Physics 8 — Wednesday, December 4, 2019

- Practice exam: If you turn it in Monday (in class, or in my office, by 5pm), I'll return it to you on Wed, Dec 11. If I don't have your exam by 5pm on Monday, Dec 9, your score is zero, no exceptions, so that I can return graded exams promptly.
- 4 previous years' exams & practice exams are at `http://positron.hep.upenn.edu/p8/files/oldexams`
- Extra credit options (until Thu, Dec 19):
  - O/K ch9 (columns)
  - Citigroup Center "structural integrity" podcast
  - Mazur ch13 (gravity), ch14 (Einstein relativity)
  - Code something in Processing or Py.Processing
  - Go through tutorials to learn Wolfram Mathematica
  - Go through Prof. Nelson's python data modeling book
  - You can suggest something else!

- Worth remembering: natural frequency for a mass on a spring

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- double k $\rightarrow$ multiply f by $\sqrt{2}$
- double m $\rightarrow$ divide f by $\sqrt{2}$

- For a wide range of equilibrium situations in which the restoring force is provided by some form of elasticity,
  - more stiffness $\rightarrow$ higher f
  - more mass $\rightarrow$ lower f
- See same $\sqrt{\frac{\text{stiffness}}{\text{inertia}}}$ trend in beams, skyscrapers, etc.
- But pendulum is an exception, because restoring force $\propto$ m. We'll see in a moment.
- Another surprising result: frequency of oscillation is independent of amplitude
- Unfortunately, with A6 booked for back-to-back classes, I didn't have occasion to figure out why Monday's measured period for me-on-spring was 15% larger than our prediction.
- Let's make sure we understand how we got the prediction.

(From 2017 practice exam): Your physics teacher Bill gets the crazy idea that he himself will be the "mass" bobbing up and down on the end of a stiff spring that is attached to the ceiling of DRL room A2. Bill first hangs the spring from the ceiling and measures its relaxed length to be 0.85 meters. Then he climbs the ladder, gradually applies his full weight to the lower end of the spring (by sitting on a little attached bar), and measures the spring's new equilibrium length (the length of the spring when Bill is in static equilibrium) to be 1.55 meters.

[a] If Bill's mass is 70 kg, what is the spring constant k of the spring?

(A) $(70)/(1.55 - 0.85) \;=\; 100\,\mathrm{N/m}$

(B) $(1.55 - 0.85)/(70 \times 9.8) \;=\; 1.02 \times 10^{-3}\,\mathrm{N/m}$

(C) $(70 \times 9.8)/(0.85) \;=\; 807\,\mathrm{N/m}$

(D) $(70 \times 9.8)/(1.55 - 0.85) \;=\; 980\,\mathrm{N/m}$

[b] If someone pulls down on Bill's feet until the spring's length is 1.85 meters, holds them there for a moment, then lets go (without giving any sort of push), will Bill's motion repeat itself periodically? If so, how often? If not, why not?

(A) No: the spring will return to $\ell = 1.55\,\mathrm{m}$ and stay there.

(B) Yes, with period $T = 2\pi\sqrt{70/980} = 1.7\,\mathrm{s}$

(C) Yes, with period $T = 2\pi\sqrt{70 \times 9.8/980} = 0.83\,\mathrm{s}$

(D) Yes, with period $T = 2\pi\sqrt{980/70} = 23.5\,\mathrm{s}$

(E) Yes, with period $T = \sqrt{70/980} = 0.26\,\mathrm{s}$

(F) Yes, with period $T = \sqrt{70/980}/(2\pi) = 0.043\,\mathrm{s}$

(G) Yes, with period $T = \sqrt{980/70} = 3.7\,\mathrm{s}$

(H) Yes, with period $T = \sqrt{980/70}/(2\pi) = 0.60\,\mathrm{s}$

[c] Sketch a graph of the length of the spring as a function of time, where $t = 0$ is where the person lets go of Bill's feet. Be sure to label the important features of the graph, e.g. period and amplitude.

[d] If the person instead pulls down on Bill's feet until the spring's length is 1.70 meters, then lets go, how will the period of the motion be affected? (State what the period will be.)

[e] How will the amplitude of the motion be affected? (State what the amplitude will be.)

[f] If Bill somehow managed to hold a 70 kg medicine ball while sitting on this same spring, thus effectively doubling his mass, would the natural period of the motion be affected? (State what the period would be.)

If the amplitude of simple harmonic motion doubles, what happens to the frequency (i.e. the natural frequency) of the system?

(A) The frequency is $1/2$ as large.
(B) The frequency is $1/\sqrt{2}$ as large.
(C) The frequency is unchanged.
(D) The frequency is $\sqrt{2}$ times as large.
(E) The frequency is 2 times as large.

If the amplitude of simple harmonic motion doubles, what happens to the energy of the system?

(A) The energy is unchanged.
(B) The energy is $\sqrt{2}$ times as large.
(C) The energy is 2 times as large.
(D) The energy is 4 times as large.

If the amplitude of simple harmonic motion doubles, what happens to the energy of the system?

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One way to see that **(D)** is correct is to write

$$x = A\sin(\omega t) \qquad v_x = \omega A\cos(\omega t)$$

and then write out the energy

$$\frac{1}{2}mv^2 \ + \ \frac{1}{2}kx^2$$

and see that energy is proportional to A^2.

Pendulum: gravity provides restoring torque

$$\tau = I\alpha \quad \Rightarrow \quad \alpha = \frac{\tau}{I}$$

$$\tau = -mg\ell \sin\theta$$

$$I = m\ell^2$$

Using $\sin\theta = \theta - \theta^3/6 + \theta^5/120 + - \cdots$,

$$\alpha = -\frac{g \sin\theta}{\ell} \approx -\frac{g\theta}{\ell}$$

for small θ, so

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{\ell}\ \theta$$

So for a pendulum (a point mass on a string, small amplitude),

$$\omega_0 = \sqrt{\frac{g}{\ell}} \qquad \boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}}$$

(Mazur also generalizes this to objects with more complicated rotational inertias. That's only relevant for XC problems.)

Remember: oscillator period is **independent** of the amplitude

Mass on spring (use "0" to mean "natural"):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T_0 = 2\pi\sqrt{\frac{m}{k}}$$

Simple pendulum (small heavy object at end of "massless" cable):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \qquad T_0 = 2\pi\sqrt{\frac{\ell}{g}}$$

For a pendulum, the period is also independent of the mass, because the restoring force (due to gravity) is proportional to mass, so the mass cancels out.

Let's measure the oscillation period T_0 for this ball on a string, for a few different values of ℓ.

Remember,

$$T_0 = \frac{1}{f_0} = 2\pi\sqrt{\frac{\ell}{g}}$$

To speed us up, I've pre-calculated everything but ℓ:

$$T_0 \;=\; 2\pi\sqrt{\frac{\ell}{g}} \;=\; \left(2\pi\,\sqrt{\frac{1\text{ meter}}{g}}\right)\sqrt{\frac{\ell}{1\text{ meter}}}$$

$$T_0 \;=\; (2.01\text{ seconds}) \;\times\; \sqrt{\frac{\ell}{1\text{ meter}}}$$

Does it depend on amplitude? Does it depend on mass?

Imagine your old playground swing set. I'll bet you remember everybody going back and forth at about the same time interval, even if some kids had different amplitudes, different phases, or even different masses!

You probably also remember the time between swings to be something like a few seconds:

$$T_0 = 2\pi\sqrt{\frac{\ell}{g}} \approx 2\pi\sqrt{\frac{3.1 \text{ m}}{9.8 \text{ m/s}^2}} \approx 3.5 \text{ s}$$

Notice that this is independent of the mass of the kid.

Let's try this for a very lightweight and a much heavier "kid!"

By the way, how often should I "kick" if I want to make the swing go as high as possible? Time kicks to swing's natural motion!

Most important points about periodic motion

- Meaning of amplitude, period, frequency
- Drawing or interpreting a graph of periodic motion
- Don't confuse angular frequency vs. frequency ($\omega = 2\pi f$)
- Any system that is in stable equilibrium can undergo vibrations w.r.t. that stable position.
- Mass on spring: (natural) frequency is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- Pendulum: (natural) frequency is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

- For a given mass, a larger restoring force (more stiffness) increases f_0.
- If the restoring force is elastic (not gravitational), then a bigger mass decreases f_0. For pendulum, f_0 doesn't depend on mass, because restoring force is gravitational.

We often write $x(t)$ in terms of ω = "natural angular frequency:"

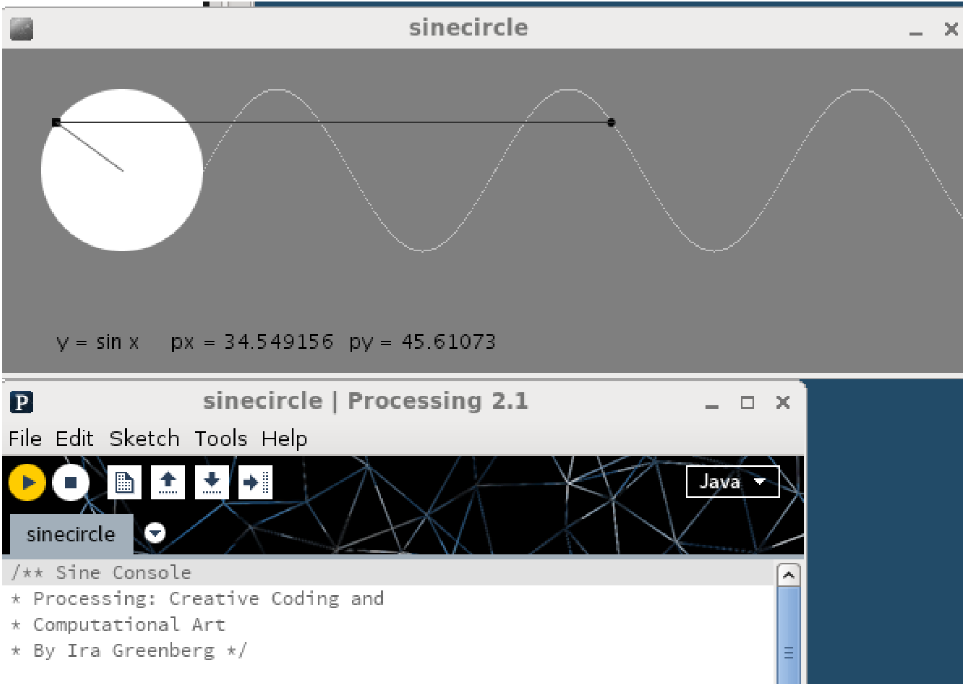
$$x = A\cos(\omega t + \phi_i)$$

but we can equivalently used f = "natural frequency:"

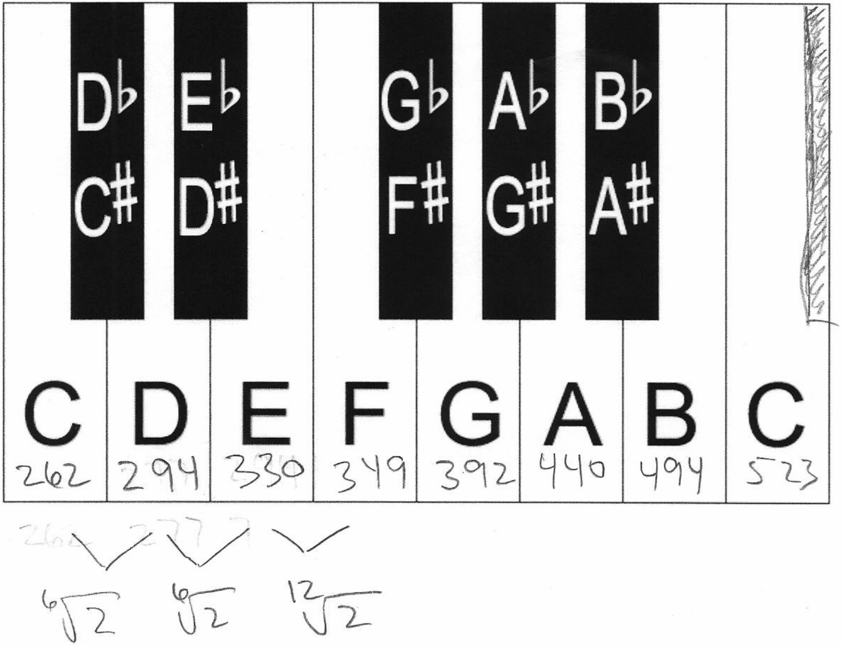
$$x = A\cos(2\pi f\ t + \phi_i)$$

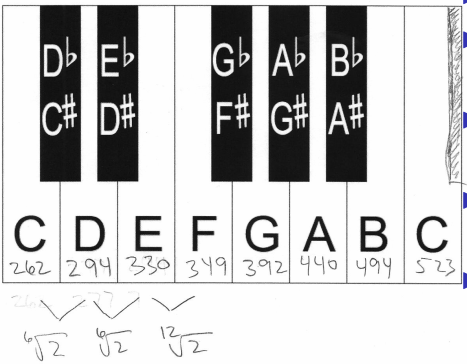
- f = *frequency*, measured in cycles/sec, or Hz (hertz)
- $\omega = \frac{f}{2\pi}$ is *angular frequency*, measured in radians/sec, or s^{-1}
- The frequency $f = 2\pi\omega$ is much more intuitive than ω
- Using ω keeps the equations cleaner — can be helpful for derivations, etc., so that you don't have to keep writing 2π

“angular velocity” ω is our old friend from studying circular motion:



"frequency" $f = \frac{\omega}{2\pi}$ is more familiar from music, etc.





- A above middle C: 440 Hz
- Middle C: 261.63 Hz
- $440 \times (\frac{1}{2})^{\frac{3}{4}} = 261.63$
- Octave = factor of 2 in frequency f
- Half step = factor of $\sqrt[12]{2}$ in frequency
- Whole step = factor of $\sqrt[6]{2}$ in frequency
- Major scale (white keys, starting from C) = (root) W W H W W W H
- Minor scale (white keys, starting from A) = (root) W H W W H W W

resonance: Tacoma Narrows Bridge collapse

http://www.youtube.com/watch?v=j-zczJXSxnw

- We may not have time for this 6-minute video in class. If you've never seen it, I highly recommend watching it!
- This one has no audio. There's another version of this video out there that has newsreel-style audio.

Let's return to our two favorite examples of oscillating systems.

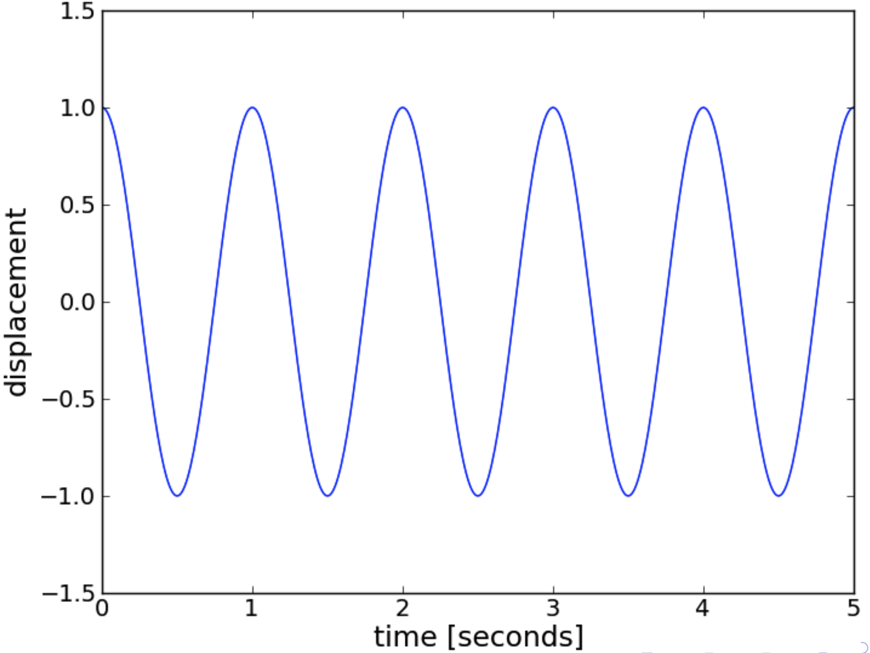
Natural frequency & period for mass on spring:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T_0 = 2\pi\sqrt{\frac{m}{k}}$$

Natural frequency & period for simple pendulum (small heavy object at end of "massless" cable):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \qquad T_0 = 2\pi\sqrt{\frac{\ell}{g}}$$

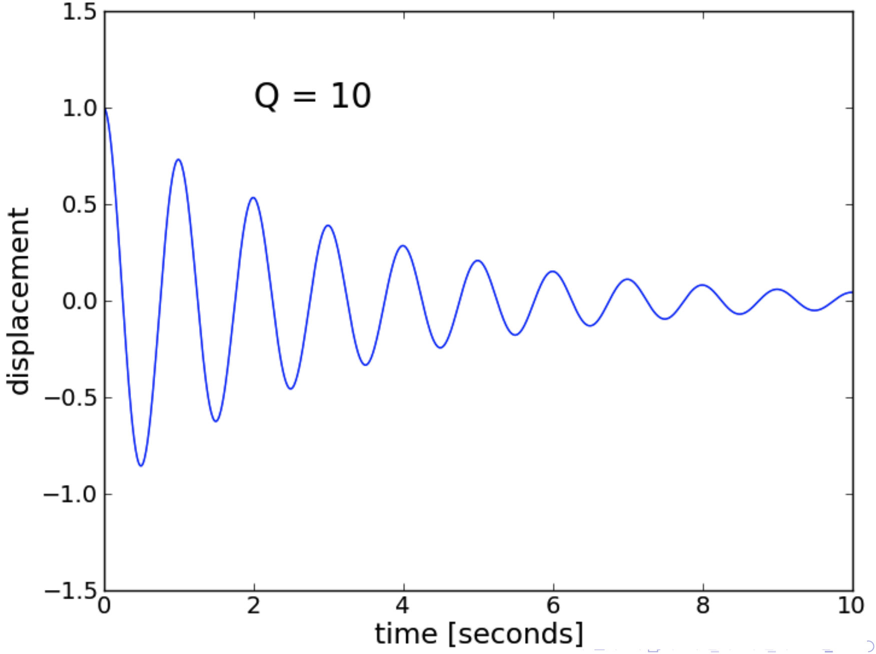
Oscillation: $v_{\mathrm{initial}} = 0$, $x_{\mathrm{initial}} \neq 0$



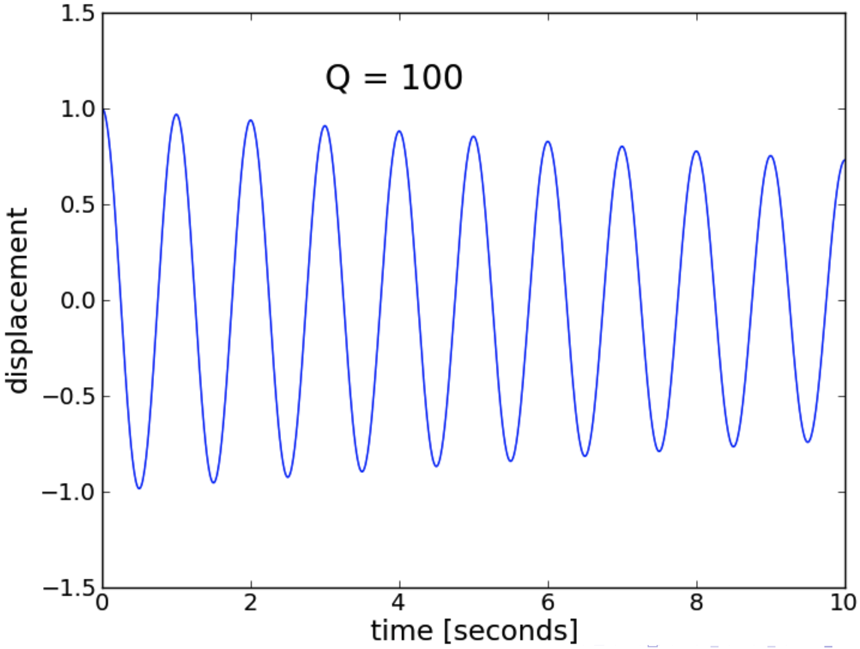
Missing from previous picture: **damping**

- Without some kind of external push, a swingset eventually slows to a stop, right? Eventually the mechanical energy is dissipated by friction, air resistance, etc.
- A piano wire doesn't vibrate forever, does it?
- Normally once you hit a key, the sound dies out after about half a second or so.
- If your foot is on the sustain pedal, the sound lasts several seconds.
- What is the difference?
- It's the felt *damper* touching the strings!

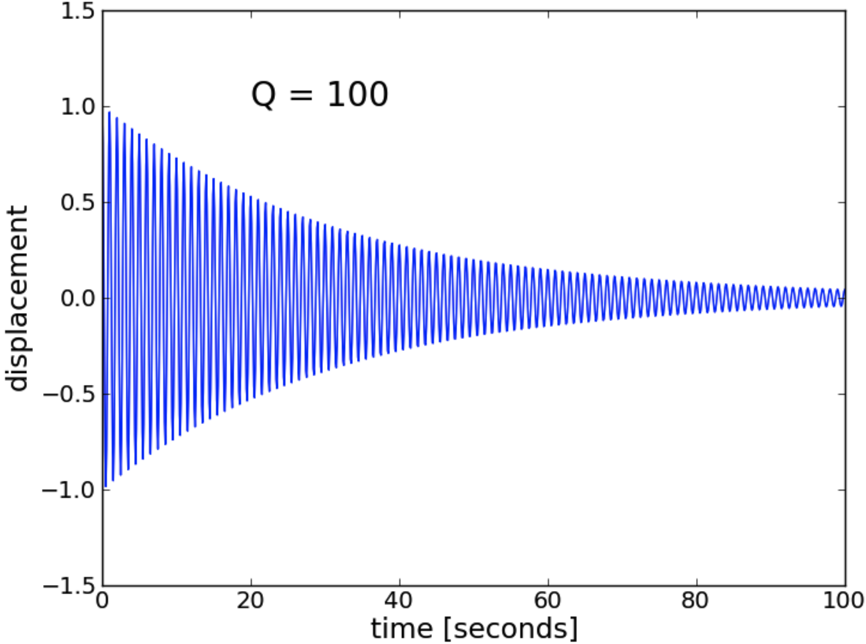
Oscillation (damped, Q=10, f=1 Hz)



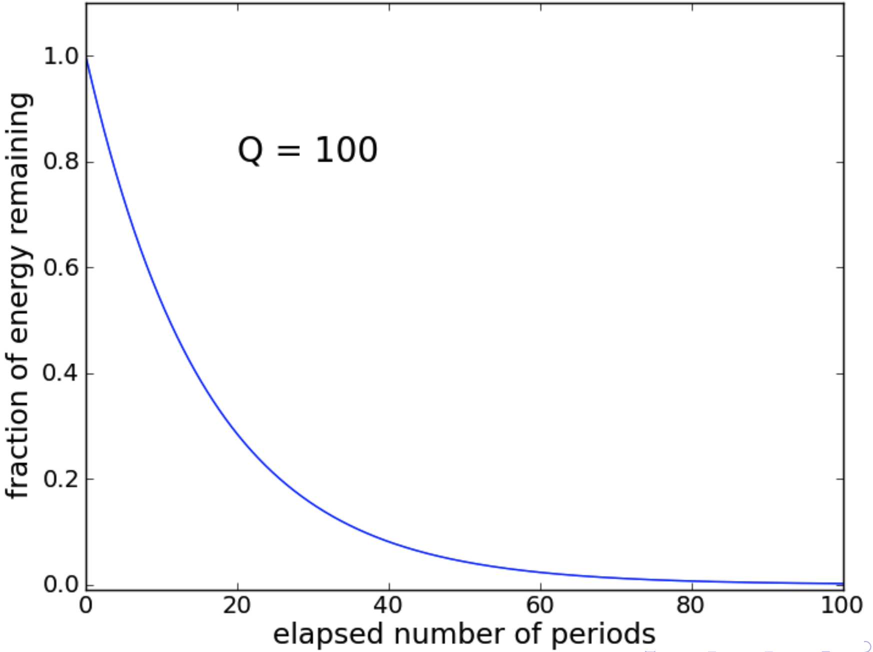
Oscillation (damped, Q=100, f=1 Hz)



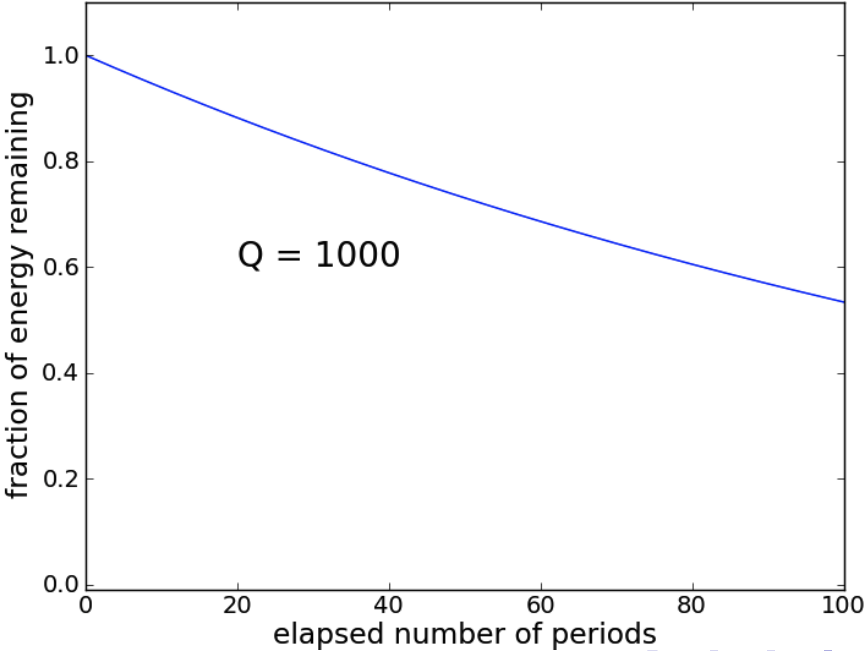
Oscillation (damped, Q=100, f=1 Hz)



Damped, Q=100, f=1 Hz: energy vs time (in periods)



Damped, Q=1000, f=1 Hz: energy vs time (in periods)



For a given frequency f,

- Less damping $\leftrightarrow$ higher Q
- More damping $\leftrightarrow$ lower Q

- $Q = \omega\tau$ is number of radians after which energy has decreased by a factor $e^{-1} \approx 0.37$
- Equivalently, $Q = 2\pi f \tau$ is number of cycles after which energy has decreased by a factor $e^{-2\pi} \approx 0.002$

- More simply, Q is roughly the number of periods after which nearly all of the energy has been dissipated.

- "Tinny" sound of frying pan $\leftrightarrow$ low Q (fast dissipation)
- Smooth, enduring sound of a gong, or a bell tower $\leftrightarrow$ high Q (slow dissipation)

Suppose you want to go for a long time on a swing set.

Dissipation is continuously removing energy.

If you're going to keep going for many minutes, you need some way of continuously putting energy back in.

If you're a big kid, you swing your feet. If you're a little kid, your parent or older sibling pushes you.

The push of parent or swing of feet has to be at approximately the natural frequency of the swingset, or else you don't get anywhere!

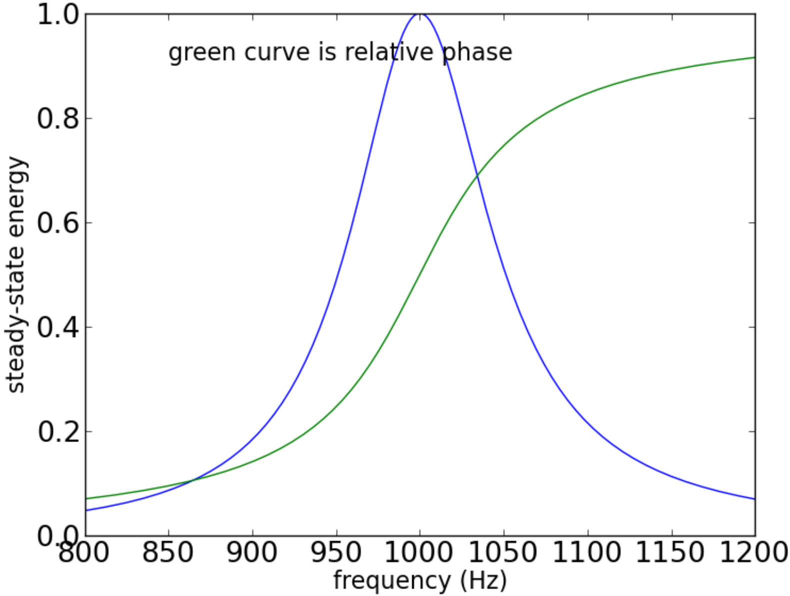
But if your pushes are close to the right interval, the amplitude gets larger and larger with each successive push, until eventually the rate at which the push is adding energy equals the rate at which dissipation is removing energy.

Hitting the right frequency is called resonance

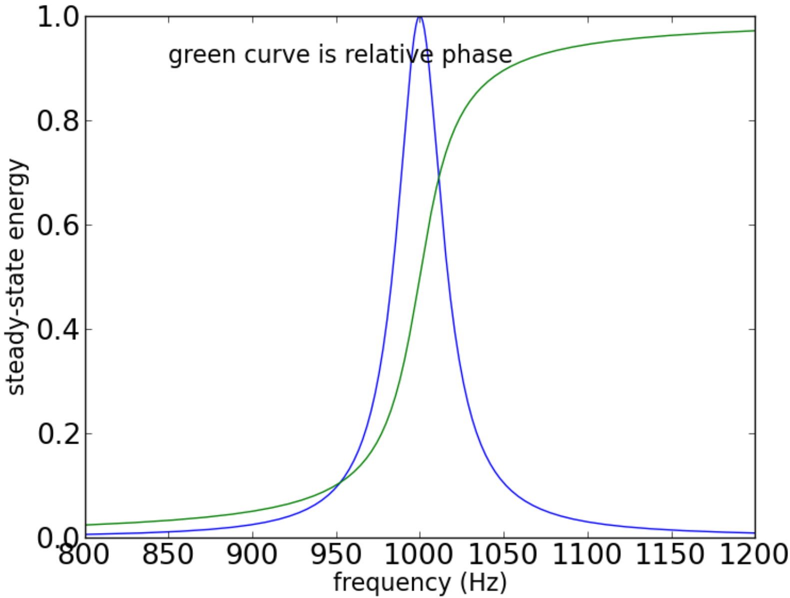
The higher the Q (i.e. slower dissipation), the more periods you have available for building up energy. A high Q makes it easy to build up a really big amplitude!

But the higher the Q, the closer you have to get to the right frequency in order to get the thing moving.

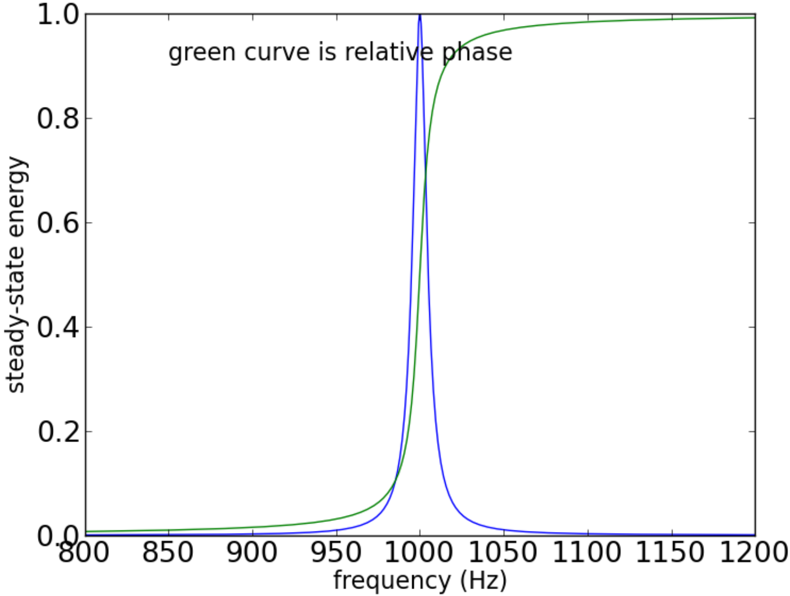
$f_0 = 1000$ Hz, $Q = 10$: energy and phase vs. f_{push}



$f_0 = 1000$ Hz, $Q = 30$: energy and phase vs. f_{push}



$f_0 = 1000$ Hz, $Q = 100$: energy and phase vs. f_{push}



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