If you don't like my videos, you can just read Mazur chapter 5, and maybe quickly glance over my PDF slides at `phys8_slides_05.pdf` . If you don't like to learn by reading, you can just watch this video (consider 1.5× or 2× speed, slowing down for parts that need more thinking). If you do read/skim chapter 5, I suggest trying to do the checkpoints, but you can gloss over most of the equations.

Checkpoint 5.13 typo (in PDF: printed book is good)

5.13 Yes; cart 1 gets twice as much energy as cart 2: $K_{1f} = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2}(0.25 \text{ kg})(2.0 \text{ m/s})^2 = 0.50 \text{ J}$, $K_{2f} = \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2}(0.50 \text{ kg})(1.0 \text{ m/s})^2 = 0.25 \text{ J}$. The reason is that the system's final momentum needs to be zero, and so v_{1f} must be $2v_{2f}$. Because $m_2 = 2m_1$, you have $K_{2f} = \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}(2m_1)(\frac{1}{2}v_{1f})^2 = \frac{1}{4}m_1 v_{1f}^2 = \frac{1}{2}K_{1f}$.

He means "no" here.

Most of this answer is fine, but when he writes, "Yes" at the beginning, he really means to write, "No." (Even Harvard professors make mistakes once in a while!)

- Ch04 introduced our first example of a **conserved quantity** in physics: momentum, $\vec{p} = m\,\vec{v}$.
- Ch05 introduces a second conserved quantity: $\boxed{\textbf{energy}}$, which is a scalar, while momentum is a vector.
- Energy can be transferred from one object to another (or to/from the environment surrounding a system), but it cannot be created or destroyed. We say that a **closed** system exchanges no energy with its environment. Because energy is conserved, the energy of a closed system is constant.
- In physics, the SI unit of energy is the joule, J. Much of our everyday experience with energy relates to chemical energy stored in fuel and in food.
- If you took high-school chemistry, you may remember that a calorie is $1\,\text{cal} \;=\; 4.18\,\text{J}$. In nutrition, a "food Calorie" is $1\,\text{Cal} \;=\; 4180\,\text{J}$. Outside the US, soda cans often list kcal.
- The first form of energy we will consider in physics is **kinetic energy**, the energy of motion.

$$K = \frac{1}{2}mv^2$$

What is the expression for the kinetic energy of an object of mass m that is moving at speed v ?

(Assume the object is not rotating — we'll deal with that later.)

- When an isolated system of two objects collides, the momentum of the two-object system is constant — the total momentum remains unchanged.
- But the momentum law does not tell us the whole story when two objects collide. Watching two objects collide on a track, we can identify 4 distinct types of collision. What distinguishes these 4 types of collision is whether/how kinetic energy is transformed to/from other forms in the collision.
- In an **elastic collision**, the total kinetic energy is constant.
- In an **inelastic collision**, some of the two objects' total kinetic energy is transformed into other forms. The total K.E. of the colliding objects is smaller after the collision vs before.
- In a **totally inelastic collision**, the two objects stick together.
- In an **explosive separation**, two objects initially stuck together push apart from one another.
- Your eye can most readily distinguish the 4 types of collision by observing the **relative speed** of the two objects after vs before the collision (or explosion).

- The **relative velocity** of objects 1 and 2 is the difference in the two objects' velocities: $\boxed{\vec{v}_1 - \vec{v}_2}$. Considering only x components for now, the x component of the relative velocity of objects 1 and 2 is $\boxed{v_{1x} - v_{2x}}$.
- The **relative speed** of two objects is the magnitude of their relative velocity: $\boxed{|\vec{v}_1 - \vec{v}_2|}$. In one dimension, the relative speed is $\boxed{|v_{1x} - v_{2x}|}$.
- In a totally inelastic collision (objects stick together), the relative speed of the two objects is **zero** after the collision.
- In an explosive separation, the relative speed of the two objects is zero **before** the collision but nonzero afterward.
- It turns out that for an **elastic** collision, the relative speed of the two objects is unchanged by the collision.
- In fact, for an elastic collision along the x axis, the relative velocity simply changes sign (Mazur Eqn 5.4):

$$(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i})$$

$$\frac{1}{2}m_1 v_{1xi}^2 + \frac{1}{2}m_2 v_{2xi}^2 \stackrel{?}{=} \frac{1}{2}m_1 v_{1xf}^2 + \frac{1}{2}m_2 v_{2xf}^2$$

$$K_f - K_i = \frac{1}{2}m_1\left(v_{1xf}^2 - v_{1xi}^2\right) + \frac{1}{2}m_2\left(v_{2xf}^2 - v_{2xi}^2\right)$$

$$= \frac{1}{2}m_1\left(v_{1xf} + v_{1xi}\right)\left(v_{1xf} - v_{1xi}\right) + \frac{1}{2}m_2\left(v_{2xf} + v_{2xi}\right)\left(v_{2xf} - v_{2xi}\right)$$

$$v_{1xf} - v_{2xf} = v_{2xi} - v_{1xi}$$

$$v_{1xf} + v_{1xi} = v_{2xf} + v_{2xi}$$

$$= \frac{1}{2}\left(v_{1xf} + v_{1xi}\right)\left[m_1 v_{1xf} - m_1 v_{1xi} + m_2 v_{2xf} - m_2 v_{2xi}\right]$$

$$= \frac{1}{2}\left(v_{1xf} + v_{1xi}\right)\left[\left(m_1 v_{1xf} + m_2 v_{2xf}\right) - \left(m_1 v_{1xi} + m_2 v_{2xi}\right)\right]$$

$= 0$ since $P_{xf} = P_{xi}$

- We just showed that for a collision along the x axis between two objects that form an isolated system (ie, momentum is constant), the statement that total K.E. is unchanged by the collision is equivalent to the statement that relative velocity flips sign in the collision.
- A much less interesting way for total K.E. to be unchanged is if relative velocity is unchanged by the collision, ie doesn't even flip sign, eg if the two objects do not collide.
- So instead of talking about changes in K.E., it turns out to be much easier (and equivalent) to consider how the **relative speed** of the two objects changes in the collision.
- We will define the **coefficient of restitution**, $\boxed{e}$, to be the ratio of relative speed after vs before a collision.

$$(v_{1x,f} - v_{2x,f}) = -e\,(v_{1x,i} - v_{2x,i})$$

- $e = 1$ for an elastic collision
- $e = 0$ for a totally inelastic collision (stick together)
- $0 \leq e < 1$ for an inelastic collision (some KE is transformed)
- It works out that $e = \infty$ for an explosive separation!

Kinetic energy

$$K = \frac{1}{2}mv^2$$

- is the energy of *motion*.
- is unchanged (in total) in an *elastic* collision.

e.g.

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

but it's much easier in practice to write (equivalently)

$$|v_{12,i}| = |v_{12,f}|$$

i.e. relative *speed* is the same before and after an elastic collision

$$(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i}) \qquad \text{[Eqn. 5.4]}$$

What are 4 types of collision? What distinguishes them?

Types of collisions

- Elastic collision: objects recoil with same *relative speed* as before they collided. Kinetic energy $K_i = K_f$.

$$(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i}) \qquad \text{[Eqn. 5.4]}$$

- Totally inelastic collision: objects stick together.

$$(v_{1x,f} - v_{2x,f}) = 0$$

- Inelastic collision: objects recoil, but with a reduction in relative speed

$$(v_{1x,f} - v_{2x,f}) = -e(v_{1x,i} - v_{2x,i}) \qquad \text{with } 0 < e < 1$$

- Explosive separation: imagine T.I.C. movie played in reverse.

$$(v_{1x,i} - v_{2x,i}) = 0$$

$$(v_{1x,f} - v_{2x,f}) \neq 0$$

Q (tricky): what value of e describes an explosive separation?!

If I play in reverse a movie of an elastic collision, what sort of collision would I appear to see?

(a) elastic
(b) inelastic
(c) totally inelastic
(d) explosive separation
(e) it depends!

When we collide (on a low-friction track) two carts whose masses and initial velocities are known, conservation of momentum allows us to write

$$m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

We have one equation, but two unknowns. Knowing something about energy gives us a second equation. Relative speed = key.

- elastic: $(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i})$
- totally inelastic: $(v_{1x,f} - v_{2x,f}) = 0$
- if e is given: $(v_{1x,f} - v_{2x,f}) = -e(v_{1x,i} - v_{2x,i})$
- if change in internal energy is given:

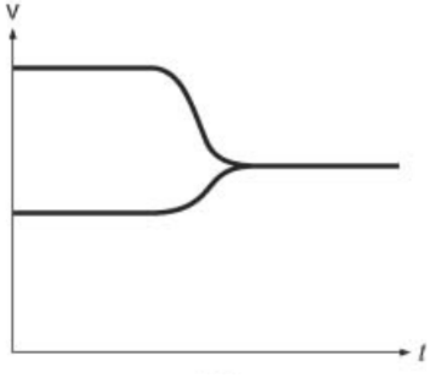
$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 + \Delta E_{\mathrm{internal}}$$

(or equivalently)

$$K_{1i} + K_{2i} + E_{i,\mathrm{internal}} = K_{1f} + K_{2f} + E_{f,\mathrm{internal}}$$

(Maybe skim ch05 re: internal energy, energy diagrams, etc.)

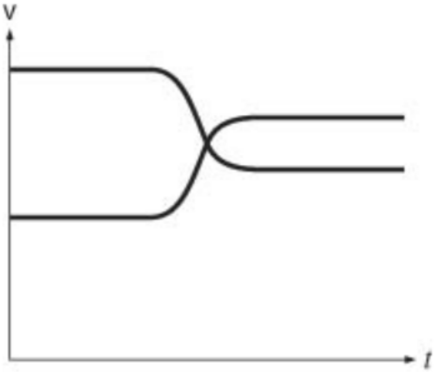
What sort of collision is illustrated by this velocity-vs-time graph?



(A) elastic
(B) inelastic
(C) totally inelastic
(D) explosive separation
(E) can't tell from given information

(By the way, can you infer the ratio of masses?)

What sort of collision is illustrated by this velocity-vs-time graph?



(A) elastic
(B) inelastic
(C) totally inelastic
(D) explosive separation
(E) can’t tell from given information

(By the way, can you infer the ratio of masses?)

Suppose you find an isolated system in which two objects about to collide have equal and opposite momenta. If the collision is totally inelastic, what can you say about the motion after the collision?

Imagine making two springy devices, each made up of a dozen or so metal blocks loosely connected by springs, and then colliding the two head-on. Do you expect the collision to be elastic, inelastic, or totally inelastic? (Think about what happens to the kinetic energy.)

(A) elastic

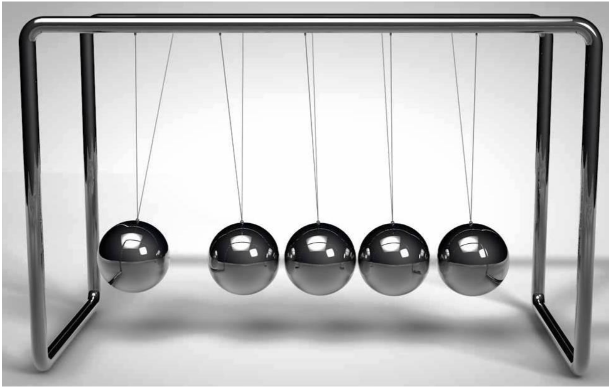
(B) inelastic

(C) totally inelastic

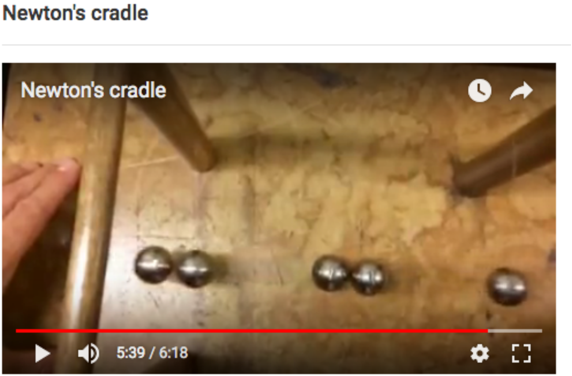
`http://youtu.be/SJIKCmg2Uzg`

"Newton's cradle:" what do you expect to happen if I pull back two of the spheres and release them?

What do you expect to happen if I put a piece of play dough between two of the spheres?

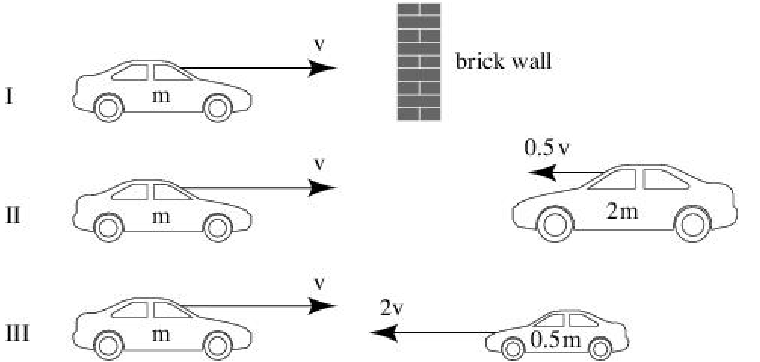


Newton’s cradle (slow motion video from my smartphone)



https://youtu.be/rrrs81pl_DU

If all three collisions in the figure shown here are totally inelastic, which bring(s) the car on the left to a halt?



(A) I

(B) II

(C) II,III

(D) all three

(E) III

Which of these systems are isolated?

(1) While slipping on ice, a car collides totally inelastically with another car. System: both cars (ignore friction)

(2) Same situation as in (a). System: slipping car

(3) A single car slips on a patch of ice. System: car

(4) A car brakes to a stop on a road. System: car

(5) A ball drops to Earth. System: ball

(6) A billiard ball collides elastically with another billiard ball on a pool table. System: both balls (ignore friction)

(A) (1) only

(B) (6) only

(C) (1) + (2) + (3) + (4) + (5)+ (6)

(D) (1) + (2) + (3) + (4) + (6)

(E) (1) + (3) + (6)

We've now spent a week watching two carts collide on low-friction tracks. Conservation of momentum lets us write one equation:

$$m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

Often we know both initial velocities, but we don't know either of the two unknown final velocities. So we have two unknowns. Energy adds a second equation, which usually involves **relative speed** $|v_{1x} - v_{2x}|$ of the two carts.

- elastic: $(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i})$
- totally inelastic: $(v_{1x,f} - v_{2x,f}) = 0$
- if e is given: $(v_{1x,f} - v_{2x,f}) = -e(v_{1x,i} - v_{2x,i})$
- if change in internal energy is given:

$$K_{1i} + K_{2i} + E_{i,\mathrm{internal}} = K_{1f} + K_{2f} + E_{f,\mathrm{internal}}$$

Let's try using these results.

Two carts, of inertias (masses) $m_1 = 1.0\ \mathrm{kg}$ and $m_2 = 1.0\ \mathrm{kg}$, collide head-on on a low-friction track. Before the collision, which is elastic, cart 1 is moving to the right at 1.0 m/s and cart 2 is at rest. What are the two carts' final velocities?

```
ClearAll["Global`*"];
m1 = 1.0; m2 = 1.0; v1xi = +1.0; v2xi = 0.0;
Reduce[{
   m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
   (v1xf - v2xf) == -(v1xi - v2xi)
  }]

v2xf == 1. && v1xf == 0.
```

Two carts, of inertias $m_1 = 1.0\ \mathrm{kg}$ and $\boxed{m_2 = 9.0\ \mathrm{kg}}$, collide head-on on a low-friction track. Before the collision, which is elastic, cart 1 is moving to the right at 1.0 m/s and cart 2 is at rest. What are the two carts' final velocities?

```
ClearAll["Global`*"];
m1 = 1.0; m2 = 9.0; v1xi = +1.0; v2xi = 0.0;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  (v1xf - v2xf) == -(v1xi - v2xi)
 }]

v2xf == 0.2 && v1xf == -0.8
```

Digression: notice what happens if I change the 1:9 ratio of masses into a 1:14 ratio, which you may see in an upcoming worksheet. (You'll only need to sketch, not solve with equations).

```
ClearAll["Global`*"];
m1 = 1; m2 = 14; v1xi = +1; v2xi = 0;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  (v1xf - v2xf) == -(v1xi - v2xi)
 }]
```

$$v2xf == \frac{2}{15} \;\&\&\; v1xf == -\frac{13}{15}$$

```
N[%]
```

v2xf == 0.133333 && v1xf == −0.866667

Two carts, of inertias $m_1 = 1.0 \text{ kg}$ and $m_2 = 9.0 \text{ kg}$, collide head-on on a low-friction track. Before the collision, which is **totally inelastic**, cart 1 is moving to the right at 1.0 m/s and cart 2 is at rest. What are the two carts' final velocities?

```
ClearAll["Global`*"];
m1 = 1.0; m2 = 9.0; v1xi = +1.0; v2xi = 0.0;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  (v1xf - v2xf) == 0
 }]

v2xf == 0.1 && v1xf == 0.1
```

Two carts, of inertias $m_1 = 1.0\ \mathrm{kg}$ and $m_2 = 1.0\ \mathrm{kg}$, collide head-on on a low-friction track. Before the collision, cart 1 is moving to the right at 2.0 m/s and cart 2 is moving to the left at 2.0 m/s. After the collision, cart 1 is moving to the left at 1.0 m/s and cart 2 is moving to the right at 1.0 m/s.

Let "the system" be cart 1 + cart 2. With the given values, is the system's total momentum the same before and after the collision?

What is the coefficient of restitution, e, for this collision?

Initial and final momentum are both zero, as you can verify. The relative speed of the two objects is reduced by a factor $e = 0.5$.

A system consists of two 1.00 kg carts attached to each other by a compressed spring. Initially, the system is at rest on a low-friction track. When the spring is released, internal energy that was initially stored in the spring is converted into kinetic energy of the carts. The change in the spring's internal energy during the separation is 4.00 joules. What are the two carts' final velocities?

```
ClearAll["Global`*"];
m1 = 1.00; m2 = 1.00; v1xi = 0.0; v2xi = 0.0;
Eispring = 4.00; Efspring = 0.00;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  1/2 m1 v1xi^2 + 1/2 m2 v1xi^2 + Eispring == 1/2 m1 v1xf^2 + 1/2 m2 v2xf^2 + Efspring,
  v2xf > 0
 }]

v2xf == 2. && v1xf == -2.
```

A system consists of a 2.00 kg cart and a 1.00 kg cart attached to each other by a compressed spring. Initially, the system is at rest on a low-friction track. When the spring is released, an explosive separation occurs at the expense of the internal energy of the compressed spring. If the decrease in the spring's internal energy during the separation is 10.0 J, what is the speed of each cart right after the separation?

Since the two-cart system is isolated, what equation can we write down?

Since the spring's internal energy is converted into the carts' kinetic energies, we can account for the initial and final energies of the cart + spring + cart system and can see that this system is closed. (No energy goes in or out of the system.) What second equation can we write down?

```
ClearAll["Global`*"];
m1 = 2.0; m2 = 1.0;
Reduce[{
  10.0 == 0.5 m1 v1xf^2 + 0.5 m2 v2xf^2,
  0 == m1 v1xf + m2 v2xf,
  v2xf > 0
 }]
```

```
v2xf == 3.65148 && v1xf == -1.82574
```

A battery-powered car, with bald tires, sits on a sheet of ice. Friction between the bald tires and the ice is negligible. The driver steps on the accelerator, but the wheels just spin (frictionlessly) on the ice without moving the car. Is the car an isolated system (considering only the coordinate along the car's axis) — i.e. does nothing outside the system push/pull on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

(A) Closed but not isolated.

(B) Isolated but not closed.

(C) Both closed and isolated.

(D) Isolated: yes. Closed: very nearly so, yes.

(E) Neither closed nor isolated.

A battery-powered Aston Martin car, with James-Bond-like spiked tires, sits on a sheet of ice. Agent 007 (or maybe it is really Austin Powers?) steps on the pedal, and the car accelerates forward. Is the car an isolated system (considering only the coordinate along the car's axis), i.e. nothing outside the system pushes/pulls on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

(A) Closed but not isolated.

(B) Isolated: no. Closed: very nearly so, yes.

(C) Isolated but not closed.

(D) Both closed and isolated.

(E) Neither closed nor isolated.

A battery-powered Aston Martin car, with James-Bond-like spiked tires, sits on a sheet of ice. Agent 007 steps on the accelerator, and the car accelerates forward. All the while, a high-tech solar panel on the car's roof rapidly charges the car's battery. Is the car an isolated system (considering only the coordinate along the car's axis), i.e. nothing outside the system pushes/pulls on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

(A) Closed but not isolated.

(B) Isolated but not closed.

(C) Both closed and isolated.

(D) Neither closed nor isolated.

A battery-powered Aston Martin, with James-Bond-like spiked tires, sits atop an iceberg that floats in the North Sea. Agent 007 steps on the accelerator, and the car accelerates forward. (What happens to the iceberg?) All the while, a high-tech solar panel on the car's roof rapidly charges the car's battery. Ignore any friction (or viscosity, drag, etc.) between the water and the iceberg. Which statement is true?

(A) "Car alone" system is isolated but not closed.

(B) "Car + iceberg" system is isolated but not closed.

(C) "Iceberg alone" system is isolated but not closed.

(D) "Car alone" system is isolated and closed.

(E) "Car + iceberg" system is isolated and closed.

(F) "Iceberg alone" system is isolated and closed.

(G) None of the above.

- An **isolated** system has no mechanism for momentum to get in/out of the system from/to outside of the system. This means nothing outside of the system can push/pull on anything inside of the system. (Later this week, we'll say: "no external forces act on the system.")
- This will make more sense when we discuss *forces*, next time!
- A hugely important idea in physics is that if the parts of a system interact only with each other (do not push/pull on anything outside of the system), then the total momentum of that system does not change.
- A **closed** system has no mechanism for energy to get in/out of the system. Examples so far are contrived, but soon we will learn to calculate energy stored in springs, energy stored in Earth's gravitational field, etc. The concept of a closed system is much more useful once we learn how to account for the many ways energy can be stored.
- Accounting for movement of energy in/out of a system will make more sense when we discuss *work,* just after forces.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. What system is isolated?

(A) One cart.

(B) Cart + spring + other cart.

(C) One cart plus the spring.

(D) None of the above.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. Is the cart + spring + other cart system closed?

(A) Yes, for all practical purposes, because the system's total energy $K_1 + K_2 + E_{\mathrm{spring}}$ is the same before and after releasing the spring, and other tiny transfers of energy (escaping sound, etc.) are negligible by comparison.

(B) No.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. Is the spring alone a closed system?

(A) Yes.

(B) No, because it transferred energy to the carts, which are outside of what you're now calling "the system."

I put two carts on a low-friction track, with a lighted firecracker between them. The firecracker explodes, which sets the carts moving apart. Is the cart + firecracker + other cart system closed?

(A) Yes, by analogy with the cart + spring + cart system.

(B) Yes, for some other reason.

(C) No, because realistically, some of the firecracker's energy will escape in the form of heat, flying debris, etc. So really energy conservation only provides an upper limit on $K_1 + K_2$ after the explosion, because accounting for where the energy goes is more difficult here than for a simple spring.

(D) No, for some other reason.

(E) I still don't understand what "closed" means.