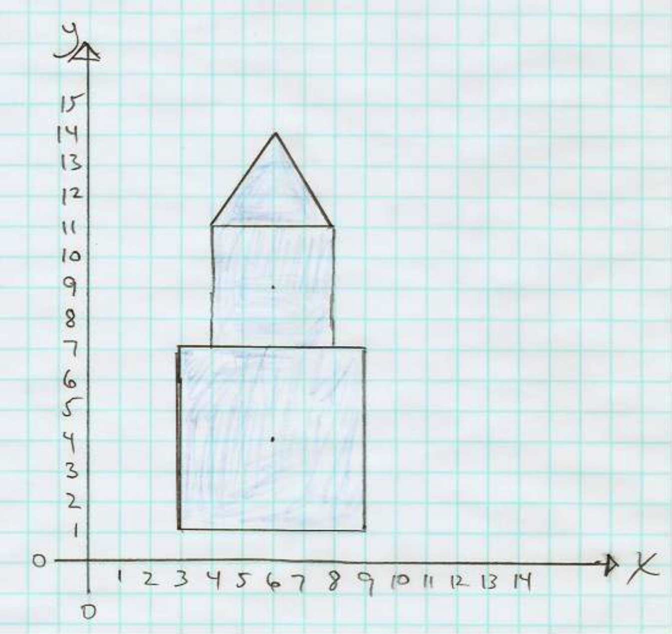
Physics 8 — Friday, November 15, 2019

- Turn in HW10. Pick up HW11 handout. HW11 is "due" next Friday, but you can turn it in on Monday, Nov 25, just in case it takes us an extra day to get through the material on beams.
- This week, you read Ch6 (cross-sectional properties) and Ch7 (simple beams). Next week, you'll read Ch8 (more details on beams).

- The idea of computing centroids of simple and composite shapes is very, very briefly introduced in O/K ch3 (in the context of “distributed loads”), and is discussed in much more detail in O/K ch6 (you read this week, for Monday).
- Let’s go through one example using rectangles and triangles. It will help you in cases when you need to solve for the “reaction forces” on a beam that carries distributed loads. (Example coming up next.)

What is X_{centroid} for the shaded area?



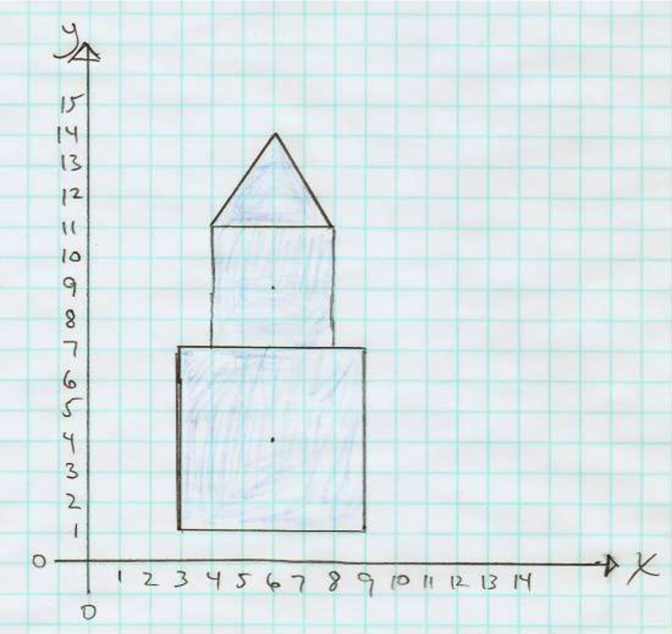
(A) 0

(B) 3

(C) 6

(D) 9

What are the areas of the three individual polygons?



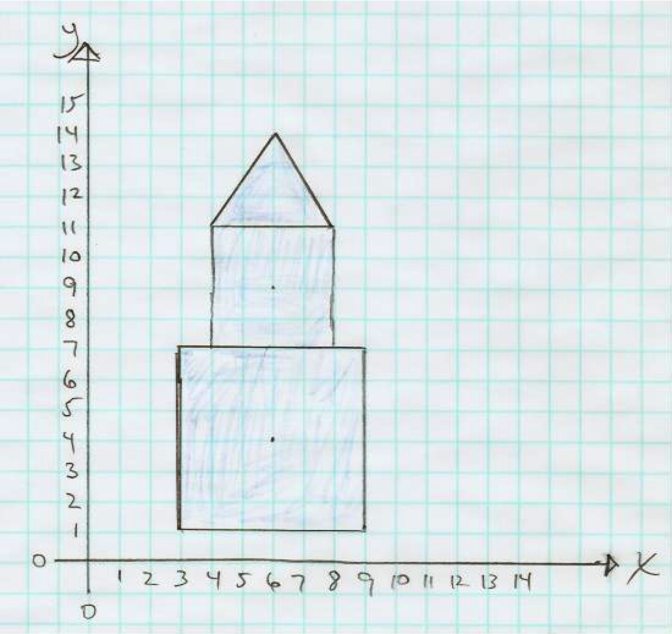
(A) 36, 16, 16

(B) 36, 16, 12

(C) 36, 16, 8

(D) 36, 16, 6

What are the Y_{centroid} values of the three individual polygons?



(A) 4, 9, 11

(B) 4, 9, 11.667

(C) 4, 9, 12

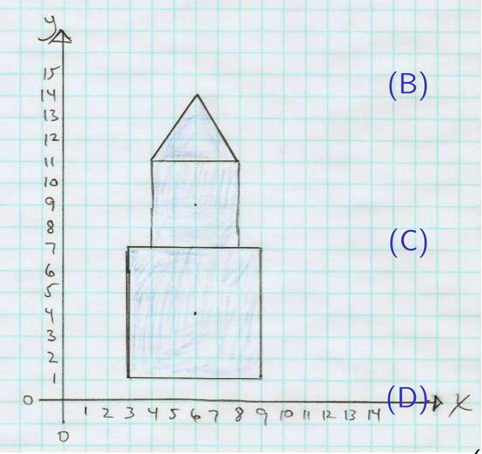
(D) 4, 9, 12.333

(E) 4, 9, 12.5

(F) 4, 9, 13

(G) 4, 9, 14

What is Y_{centroid} for the whole shaded area?



(A)

$$\frac{4+9+12}{3} = 8.33$$

(B)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{36+16+6} = 6.21$$

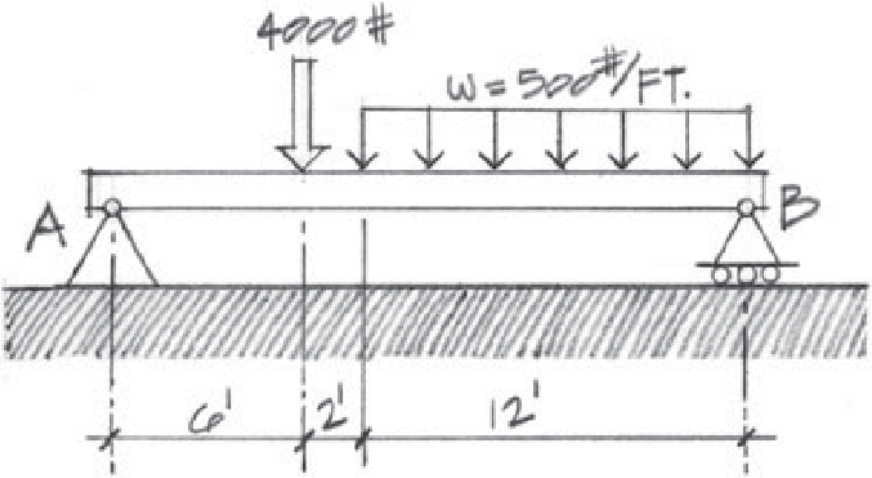
(C)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{4+9+12} = 14.4$$

(D)

$$\frac{(4^2)(36)+(9^2)(16)+(12^2)(6)}{36+16+6} = 47.2$$

There was one problem similar to this (but using metric units) on HW10: Determine the support reactions at A and B.



```
ClearAll["Global`*"];
load1Force = 4000.0 pound;
load1X = 6.0 foot;  Measure positions w.r.t. support A;
load2Force = (500.0 pound / foot) * (12 foot)
```

```
6000. pound
```

```
Find centroid of distributed load, for equivalent concentrated load;
load2X = (6.0 + 2.0 + 12.0 / 2) foot
```

```
14. foot
```

```
Evaluate moments about pivot A;
Solve[0 == By (6.0 + 2.0 + 12.0) foot - load1Force * load1X -
   load2Force * load2X, By]
```

```
{{By → 5400. pound}}
```

```
By = By /. First[%]
```

```
5400. pound
```

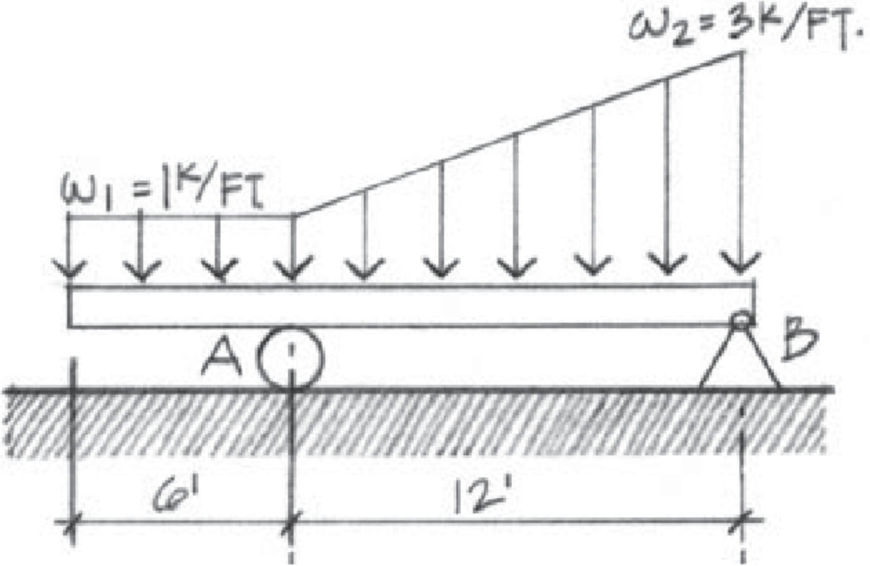
```
Solve[0 == Ay + By - load1Force - load2Force, Ay]
```

```
{{Ay → 4600. pound}}
```

```
Solve[0 == Ax]
```

```
{{Ax → 0}}
```

This one is harder than your homework, because the distributed load is non-uniform: Determine the support reactions at A and B.



```
ClearAll["Global`*"];
foot = Quantity[1.0, "foot"];
pound = Quantity[1.0, "pound"];
Rectangle for uniform 1k/foot load spanning entire beam.;
load1X = 0.5 (6 foot + 12 foot)
```

```
9. ft
```

```
load1Force = (1000 pound / foot) * (18.0 foot)
```

```
18 000. lb
```

```
Trianglular load that sits above uniform load.;
load2X = 18.0 foot - (12.0 foot) / 3
```

```
14. ft
```

```
load2Force = 0.5 * (12.0 foot) * (2000 pound / foot)
```

```
12 000. lb
```

```
Moments about B;
L = 18.0 foot;
Solve[
 0 == load1Force * (L - load1X) + load2Force * (L - load2X) -
   Ay * (12 foot), Ay]
```

```
{{Ay → 17 500. lb}}
```

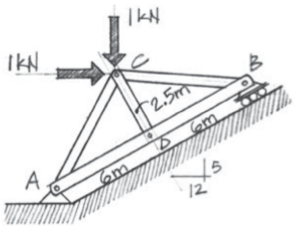
```
Ay = Ay /. First[%]
```

```
17 500. lb
```

```
Solve[0 == Ay + By - load1Force - load2Force, By]
```

```
{{By → 12 500. lb}}
```

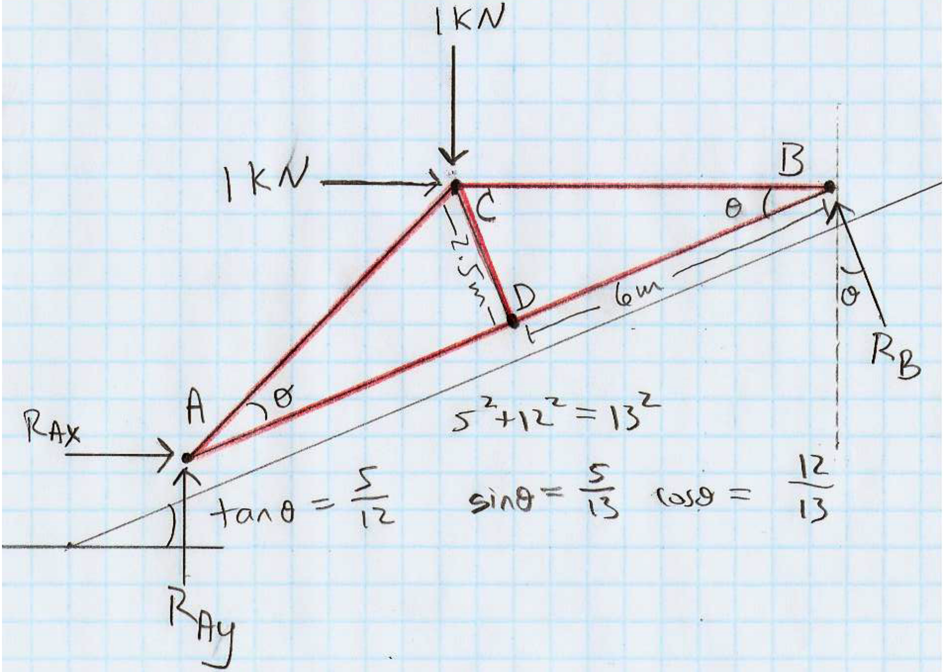
2.38 An inclined king-post truss supports a vertical and horizontal force at *C*. Determine the support reactions developed at *A* and *B*.



This is not really a "truss problem," since we're not asked to solve for the internal forces in the truss, but it is an example of a pretty tricky equilibrium problem.

Let's try working through this together in class. (I think it's deviously tricky!)

Notice, from the given dimensions: the angle of the incline is the same as the interior angle at joints A and B of the truss. Also notice pin/hinge support at A and roller support at B.



$$5^2+12^2=13^2$$

$$\tan\theta = \frac{5}{12} \qquad \sin\theta = \frac{5}{13} \qquad \cos\theta = \frac{12}{13}$$

$$① \quad 0 = \sum_{\text{on truss}} F_x = R_{Ax} + 1\,kN - R_B \sin\theta$$

$$\Rightarrow R_{Ax} = R_B \sin\theta - 1\,kN$$

$$② \quad 0 = \sum_{\text{on truss}} F_y = R_{Ay} - 1\,kN + R_B \cos\theta$$

$$\Rightarrow R_{Ay} = 1\,kN - R_B \cos\theta$$

$$③ \quad 0 = \sum_{\circlearrowleft} M_A = R_B(12m) - 1\,kN(12m \sin\theta) - 1\,kN(6.5m \cos 2\theta)$$

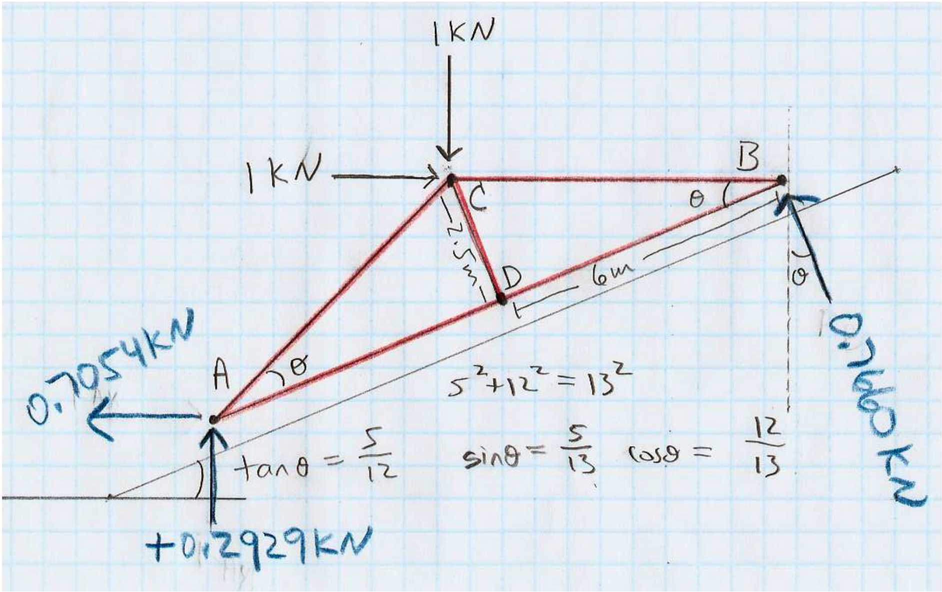
$$\Rightarrow R_B = 1\,kN(\sin\theta) + \frac{6.5}{12}\,kN(\cos 2\theta) = 0.7660\,kN$$

$$(R_B)_x = -R_B \sin\theta = -0.2946\,kN$$

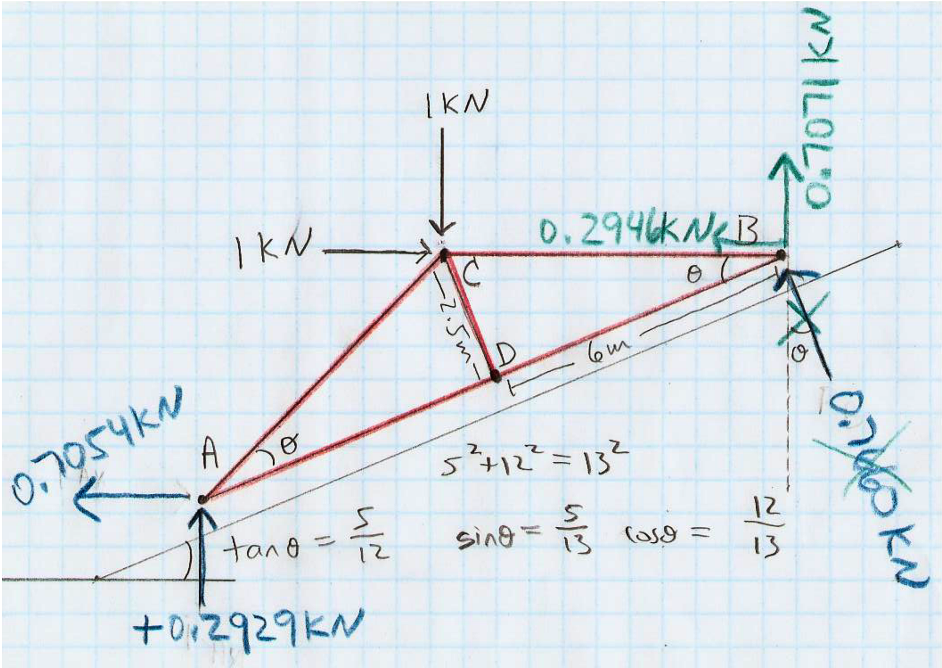
$$(R_B)_y = R_B \cos\theta = 0.7071\,kN$$

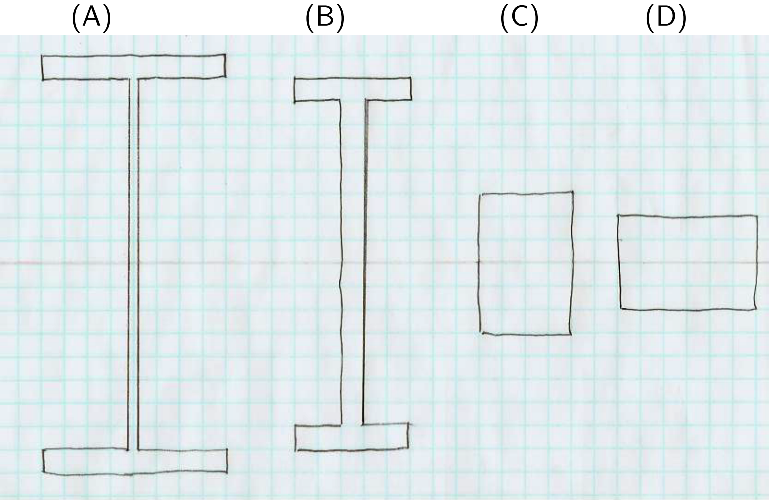
$$R_{Ax} = (0.2946 - 1)\,kN = -0.7054\,kN$$

$$R_{Ay} = (1 - 0.7071)\,kN = 0.2929\,kN$$

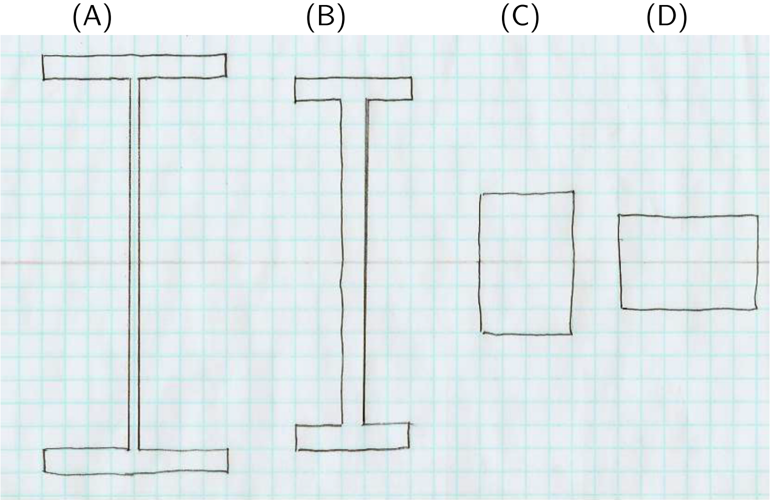


1KN
1KN
B
C
θ
2.5m
D
6m
θ
0.7054KN
A
θ
0.7660 KN
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
+0.2929KN

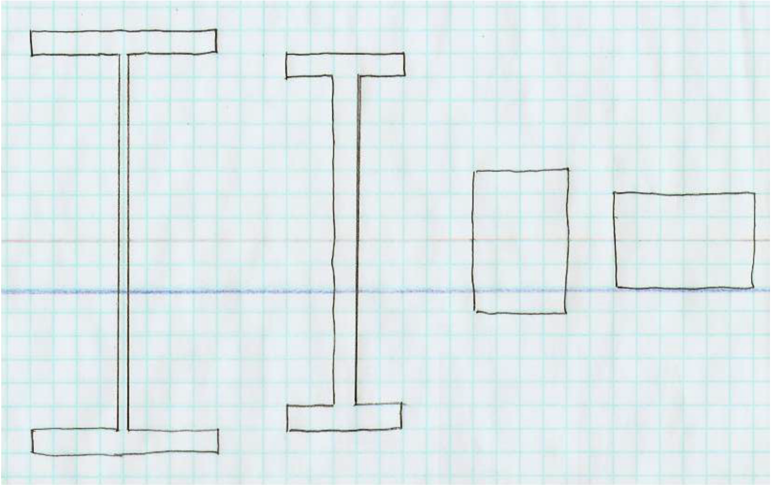

1KN
1KN
0.2946kN
B
0.7071 kN
C
θ
2.5m
D
6m
θ
A
θ
0.7054kN
5²+12²=13²
tanθ = 5/12
sinθ = 5/13
cosθ = 12/13
+0.2929kN



Each shape has the same area: 24 squares. Which shape has the largest $I_x = \int y^2 \mathrm{d}A$ ("second moment of area about the x-axis"), with $y = 0$ given by the faint horizontal red line at the center?

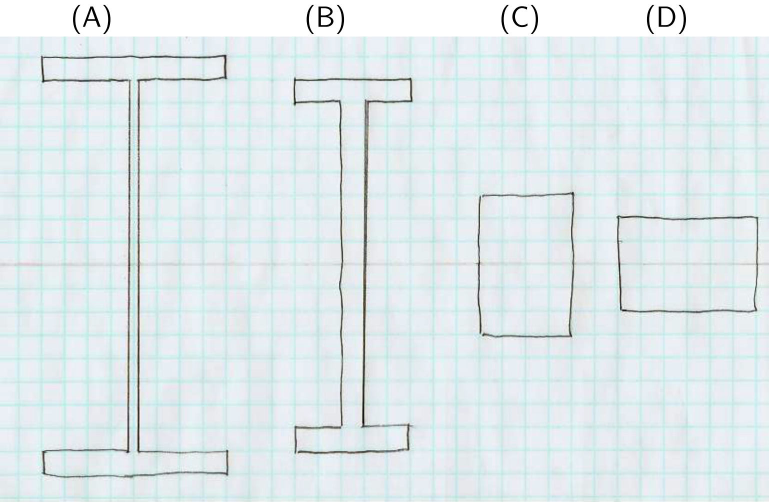


Each shape has the same area: 24 squares. Which shape has the **smallest** $I_x = \int y^2 \mathrm{d}A$ ("second moment of area about the x-axis"), with $y = 0$ given by the faint horizontal red line at the center?



If you moved the x-axis down by a couple of grid units, what would happen to $I_x = \int y^2 \mathrm{d}A$ for each shape? Would I_x change? Would I_x change by the same amount for each shape?

(Think: "parallel-axis theorem.")



Given that $I_x = \int y^2 \mathrm{d}A = \frac{1}{12} b h^3$ for a rectangle centered at $y = 0$, let's use the parallel-axis theorem to calculate I_x for shapes A, B, C, and D. For definiteness, let each graph-paper box be $1\ \mathrm{cm} \times 1\ \mathrm{cm}$. So the units will be cm^4.

Let's do the two rectangular shapes first, since they're quick.

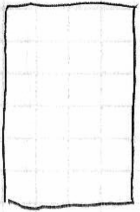
Then, the trick for the non-rectangular shapes is to use (from O/K §6.3) the "parallel-axis theorem:"

$$I_x = \sum I_{xc} + \sum A\, d_y^2$$

where each sum is over the simple shapes that compose the big shape.

- I_{xc} is the simple shape's own I_x value about its own centroid (which is $bh^3/12$ for a rectangle),
- A is the simple shape's area, and
- d_y is the vertical displacement of the simple shape's centroid from $y = 0$ (which should be the centroid of the big shape).

(C)



$b = 4\,cm$ $h =$

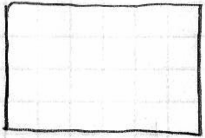
$h = 6\,cm$

$A = 24\,cm^2$

$y_c = 0$

$\frac{1}{12} b h^3 = \boxed{72\,cm^4}$

(D)



$b = 6\,cm$

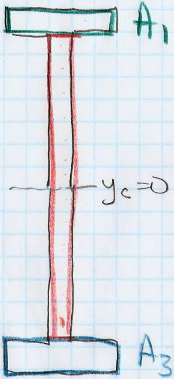
$h = 4\,cm$

$A = 24\,cm^2$

$y_c = 0$

$\frac{1}{12} b h^3 = \boxed{32\,cm^4}$

(B)



$A_1 = 5\,cm^2 \quad b_1 = 5\,cm \quad h_1 = 1\,cm$

$y_{c_1} = +7.5\,cm$

$A_2 = 14\,cm^2 \quad b_2 = 1\,cm$

$h_2 = 14\,cm$

$y_{c_2} = 0$

$A_3 = 5\,cm^2 \quad b_3 = 5\,cm \quad h_3 = 1\,cm$

$y_{c_3} = -7.5\,cm$

$\frac{1}{12} b_1 h_1^3 = 0.417\,cm^4$

$A_1 y_{c_1}^2 = 281.25\,cm^4$

$\frac{1}{12} b_2 h_2^3 = 228.67\,cm^4$

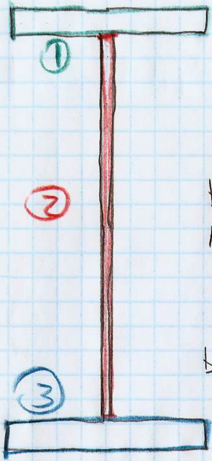
$A_2 y_{c_2}^2 = 0$

$\frac{1}{12} b_3 h_3^3 = 0.417\,cm^4$

$A_3 y_{c_3}^2 = 281.25\,cm^4$

$$I_B = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 + A_1 y_{c_1}^2 + A_2 y_{c_2}^2 + A_3 y_{c_3}^2 = \boxed{792\,cm^4}$$

(A)



$b_1 = 8\,\text{cm} \quad h_1 = 1\,\text{cm}$

$A_1 = 8\,\text{cm}^2 \quad y_{c_1} = +8.5\,\text{cm}$

$\frac{1}{12} b_1 h_1^3 = 0.67\,\text{cm}^4$

$A_1 y_{c_1}^2 = 578\,\text{cm}^4$

$b_2 = 0.5\,\text{cm} \quad h_2 = 16\,\text{cm}$

$A_2 = 8\,\text{cm}^2 \quad y_{c_2} = 0$

$\frac{1}{12} b_2 h_2^3 = 170.67\,\text{cm}^4$

$A_2 y_{c_2}^2 = 0$

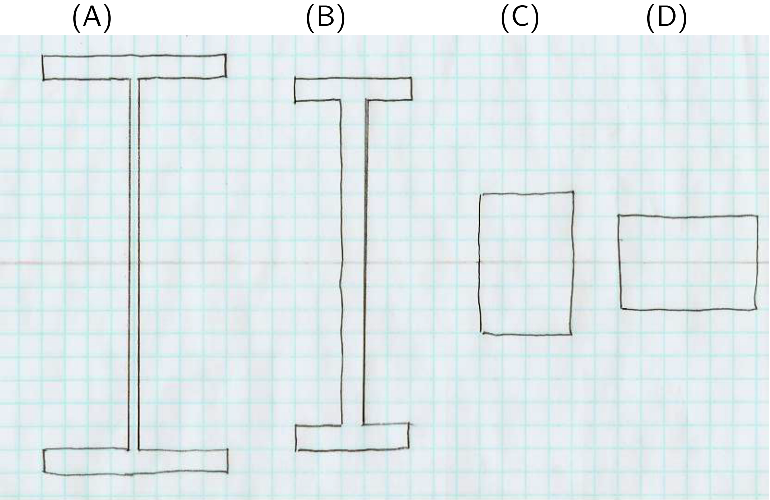
$b_3 = 8\,\text{cm} \quad h_3 = 1\,\text{cm}$

$A_3 = 8\,\text{cm}^2 \quad y_{c_3} = -8.5\,\text{cm}$

$\frac{1}{12} b_3 h_3^3 = 0.67\,\text{cm}^4$

$A_3 y_{c_3}^2 = 578\,\text{cm}^4$

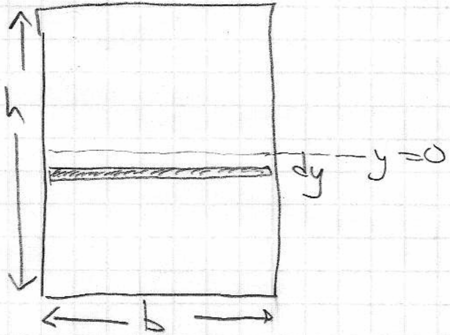
$$I_A = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 + A_1 y_{c_1}^2 + A_2 y_{c_2}^2 + A_3 y_{c_3}^2 = \boxed{1328\,\text{cm}^4}$$



Each shape has same area $A = 24\ \mathrm{cm}^2$, but "second moment of area" is $I_A = 1328\ \mathrm{cm}^4$, $I_B = 792\ \mathrm{cm}^4$, $I_C = 72\ \mathrm{cm}^4$, $I_D = 32\ \mathrm{cm}^4$. That's the motivation for the "I" shape of an I-beam: to get a large "second moment of area," $I = \int y^2\,\mathrm{d}A$. The deflection of a beam under load is inversely proportional to I.

rectangle

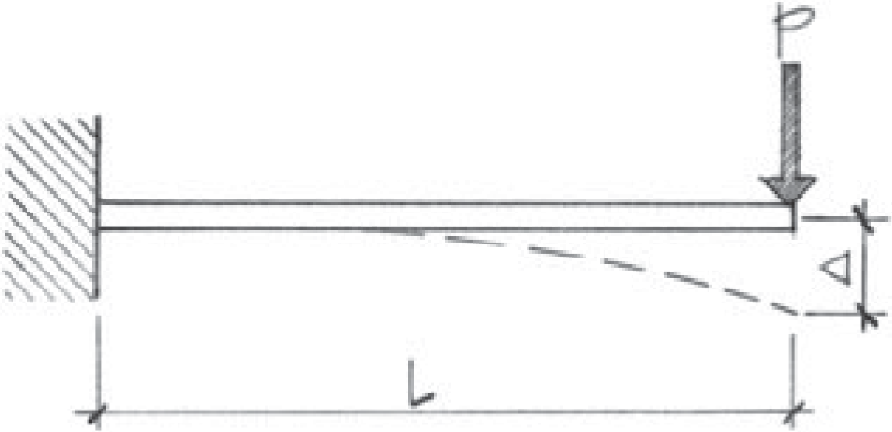
$$dA = b\,dy$$



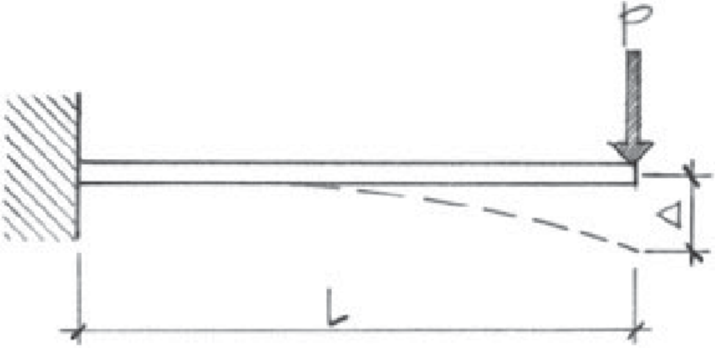
$$I = \int y^2\,dA = \int_{y=-\frac{h}{2}}^{y=h/2} y^2\, b\,dy = \left[\frac{b y^3}{3}\right]_{y=-\frac{h}{2}}^{h/2}$$

$$= \frac{b(h/2)^3}{3} - \frac{b(-h/2)^3}{3} = \frac{bh^3}{12}$$

We can use the Method of Sections to study the internal forces and torques ("moments") within a beam. Consider this cantilever beam (whose own weight we neglect here) supporting a concentrated "load" force P at the far end. The left half is what holds up the right half. What force and torque ("moment") does the left half exert on the right half? Does the answer depend on where we "section" the beam?



We draw "load diagram" (basically a FBD for the beam), then the "shear (V) diagram" below that, then the "moment (M) diagram" below that. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



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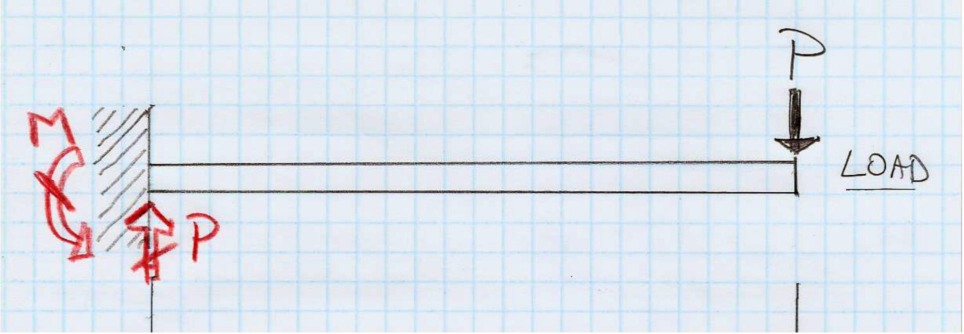

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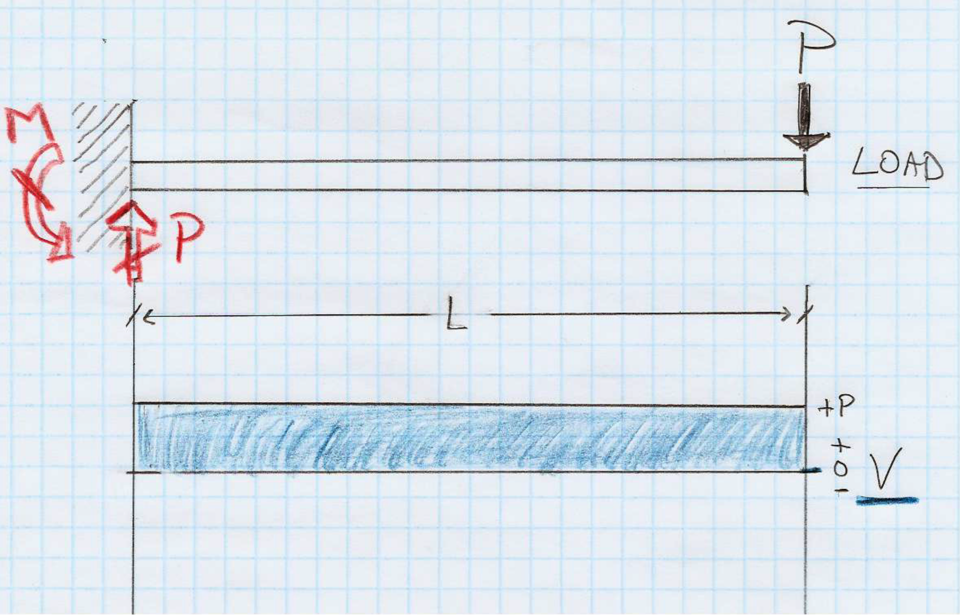
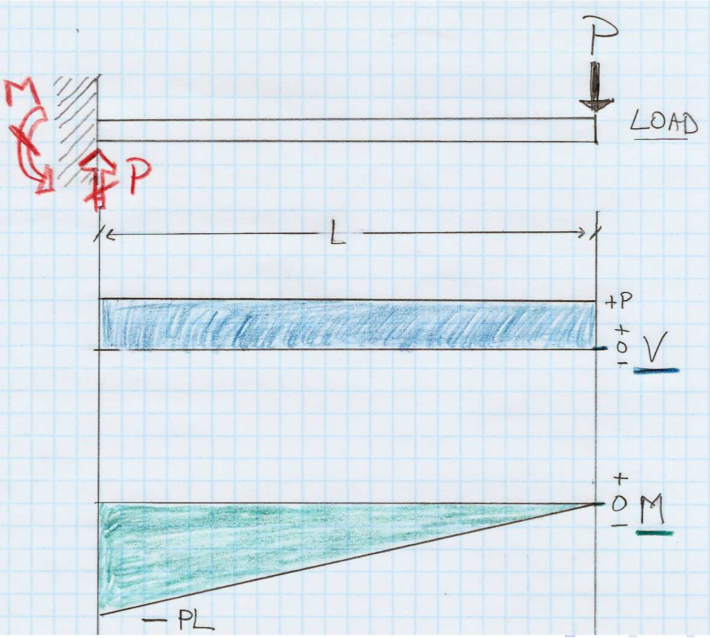
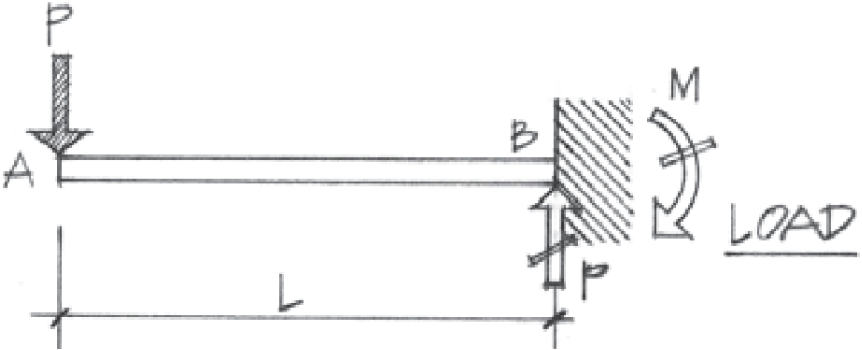


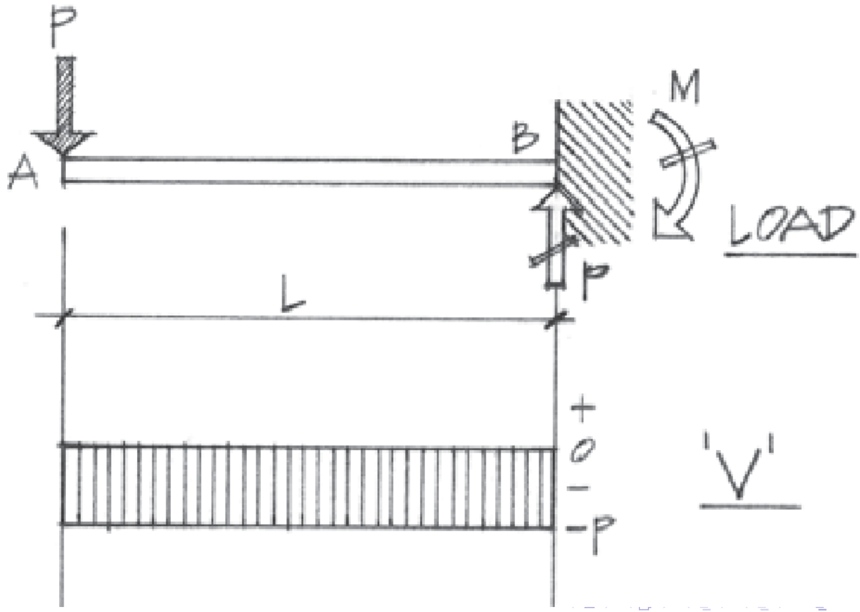
Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Let's try a mirror image of the same cantilever beam. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



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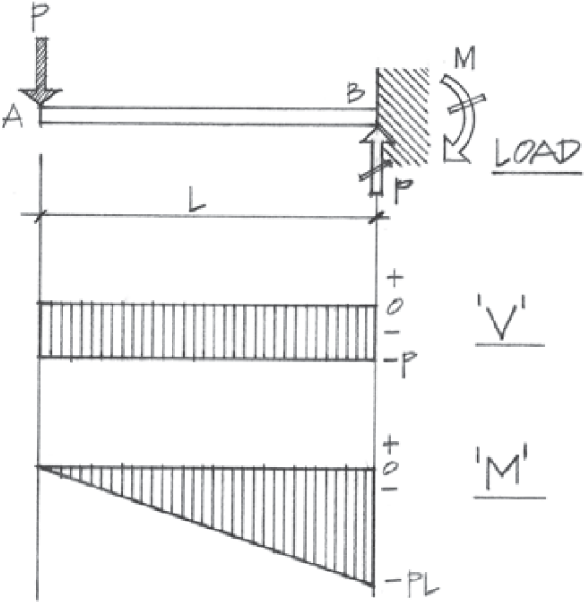

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