L}{2}\cos\theta = 0$

(G) $F^W L\sin\theta - mgL\cos\theta = 0$

(H) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?



What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

Let's answer the original question:

What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Suppose we add to this picture a woman of mass M who has climbed up a distance d along the length of the ladder. Now how do we write the moment equation $\sum M_z = 0$?

(A) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\cos\theta = 0$

(B) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\sin\theta = 0$

(C) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mgd\cos\theta = 0$

(D) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mgd\sin\theta = 0$

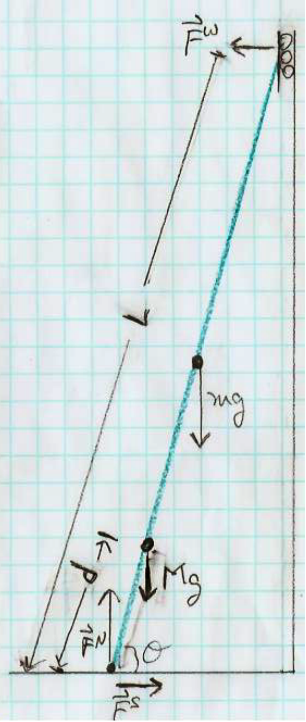
(E) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\cos\theta = 0$

(F) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\sin\theta = 0$

(G) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mgd\cos\theta = 0$

(H) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mgd\sin\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

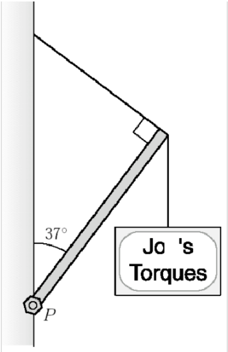


What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

For a given θ, how far up can she climb before the ladder slips?

Here's a trickier equilibrium problem:

4*. You want to hang a 22 kg sign (shown at right) that advertises your new business. To do this, you attach a 7.0 kg beam of length 1.0 m to a wall at its base by a pivot P. You then attach a thin cable to the beam and to the wall in such a way that the cable and beam are perpendicular to each other. The beam makes an angle of 37° with the vertical. You hang the sign from the end of the beam to which the cable is attached. (a) What must be the minimum tensile strength of the cable (the amount of tension it can sustain) if it is not to snap? (b) Determine the horizontal and vertical components of the force the pivot exerts on the beam.



What forces act on the beam?

What 3 equations can we write for the beam?

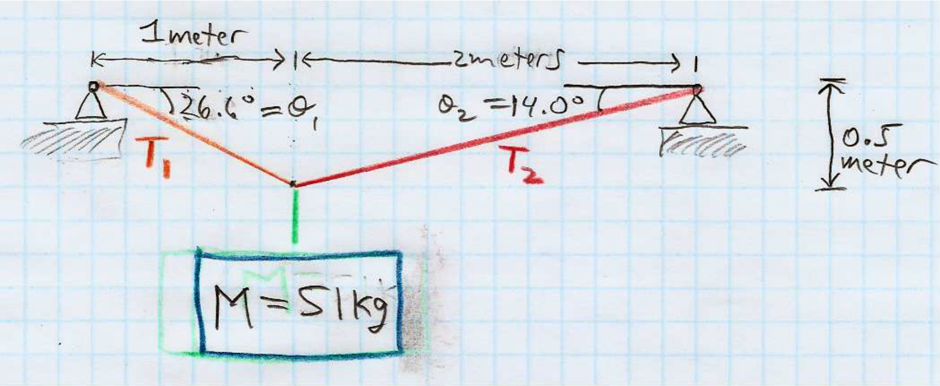
A tightly stretched "high wire" has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

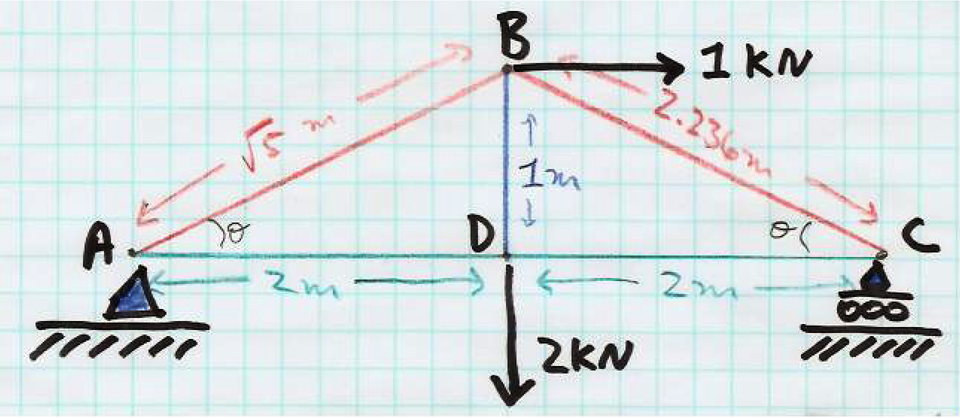
What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



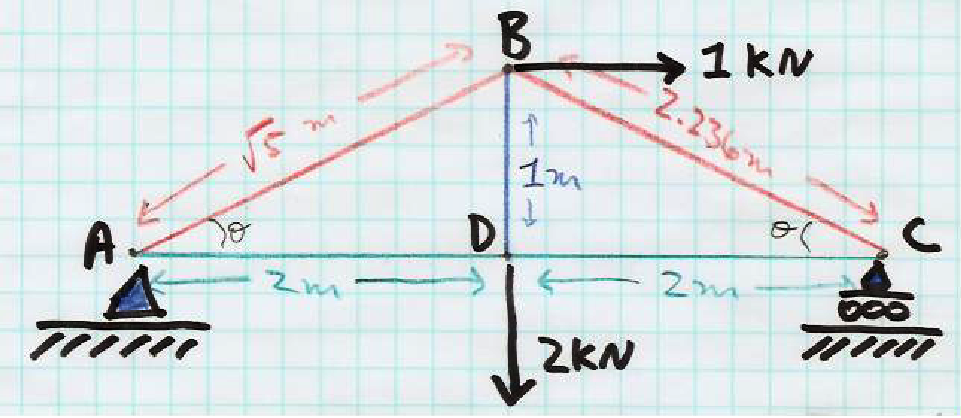
How would you find the tensions T_1 and T_2?

Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?



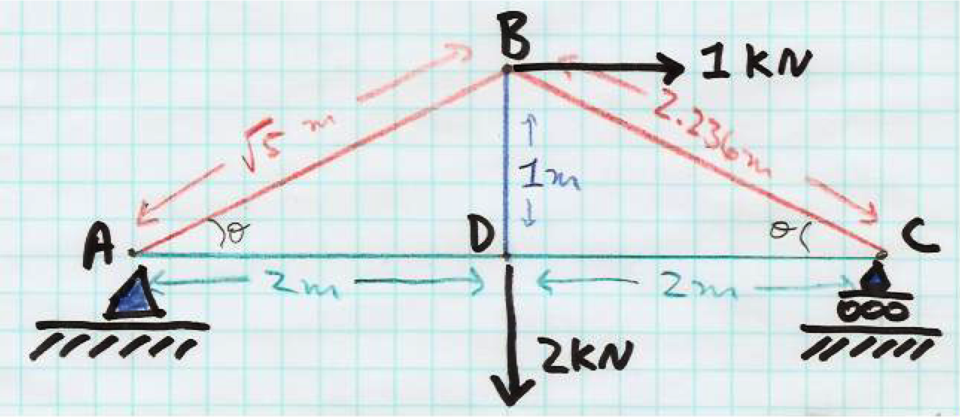
How many equations does the "method of joints" allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15



How many unknown internal forces (tensions or compressions) do we need to determine when we "solve" this truss?

(A) 4 (B) 5 (C) 6 (D) 7



This is a "simply supported" truss. How many independent "reaction forces" do the two supports exert on the truss? (If there are independent horizontal and vertical components, count them as separate forces.)

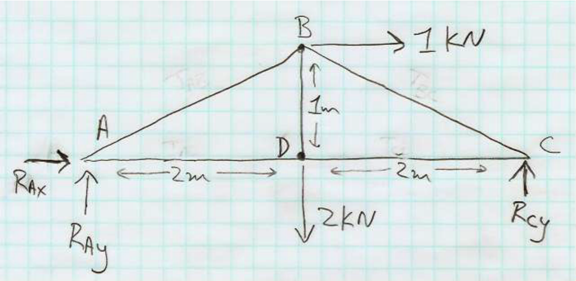
(A) 2 (B) 3 (C) 4 (D) 6

Notice that $8 = 5 + 3$.

For a planar truss that is stable and that you can solve using the equations of static equilibrium,

$$2N_{\mathrm{joints}} = N_{\mathrm{bars}} + 3$$

You get two force equations per joint. You need to solve for one unknown tension/compression per bar plus three support "reaction" forces.



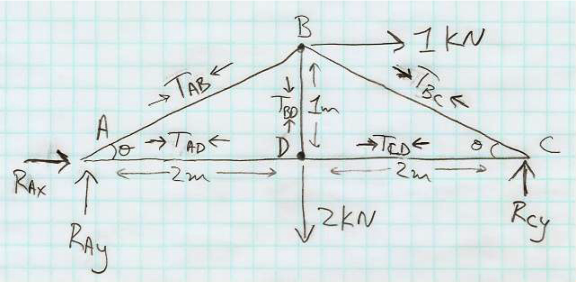
What do we learn by writing $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$ for the truss as a whole? (Use joint **A** as pivot.)

(I write R_{Ax}, R_{Ay}, R_{Cy} for the 3 "reaction forces" exerted by the supports on the truss.)

(A) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(2\ \mathrm{m}) = 0$

(B) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(1\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$

(C) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$



What two equations does the "method of joints" let us write for joint **C** ?

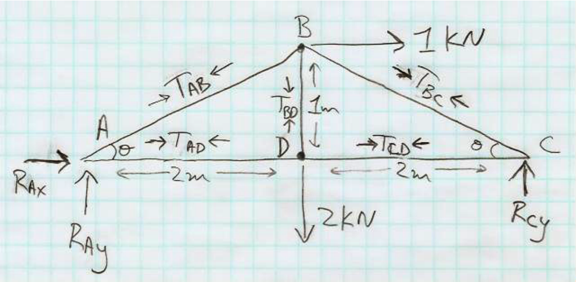
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $T_{CD} - T_{BC}\cos\theta = 0$
$R_{Cy} - T_{BC}\sin\theta = 0$

(B) $T_{CD} - T_{BC}\sin\theta = 0$
$R_{Cy} - T_{BC}\cos\theta = 0$

(C) $T_{CD} + T_{BC}\cos\theta = 0$
$R_{Cy} + T_{BC}\sin\theta = 0$

(D) $T_{CD} + T_{BC}\sin\theta = 0$
$R_{Cy} + T_{BC}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **A** ?

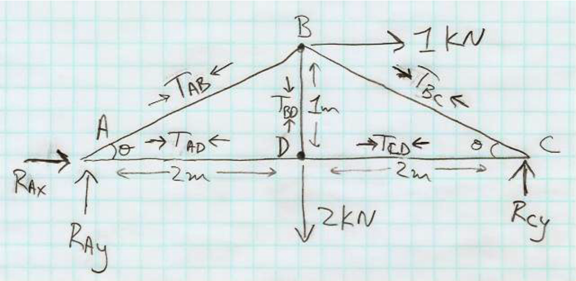
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $R_{Ax} - T_{AD} - T_{AB}\cos\theta = 0$
$R_{Ay} - T_{AB}\sin\theta = 0$

(B) $R_{Ax} - T_{AD} - T_{AB}\sin\theta = 0$
$R_{Ay} - T_{AB}\cos\theta = 0$

(C) $R_{Ax} + T_{AD} + T_{AB}\cos\theta = 0$
$R_{Ay} + T_{AB}\sin\theta = 0$

(D) $R_{Ax} + T_{AD} + T_{AB}\sin\theta = 0$
$R_{Ay} + T_{AB}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **D** ?

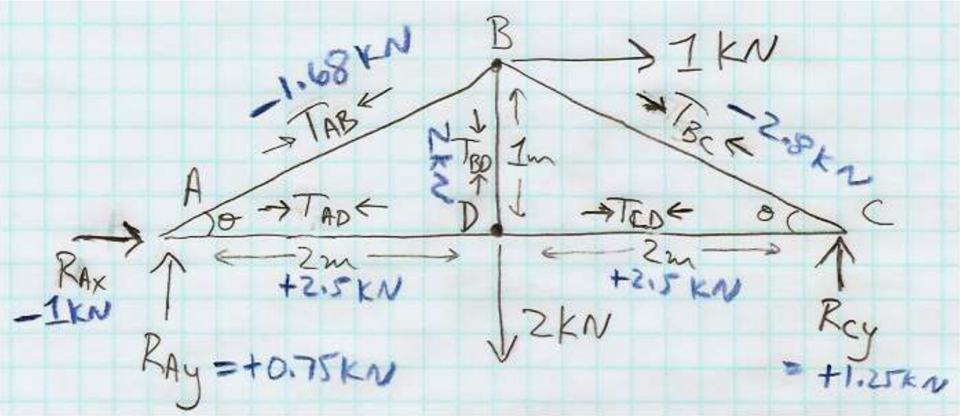
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

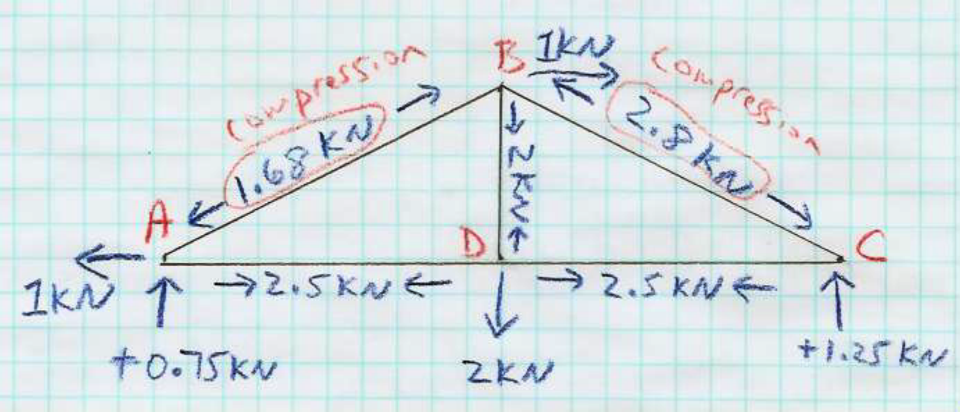
(B) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$

(C) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

(D) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$



I named each member force T_{ij} (for "tension") and let $T_{ij} > 0$ mean tension and $T_{ij} < 0$ mean compression. Once you've solved the truss, it's best to draw the arrows with the correct signs for clarity. (Next page.)



Forces redrawn with arrows in correct directions, now that we know the sign of each force. Members **AB** and **BC** are in compression. All other members are in tension.

Another option is to write down all $2J$ equations at once and to type them into **Mathematica**, Maple, Wolfram Alpha, etc.

```
In[92] eq := {

RAx + TAB*cos + TAD == 0,
RAy + TAB*sin == 0,
-TAB*cos+TBC*cos+1 == 0,
-TBD-TAB*sin-TBC*sin == 0,
-TAD+TCD == 0,
-2 + TBD == 0,
-TCD - TBC*cos == 0,
RCy + TBC*sin == 0,

sin==1.0/Sqrt[5.0],
cos==2.0/Sqrt[5.0]

}
```

```
In[93] Solve[eq]

Out[93] {

RAx → -1.,
RAy → 0.75,
RCy → 1.25,
TAB → -1.67705,
TAD → 2.5,
TBC → -2.79508,
TBD → 2.,
TCD → 2.5,

cos → 0.894427,
sin → 0.447214

}
```

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