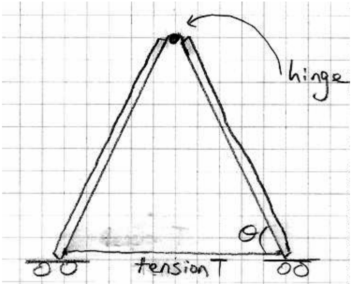
Physics 8 — Wednesday, November 6, 2019

- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane.
- HW9 due Friday. HW help: (Bill) Wed 4-6:30pm DRL 3C4, (Grace/Brooke) Thu 6-8pm DRL 2C4.
- I should have paused, after we worked out the hinged arch last Friday, to talk about the torques (moments) due to the vertical vs. horizontal forces and how they vary vs. θ.



Another equilibrium problem!

The top end of a ladder of inertia m rests against a smooth (i.e. slippery) wall, and the bottom end rests on the ground. The coefficient of static friction between the ground and the ladder is μ_s. What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Let's start by drawing an EFBD for the ladder.

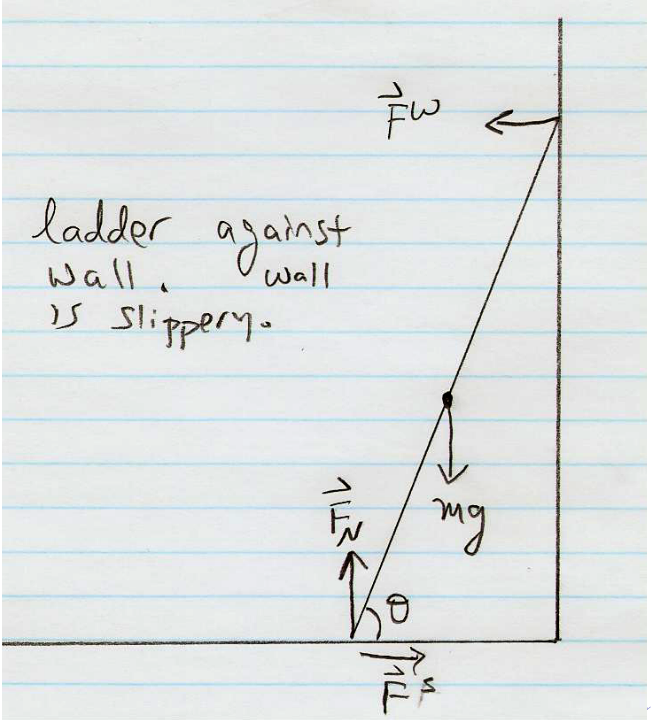
Why must we say the wall is slippery?

Is the slippery wall more like a pin or a roller support?

What plays the role here that string tension played in the previous problem?

Does the combination of two forces at the bottom act more like a pin or a roller support?

Which forces would an engineer call "reaction" forces?

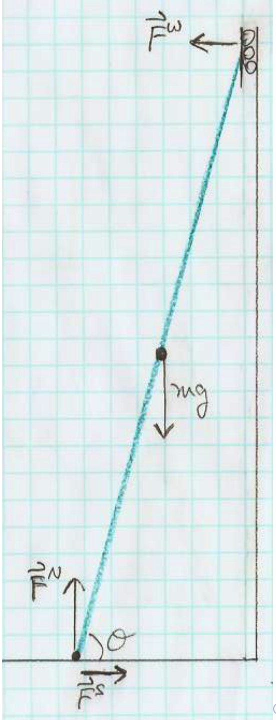


Which choice of pivot axis will give us the simplest equation for $\sum M_z = 0$? (We'll get an equation involving only two forces if we choose this axis.)

(A) Use bottom of ladder as pivot axis.

(B) Use center of ladder as pivot axis.

(C) Use top of ladder as pivot axis.



How would I write $\sum M_z = 0$ about the bottom end of the ladder? (Take length of ladder to be L.)

(A) $F^W L\cos\theta + mgL\sin\theta = 0$

(B) $F^W L\cos\theta + mg\frac{L}{2}\sin\theta = 0$

(C) $F^W L\cos\theta - mgL\sin\theta = 0$

(D) $F^W L\cos\theta - mg\frac{L}{2}\sin\theta = 0$

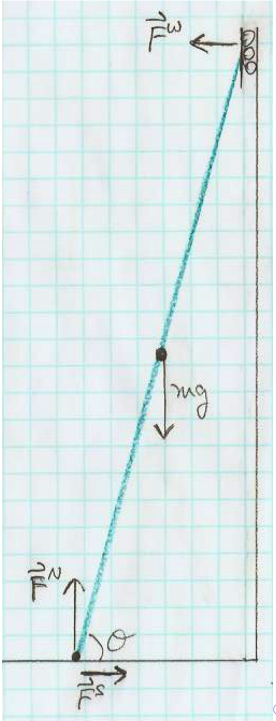
(E) $F^W L\sin\theta + mgL\cos\theta = 0$

(F) $F^W L\sin\theta + mg\frac{L}{2}\cos\theta = 0$

(G) $F^W L\sin\theta - mgL\cos\theta = 0$

(H) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?



What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

Let's answer the original question:

What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Suppose we add to this picture a woman of mass M who has climbed up a distance d along the length of the ladder. Now how do we write the moment equation $\sum M_z = 0$?

(A) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\cos\theta = 0$

(B) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\sin\theta = 0$

(C) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mgd\cos\theta = 0$

(D) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mgd\sin\theta = 0$

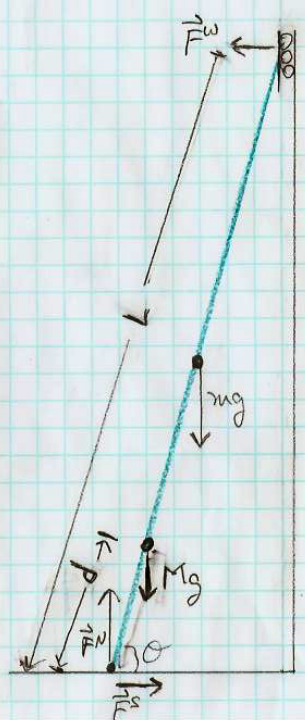
(E) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\cos\theta = 0$

(F) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\sin\theta = 0$

(G) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mgd\cos\theta = 0$

(H) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mgd\sin\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

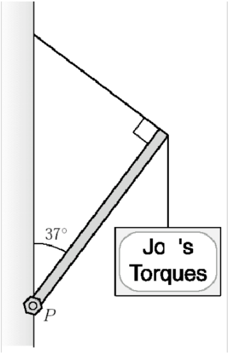


What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

For a given θ, how far up can she climb before the ladder slips?

Here's a trickier equilibrium problem:

4*. You want to hang a 22 kg sign (shown at right) that advertises your new business. To do this, you attach a 7.0 kg beam of length 1.0 m to a wall at its base by a pivot P. You then attach a thin cable to the beam and to the wall in such a way that the cable and beam are perpendicular to each other. The beam makes an angle of 37° with the vertical. You hang the sign from the end of the beam to which the cable is attached. (a) What must be the minimum tensile strength of the cable (the amount of tension it can sustain) if it is not to snap? (b) Determine the horizontal and vertical components of the force the pivot exerts on the beam.



What forces act on the beam?

What 3 equations can we write for the beam?

(We stopped after this.)

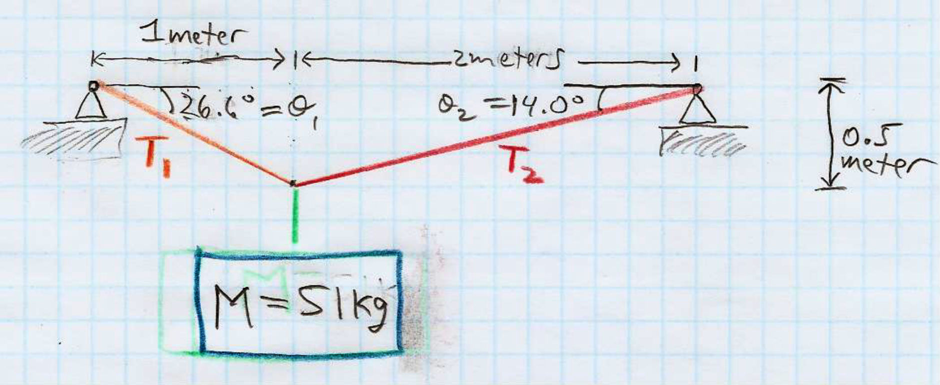
A tightly stretched "high wire" has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

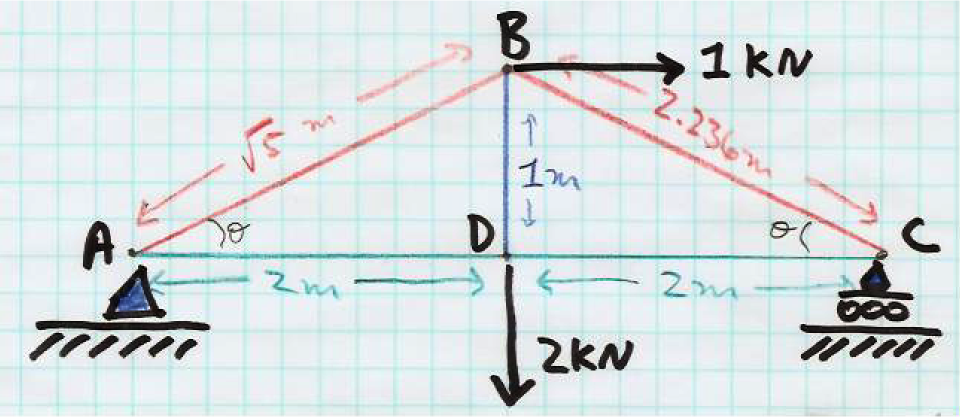
What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



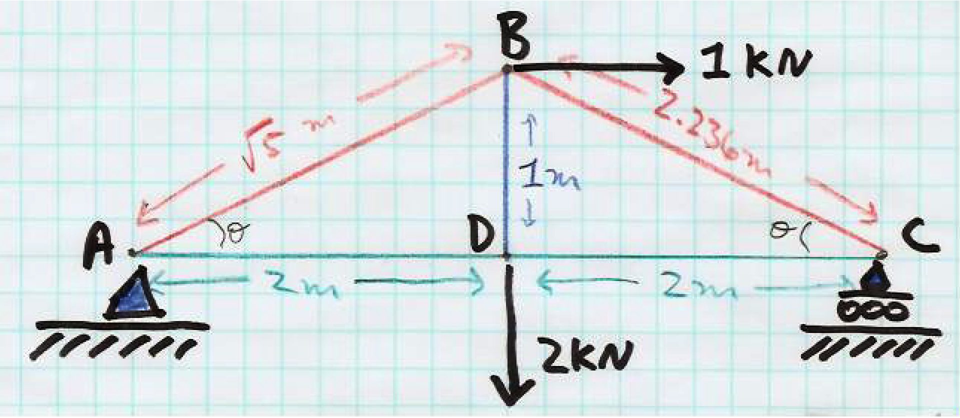
How would you find the tensions T_1 and T_2?

Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?



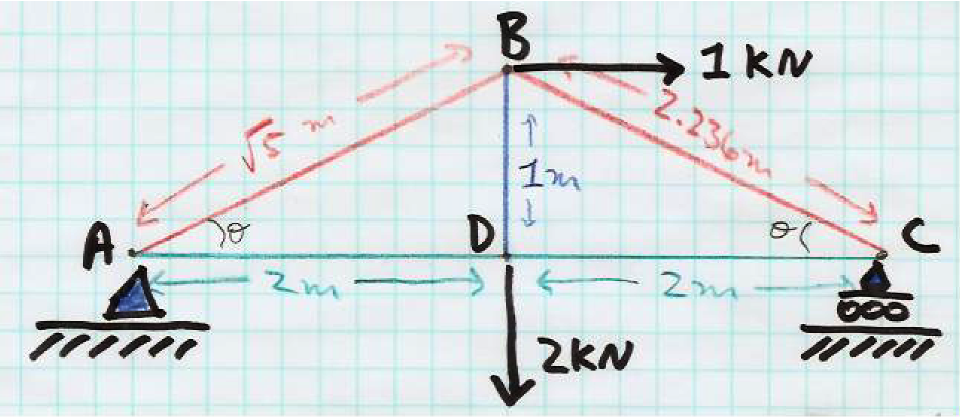
How many equations does the "method of joints" allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15



How many unknown internal forces (tensions or compressions) do we need to determine when we "solve" this truss?

(A) 4 (B) 5 (C) 6 (D) 7



This is a “simply supported” truss. How many independent “reaction forces” do the two supports exert on the truss? (If there are independent horizontal and vertical components, count them as separate forces.)

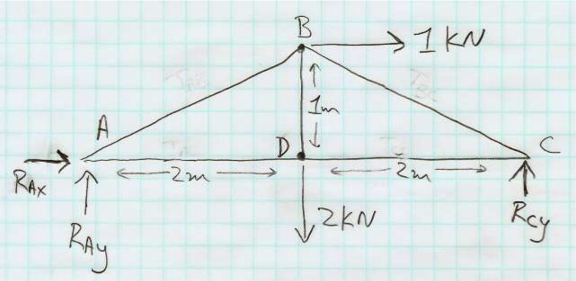
(A) 2 (B) 3 (C) 4 (D) 6

Notice that $8 = 5 + 3$.

For a planar truss that is stable and that you can solve using the equations of static equilibrium,

$$2N_{\text{joints}} \ = \ N_{\text{bars}} + 3$$

You get two force equations per joint. You need to solve for one unknown tension/compression per bar plus three support "reaction" forces.



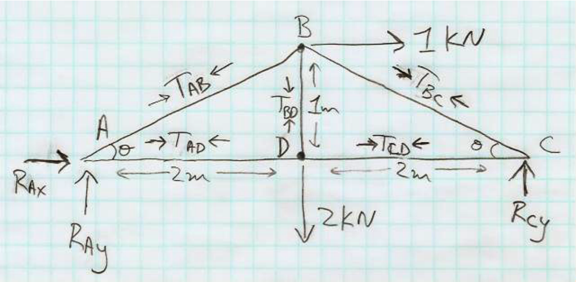
What do we learn by writing $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$ for the truss as a whole? (Use joint **A** as pivot.)

(I write R_{Ax}, R_{Ay}, R_{Cy} for the 3 "reaction forces" exerted by the supports on the truss.)

(A) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$
$R_{Ax} + 1\ \mathrm{kN} = 0,$
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(2\ \mathrm{m}) = 0$

(B) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$
$R_{Ax} + 1\ \mathrm{kN} = 0,$
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(1\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$

(C) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$
$R_{Ax} + 1\ \mathrm{kN} = 0,$
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$



What two equations does the "method of joints" let us write for joint **C** ?

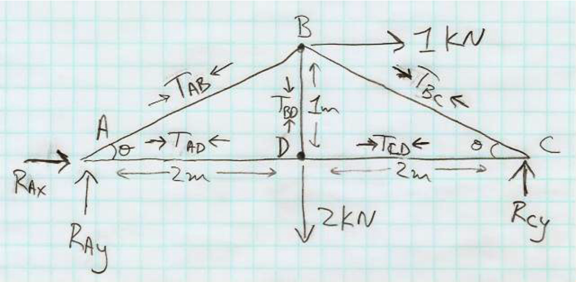
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $T_{CD} - T_{BC}\cos\theta = 0$
$R_{Cy} - T_{BC}\sin\theta = 0$

(B) $T_{CD} - T_{BC}\sin\theta = 0$
$R_{Cy} - T_{BC}\cos\theta = 0$

(C) $T_{CD} + T_{BC}\cos\theta = 0$
$R_{Cy} + T_{BC}\sin\theta = 0$

(D) $T_{CD} + T_{BC}\sin\theta = 0$
$R_{Cy} + T_{BC}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **A** ?

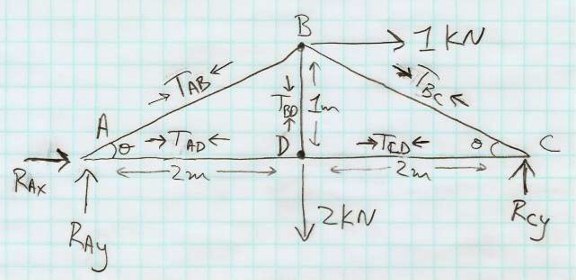
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $R_{Ax} - T_{AD} - T_{AB}\cos\theta = 0$
$R_{Ay} - T_{AB}\sin\theta = 0$

(B) $R_{Ax} - T_{AD} - T_{AB}\sin\theta = 0$
$R_{Ay} - T_{AB}\cos\theta = 0$

(C) $R_{Ax} + T_{AD} + T_{AB}\cos\theta = 0$
$R_{Ay} + T_{AB}\sin\theta = 0$

(D) $R_{Ax} + T_{AD} + T_{AB}\sin\theta = 0$
$R_{Ay} + T_{AB}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **D** ?

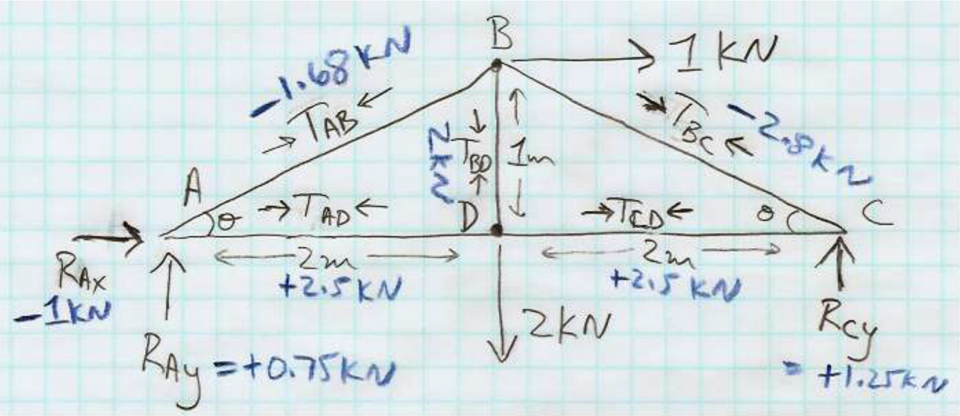
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

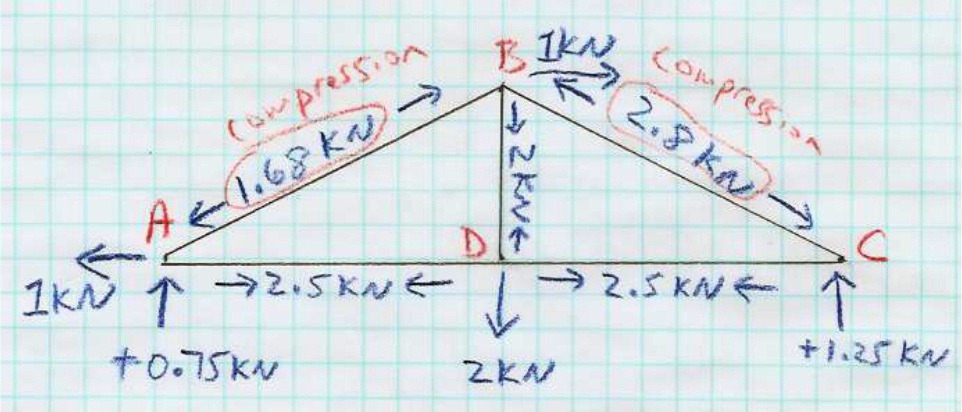
(B) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$

(C) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

(D) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$



I named each member force T_{ij} (for "tension") and let $T_{ij} > 0$ mean tension and $T_{ij} < 0$ mean compression. Once you've solved the truss, it's best to draw the arrows with the correct signs for clarity. (Next page.)

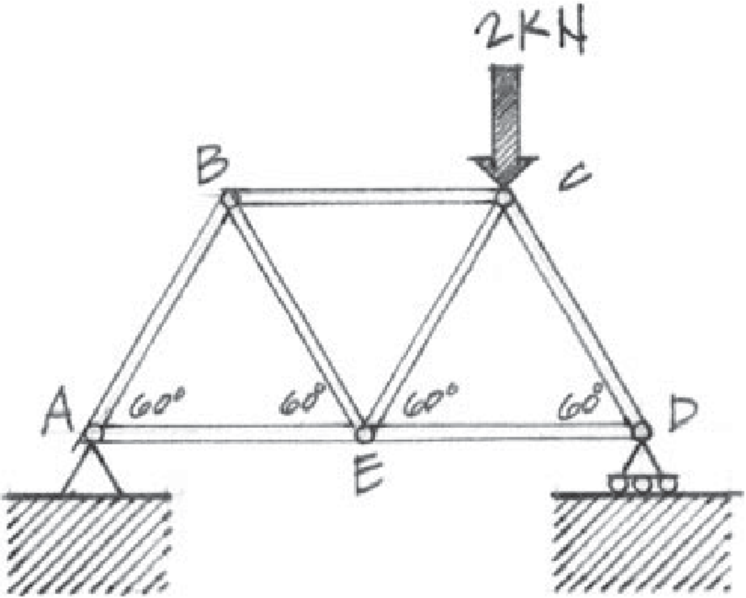


Forces redrawn with arrows in correct directions, now that we know the sign of each force. Members **AB** and **BC** are in compression. All other members are in tension.

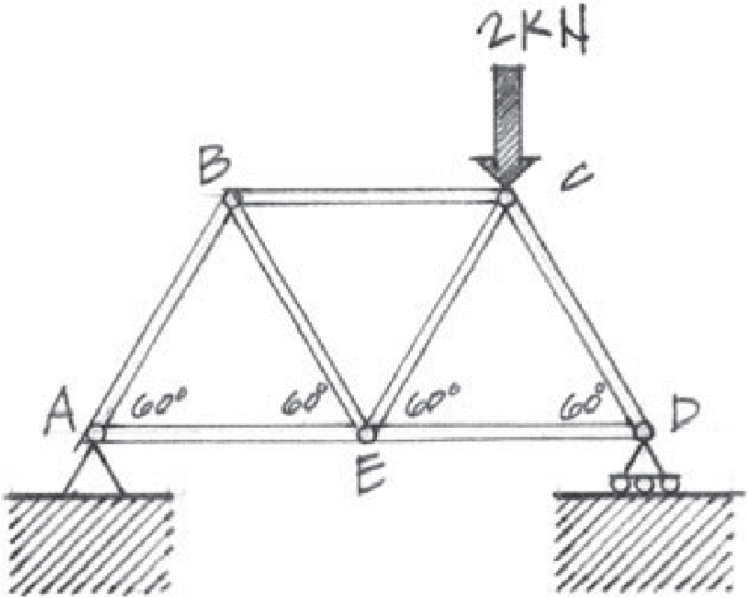
Another option is to write down all $2J$ equations at once and to type them into **Mathematica**, Maple, Wolfram Alpha, etc.

```
In[92] eq := {

RAx + TAB*cos + TAD == 0,
RAy + TAB*sin == 0,
-TAB*cos+TBC*cos+1 == 0,
-TBD-TAB*sin-TBC*sin == 0,
-TAD+TCD == 0,
-2 + TBD == 0,
-TCD - TBC*cos == 0,
RCy + TBC*sin == 0,

sin==1.0/Sqrt[5.0],
cos==2.0/Sqrt[5.0]

}
```

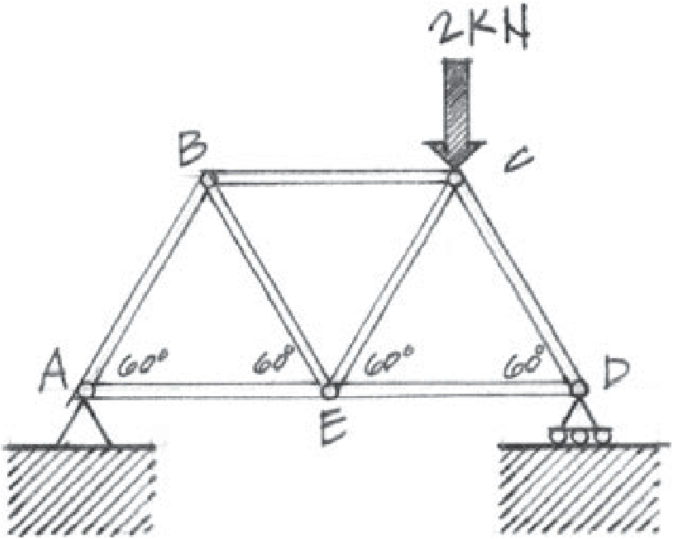
```
In[93] Solve[eq]

Out[93] {

RAx → -1.,
RAy → 0.75,
RCy → 1.25,
TAB → -1.67705,
TAD → 2.5,
TBC → -2.79508,
TBD → 2.,
TCD → 2.5,

cos → 0.894427,
sin → 0.447214

}
```



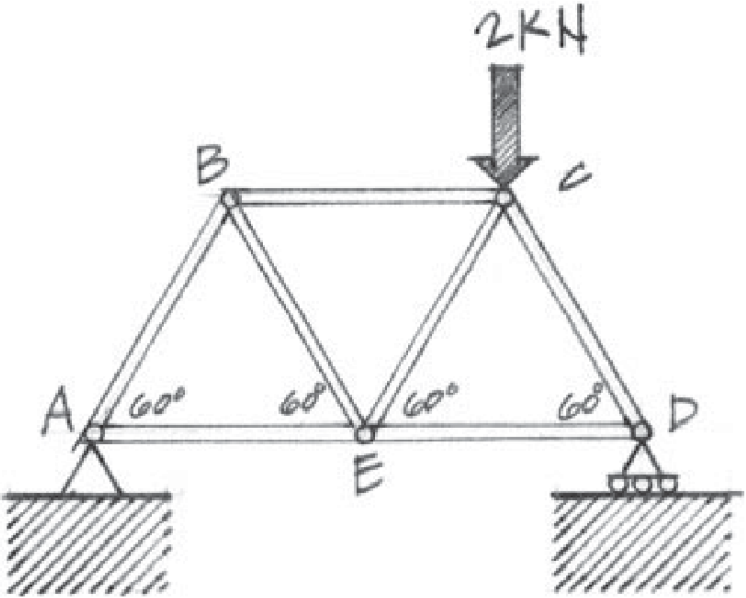
How many "reaction forces" are exerted by the supports (i.e. exerted on the truss by the supports)?



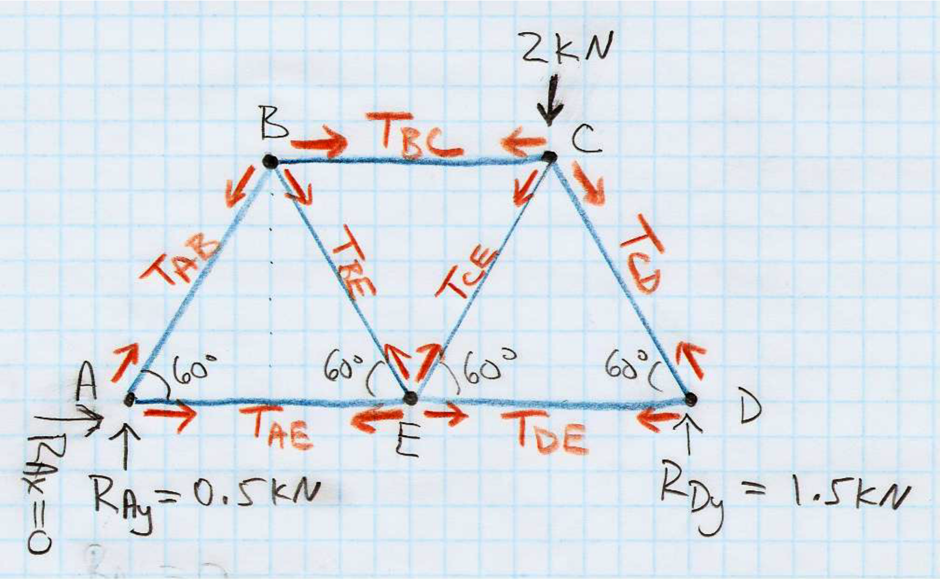
How many internal forces (tensions or compressions in the members) do we need to solve for to “solve” this truss?



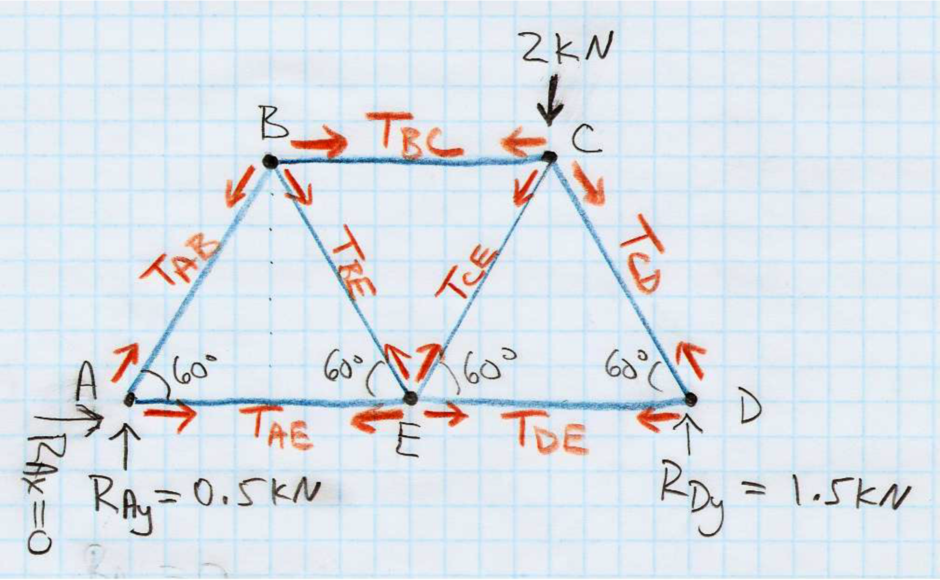
Do you see any joint at which there are ≤ 2 unknown forces? If so, we can start there. If not, we need to start with an EFBD for the truss as a whole.



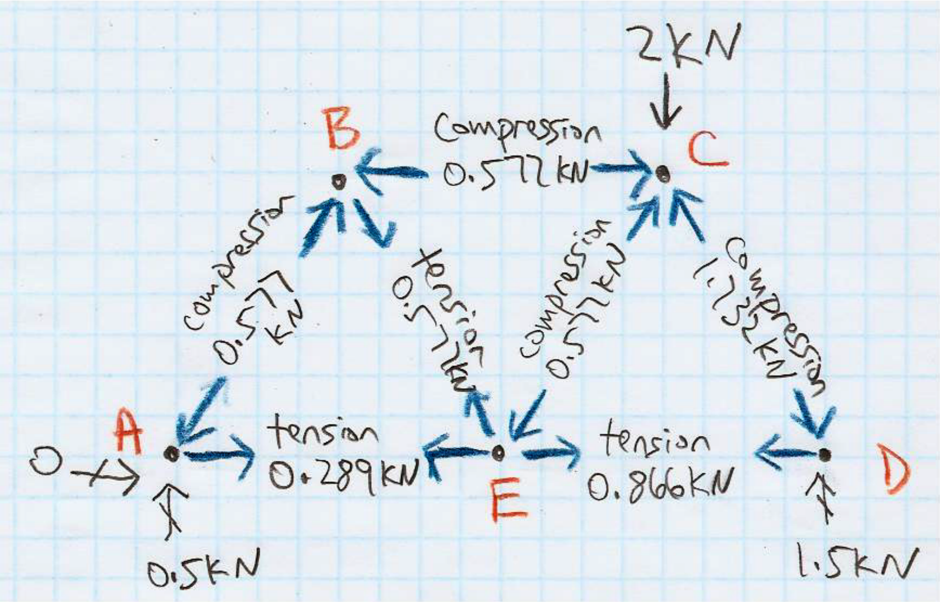
Try to guess $R_{A,x}$, $R_{A,y}$, and $R_{D,y}$ by inspection. Then let's check with the usual equations.



Now start from a joint having ≤ 2 unknown forces. In this case, I just went through the joints alphabetically. You can make your life easier by seeking out equations having just 1 unknown.



$T_{AB} = -0.577$ kN, $T_{AE} = +0.289$ kN, $T_{BE} = +0.577$ kN,
$T_{BC} = -0.577$ kN, $T_{CE} = -0.577$ kN, $T_{CD} = -1.732$ kN,
$T_{DE} = +0.866$ kN. My notation: tension > 0, compression < 0.


2KN
B
C
Compression
0.577kN
compression
0.577 kN
tension
0.577kN
compression
0.577kN
compression
1.732kN
A
tension
0.289kN
E
tension
0.866kN
D
0
0.5KN
1.5KN

$\cos 60^\circ = 0.5, \quad \sin 60^\circ = 0.866$

Joint A: $0 + T_{AE} + T_{AB}\cos 60^\circ = 0$

$0.5\,kN + T_{AB}\sin 60^\circ = 0 \Rightarrow \boxed{T_{AB} = -0.577\,kN}$

$\boxed{T_{AE} = +0.289\,kN}$

Joint B: $-T_{AB}\cos 60^\circ + T_{BE}\cos 60^\circ + T_{BC} = 0$

$-T_{AB}\sin 60^\circ - T_{BE}\sin 60^\circ = 0 \Rightarrow \boxed{T_{BE} = +0.577\,kN}$

$\boxed{T_{BC} = -0.577\,kN}$

Joint C: $-T_{BC} - T_{CE}\cos 60^\circ + T_{CD}\cos 60^\circ = 0 \Rightarrow T_{CD} = T_{CE} - 1.155\,kN$

(could do C, D, or E at this point)

$-2\,kN - T_{CE}\sin 60^\circ - T_{CD}\sin 60^\circ = 0$

$-2\,kN - T_{CE}\sin 60^\circ - (T_{CE} - 1.155\,kN)\sin 60^\circ = 0$

$2T_{CE}\sin 60^\circ = -2\,kN + 1.0\,kN = -1\,kN$

$\boxed{T_{CE} = -0.577\,kN}$

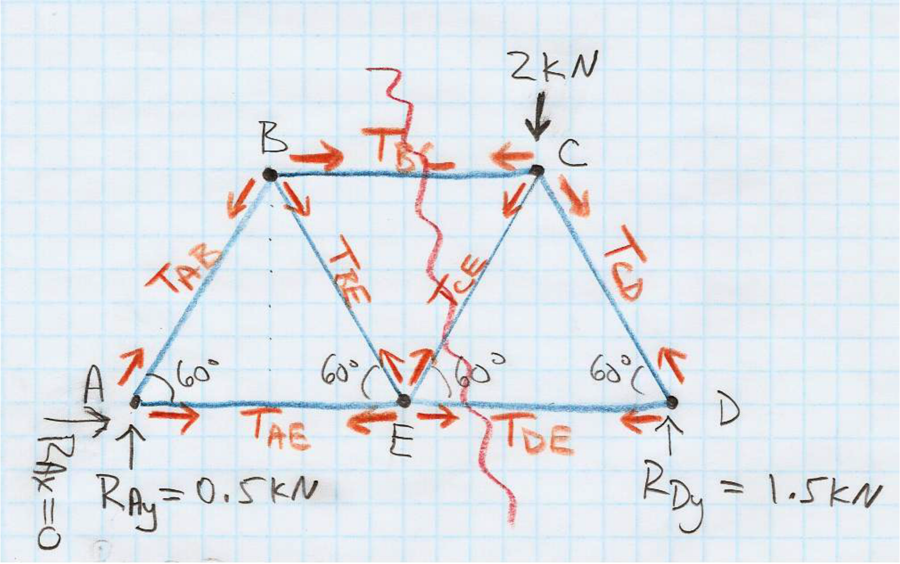
$T_{CD} = T_{CE} - 1.155\,kN = \boxed{-1.732\,kN = T_{CD}}$

Joint D: $-T_{DE} - T_{CD}\cos 60^\circ = 0 \Rightarrow \boxed{T_{DE} = +0.866\,kN}$

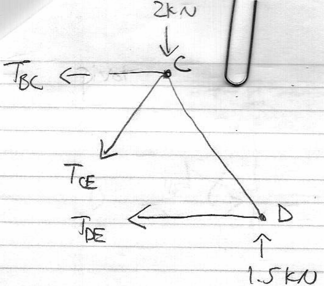
(check): $1.5\,kN + T_{CD}\sin 60^\circ = 0$ ✓

(check Joint E): $-T_{AE} + T_{DE} - T_{BE}\cos 60^\circ + T_{CE}\cos 60^\circ = 0$ ✓

$+T_{BE}\sin 60^\circ + T_{CE}\sin 60^\circ = 0$ ✓



Let's try drawing an EFBD for the **right** side of the cut ("section").



$$\sum F_x = 0 \Rightarrow -T_{BC} - T_{CE}\cos 60^\circ - T_{DE} = 0$$

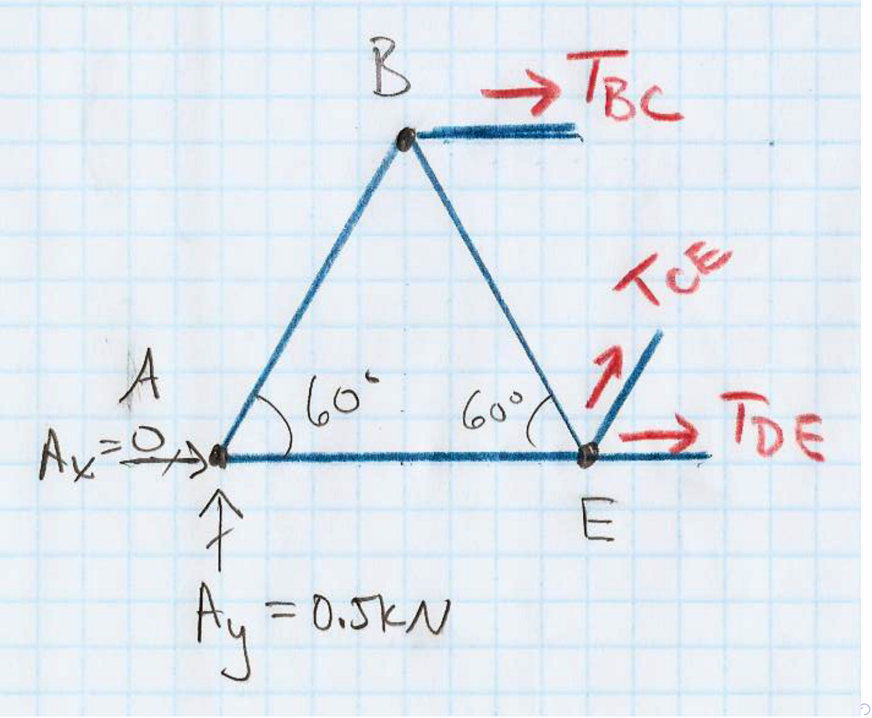
$$\sum F_y = 0 \Rightarrow -2kN + 1.5kN - T_{CE}\sin 60^\circ = 0$$

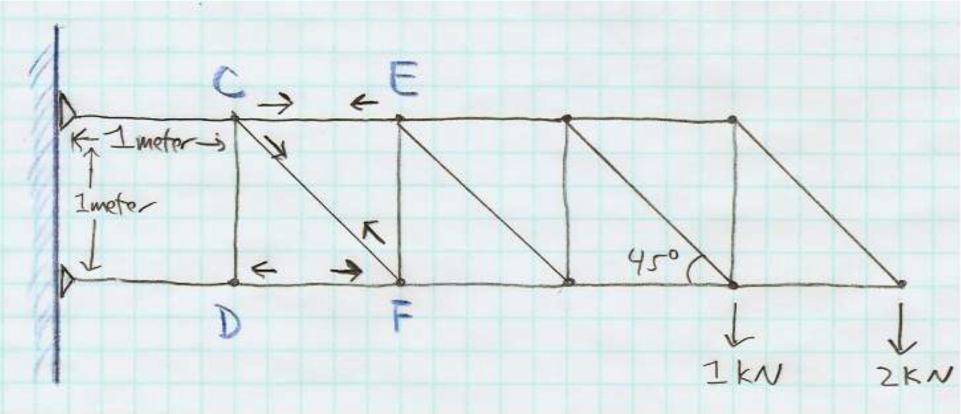
$$\sum M_C = 0 \Rightarrow +(1.5kN)\left(\frac{L}{2}\right) - T_{DE}\, L\sin 60^\circ = 0$$

$$\to T_{DE} = \frac{1.5kN}{2\sin 60^\circ} = +0.866kN$$

$$\to T_{CE} = \frac{-0.5kN}{\sin 60^\circ} = -0.577kN$$

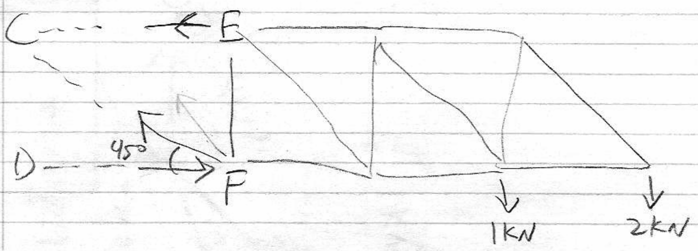
$$\to T_{BC} = -(T_{CE}\cos 60^\circ + T_{DE}) = -0.577$$


B
T_{BC}
T_{CE}
T_{DE}
A
60°
60°
$A_x = 0$
E
$A_y = 0.5kN$



Here's another truss problem that you can solve using the Method of Sections. Find forces in members **CE**, **CF**, and **DF**, with assumed force directions as shown.

- What happens if an assumed force direction is backwards?
- Where should we "section" the truss?
- Then what do we do next?



$$\sum F_x = 0 = -CE - CF\cos 45^\circ + DF = 0$$

$$\sum F_y = 0 = CF\sin 45^\circ = 3\,kN \Rightarrow \boxed{CF = 3\,kN\sqrt{2}}$$

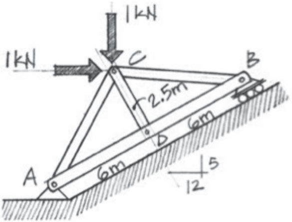
$$\sum M_{DF} = 0 = CE(1) - (1\,kN)(2) - (2\,kN)(3)$$

$$\Rightarrow \boxed{CE = 8\,kN}$$

$$DF = CE + CF\cos 45^\circ = 8\,kN + 3\,kN$$

$$\boxed{DF = 11\,kN}$$

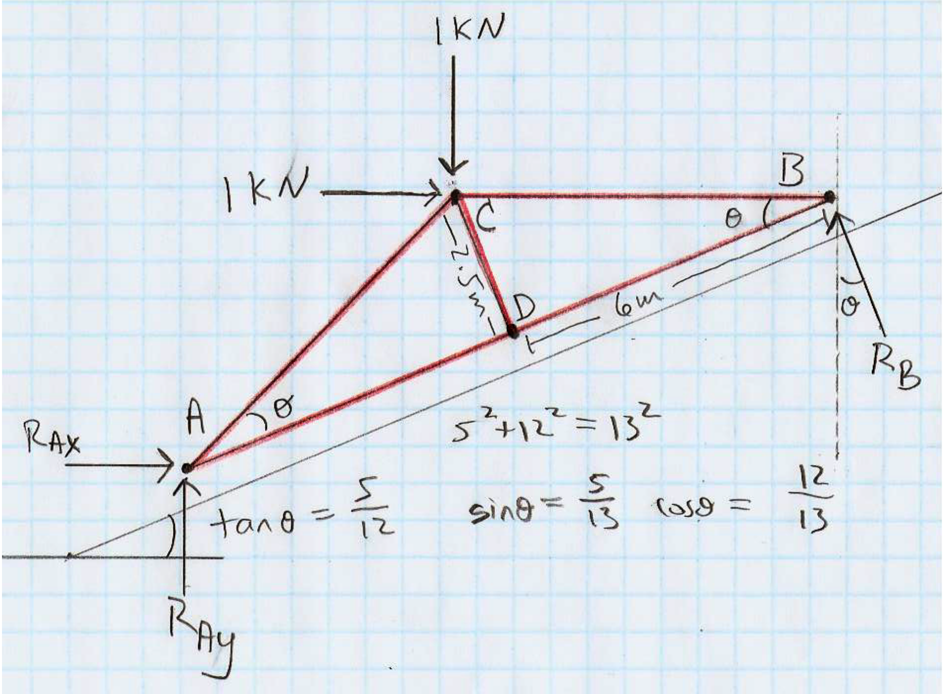
2.38 An inclined king-post truss supports a vertical and horizontal force at *C*. Determine the support reactions developed at *A* and *B*.



This is not really a "truss problem," since we're not asked to solve for the internal forces in the truss, but it is an example of a pretty tricky equilibrium problem.

Let's try working through this together in class.

Notice, from the given dimensions: the angle of the incline is the same as the interior angle at joints A and B of the truss. Also notice pin/hinge support at A and roller support at B.


1 kN
1 kN
A
B
C
D
θ
θ
θ
2.5m
6m
R_{Ax}
R_{Ay}
R_B
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$

① $0 = \sum_{\text{on truss}} F_x = R_{Ax} + 1\,kN - R_B \sin\theta$

$$\Rightarrow R_{Ax} = R_B \sin\theta - 1\,kN$$

② $0 = \sum_{\text{on truss}} F_y = R_{Ay} - 1\,kN + R_B \cos\theta$

$$\Rightarrow R_{Ay} = 1\,kN - R_B \cos\theta$$

③ $0 = \sum_{\circlearrowleft} M_A = R_B(12m) - 1\,kN(12m \sin\theta) - 1\,kN(6.5m \cos 2\theta)$

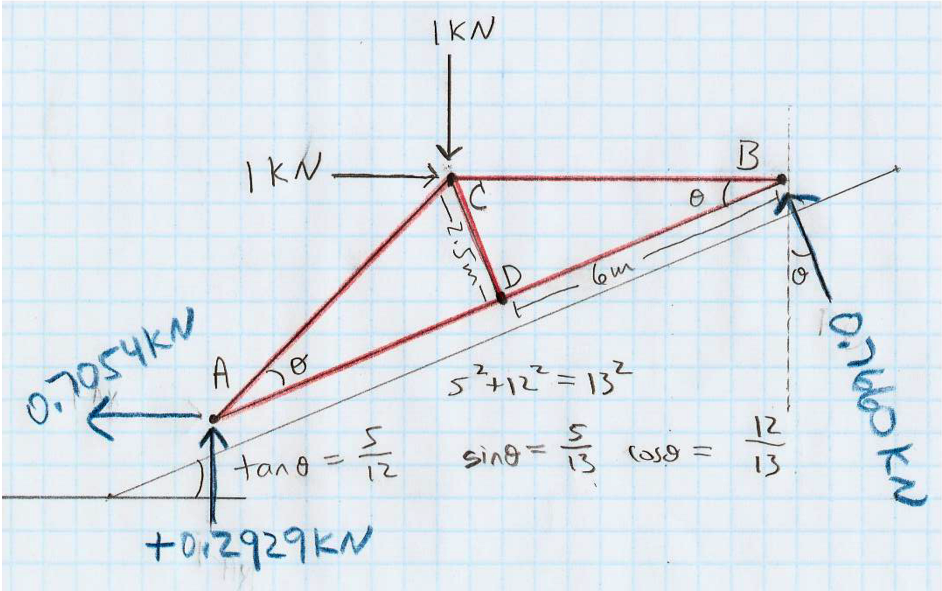
$$\Rightarrow R_B = 1\,kN(\sin\theta) + \frac{6.5}{12}\,kN(\cos 2\theta) = 0.7660\,kN$$

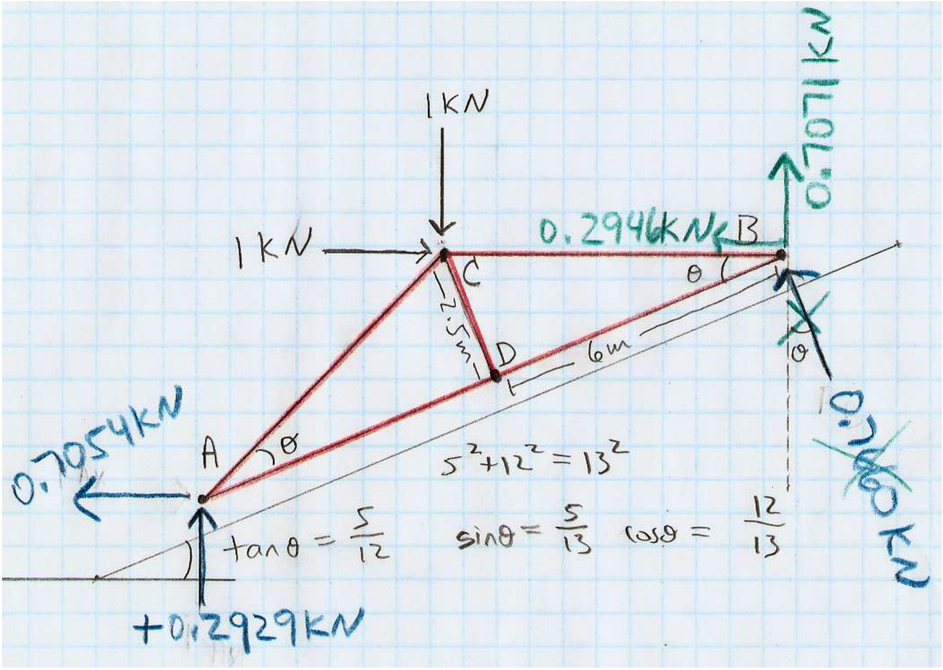
$$(R_B)_x = -R_B \sin\theta = -0.2946\,kN$$

$$(R_B)_y = R_B \cos\theta = 0.7071\,kN$$

$$R_{Ax} = (0.2946 - 1)\,kN = -0.7054\,kN$$

$$R_{Ay} = (1 - 0.7071)\,kN = 0.2929\,kN$$

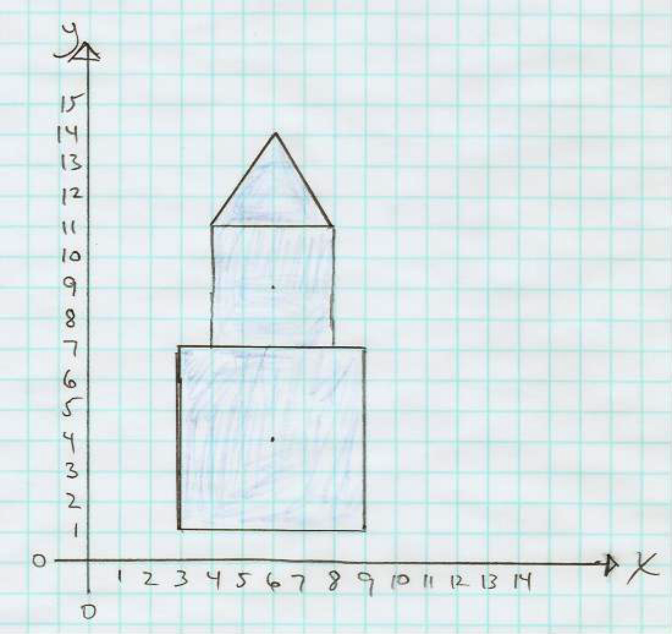

1KN
1 KN
B
C
D
A
θ
2.5m
6m
0.7054KN
+0.2929KN
0.7660 KN
5²+12²=13²
tanθ = 5/12
sinθ = 5/13
cosθ = 12/13



1KN
1KN
0.2946kN
B
C
D
A
θ
2.5m
6m
0.7071 kN
0.7054kN
+0.2929kN
5²+12²=13²
tan θ = 5/12
sin θ = 5/13
cos θ = 12/13

- The idea of computing centroids of simple and composite shapes is very, very briefly introduced in O/K ch3 (in the context of "distributed loads"), and will be discussed in much more detail in O/K ch6 (for next Monday).
- Let's go through one example using rectangles and triangles. It will help you in cases when you need to solve for the "reaction forces" on a beam that carries distributed loads. (Example coming up next.)

What is X_{centroid} for the shaded area?



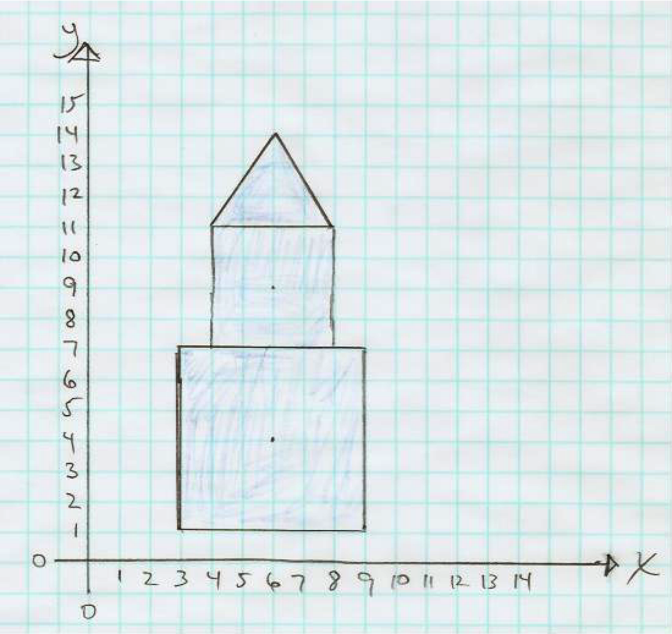
(A) 0

(B) 3

(C) 6

(D) 9

What are the areas of the three individual polygons?



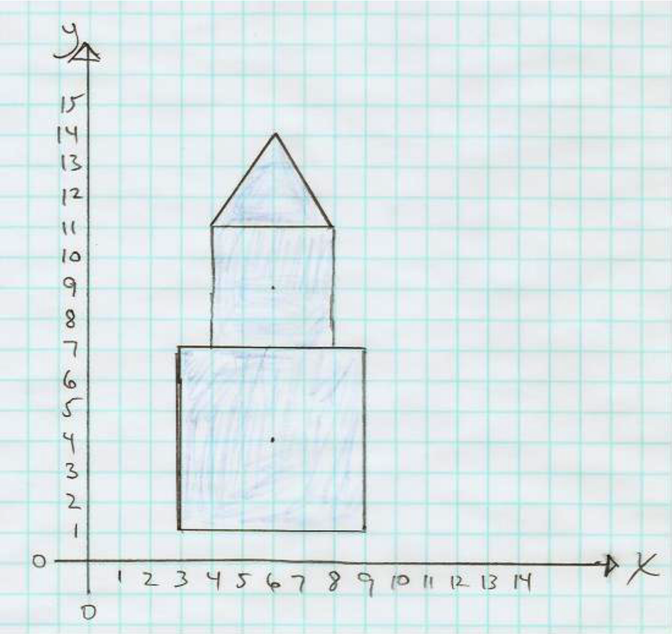
(A) 36, 16, 16

(B) 36, 16, 12

(C) 36, 16, 8

(D) 36, 16, 6

What are the Y_{centroid} values of the three individual polygons?



(A) 4, 9, 11

(B) 4, 9, 11.667

(C) 4, 9, 12

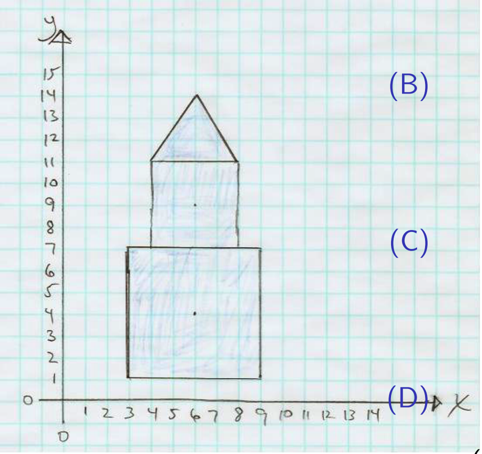
(D) 4, 9, 12.333

(E) 4, 9, 12.5

(F) 4, 9, 13

(G) 4, 9, 14

What is Y_{centroid} for the whole shaded area?



(A)

$$\frac{4+9+12}{3} = 8.33$$

(B)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{36+16+6} = 6.21$$

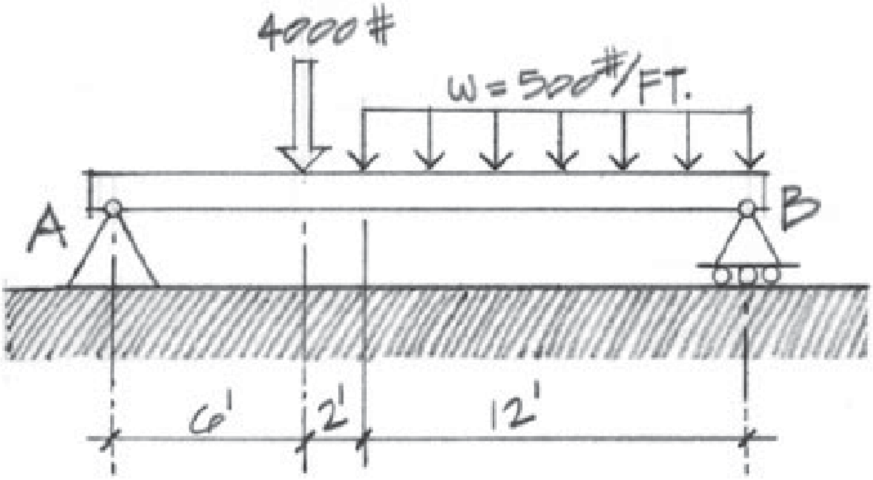
(C)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{4+9+12} = 14.4$$

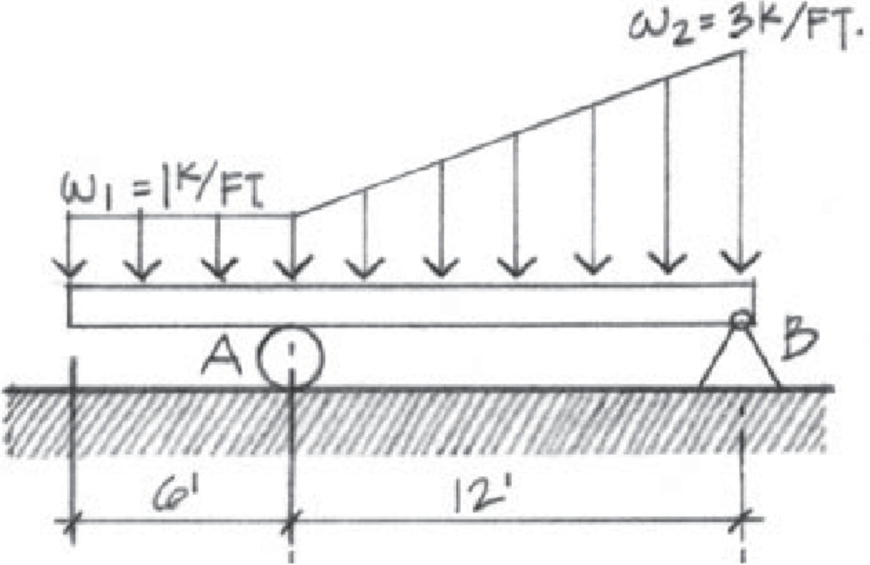
(D)

$$\frac{(4^2)(36)+(9^2)(16)+(12^2)(6)}{36+16+6} = 47.2$$

There's one problem similar to this (but using metric units) on HW(?): Determine the support reactions at *A* and *B*.



This one is harder than your homework, because the distributed load is non-uniform: Determine the support reactions at *A* and *B*.



Physics 8 — Wednesday, November 6, 2019

- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane.
- HW9 due Friday. HW help: (Bill) Wed 4-6:30pm DRL 3C4, (Grace/Brooke) Thu 6-8pm DRL 2C4.