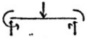
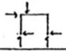
Physics 8 — Monday, November 25, 2019

- Turn in HW11.
- Today is our last day on beams. Then we'll spend a week on oscillation / vibration / periodic motion.
- I'll put the PDF of the take-home practice exam online before Thanksgiving. I intend for it to be similar in coverage to the in-class final exam (Dec 12, noon, A1), though the in-class exam will be shorter than the take-home. If you turn it in on Friday, Dec 6, then I will email it back to you after class on Monday, Dec 9. If you turn it in on Monday, Dec 9, then I will give it back to you at the Wednesday (Dec 11) review session.
- I plan to do a review session on Dec 11, time/place TBD.
- Prof. Farley plans to join us today. Stop me before it gets too late!
- Wednesday, you don't have to show up, but if you do, you'll get a bonus point. I will spend the hour introducing "Python Mode for Processing" (`py.processing.org`) — a visual-arts-oriented approach to writing code to do drawing and animation. Last time we coded a simplified "breakout" video game. This year we may code a highly simplified "asteroids" video game, if all goes well.

ELEMENTS OF STRUCTURE

	1D in 1D	Beam
	1D in 1D	Column
	1D in 2D	Frame
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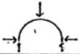
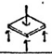
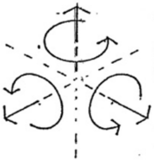
	1D in 2D	Arch
	2D in 2D	Slab Plate
	2D in 2D	Wall
	1D in 3D	Spaceframe
	2D in 3D	Vault Shell
	2D in 3D	Dome Membrane
	3D in 3D	Foundations Soils

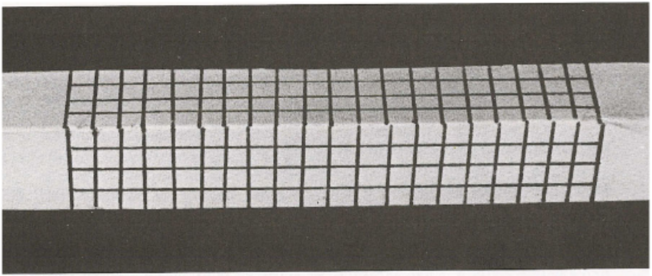
DEGREES OF FREEDOM



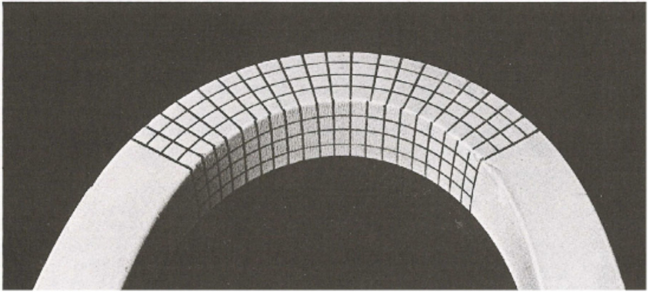
IDEAS OF STRUCTURE

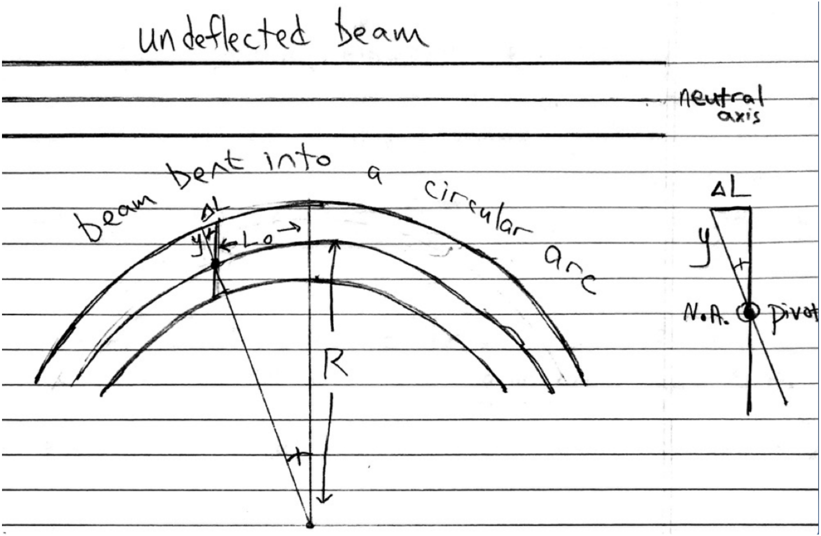
- Force
- Restraint
- Equilibrium
- Stress
- Deformation
- Elasticity
- Geometry
- Strength
- Stiffness
- Scale
- Continuity
- Stability
- Safety

rubber beam with grid lines subjected to pure bending.



(a) Before Loading





By similar triangles, $\frac{\Delta L}{y} = \frac{L_0}{R} \quad \Rightarrow \quad$ strain $e = \frac{\Delta L}{L_0} = \frac{y}{R}$

Hooke's law: $f = eE \quad \Rightarrow \quad \boxed{f = \frac{Ey}{R}}$ (1)

f = stress = force per unit area. E = Young's modulus.

Imagine a fiber running along the length of the bent beam. Let the fiber have cross-section area $\mathrm{d}A$ and height y above the neutral surface. The tension (force) in the fiber is

$$\mathrm{d}F \;=\; f\,\mathrm{d}A \;=\; \frac{E}{R}\,y\,\mathrm{d}A$$

Pivoting about the neutral axis, the moment (torque) exerted by this fiber is (since y is the lever arm from the pivot)

$$\mathrm{d}M \;=\; y\,\mathrm{d}F \;=\; \frac{E}{R}\,y^2\,\mathrm{d}A$$

To find the total bending moment exerted by this cross-section of beam, we add up all of the fibers over the entire cross-section:

$$M \;=\; \frac{E}{R}\int y^2\,\mathrm{d}A \;=\; \frac{EI}{R} \qquad \text{where} \quad \boxed{I \;=\; \int y^2\,\mathrm{d}A} \quad (2)$$

- One factor of y comes from strain $\Delta L/L_0 \propto y$.
- The second factor of y is lever arm above the N.A.

So the beam's radius of curvature is $\boxed{R = \frac{EI}{M}}$ (3) (illustrate).

Combine (1) + (3) $\Rightarrow$ bending stress $f = \frac{Ey}{EI/M} = \boxed{\frac{My}{I} = f}$

The maximum bending stress is

$$f_{\max} = \frac{|M|_{\max}|y|_{\max}}{I} = \boxed{\frac{|M|_{\max}}{S} = f_{\max}}$$

where S is the "section modulus" $\boxed{S = \frac{I}{|y|_{\max}}}$

- know load & span $\rightarrow$ find $|M|_{\max}$
- know type of material $\rightarrow$ allowable $f_{\max}$

$$S_{\text{required}} \geq \frac{|M|_{\max}}{f_{\text{allowable}}}$$

tells you how "big" a beam cross-section you need for this load, span, & material, to meet the maximum-bending-stress criterion, which is a "strength" criterion (not a "stiffness" criterion).

In calculus, $\frac{1}{R}$ quantifies the "curvature" of a function $Y(x)$

$$\text{curvature} \;=\; \frac{1}{R} \;=\; \frac{Y''(x)}{[1+Y'(x)^2]^{3/2}} \;\approx\; Y''(x)$$

The curvature of a function is closely related to its second derivative $Y''(x)$. If the slope $|Y'(x)| \ll 1$, as is true for beams used in structures, then $\frac{1}{R} = Y''(x)$.

For clarity, I'll write $Y(x)$ for the shape of the deflected beam, and reserve y to denote height above the neutral surface.

$$Y''(x) \;=\; \frac{1}{R} \;=\; \frac{M(x)}{EI}$$

$$\text{slope} \;\; Y'(x) \;=\; \frac{1}{EI}\int M(x)\,\mathrm{d}x$$

$$Y(x) \;=\; \frac{1}{EI}\int \mathrm{d}x \int M(x)\,\mathrm{d}x$$

deflection under load $\Delta(x) = -Y(x)$. This is where you get $\Delta_{\max} = \frac{5wL^4}{384EI}$ for simply-supported beam with uniform load w, etc.

This Onouye/Kane figure writes "y" here for deflection, but I wrote "Y" for deflected beam shape, because we were already using y for "distance above the neutral surface."

So you integrate $M(x)/EI$ twice w.r.t. x to get the deflected beam shape $Y(x)$.

The bending moment $M(x) = EI\,\mathrm{d}^2Y/\mathrm{d}x^2$, where E is Young's modulus and I is second moment of area.

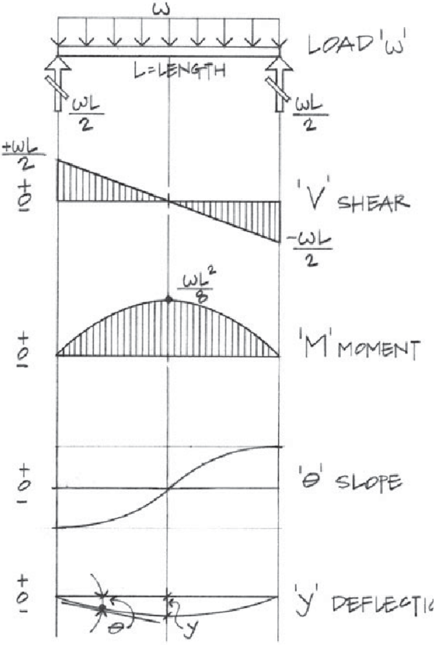
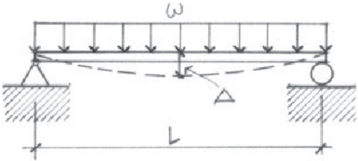
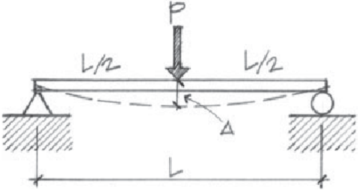
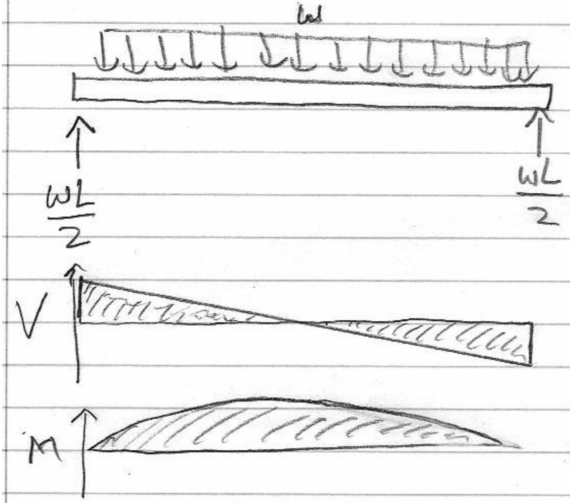


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

The most common deflection results can be found in tables.

Beam Load and Support	**Actual Deflection***
 (a) Uniform load, simple span	$\Delta_{max} = \dfrac{5\omega L^4}{384EI}$ (at the centerline)
 (b) Concentrated load at midspan	$\Delta_{max} = \dfrac{PL^3}{48EI}$ (at the centerline)

FYI, here's where that crazy $(5wL^4)/(384EI)$ comes from!



$$V(x) = \frac{wL}{2} - wx$$
$$= w\left(\frac{L}{2} - x\right)$$

$$M(x) = \frac{wLx}{2} - \frac{wx^2}{2}$$
$$= \frac{w}{2}\left(Lx - x^2\right)$$

(continued on next page)

Here's where that crazy $(5wL^4)/(384EI)$ comes from!

$$\Delta(x) = -\frac{1}{EI}\int dx \int M(x)\,dx$$

$$= -\frac{w}{2EI}\int dx \int (Lx - x^2)\,dx = -\frac{w}{2EI}\int dx \left(\frac{Lx^2}{2} - \frac{x^3}{3} + C_1\right)$$

$$= -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} + C_1 x + C_2\right)$$

$$\Delta(0) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$\Delta(L) = 0 \Rightarrow \frac{L^4}{6} - \frac{L^4}{12} + C_1 L = 0 \Rightarrow \boxed{C_1 = -\frac{L^3}{12}}$$

$$\Delta(x) = -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} - \frac{L^3 x}{12}\right)$$

$$\Delta_{max} = \Delta\left(\frac{L}{2}\right) = -\frac{w}{2EI}\left(\frac{L(L/2)^3}{6} - \frac{(L/2)^4}{12} - \frac{L^3(L/2)}{12}\right)$$

$$= -\frac{wL^4}{2EI}\left(\frac{1}{48} - \frac{1}{192} - \frac{1}{24}\right) = -\frac{wL^4}{2EI}\left(-\frac{5}{192}\right) = \boxed{\frac{5wL^4}{384EI}}$$

The 2 integration constants can be tricky. Simply supported: $\Delta(0) = \Delta(L) = 0$. (For cantilever, $\Delta(0) = \Delta'(0) = 0$ instead.)

Maximum deflection is one of several beam-design criteria. $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For uniform load w on simply-supported beam, you get

$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them. But I had great fun calculating the 5/384 myself!

Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

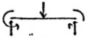
- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Notice that putting a column in the middle of a long, uniformly loaded beam reduces $\Delta_{\max}$ by a factor of $2^4 = 16$. Alternatively, if you want to span a large, open space without intermediate columns or bearing walls, you need beams with large I.

Beam criteria:

- Normal stress in the extreme fibers of the beam (farthest from neutral surface) must be smaller than the allowable bending stress, F_b, which depends on the material (wood, steel, etc.).
- This happens where $M(x)$ has largest magnitude.
- Shear stress (in both y ("transverse") and x ("longitudinal") must be smaller than the allowable shear stress, F_v, which is also a property of the material (wood, steel, etc.).
- This happens where $V(x)$ has largest magnitude, and (surprisingly) is largest near the neutral surface.
- The above two are "strength" criteria. The third one is a "stiffness" criterion:
- The maximum deflection under load must satisfy the building code: typically $\Delta y_{\max} < L/360$.
- For a uniform load, this happens farthest away from the supports. If deflection is too large, plaster ceilings develop cracks, floors feel uncomfortably bouncy or sloped.
- The book also notes buckling as a beam failure mode.

ELEMENTS OF STRUCTURE

	1D in 1D	Beam
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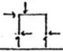
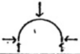
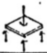
	1D in 2D	Frame
	1D in 2D	Truss
	1D in 2D	Arch
	2D in 2D	Slab Plate
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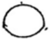
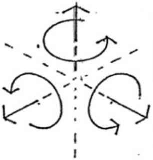
	2D in 3D	Dome Membrane
	3D in 3D	Foundations Soils

DEGREES OF FREEDOM



IDEAS OF STRUCTURE

- Force
- Restraint
- Equilibrium
- Stress
- Deformation
- Elasticity
- Geometry
- Strength
- Stiffness
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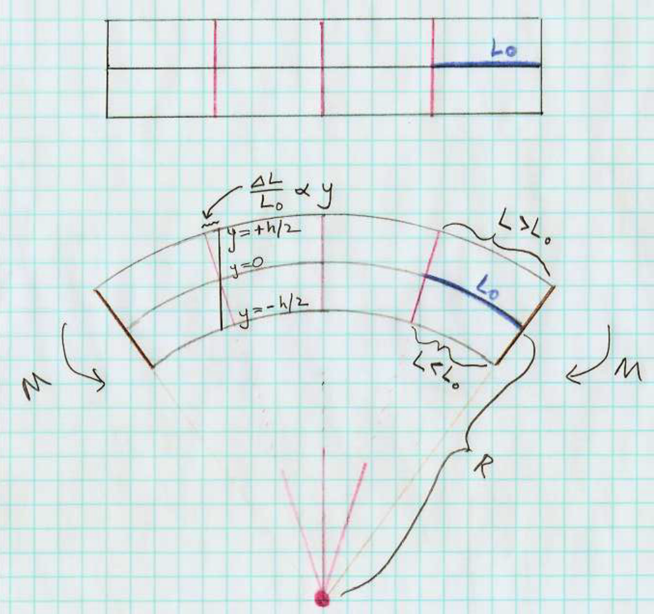
- Continuity
- Stability
- Safety

Key idea: bending moment $M \propto \frac{1}{R}$, where R is the radius of curvature of the beam. For constant M, vertical lines converge toward common center of curvature.



$$\text{strain} = \frac{\Delta L}{L_0} = \frac{y}{R}$$

where $y = 0$ is the neutral surface.

So in this case $y > 0$ is in tension and $y < 0$ is in compression.

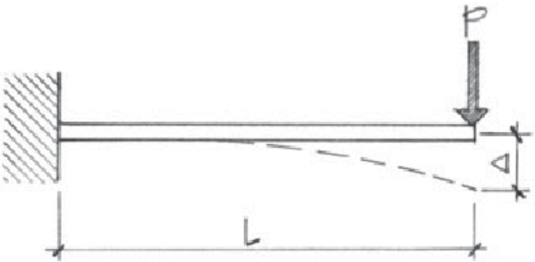
$$M = \frac{E}{R} \int y^2 \mathrm{d}A = \frac{EI}{R}$$

Meanwhile, the vertical *deflection* Δ of a point along the beam is related to its curvature by (in limit where $\Delta \ll R$)

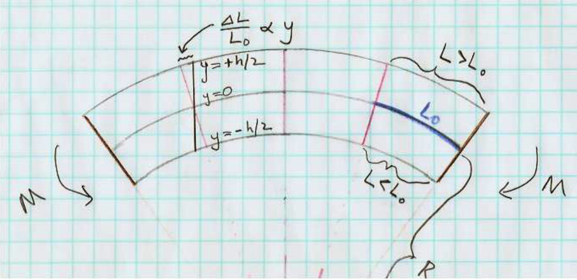
$$\frac{1}{R} \approx -\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

so you can integrate the $M(x)$ curve twice to get deflection

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2} = -\frac{M}{EI} \quad \Rightarrow \quad \Delta(x) = -\frac{1}{EI} \int \mathrm{d}x \int M(x)\, \mathrm{d}x$$



Another beam-design criterion is maximum bending stress: the fibers farthest from the neutral surface experience the largest tension or compression, hence largest bending stress.

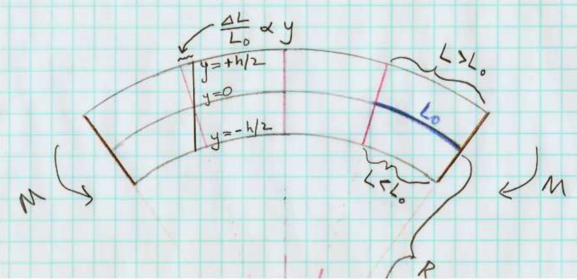


When we section the beam at x, bending moment $M(x)$ is

$$M = \frac{EI}{R}$$

which we can solve for the radius of curvature $R = EI/M$. Then the stress a distance y above the neutral surface is

$$f = Ee = E\,\frac{y}{R} = \frac{E\,y}{(EI/M)} = \frac{M\,y}{I}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\max} \equiv c$.

$$f_{\max} = \frac{M\,c}{I} = \frac{M}{(I/c)} = \frac{M}{S}$$

The ratio $S = I/c$ is called "section modulus."

Bending stress in fibers farthest from neutral surface:

$$f_{\max} = \frac{M}{(I/c)} = \frac{M}{S}$$

So you sketch your load, V, and M diagrams, and you find $M_{\max}$, i.e. the largest magnitude of $M(x)$.

Then, the material you are using for beams (wood, steel, etc.) has a maximum allowable bending stress, F_b.

So then you look in your table of beam cross-sections and choose

$$S \geq S_{\text{required}} = \frac{M_{\max}}{F_b}$$

Maximum deflection is one of several beam-design criteria. $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For uniform load w on simply-supported beam, you get

$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them. But I had great fun calculating the $5/384$ myself!

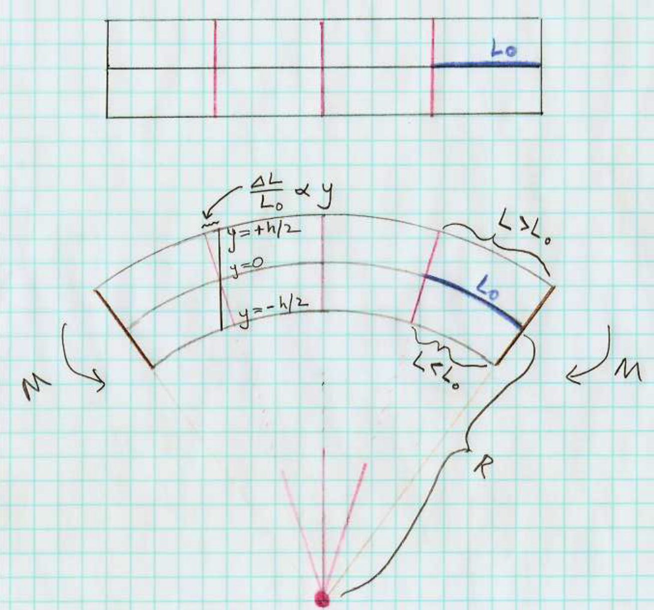
Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Notice that putting a column in the middle of a long, uniformly loaded beam reduces $\Delta_{\max}$ by a factor of $2^4 = 16$. Alternatively, if you want to span a large, open space without intermediate columns or bearing walls, you need beams with large I.

Bending beam into circular arc of radius R gives $\boxed{e = \frac{\Delta L}{L_0} = \frac{y}{R}}$,

strain e vs. distance y above the neutral surface.



Hooke's Law $\boxed{f = Ee}$

gives stress $\boxed{f = \frac{E\,y}{R}}$

Torque exerted by fibers of beam is

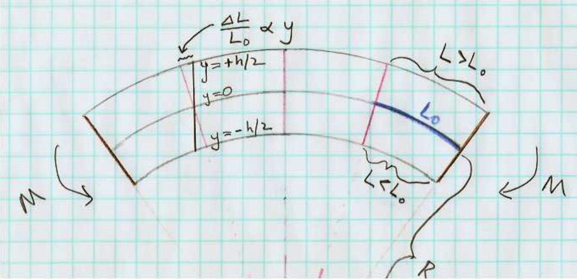
$$M = \int y\,(f\,\mathrm{d}A) =$$

$$y\,\frac{E\,y}{R}\,\mathrm{d}A = \frac{E}{R}\,y^2\,\mathrm{d}A$$

$$\boxed{M = \frac{EI}{R}}$$

Eliminate R $\Rightarrow$

$$\boxed{f = \frac{M\,y}{I} = \frac{M}{I/y}}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\max} \equiv c$.

$$f_{\max} = \frac{M_{\max}\,c}{I} = \frac{M_{\max}}{(I/c)} = \frac{M_{\max}}{S}$$

The ratio $S = I/c$ is called "section modulus." The load diagram gives you $M_{\max}$. Each material (wood, steel, etc.) has allowed bending stress $f_{\max}$. Then $S_{\min}$ tells you how big a beam you need.

(The next few slides contain beam-design examples. Let's skip ahead to slide 35)

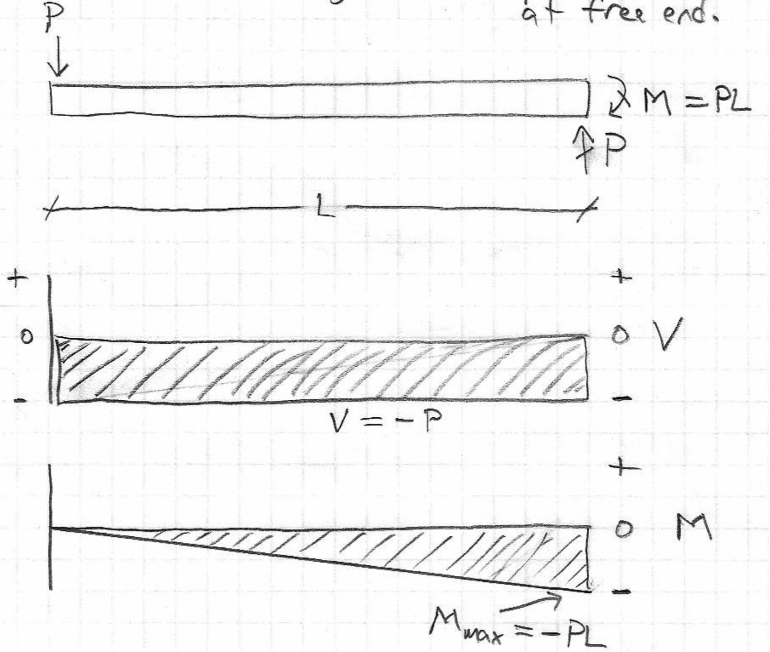
Example (using metric units!): A cantilever beam has a span of 3.0 m with a single concentrated load of 100 kg at its unsupported end. If the beam is made of timber having allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$ (was 1600 psi in US units), what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width = 1.5 inch = 0.038 m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\mathrm{max}} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\mathrm{max}} = PL^3/(3EI)$ for a cantilever with concentrated load at end. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.

Cantilever of length L with point load P at free end.



$$S_{\text{min}} = \frac{|M_{\text{max}}|}{f_{\text{allowed}}} = \frac{PL}{F_b} = \frac{(980\,\text{N})(3\,\text{m})}{1.1 \times 10^7\,\text{N/m}^2} = 26.7 \times 10^{-5}\,\text{m}^3$$

$$\Delta_{\text{allowed}} = \frac{L}{240} = \frac{3.0\,\text{m}}{240} = 0.0125\ \text{m}$$

$$\Delta_{\text{max}} = \frac{PL^3}{3EI} \quad \Rightarrow \quad I_{\text{min}} = \frac{PL^3}{3E\Delta_{\text{allowed}}} = 64.2 \times 10^{-6}\,\text{m}^4$$

I worked out b, h, I, and $S = I/c$ values in metric units for standard "2×" dimensional lumber.

	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	.038 m	3.5 in	.089 m	$2.23 \times 10^{-6}\ \mathrm{m}^4$	$5.02 \times 10^{-5}\ \mathrm{m}^3$
2×6	1.5 in	.038 m	5.5 in	.140 m	$8.66 \times 10^{-6}\ \mathrm{m}^4$	$12.4 \times 10^{-5}\ \mathrm{m}^3$
2×8	1.5 in	.038 m	7.5 in	.191 m	$21.9 \times 10^{-6}\ \mathrm{m}^4$	$23.0 \times 10^{-5}\ \mathrm{m}^3$
2×10	1.5 in	.038 m	9.5 in	.241 m	$44.6 \times 10^{-6}\ \mathrm{m}^4$	$37.0 \times 10^{-5}\ \mathrm{m}^3$
2×12	1.5 in	.038 m	11.5 in	.292 m	$79.1 \times 10^{-6}\ \mathrm{m}^4$	$54.2 \times 10^{-5}\ \mathrm{m}^3$

The numbers are nicer if you use centimeters instead of meters, but then you have the added hassle of remembering to convert back to meters in calculations.

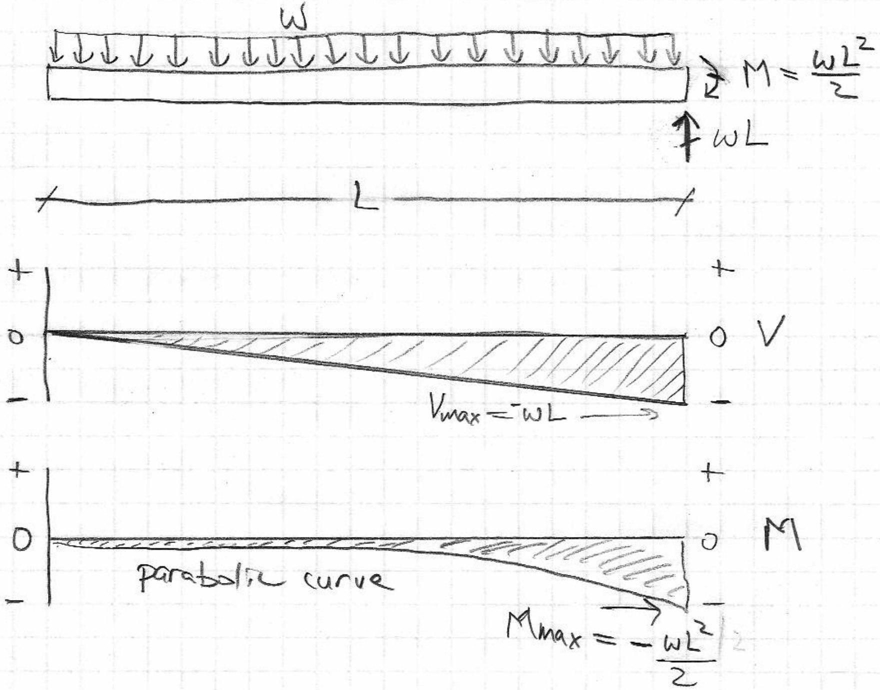
	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	3.8 cm	3.5 in	8.9 cm	$223\ \mathrm{cm}^4$	$50.2\ \mathrm{cm}^3$
2×6	1.5 in	3.8 cm	5.5 in	14.0 cm	$866\ \mathrm{cm}^4$	$124\ \mathrm{cm}^3$
2×8	1.5 in	3.8 cm	7.5 in	19.1 cm	$2195\ \mathrm{cm}^4$	$230\ \mathrm{cm}^3$
2×10	1.5 in	3.8 cm	9.5 in	24.1 cm	$4461\ \mathrm{cm}^4$	$370\ \mathrm{cm}^3$
2×12	1.5 in	3.8 cm	11.5 in	29.2 cm	$7913\ \mathrm{cm}^4$	$542\ \mathrm{cm}^3$

Minor variation on same problem: A cantilever beam has a span of $3.0\ \mathrm{m}$ with a uniform distributed load of $33.3\ \mathrm{kg/m}$ along its entire length. If we use timber with allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$, what minimum section modulus is required?

What is the smallest "$2\times$" dimensional lumber (width $= 1.5$ inch $= 0.038$ m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\mathrm{max}} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "$2\times$" cross-section is needed instead?

$\Delta_{\mathrm{max}} = wL^4/(8EI)$ for a cantilever with uniform load. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.



w
M = wL^2/2
wL
L
+
0
−
V
Vmax = −wL
M
parabolic curve
Mmax = −wL^2/2

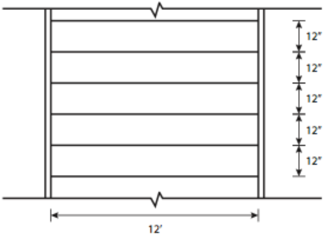
$$S_{\min} = \frac{|M_{\max}|}{f_{\text{allowed}}} = \frac{wL^2/2}{F_b} = \frac{(326\,\mathrm{N/m})(3\,\mathrm{m})^2/2}{1.1 \times 10^7\,\mathrm{N/m^2}} = 13.3 \times 10^{-5}\,\mathrm{m}^3$$

$$\Delta_{\text{allowed}} = \frac{L}{240} = \frac{3.0\,\mathrm{m}}{240} = 0.0125\ \mathrm{m}$$

$$\Delta_{\max} = \frac{wL^4}{8EI} \quad \Rightarrow \quad I_{\min} = \frac{wL^4}{8E\Delta_{\text{allowed}}} = 24.0 \times 10^{-6}\,\mathrm{m}^4$$

2) Size a wood joist for a row house floor which spans 12 feet. Joists are spaced at 16 inches on center.

f = 1,300 psi
f = 85 psi
E = 1.7 x 10 psi
LL = 60 psf
DL = 30 psf



Plan View

Hint: remember that a "2 x 4" wood joist is only nominal; its true dimensions are "1.5 x 3.5" inches. (4 = 1.5, 6 = 5.5, 8 = 7.25, 10 = 9.25 inches)

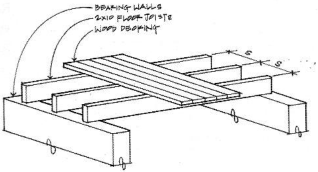
(Here's a homework problem from ARCH 435.)

Actually, Home Depot's 2×10 really is 9.5 inches deep, not 9.25 inches, and 2×12 really is 11.5 inches deep.

A timber floor system uses joists made of "2×10" dimensional lumber. Each joist spans a length of 4.27 m (simply supported). The floor carries a load of 2400 $\mathrm{N/m^2}$. At what spacing should the joists be placed, in order not to exceed allowable bending stress $F_b = 10000\ \mathrm{kN/m^2}$ ($1.0 \times 10^7\ \mathrm{N/m^2}$)?

(We should get an answer around 24 inches $=$ 0.61 meters.)

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



Solution:

Based on the allowable stress criteria:

$$f = \frac{Mc}{I} = \frac{M}{S}$$

$$M_{max} = S \times f_b = (21.4 \text{ in.}^3)(1.45 \text{ k/in.}^3) = 31 \text{ k-in.}$$

$$M = \frac{31 \text{ k-in.}}{12 \text{ in./ft.}} = 2.58 \text{ k-ft.}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$

Substituting for M obtained previously,

$$\omega = \frac{8(2.58 \text{ k-ft.})}{(14')^2} = 0.105 \text{ k/ft.} = 105 \text{ \#/ft.}$$

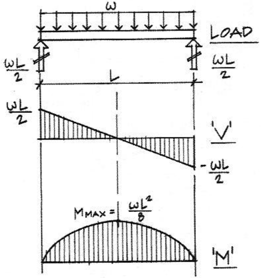
But

$$\omega = \text{\#/ft.}^2 \times \text{tributary width (joist spacing } s)$$

$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

$$s = 25'' \text{ spacing}$$

Use 24″ o.c. spacing.



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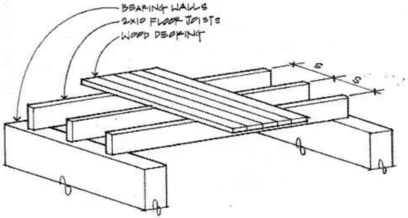

Solution:

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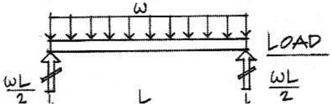
$$M = \frac{31 \text{ k-in.}}{12 \text{ in./ft.}} = 2.58 \text{ k-ft.}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$



Therefore,

$$\omega = \frac{8M}{L^2}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$

Therefore,

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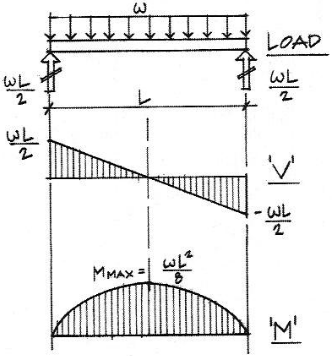


Substituting for M obtained previously,

$$\omega = \frac{8(2.58 \text{ k-ft.})}{(14')^2} = 0.105 \text{ k/ft.} = 105 \text{ \#/ft.}$$

But

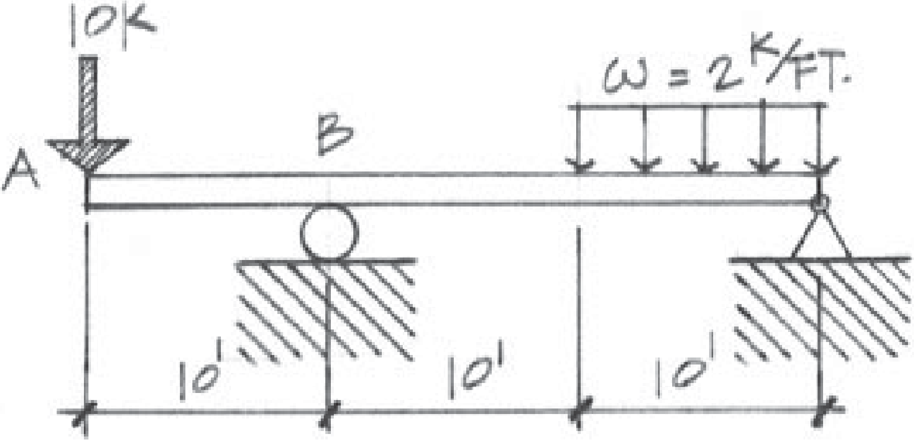
$$\omega = \text{\#/ft.}^2 \times \text{tributary width (joist spacing } s)$$

$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

$$s = 25'' \text{ spacing}$$

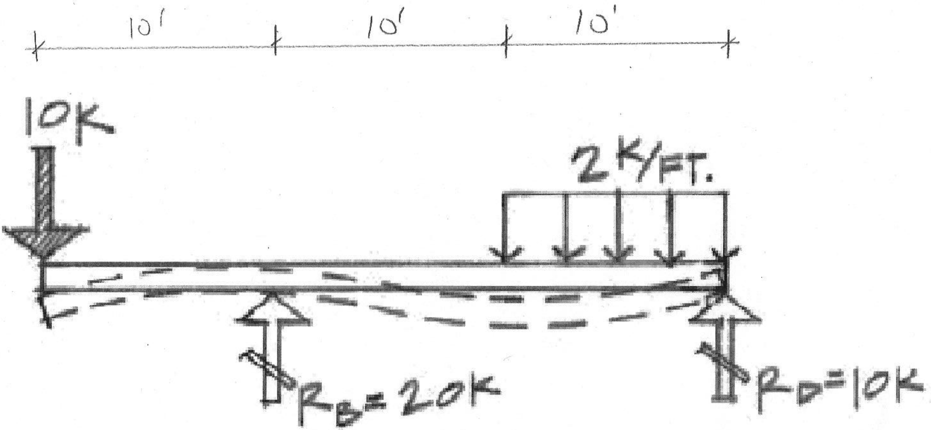
Use 24″ o.c. spacing.

Note: *Spacing is more practical for plywood subflooring, based on a 4 ft. module of the sheet.*

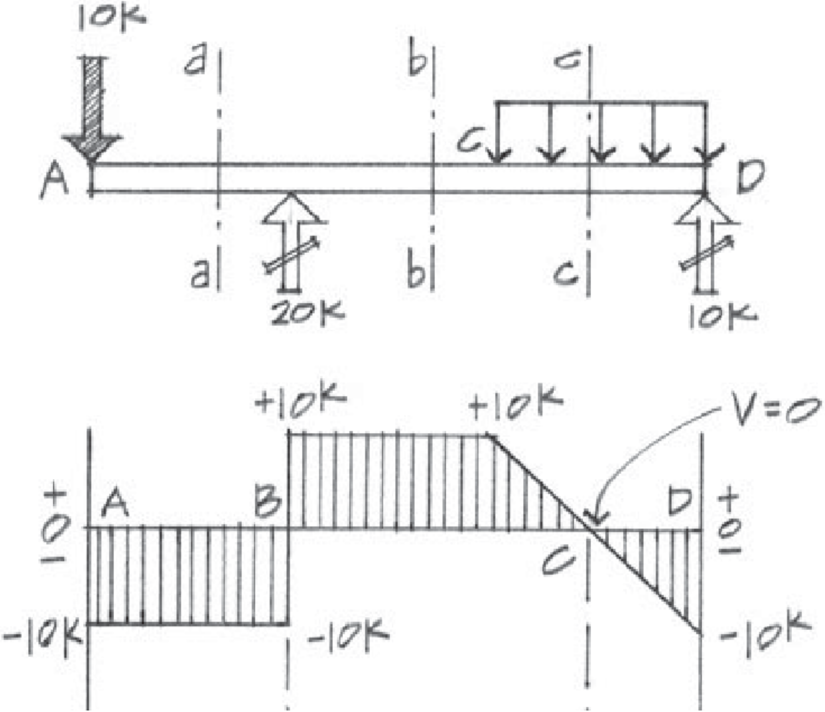


Draw shear (V) and moment (M) diagrams for this beam! Tricky! First one needs to solve for the support ("reaction") forces.

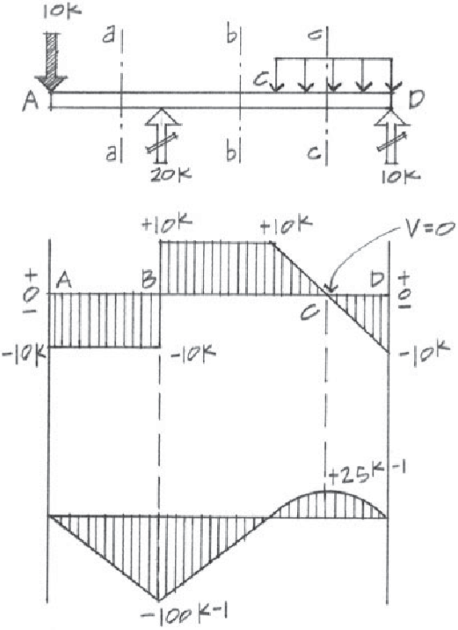
Note: in solving for the support forces, you replace distributed load w with equivalent point load. But when you draw the load diagram to find V and M, you need to keep w in its original form.



Remember that $V(x)$ is the running sum, from LHS to x, of vertical forces acting on the beam, with upward=positive.



10K
a
b
c
a
b
c
A
C
D
20K
10K
+10K
+10K
V=0
+
0
−
A
B
D
+
0
−
C
−10K
−10K
−10K



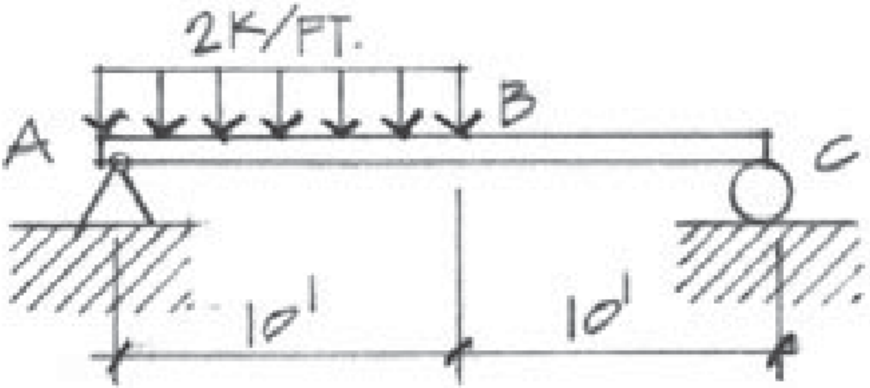
Load diagram.

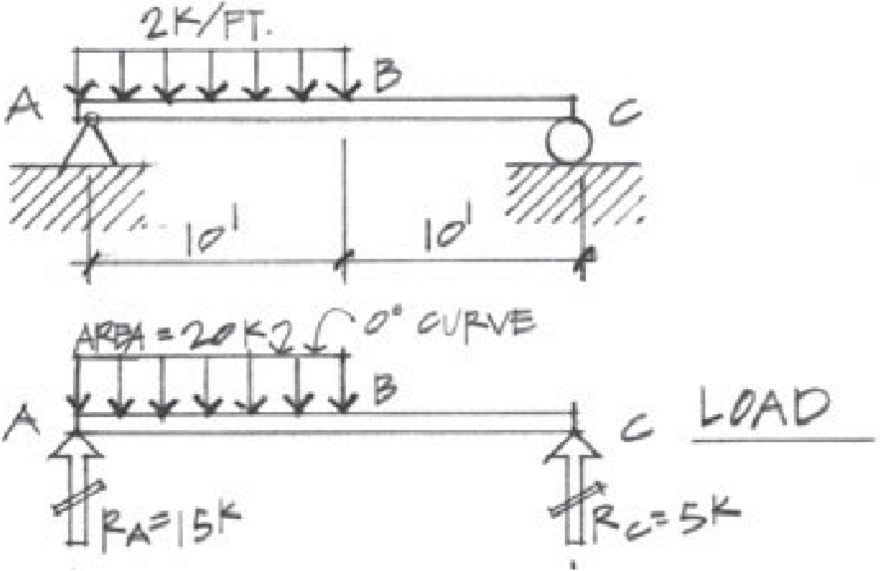
Shear diagram.

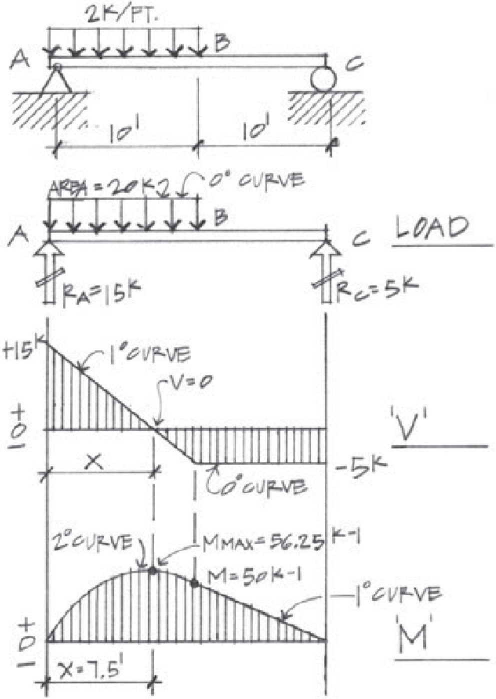
Moment diagram.

Neat trick: $M_2 - M_1 = (V_{1\to 2}^{\text{average}})(x_2 - x_1)$

Draw load, V, and M diagrams for this simply supported beam with a partial uniform load.

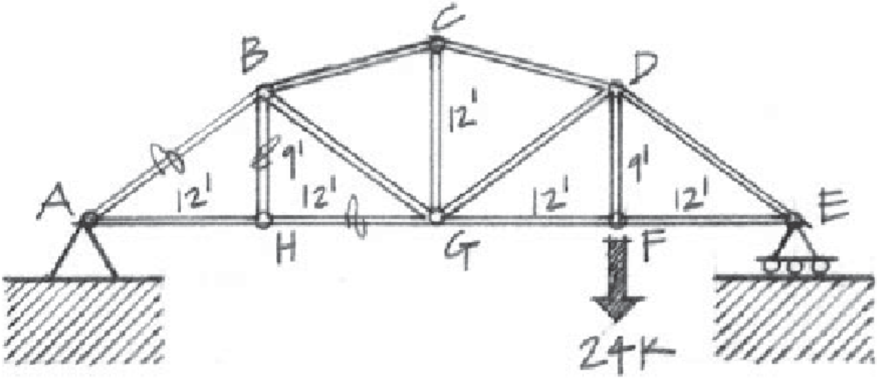



2K/FT.
A
B
C
10'
10'
AREA = 20K
2
0° CURVE
A
B
C
LOAD
RA=15K
RC=5K



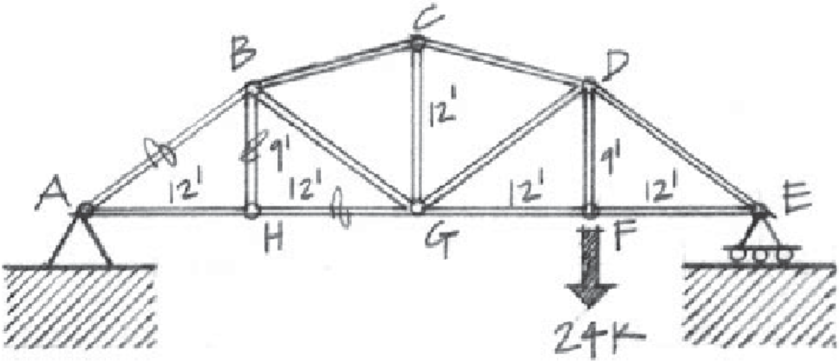
2K/FT.
A
B
C
10'
10'
AREA = 20K
0° CURVE
LOAD
R_A=15K
R_C=5K
+15K
1° CURVE
V=0
+
0
−
'V'
X
-5K
0° CURVE
2° CURVE
MMAX=56.25 K-1
M=50K-1
1° CURVE
'M'
X=7.5'

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



If we have time left, let's solve this truss problem together. It's actually pretty quick, using method of sections. First solve for vertical support force at A, then analyze left side of section.

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



XC2. **(I haven't checked this with anyone else yet.)** For the truss as a whole $\sum F_x = 0$ gives $\boxed{R_{Ax} = 0}$. Then $\sum M_A = 0 = R_{Ey}(48') - 24\text{k}(36')$ gives $\boxed{R_{Ey} = 18\text{k}}$. Then $\sum F_y = 0 = R_{Ay} + R_{Ey} - 24\text{k}$ gives $\boxed{R_{Ay} = 6\text{k}}$. Now section the truss through members AB, BH, and GH and analyze the left-hand side. Then $\sum M_A = 0 = T_{BH}(12')$ gives $\boxed{T_{BH} = 0}$, which one can see by inspection of the vertical forces at joint H: bar BH is a "zero-force member." Then $\sum F_y = 0 = +6\text{k} + (3/5)T_{AB} + T_{BH}$ gives $\boxed{T_{AB} = -10\text{k}}$ (i.e. compression). Finally, $\sum F_x = 0 = (4/5)T_{AB} + T_{GH}$ gives $\boxed{T_{GH} = 8\text{k}}$.

Physics 8 — Monday, November 25, 2019

- Turn in HW11.
- Today is our last day on beams. Then we'll spend a week on oscillation / vibration / periodic motion.
- I'll put the PDF of the take-home practice exam online before Thanksgiving. I intend for it to be similar in coverage to the in-class final exam (Dec 12, noon, A1), though the in-class exam will be shorter than the take-home. If you turn it in on Friday, Dec 6, then I will email it back to you after class on Monday, Dec 9. If you turn it in on Monday, Dec 9, then I will give it back to you at the Wednesday (Dec 11) review session.
- I plan to do a review session on Dec 11, time/place TBD.
- Prof. Farley plans to join us today.
- Wednesday, you don't have to show up, but if you do, you'll get a bonus point. I will spend the hour introducing "Python Mode for Processing" (`py.processing.org`) — a visual-arts-oriented approach to writing code to do drawing and animation.