- positron.hep.upenn.edu/p8/files/ws28.pdf
- Today: 2 problems, 3 XC, and a **hands-on activity!**, all apropos oscillations. Also, in the last 15m of class, Ryan will use a loud sound to shatter a wineglass (!).
- positron.hep.upenn.edu/p8/files/exam2023.pdf
- We'll do exam at the whiteboard in my office (DRL 1W15): http://www.hep.upenn.edu/~ashmansk/drl_1w15.jpg
- Last day to submit late work to Marija is Dec **??**. Last day to submit XC to Bill is Dec 23 — maybe later if you tell me that you want to learn something interesting and write it up.

I think this slide and the next one could make a terrific "design your own XC problem" exercise. For scale in this photo, the nominal height of a CMU masonry block is 8 inches. So I think the height of the I-beam is about 10 inches. I would guess just under 6 inches for the width of each flange.

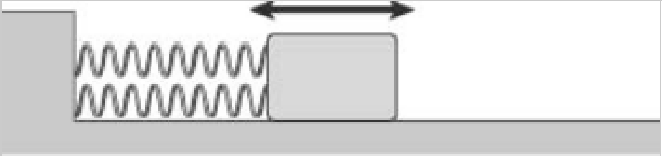
You could turn this into an amazing estimation problem using your knowledge of beams, etc.

1. Hands-on activity! Ryan is going to cook up something fun involving our two favorite examples of oscillating systems: the pendulum; and the bob suspended from a spring.

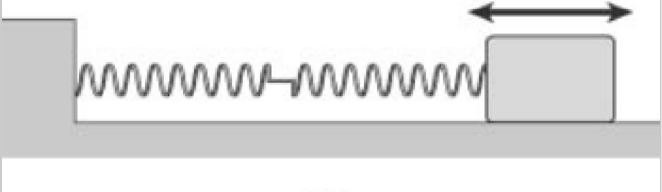
2. A pendulum is swinging with period $T = 1.0\ \mathrm{s}$ in a stationary elevator. What happens to the period when the elevator (a) accelerates upward at $a_y = +2.0\ \mathrm{m/s^2}$? (b) accelerates downward at $a_y = -2.0\ \mathrm{m/s^2}$? (c) travels downward at constant velocity $v_y = -5.0\ \mathrm{m/s}$? (d) travels downward and gradually slows to a stop ($|a_y| = 0.5\ \mathrm{m/s^2}$ — should it be positive or negative)? The easiest way to analyze this problem is to notice that when you are on an elevator, the constant "g" is effectively replaced by a value that combines g with the vertical acceleration — combines how? (Think of what happens when you are standing on a bathroom scale while riding an elevator.)

3. (a) To form a pendulum, I put a 1.50 kg mass at the end of a 0.248 m long cable. If I give the pendulum a small kick and then let it move back and forth freely, what is the period of oscillation? (b) If I replace the 1.50 kg mass with a 15.0 kg mass, how does this affect the period of oscillation? (c) If I wanted to make the period of oscillation twice as large, how long would I have to make the cable?

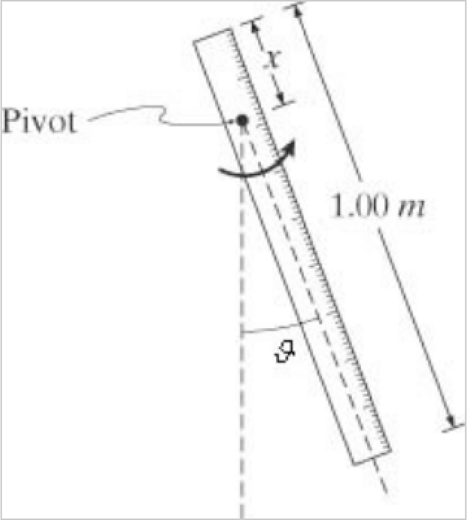
XC4*. A horizontal spring-block system made up of one block and one spring has an oscillation frequency $f = 1.5$ Hz. A second spring, identical to the first, is added to the system. (a) What is the new oscillation frequency when the two springs are connected as shown in figure **a** below? (b) What is the frequency when the springs are arranged as in figure **b** below?



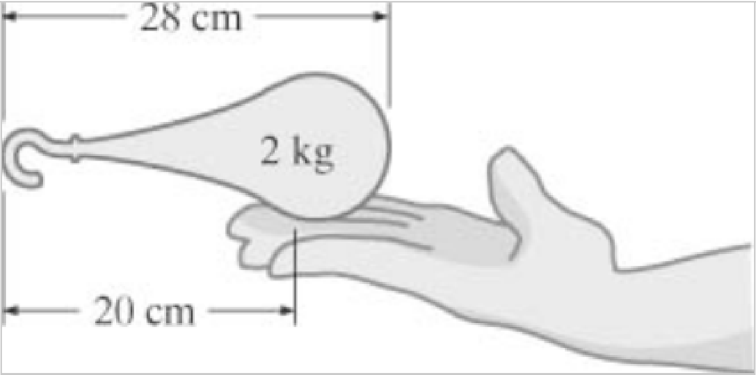
(a)



XC5*. A meter stick is free to pivot around a point located a distance x below its top end, where $0\ \mathrm{m} < x < 0.5\ \mathrm{m}$. (See figure above.) (a) What is the frequency f of its oscillation if it moves as a pendulum? (b) To what position should you move the pivot if you want to minimize the period?



XC6*. You have a teardrop-shaped 2 kg object that is 0.28 m long along its longest axis and has a hook at one end. When you try to balance it on your fingers, you find it balances when your fingers are 0.20 m from the hook end. Then you hang the object by the hook and set it into simple harmonic motion. You find that it oscillates 10 cycles in 13 s. What is its rotational inertia *I*?



We often write $x(t)$ in terms of $\omega =$ "natural angular frequency:"

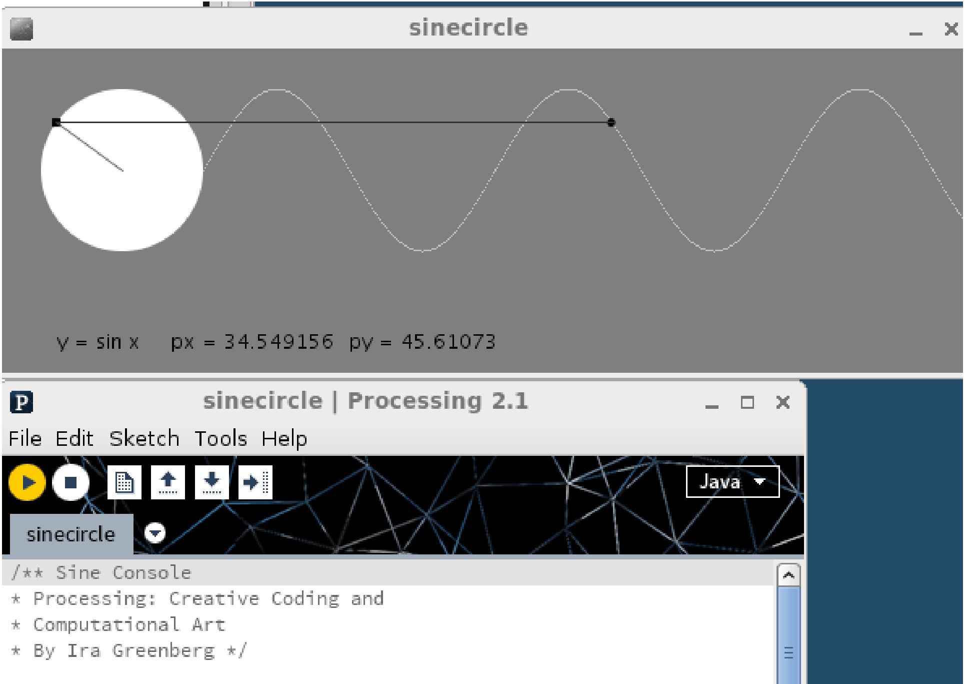
$$x = A\cos(\omega t + \phi_i)$$

but we can equivalently used $f =$ "natural frequency:"

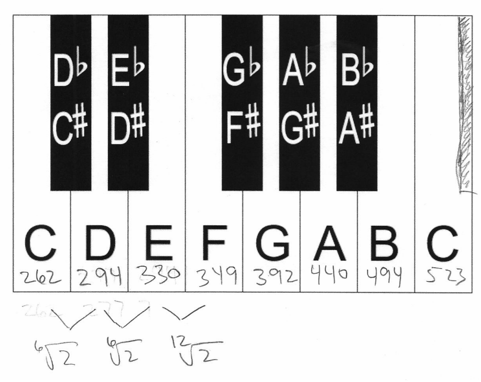
$$x = A\cos(2\pi f\ t + \phi_i)$$

- $f =$ *frequency*, measured in cycles/sec, or Hz (hertz)
- $\omega = \frac{f}{2\pi}$ is *angular frequency*, measured in radians/sec, or s^{-1}
- The frequency $f = 2\pi\omega$ is much more intuitive than ω
- Using ω keeps the equations cleaner — can be helpful for derivations, etc., so that you don't have to keep writing 2π

"angular velocity" ω is our old friend from studying circular motion:



"frequency" $f = \frac{\omega}{2\pi}$ is more familiar from music, etc.



$(\sqrt[12]{2})^4 = 1.2599 \approx \frac{5}{4}$ (major 3rd)

$(\sqrt[12]{2})^5 = 1.3348 \approx \frac{4}{3}$ (perfect 4th)

$(\sqrt[12]{2})^7 = 1.4984 \approx \frac{3}{2}$ (perfect 5th)

$(\sqrt[12]{2})^{12} = 2$ (an octave!)

- A above middle C: 440 Hz
- Middle C: 261.63 Hz
- $440 \times (\frac{1}{2})^{\frac{3}{4}} = 261.63$
- Octave = factor of 2 in frequency f
- Half step = factor of $\sqrt[12]{2}$ in frequency
- Whole step = factor of $\sqrt[6]{2}$ in frequency
- Major scale (white keys, starting from C) = (root) W W H W W W H
- Minor scale (white keys, starting from A) = (root) W H W W H W W

Let's return to our two favorite examples of oscillating systems.

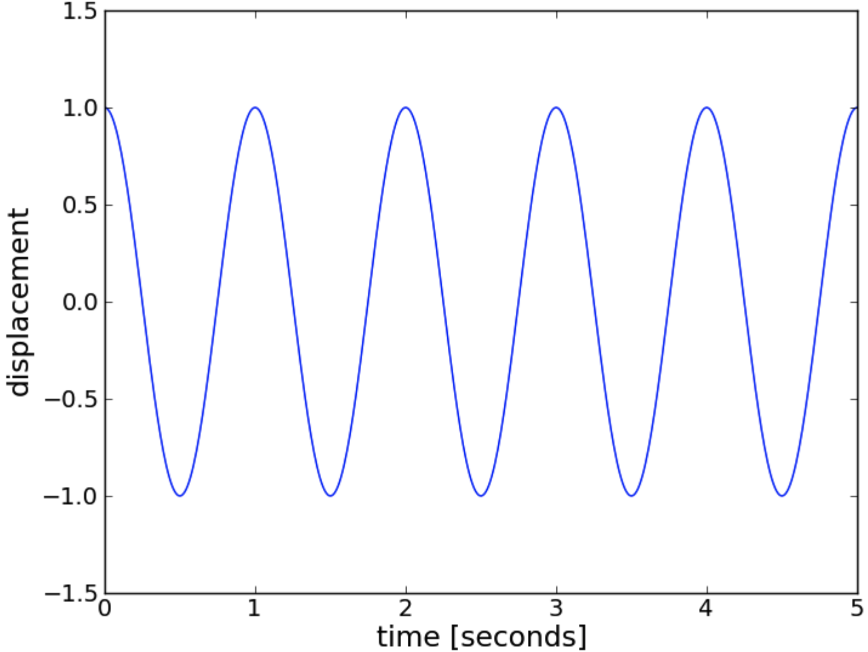
Natural frequency & period for mass on spring:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T_0 = 2\pi\sqrt{\frac{m}{k}}$$

Natural frequency & period for simple pendulum (small heavy object at end of "massless" cable):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \qquad T_0 = 2\pi\sqrt{\frac{\ell}{g}}$$

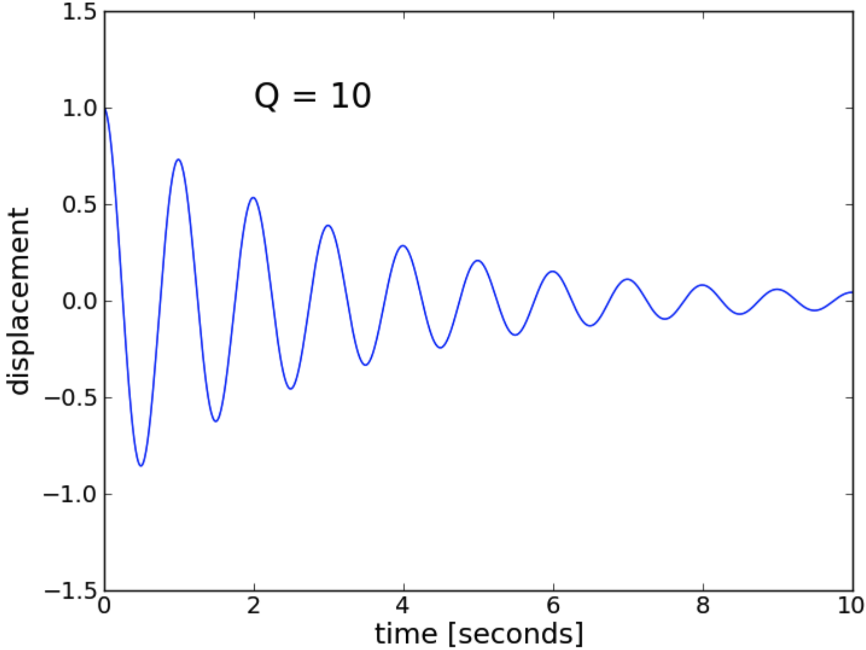
Oscillation: $v_{\mathrm{initial}} = 0$, $x_{\mathrm{initial}} \neq 0$



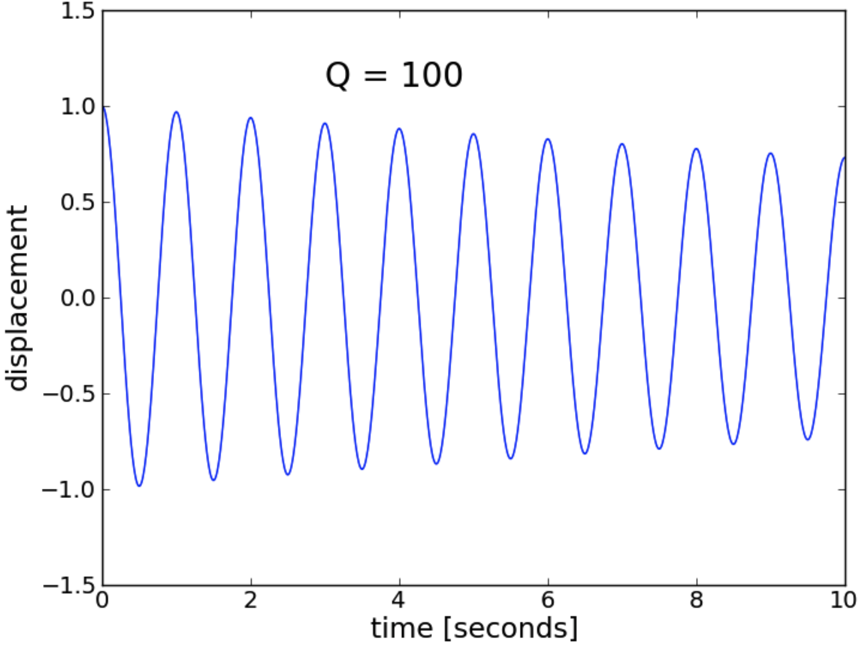
Missing from previous picture: **damping**

- Without some kind of external push, a swingset eventually slows to a stop, right? Eventually the mechanical energy is dissipated by friction, air resistance, etc.
- A piano wire doesn't vibrate forever, does it?
- Normally once you hit a key, the sound dies out after about half a second or so.
- If your foot is on the sustain pedal, the sound lasts several seconds.
- What is the difference?
- It's the felt *damper* touching the strings!

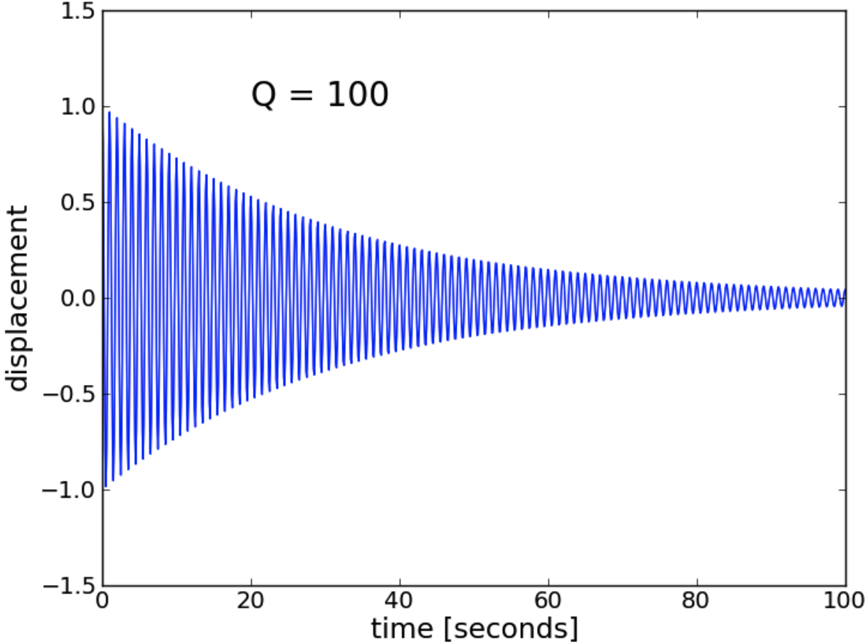
Oscillation (damped, Q=10, f=1 Hz)



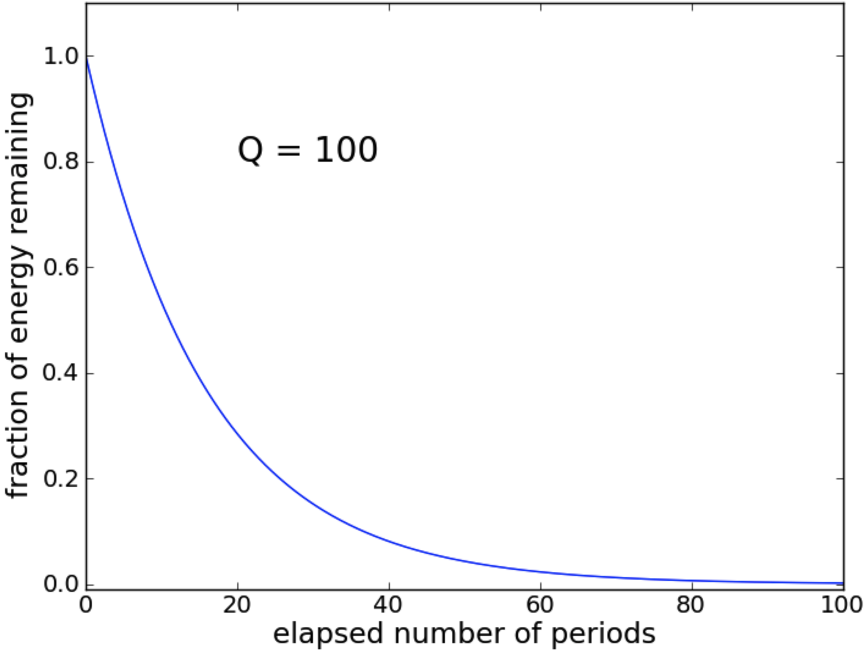
Oscillation (damped, Q=100, f=1 Hz)



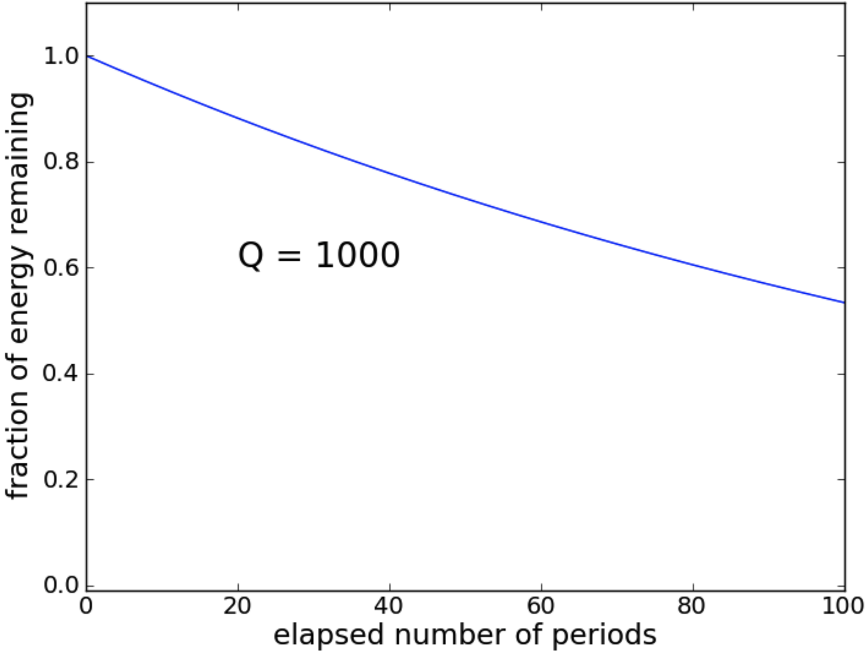
Oscillation (damped, Q=100, f=1 Hz)



Damped, Q=100, f=1 Hz: energy vs time (in periods)



Damped, Q=1000, f=1 Hz: energy vs time (in periods)



For a given frequency f,

- Less damping $\leftrightarrow$ higher Q
- More damping $\leftrightarrow$ lower Q

- $Q = \omega\tau$ is number of radians after which energy has decreased by a factor $e^{-1} \approx 0.37$
- Equivalently, $Q = 2\pi f\tau$ is number of cycles after which energy has decreased by a factor $e^{-2\pi} \approx 0.002$

- More simply, Q is roughly the number of periods after which nearly all of the energy has been dissipated.

- "Tinny" sound of frying pan $\leftrightarrow$ low Q (fast dissipation)
- Smooth, enduring sound of a gong, or a bell tower $\leftrightarrow$ high Q (slow dissipation)

Suppose you want to go for a long time on a swing set.

Dissipation is continuously removing energy.

If you're going to keep going for many minutes, you need some way of continuously putting energy back in.

If you're a big kid, you swing your feet. If you're a little kid, your parent or older sibling pushes you.

The push of parent or swing of feet has to be at approximately the natural frequency of the swingset, or else you don't get anywhere!

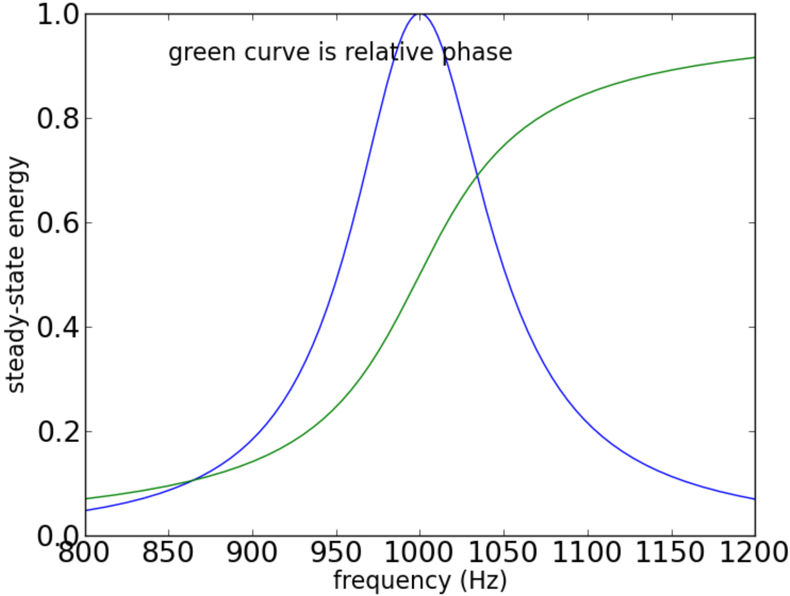
But if your pushes are close to the right interval, the amplitude gets larger and larger with each successive push, until eventually the rate at which the push is adding energy equals the rate at which dissipation is removing energy.

Hitting the right frequency is called resonance

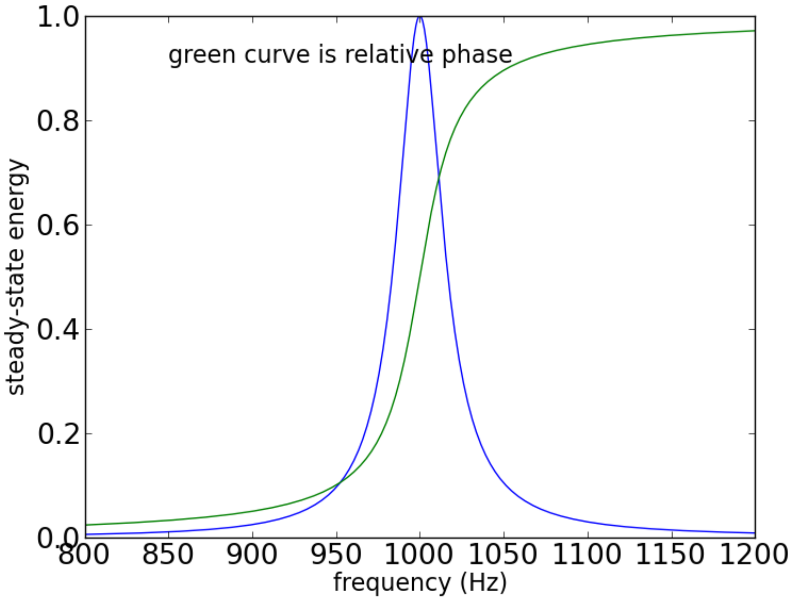
The higher the Q (i.e. slower dissipation), the more periods you have available for building up energy. A high Q makes it easy to build up a really big amplitude!

But the higher the Q, the closer you have to get to the right frequency in order to get the thing moving.

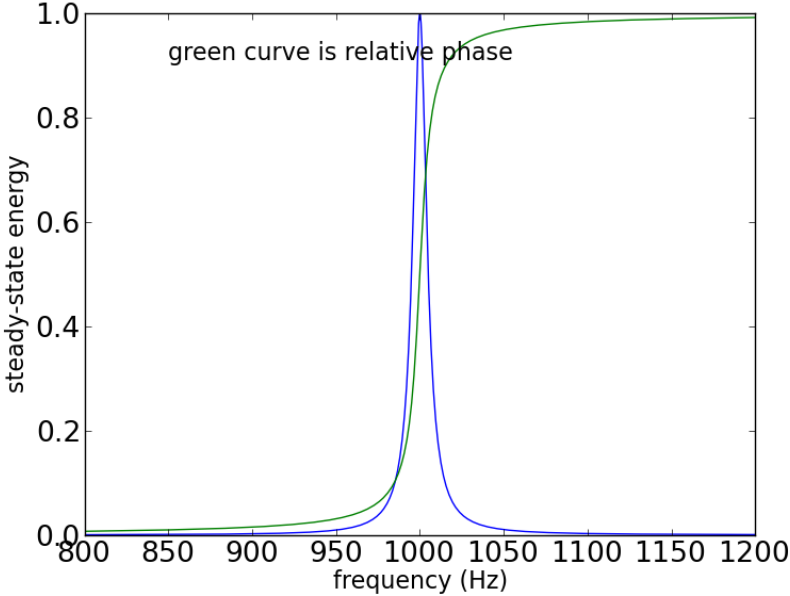
$f_0 = 1000$ Hz, $Q = 10$: energy and phase vs. f_{push}



$f_0 = 1000$ Hz, $Q = 30$: energy and phase vs. f_{push}



$f_0 = 1000$ Hz, $Q = 100$: energy and phase vs. f_{push}



(avoiding) resonance in structures

https://99percentinvisible.org/episode/supertall-101/

(avoiding) resonance in structures

Lateral Loads and Stability

Natural period of oscillation

- harmonic motion
- period of a structure is proportional to weight and inversely proportional to stiffness

Lateral stability of structures
- braced frame
- rigid frame
- shear wall

Code and Safety in Design
- factor of safety
- resilience

Design strategies
- absorb energy in the structure (flexible joints, shock absorbers) (example Bell Atlantic, west coast buildings)
- tuned mass dampers (CitiGorp)
- building shape (Burj Khalifa)

Lateral Stability in Structures