Physics 8 — Friday, November 8, 2019

- Turn in HW9. Pick up HW10 handout in back corner.
- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. Next week, you'll read Ch6 (cross-sectional properties) and Ch7 (simple beams).

A tightly stretched "high wire" has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire? Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)? What happens to the tension as we make the sag smaller and smaller?

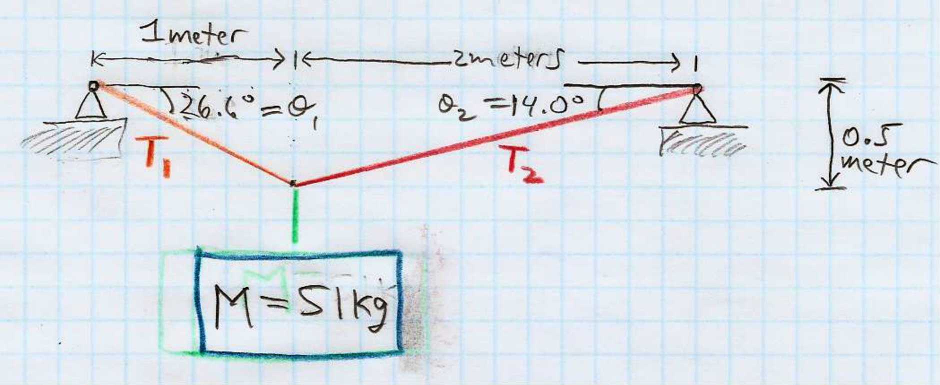
A tightly stretched "high wire" has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

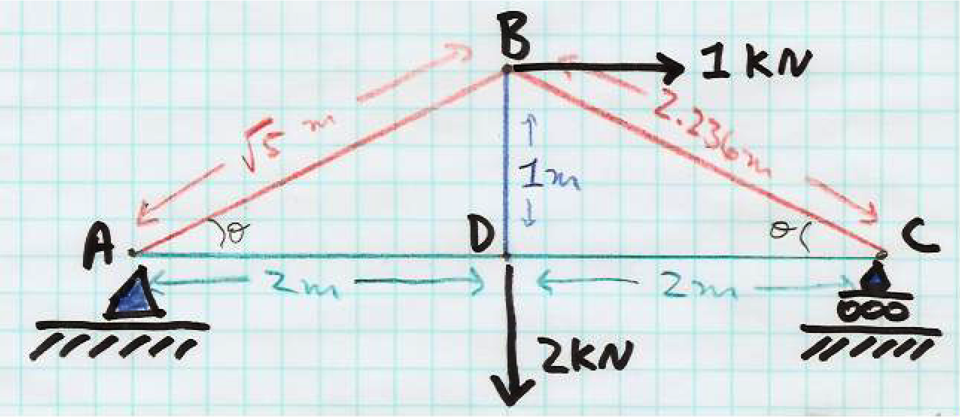
What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



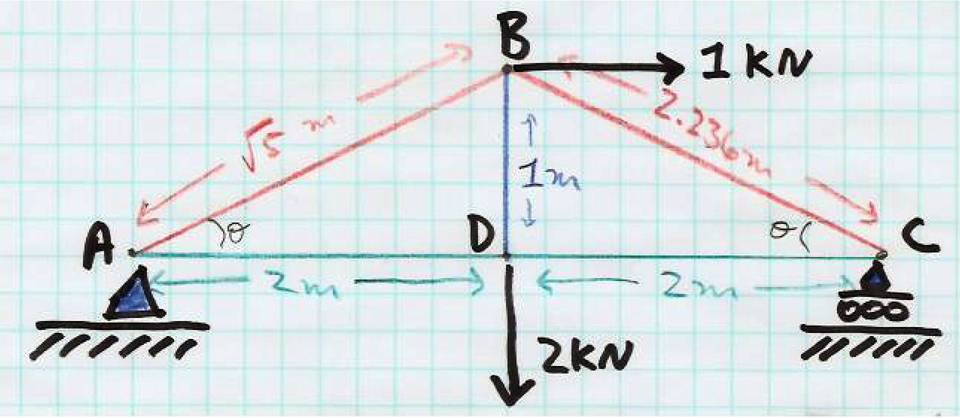
How would you find the tensions T_1 and T_2?

Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?



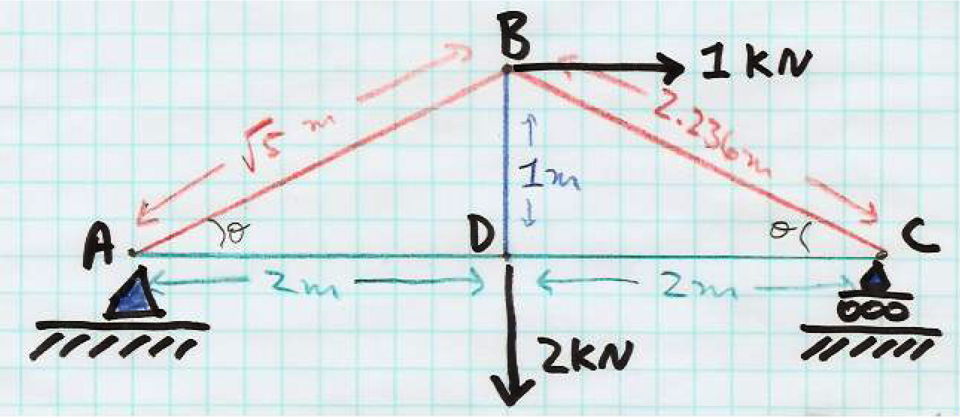
How many equations does the "method of joints" allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15



How many unknown internal forces (tensions or compressions) do we need to determine when we "solve" this truss?

(A) 4 (B) 5 (C) 6 (D) 7



This is a “simply supported” truss. How many independent “reaction forces” do the two supports exert on the truss? (If there are independent horizontal and vertical components, count them as separate forces.)

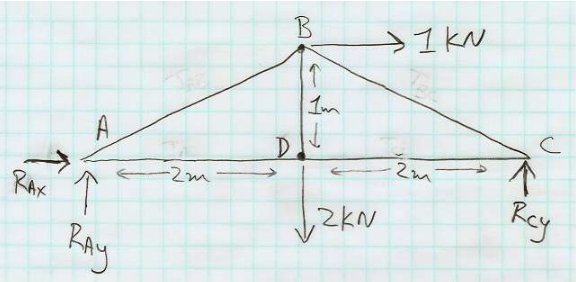
(A) 2 (B) 3 (C) 4 (D) 6

Notice that $8 = 5 + 3$.

For a planar truss that is stable and that you can solve using the equations of static equilibrium,

$$2N_{\text{joints}} \;=\; N_{\text{bars}} + 3$$

You get two force equations per joint. You need to solve for one unknown tension/compression per bar plus three support "reaction" forces.



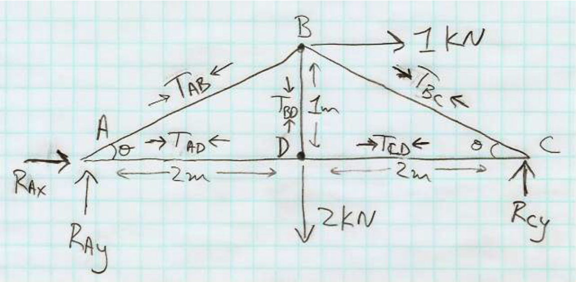
What do we learn by writing $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$ for the truss as a whole? (Use joint **A** as pivot.)

(I write R_{Ax}, R_{Ay}, R_{Cy} for the 3 "reaction forces" exerted by the supports on the truss.)

(A) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(2\ \mathrm{m}) = 0$

(B) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(1\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$

(C) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$



What two equations does the "method of joints" let us write for joint **C** ?

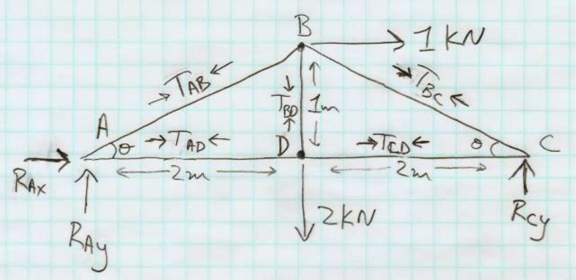
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $T_{CD} - T_{BC}\cos\theta = 0$
$R_{Cy} - T_{BC}\sin\theta = 0$

(B) $T_{CD} - T_{BC}\sin\theta = 0$
$R_{Cy} - T_{BC}\cos\theta = 0$

(C) $T_{CD} + T_{BC}\cos\theta = 0$
$R_{Cy} + T_{BC}\sin\theta = 0$

(D) $T_{CD} + T_{BC}\sin\theta = 0$
$R_{Cy} + T_{BC}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **A** ?

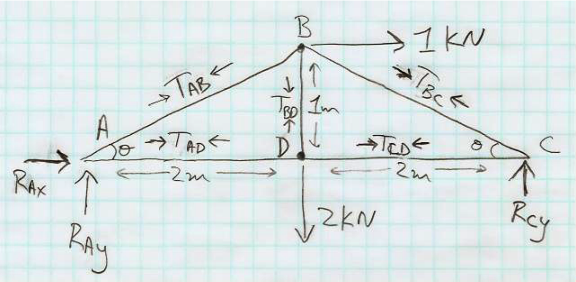
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $R_{Ax} - T_{AD} - T_{AB}\cos\theta = 0$
$R_{Ay} - T_{AB}\sin\theta = 0$

(B) $R_{Ax} - T_{AD} - T_{AB}\sin\theta = 0$
$R_{Ay} - T_{AB}\cos\theta = 0$

(C) $R_{Ax} + T_{AD} + T_{AB}\cos\theta = 0$
$R_{Ay} + T_{AB}\sin\theta = 0$

(D) $R_{Ax} + T_{AD} + T_{AB}\sin\theta = 0$
$R_{Ay} + T_{AB}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **D** ?

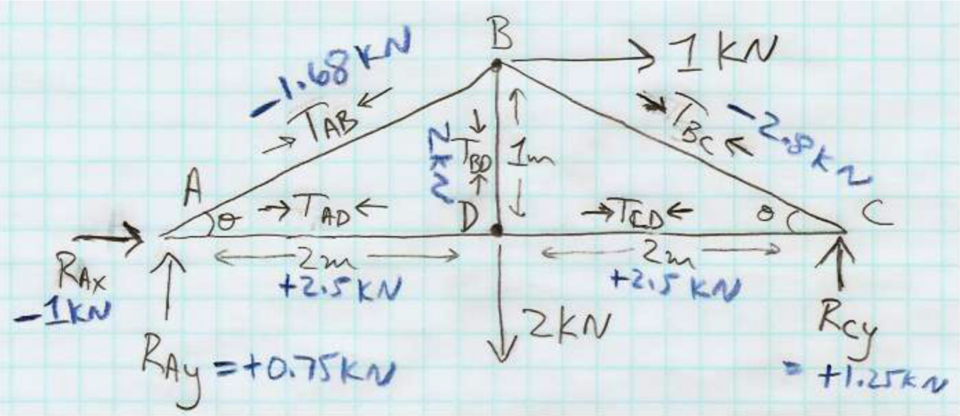
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $-2\text{ kN} + T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

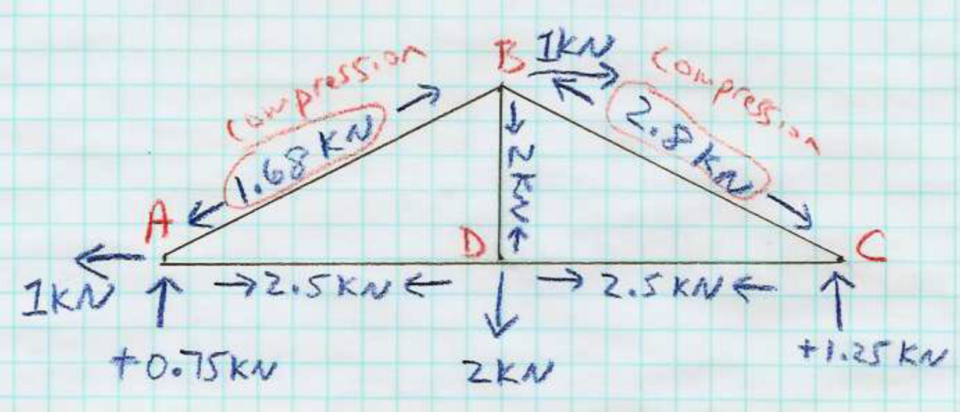
(B) $-2\text{ kN} + T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$

(C) $-2\text{ kN} - T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

(D) $-2\text{ kN} - T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$



I named each member force T_{ij} (for "tension") and let $T_{ij} > 0$ mean tension and $T_{ij} < 0$ mean compression. Once you've solved the truss, it's best to draw the arrows with the correct signs for clarity. (Next page.)

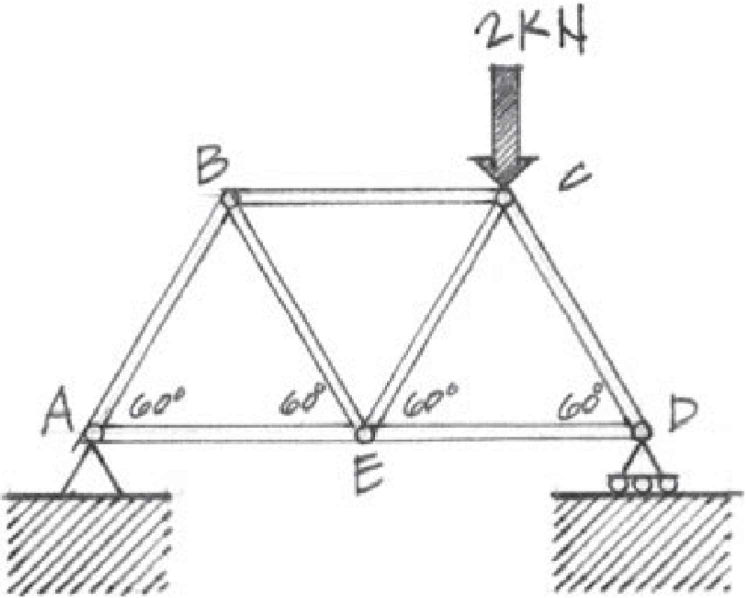


Forces redrawn with arrows in correct directions, now that we know the sign of each force. Members **AB** and **BC** are in compression. All other members are in tension.

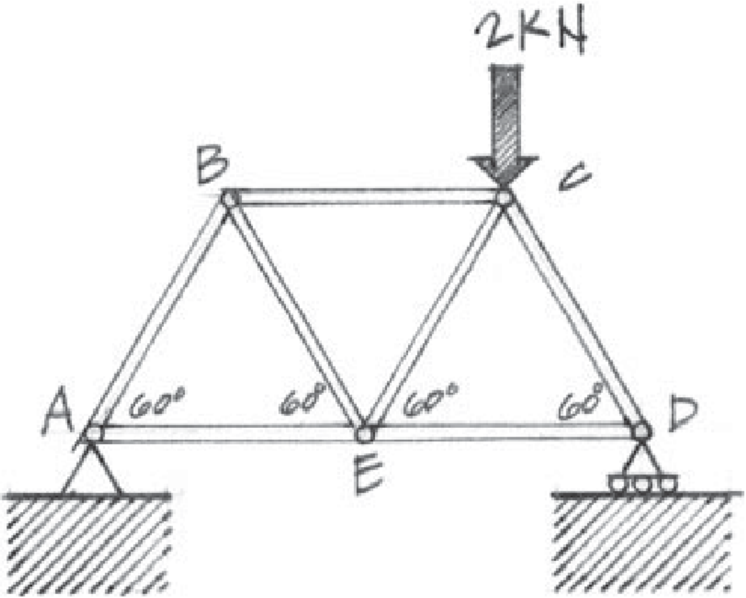
Another option is to write down all $2J$ equations at once and to type them into **Mathematica**, Maple, Wolfram Alpha, etc.

```
In[92] eq := {

RAx + TAB*cos + TAD == 0,
RAy + TAB*sin == 0,
-TAB*cos+TBC*cos+1 == 0,
-TBD-TAB*sin-TBC*sin == 0,
-TAD+TCD == 0,
-2 + TBD == 0,
-TCD - TBC*cos == 0,
RCy + TBC*sin == 0,

sin==1.0/Sqrt[5.0],
cos==2.0/Sqrt[5.0]

}
```

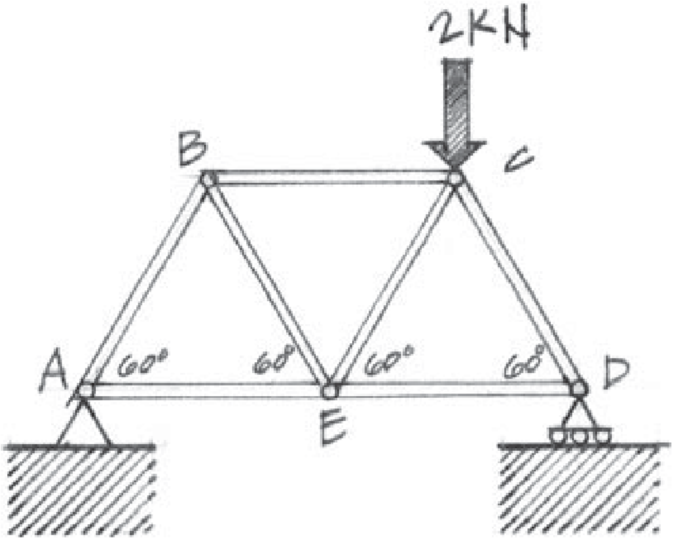
```
In[93] Solve[eq]

Out[93] {

RAx → -1.,
RAy → 0.75,
RCy → 1.25,
TAB → -1.67705,
TAD → 2.5,
TBC → -2.79508,
TBD → 2.,
TCD → 2.5,

cos → 0.894427,
sin → 0.447214

}
```



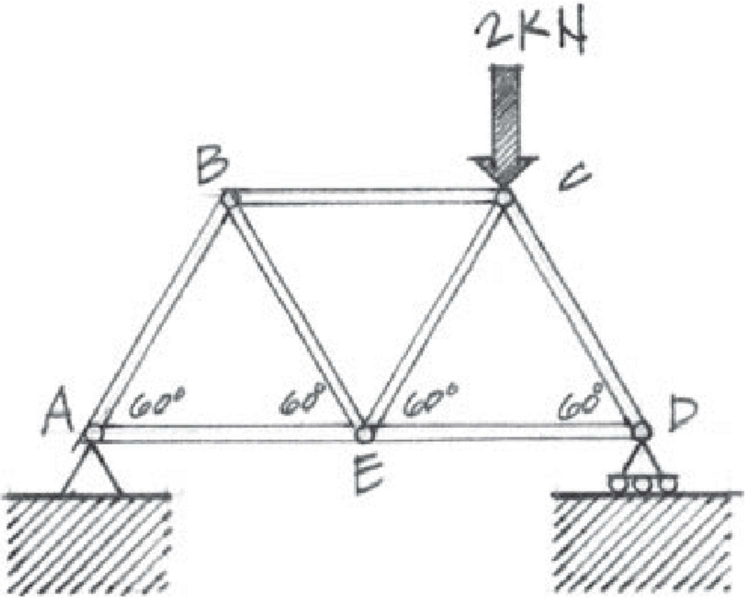
How many "reaction forces" are exerted by the supports (i.e. exerted on the truss by the supports)?



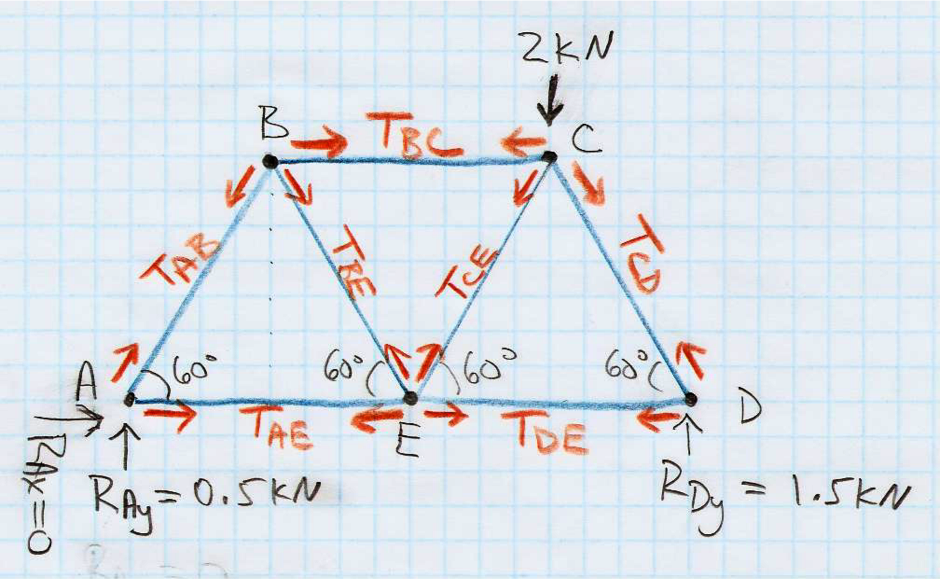
How many internal forces (tensions or compressions in the members) do we need to solve for to “solve” this truss?



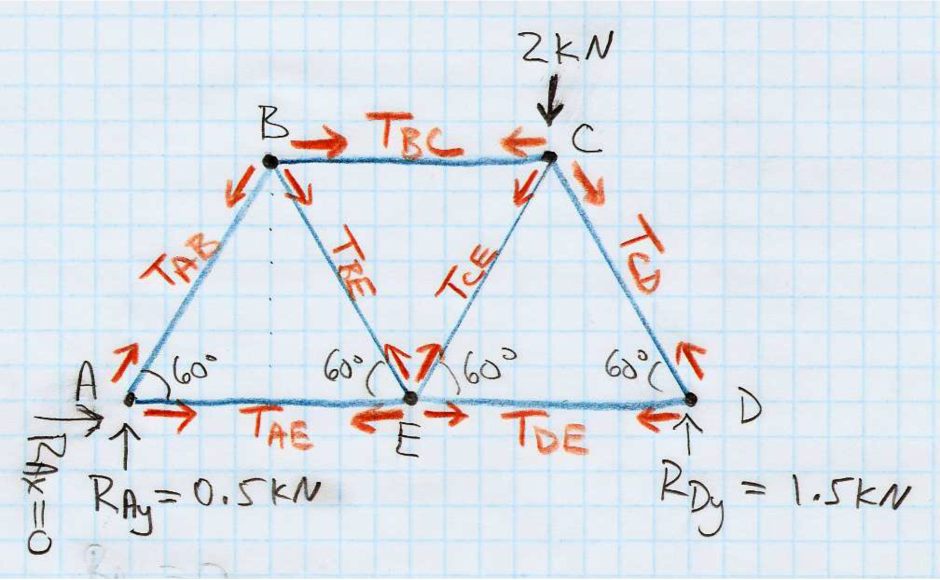
Do you see any joint at which there are ≤ 2 unknown forces? If so, we can start there. If not, we need to start with an EFBD for the truss as a whole.



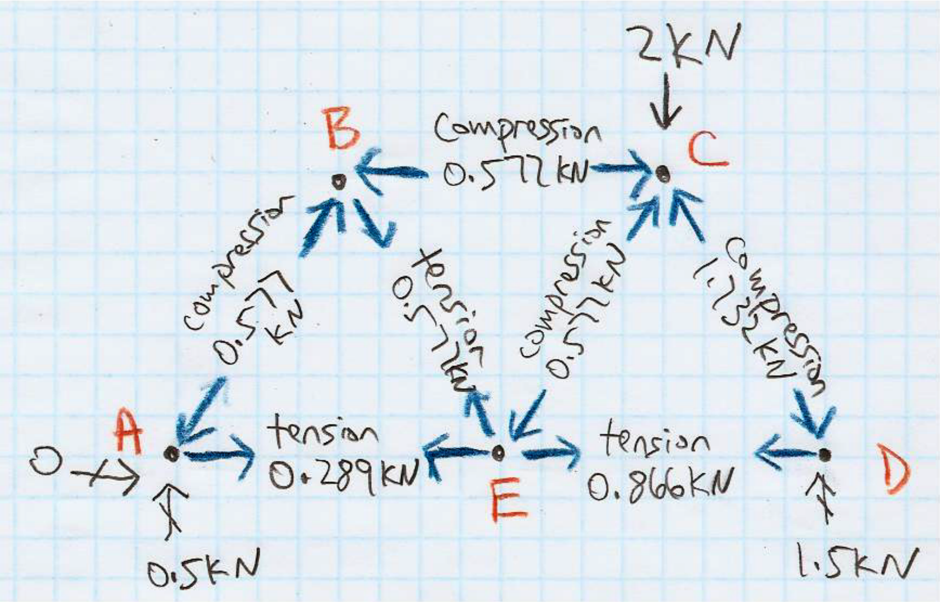
Try to guess $R_{A,x}$, $R_{A,y}$, and $R_{D,y}$ by inspection. Then let's check with the usual equations.



Now start from a joint having ≤ 2 unknown forces. In this case, I just went through the joints alphabetically. You can make your life easier by seeking out equations having just 1 unknown.



$T_{AB} = -0.577$ kN, $T_{AE} = +0.289$ kN, $T_{BE} = +0.577$ kN,
$T_{BC} = -0.577$ kN, $T_{CE} = -0.577$ kN, $T_{CD} = -1.732$ kN,
$T_{DE} = +0.866$ kN. My notation: tension > 0, compression < 0.


2KN
B
C
A
D
E
Compression
0.577kN
compression
0.577
KN
tension
0.577kN
compression
0.577kN
compression
1.732kN
tension
0.289kN
tension
0.866kN
0
0.5KN
1.5KN

$\cos 60° = 0.5, \quad \sin 60° = 0.866$

Joint A: $0 + T_{AE} + T_{AB}\cos 60° = 0$

$0.5\,kN + T_{AB}\sin 60° = 0 \Rightarrow \boxed{T_{AB} = -0.577\,kN}$

$\boxed{T_{AE} = +0.289\,kN}$

Joint B: $-T_{AB}\cos 60° + T_{BE}\cos 60° + T_{BC} = 0$

$-T_{AB}\sin 60° - T_{BE}\sin 60° = 0 \Rightarrow \boxed{T_{BE} = +0.577\,kN}$

$\boxed{T_{BC} = -0.577\,kN}$

Joint C: $-T_{BC} - T_{CE}\cos 60° + T_{CD}\cos 60° = 0 \Rightarrow T_{CD} = T_{CE} - 1.155\,kN$

(could do C,D, or E at this point)

$-2\,kN - T_{CE}\sin 60° - T_{CD}\sin 60° = 0$

$-2\,kN - T_{CE}\sin 60° - (T_{CE} - 1.155\,kN)\sin 60° = 0$

$2T_{CE}\sin 60° = -2\,kN + 1.0\,kN = -1\,kN$

$\boxed{T_{CE} = -0.577\,kN}$

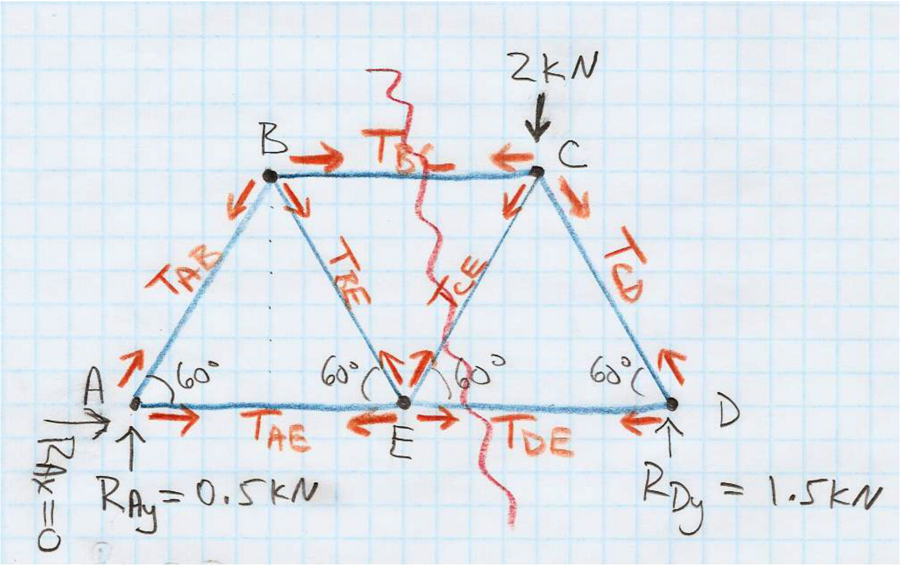
$T_{CD} = T_{CE} - 1.155\,kN = \boxed{-1.732\,kN = T_{CD}}$

Joint D: $-T_{DE} - T_{CD}\cos 60° = 0 \Rightarrow \boxed{T_{DE} = +0.866\,kN}$

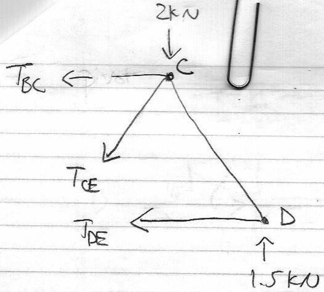
(check): $1.5\,kN + T_{CD}\sin 60° = 0$ ✓

(check Joint E): $-T_{AE} + T_{DE} - T_{BE}\cos 60° + T_{CE}\cos 60° = 0$ ✓

$+T_{BE}\sin 60° + T_{CE}\sin 60° = 0$ ✓



Let's try drawing an EFBD for the **right** side of the cut ("section").



$$\sum F_x = 0 \Rightarrow -T_{BC} - T_{CE}\cos 60^\circ - T_{DE} = 0$$

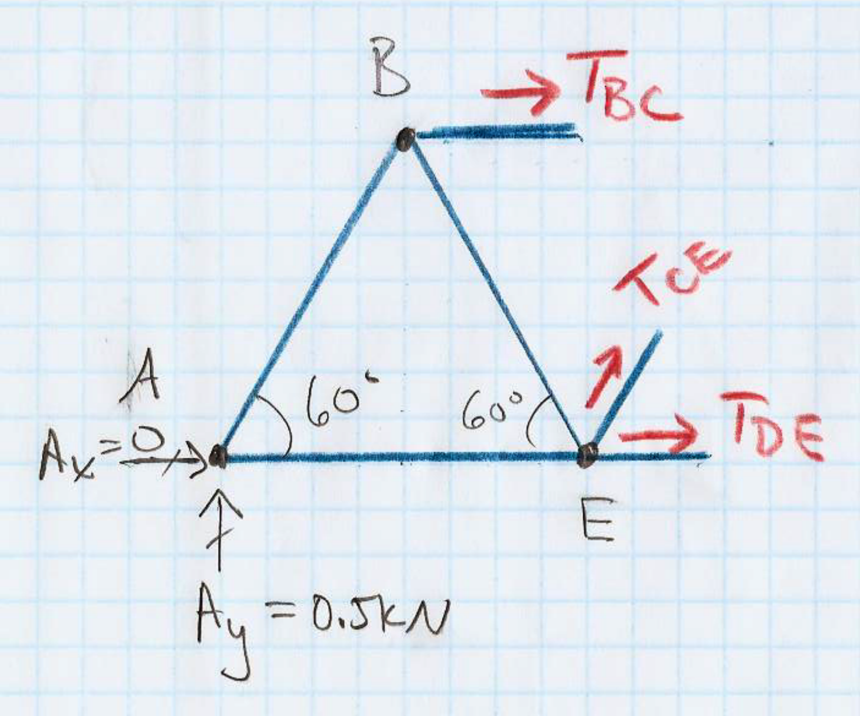
$$\sum F_y = 0 \Rightarrow -2\,\mathrm{kN} + 1.5\,\mathrm{kN} - T_{CE}\sin 60^\circ = 0$$

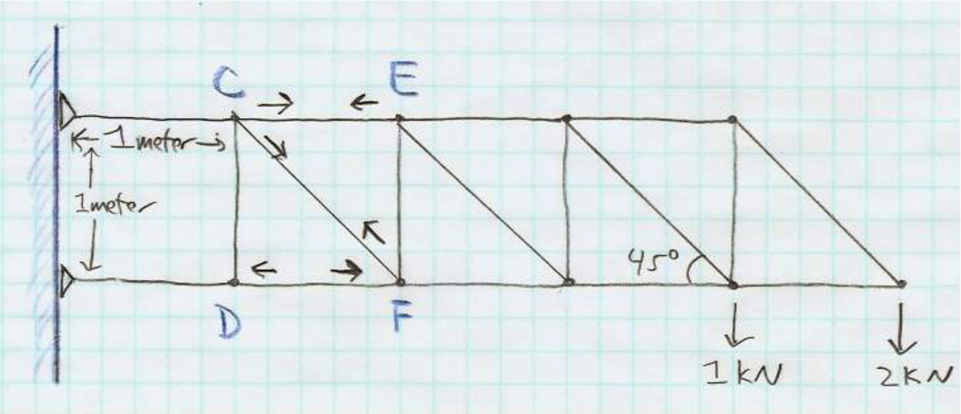
$$\sum M_c = 0 \Rightarrow +(1.5\,\mathrm{kN})\left(\frac{L}{2}\right) - T_{DE}\, L \sin 60^\circ = 0$$

$$T_{DE} = \frac{1.5\,\mathrm{kN}}{2\sin 60^\circ} = +0.866\,\mathrm{kN}$$

$$T_{CE} = \frac{-0.5\,\mathrm{kN}}{\sin 60^\circ} = -0.577\,\mathrm{kN}$$

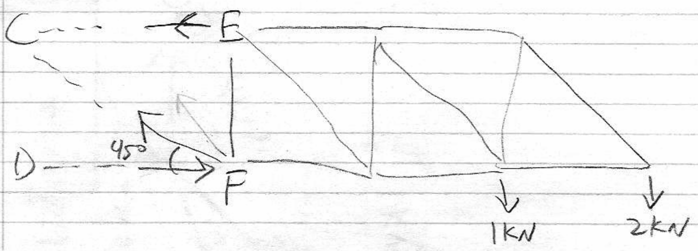
$$T_{BC} = -(T_{CE}\cos 60^\circ + T_{DE}) = -0.577$$


B
T_{BC}
T_{CE}
T_{DE}
A
60°
60°
$A_x = 0$
E
$A_y = 0.5kN$



Here's another truss problem that you can solve using the Method of Sections. Find forces in members **CE**, **CF**, and **DF**, with assumed force directions as shown.

- What happens if an assumed force direction is backwards?
- Where should we "section" the truss?
- Then what do we do next?



$$\sum F_x = 0 = -CE - CF\cos 45^\circ + DF = 0$$

$$\sum F_y = 0 = CF\sin 45^\circ = 3kN \Rightarrow \boxed{CF = 3kN\sqrt{2}}$$

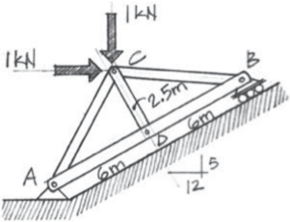
$$\sum M_{DF} = 0 = CE(1) - (1kN)(2) - (2kN)(3)$$

$$\Rightarrow \boxed{CE = 8kN}$$

$$DF = CE + CF\cos 45^\circ = 8kN + 3kN$$

$$\boxed{DF = 11kN}$$

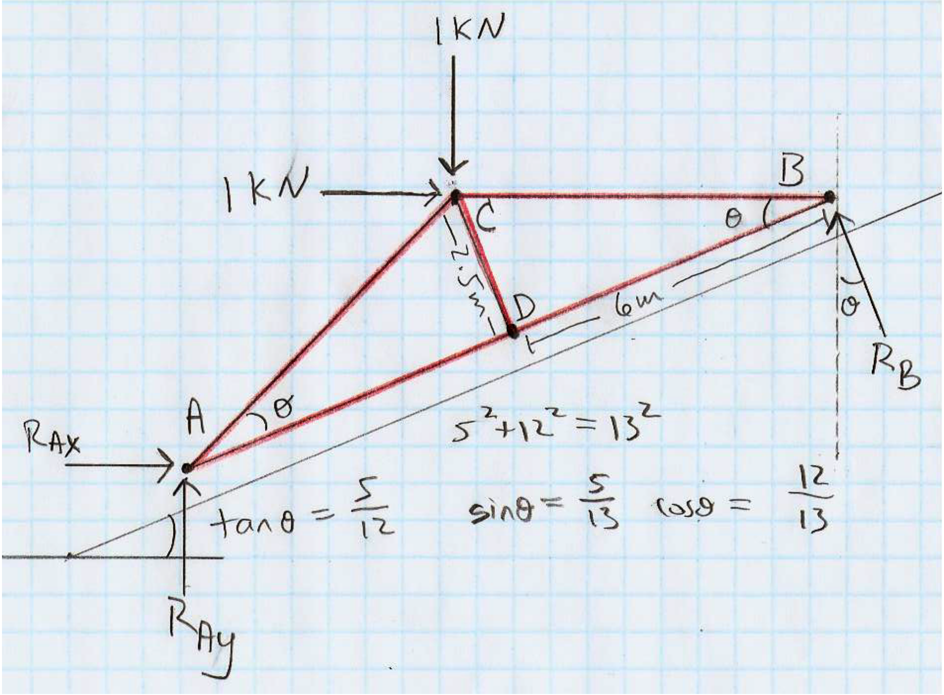
2.38 An inclined king-post truss supports a vertical and horizontal force at *C*. Determine the support reactions developed at *A* and *B*.



This is not really a "truss problem," since we're not asked to solve for the internal forces in the truss, but it is an example of a pretty tricky equilibrium problem.

Let's try working through this together in class.

Notice, from the given dimensions: the angle of the incline is the same as the interior angle at joints A and B of the truss. Also notice pin/hinge support at A and roller support at B.



$$\tan\theta = \frac{5}{12} \qquad \sin\theta = \frac{5}{13} \qquad \cos\theta = \frac{12}{13}$$

① $0 = \sum_{\text{on truss}} F_x = R_{Ax} + 1\,kN - R_B \sin\theta$

$$\Rightarrow R_{Ax} = R_B \sin\theta - 1\,kN$$

② $0 = \sum_{\substack{\text{on}\\ \text{truss}}} F_y = R_{Ay} - 1\,kN + R_B \cos\theta$

$$\Rightarrow R_{Ay} = 1\,kN - R_B \cos\theta$$

③ $0 = \sum_{\circlearrowleft} M_A = R_B(12m) - 1\,kN(12m \sin\theta) - 1\,kN(6.5m \cos 2\theta)$

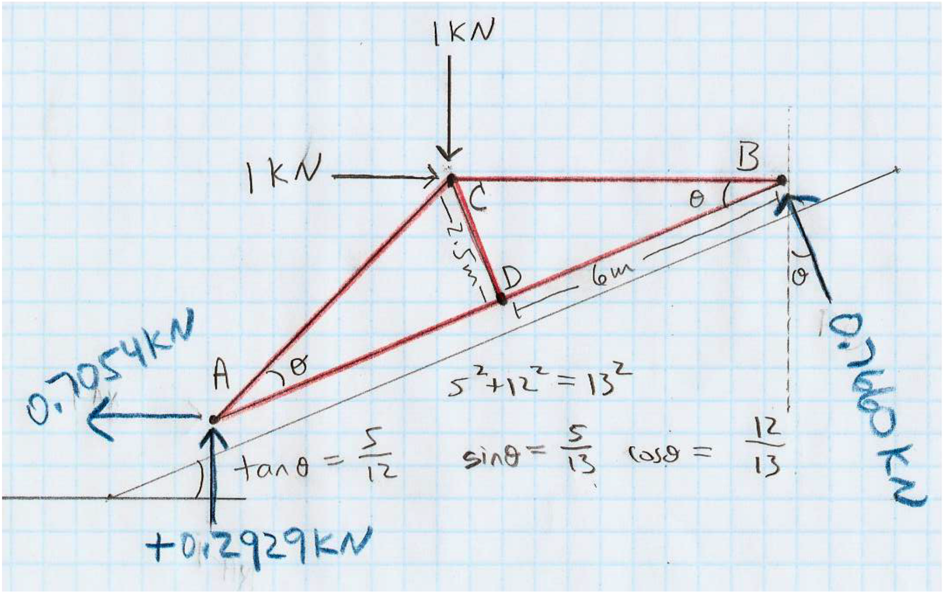
$$\Rightarrow R_B = 1\,kN(\sin\theta) + \frac{6.5}{12}\,kN(\cos 2\theta) = 0.7660\,kN$$

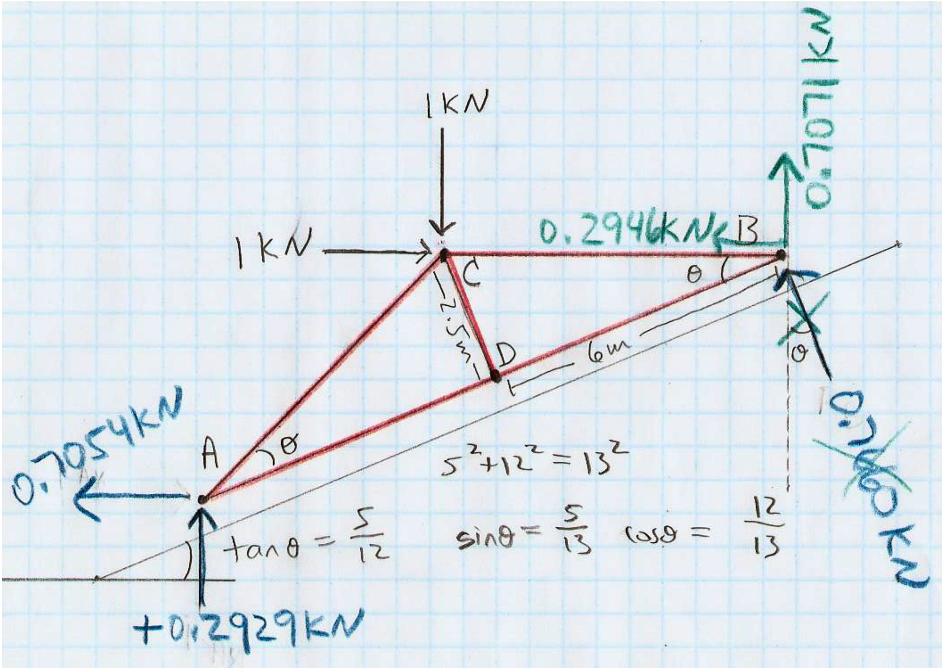
$$(R_B)_x = -R_B \sin\theta = -0.2946\,kN$$

$$(R_B)_y = R_B \cos\theta = 0.7071\,kN$$

$$R_{Ax} = (0.2946 - 1)\,kN = -0.7054\,kN$$

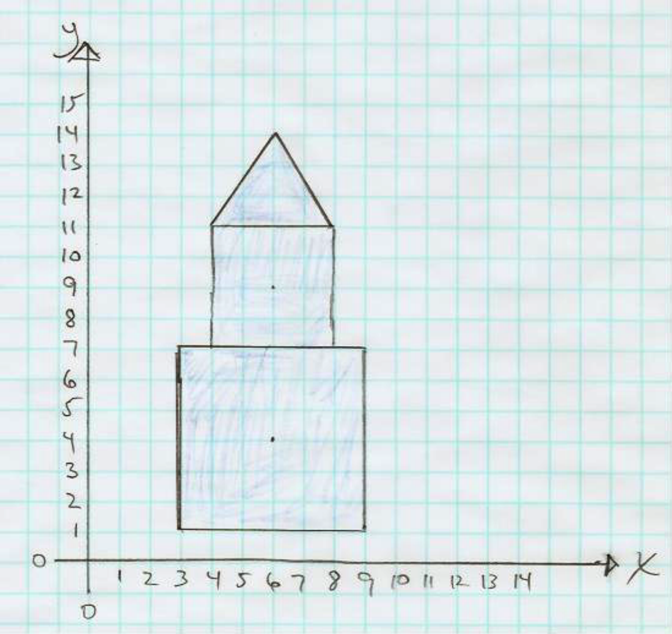
$$R_{Ay} = (1 - 0.7071)\,kN = 0.2929\,kN$$


1 KN
1 KN
B
C
θ
2.5m
D
6m
θ
A
θ
0.7054 KN
0.760 KN
+0.2929 KN
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$


1KN
1KN
0.2946kN
B
0.7071kN
C
2.5m
D
6m
θ
A
0.7054kN
+0.2929kN
5²+12²=13²
tanθ = 5/12
sinθ = 5/13
cosθ = 12/13

- The idea of computing centroids of simple and composite shapes is very, very briefly introduced in O/K ch3 (in the context of "distributed loads"), and will be discussed in much more detail in O/K ch6 (for Monday).
- Let's go through one example using rectangles and triangles. It will help you in cases when you need to solve for the "reaction forces" on a beam that carries distributed loads. (Example coming up next.)

What is X_{centroid} for the shaded area?



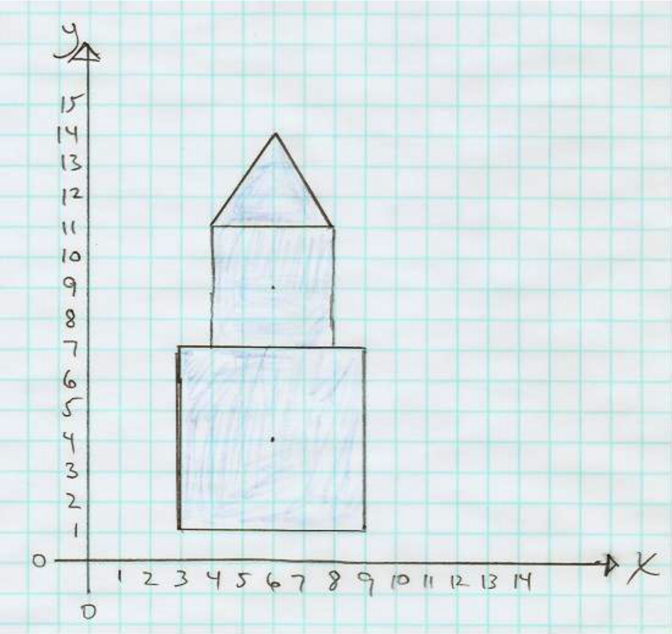
(A) 0

(B) 3

(C) 6

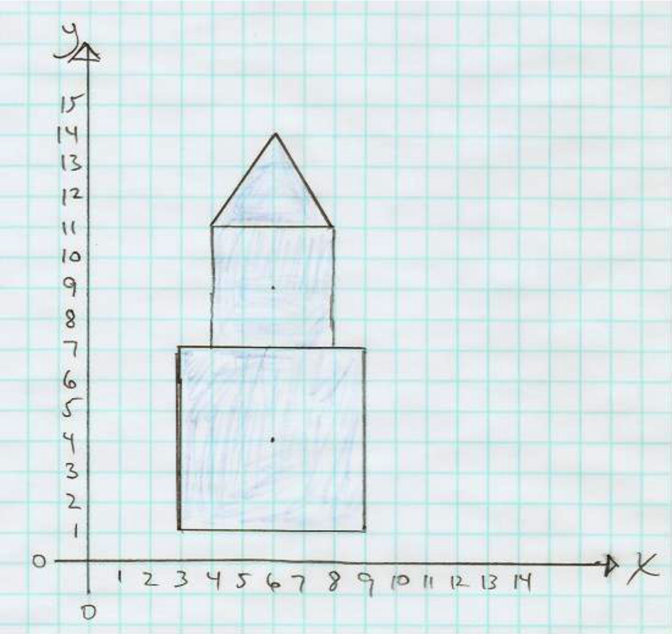
(D) 9

What are the areas of the three individual polygons?



(A) 36, 16, 16
(B) 36, 16, 12
(C) 36, 16, 8
(D) 36, 16, 6

What are the Y_{centroid} values of the three individual polygons?



(A) 4, 9, 11

(B) 4, 9, 11.667

(C) 4, 9, 12

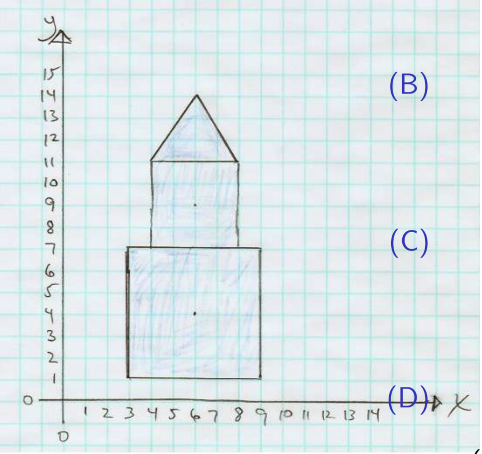
(D) 4, 9, 12.333

(E) 4, 9, 12.5

(F) 4, 9, 13

(G) 4, 9, 14

What is Y_{centroid} for the whole shaded area?



(A)

$$\frac{4+9+12}{3} = 8.33$$

(B)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{36+16+6} = 6.21$$

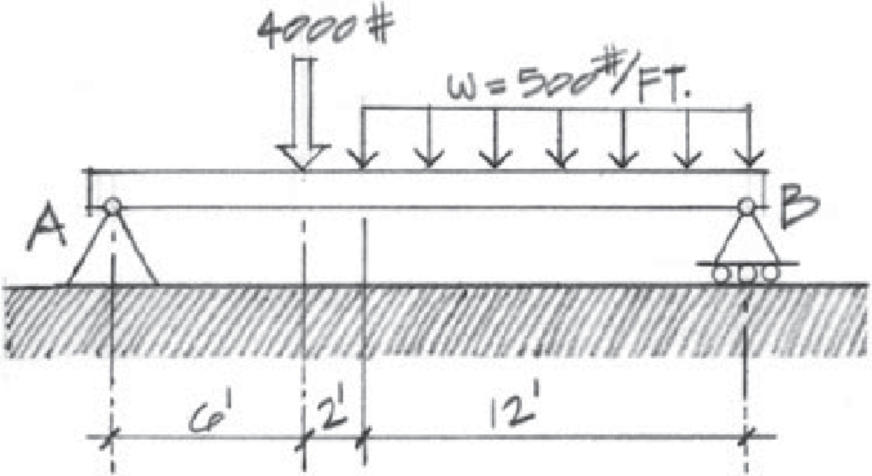
(C)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{4+9+12} = 14.4$$

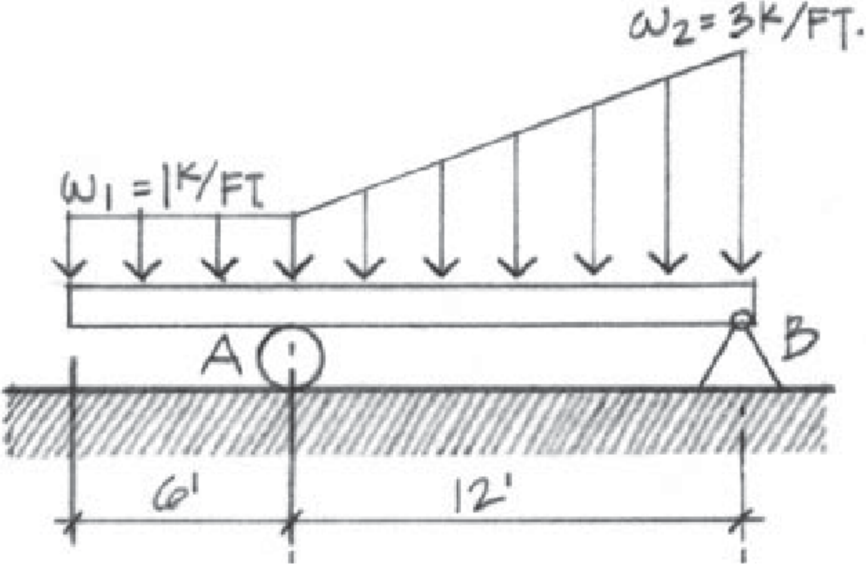
(D)

$$\frac{(4^2)(36)+(9^2)(16)+(12^2)(6)}{36+16+6} = 47.2$$

There's one problem similar to this (but using metric units) on HW(?): Determine the support reactions at A and B.



This one is harder than your homework, because the distributed load is non-uniform: Determine the support reactions at A and B.



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- Turn in HW9. Pick up HW10 handout in back corner.
- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. Next week, you'll read Ch6 (cross-sectional properties) and Ch7 (simple beams).