

- ▶ begin video preceding ws08
- ▶ If you have time, you might consider looking over Mazur ch08.

## Chapter 8: Force

- ▶ Forces **always** come in pairs: when A and B interact,

$$\vec{F}_{A \text{ on } B} = - \vec{F}_{B \text{ on } A}$$

- ▶ “Interaction pairs” have equal magnitude, opposite direction. **Always.** That’s called **Newton’s third law**. Difficult idea!
- ▶ The acceleration of object A is given by vector sum of all of the forces acting **ON** object A, divided by  $m_A$ . (Law #2.)

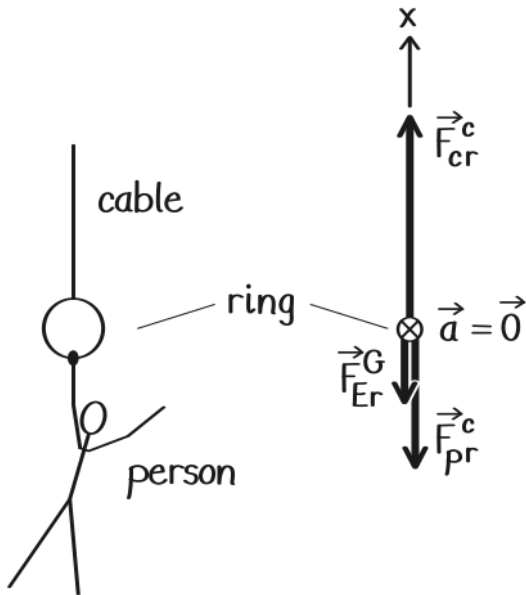
$$\vec{a}_A = \frac{1}{m_A} \sum \vec{F}_{\text{on } A}$$

- ▶ In an inertial frame of reference, object A moves at constant velocity (or stays at rest) if and only if the vector sum ( $\sum \vec{F}_{\text{on } A}$ ) equals zero. (Law #1.) #1 seems redundant?!

You push with a steady force of 25 N on a 50 kg desk fitted with (ultra-low-friction) casters on its four legs. How long does it take you (starting from rest) to get the desk across a room that is 25 m wide?

- (A) 0.71 s
- (B) 1.0 s
- (C) 1.4 s
- (D) 5.0 s
- (E) 7.1 s
- (F) 10 s
- (G) 14 s

**Free-body diagram:** A sort of visual accounting procedure for adding up the forces acting **ON** a given object. FBD for ring:



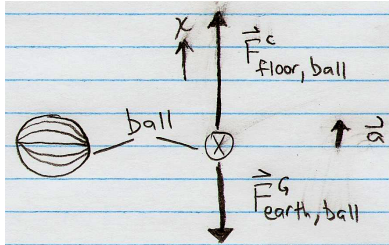
## Chapter 8 (“force”) reading Q #1

“Think about the familiar example of a basketball dropped from eye level and allowed to bounce a few times. Describe the forces acting on the basketball at its lowest point, as it is in contact with the floor and is changing direction from downward to upward motion.”

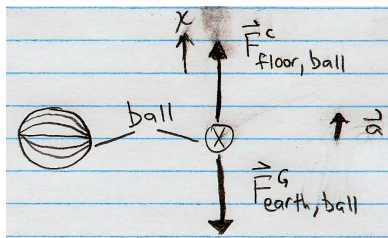
- ▶ Draw a free-body diagram of the ball at its lowest point (while it is most squished). Include all forces acting ON the ball. Indicate the direction of each force with its vector arrow. Indicate the relative magnitudes of the forces by the lengths of the arrows. Indicate the direction of the ball’s acceleration with an arrow (or a dot).
- ▶ When you finish that, draw a second free-body diagram for the ball — this time while the ball is in the air. Will the diagram be different while the ball is rising vs. falling?

**(My diagram appears on the next slide.)**

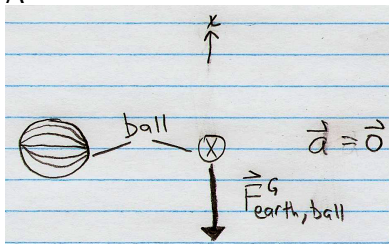
Which free-body diagram best represents the forces acting on the basketball at the *bottom* of its motion?



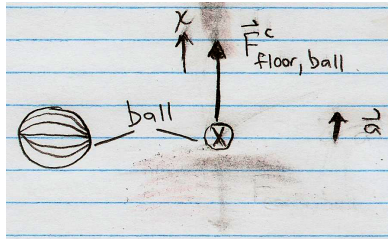
A



B

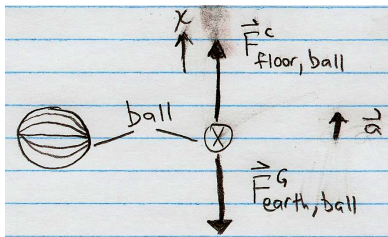


C

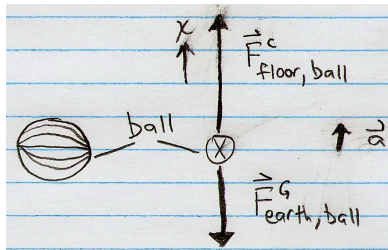


D

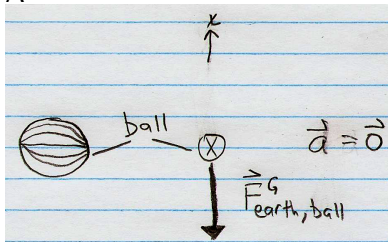
Which free-body diagram best represents the forces acting on the basketball at the *top* of its motion?



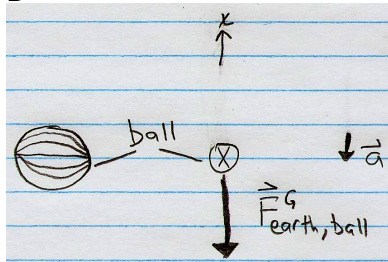
A



B



C



D

If I were to draw a free-body diagram for the basketball when it is halfway back down to the ground, that new diagram would be

- (A) the same as
- (B) slightly different from
- (C) very different from

the drawing for the basketball when it is at the top of its motion?  
(Neglect air resistance.)



## Equal and opposite forces?

Consider a car at rest on a road. We can conclude that the downward gravitational pull of Earth on the car and the upward contact force of the road on the car are equal and opposite because

- (A) the two forces form an interaction pair.
- (B) the net force on the car is zero.
- (C) neither: the two forces are not equal and opposite
- (D) both (A) and (B)

## Chapter 8 (“force”) reading Q #2

“Explain briefly in your own words what it means for the interaction between two objects to involve ‘equal and opposite’ forces. Can you illustrate this with an everyday example?”

- ▶ For instance, if I push against some object  $O$  that moves, deforms, or collapses in response to my push, is the force exerted by  $O$  on me still equal in magnitude and opposite in direction to the force exerted by me on  $O$ ?
- ▶ If every force is paired with an equal and opposite force, why is it ever possible for any object to be accelerated? Don't they all just cancel each other out?
- ▶ (I think the next example may help.)

Have you ever spotted the Tropicana juice train?!



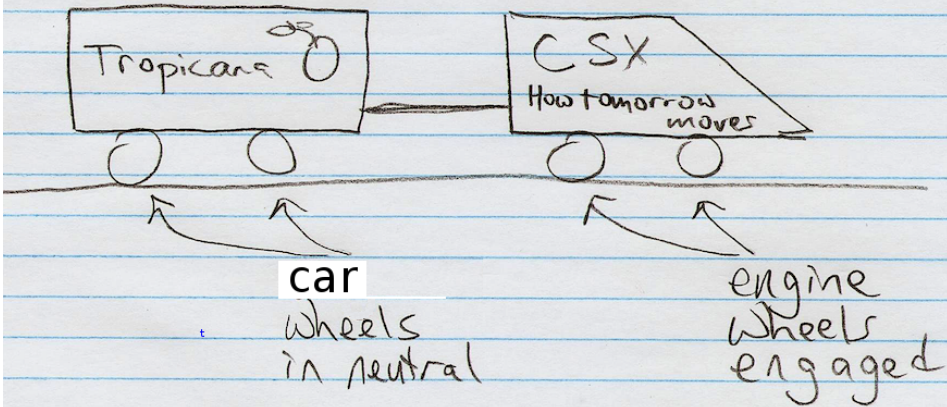
vocab: powered “locomotive” pulls the unpowered “cars”



## Equal and opposite forces?

An engine (“locomotive”) (the first vehicle of the train) pulls a series of train cars. Which is the correct analysis of the situation?

- (A) The train moves forward because the locomotive pulls forward slightly harder on the cars than the cars pull backward on the locomotive.
- (B) Because action always equals reaction, the locomotive cannot pull the cars — the cars pull backward just as hard as the locomotive pulls forward, so there is no motion.
- (C) The locomotive gets the cars to move by giving them a tug during which the force on the cars is momentarily greater than the force exerted by the cars on the locomotive.
- (D) The locomotive’s force on the cars is as strong as the force of the cars on the locomotive, but the frictional force by the track on the locomotive is forward and large while the backward frictional force by the track on the cars is small.
- (E) The locomotive can pull the cars forward only if its inertia (i.e. mass) is larger than that of the cars.



Let's see the effect of including or not including the frictional force of the tracks pushing forward on the wheels of the engine.

I'll pretend to be the engine!

Only *external* forces can accelerate a system's CoM

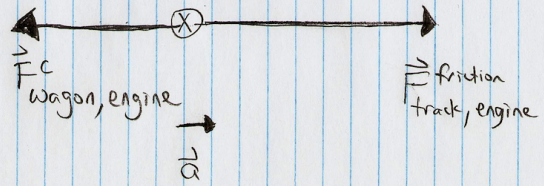
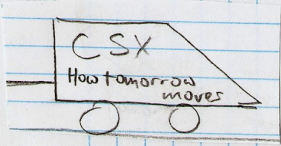
Let's define "system" to be locomotive+car.  
Remember that forces internal to system cannot accelerate system's CoM.

To change the velocity of the CoM, we need a force that is *external* to the system.

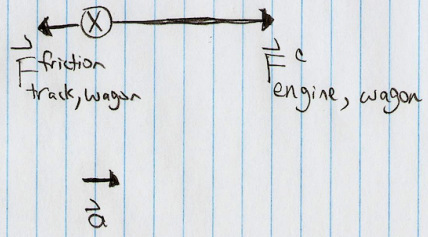
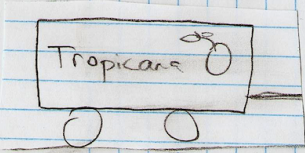
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(By the way, when you look at the two free-body diagrams on the next page, tell me if you see an "interaction pair" of forces somewhere!)

engine (a.k.a. "locomotive")



wagon (a.k.a. "car")





$$\vec{a}_{\text{CoM}} = \frac{\sum \vec{F}^{\text{external}}}{m_{\text{total}}}$$

It's useful to remember that even if the several pieces of a system are behaving in a complicated way, you can find the acceleration of the CoM of the system by considering only the **external** forces that act **on** the system.

Once again, a careful choice of “system” boundary often makes the analysis much easier. We'll see more examples of this soon. (This topic also arises in upcoming worksheets, so we'll try to practice it here first.)

(digression)

Thanks to a 2019 student, here's a neat video showing that the CoM of a dropped slinky falls at acceleration  $g$ , even though the top and bottom of the slinky do not move in unison:

<https://www.youtube.com/watch?v=eCMmmEEy000&t=43>

super-sized version (harder to see than original version):

[https://www.youtube.com/watch?v=JsytNJ\\_pSf8&t=88](https://www.youtube.com/watch?v=JsytNJ_pSf8&t=88)

## Hooke's law

- ▶ When you pull on a spring, it stretches
- ▶ When you stretch a spring, it pulls back on you
- ▶ When you compress a spring, it pushes back on you
- ▶ For an ideal spring, the pull is proportional to the stretch
- ▶ Force **by** spring, **on** load is

$$F_x = -k (x - x_0)$$

- ▶ The constant of proportionality is the “spring constant”  $k$ , which varies from spring to spring. When we talk later about properties of building materials, we'll see where  $k$  comes from.
- ▶ The minus sign indicates that if I move my end of the spring to the right of its relaxed position, the force exerted by the the spring on my finger points left.

Let's look at some examples of springs.

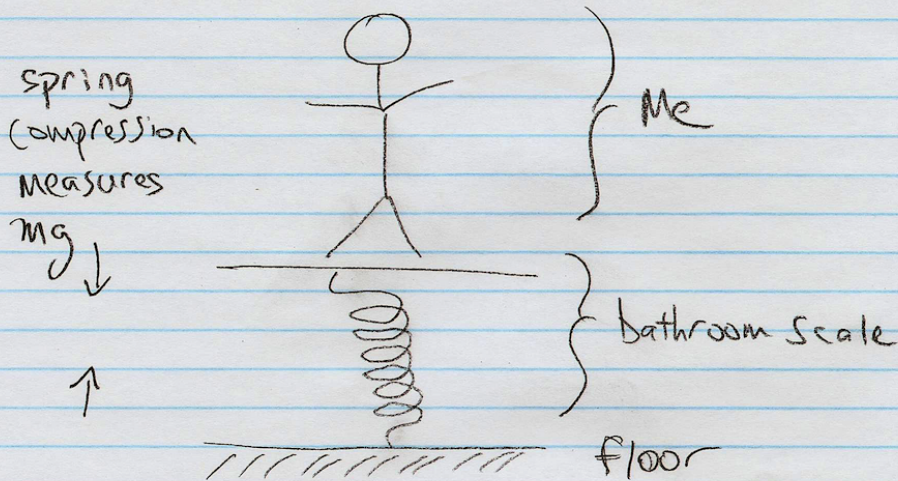
A spring hanging from the ceiling is 1.0 m long when there is no object attached to its free end. When a 4.0 kg brick is attached to the free end, the spring is 2.0 m long. (For easier math, use  $g = 10 \text{ m/s}^2 = 10 \text{ N/kg}$ .) What is the spring constant of the spring?

[Hint: draw a FBD for the brick, to figure out what magnitude force the spring must be exerting on the brick. The magnitude of the force exerted by the spring is the spring constant ( $k$ ) times how far the spring is stretched w.r.t. its relaxed length.]

- (A) 5.0 N/m
- (B) 10 N/m
- (C) 20 N/m
- (D) 30 N/m
- (E) 40 N/m

Measuring your weight ( $F = mg$ ) with a spring scale

Most bathroom scales work something like this:



Now suppose I take my bathroom scale on an elevator ...

## Bathroom scale on an accelerating elevator

A bathroom scale typically uses the compression of a spring to infer the gravitational force ( $F = mg$ ) exerted by Earth on you, which we call your *weight*.

Suppose I am standing on such a scale while riding an elevator. With the elevator parked at the bottom floor, the scale reads 700 N. I push the button for the top floor. The door closes. The elevator begins moving upward. At the moment when I can feel (e.g. in my stomach) that the elevator has begun moving upward, the scale reads

- (A) a value smaller than 700 N.
- (B) the same value: 700 N.
- (C) a value larger than 700 N.

You might want to try drawing a free-body diagram for your body, showing the downward force of gravity, the upward force of the scale pushing on your feet, **and your body's acceleration.**

## Tension vs. compression

- ▶ When a force tries to squish a spring, that is called *compression*, or a compressive force
- ▶ When a force tries to elongate a spring, that is called *tension*, or a tensile force
- ▶ We'll spend a lot of time next month talking about compression and tension in columns, beams, etc.
- ▶ For now, remember that tension is the force trying to pull apart a spring, rope, etc., and compression is the force trying to squeeze a post, a basketball, a mechanical linkage, etc.

## hooray – we are finally talking about forces

- ▶ The **force** concept quantifies interaction between two objects.
- ▶ Forces always come in “**interaction pairs.**” The force exerted by object “A” on object “B” is equal in magnitude and opposite in direction to the force exerted by B on A:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

- ▶ The acceleration of object “A” is given by the vector sum of the forces acting **on** A, divided by the mass of A:

$$\vec{a}_A = \frac{\sum \vec{F}_{(\text{on } A)}}{m_A}$$

- ▶ The vector sum of the forces acting **on** an object equals the rate of change of the object’s momentum:

$$\sum \vec{F}_{(\text{on } A)} = \frac{d\vec{p}_A}{dt}$$



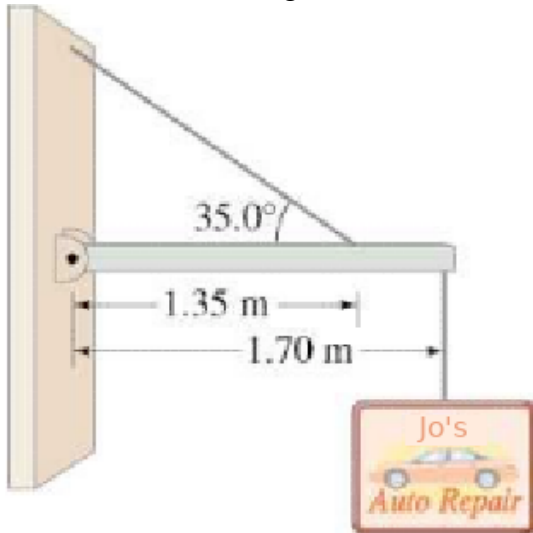
- ▶ An object whose momentum is not changing is in translational **equilibrium**. We'll see later that this will be a big deal for the members of a structure! To achieve this, we will want all forces acting **on** each member to sum vectorially to zero.
  - ▶ The unit of force is the **newton**.  $1 \text{ N} = 1 \text{ kg} \cdot \text{m}/\text{s}^2$ .
  - ▶ **Free-body diagrams** depict all of the forces acting **on** a given object. They are used all the time in analyzing structures!
- 

- ▶ The force exerted by a compressed or stretched spring is proportional to the displacement of the end of the spring w.r.t. its relaxed value  $x_0$ .  $k$  is “spring constant.”

$$F_x^{\text{spring}} = -k(x - x_0)$$

- ▶ When a rope is held taut, it exerts a force called the **tension** on each of its ends. Same magnitude  $T$  on each end.

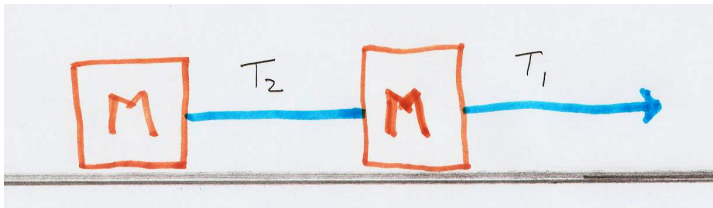
- ▶ A large category of physics problems (and even architectural structures, e.g. a suspension bridge) involves two objects connected by a rope, a cable, a chain, etc.
- ▶ These things (cables, chains, ropes) can pull but can't push. There are two cables in this figure:



## Tension in cables

- ▶ Usually the cables in physics problems are considered light enough that you don't worry about their inertia (we pretend  $m = 0$ ), and stiff enough that you don't worry about their stretching when you pull on them (we pretend  $k = \infty$ ).
- ▶ The cable's job is just to transmit a force from one end to the other. We call that force the cable's *tension*,  $T$ .
- ▶ A cable always pulls on both ends with same magnitude ( $T$ ), though in opposite directions. [Formally: we neglect the cable's mass, and the cable's acceleration must be finite.]
- ▶ E.g. hang basketball from ceiling. Cable transmits  $mg$  to ceiling. Gravity pulls ball down. Tension pulls ball up. Forces on ball add (vectorially) to zero.
- ▶ Let's try an example.

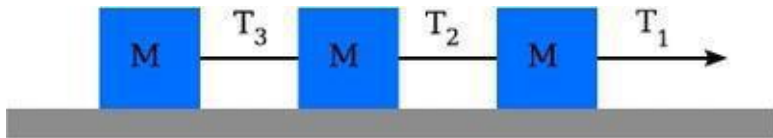
Two blocks of equal mass are pulled to the right by a constant force, which is applied by pulling at the arrow-tip on the right. The blue lines represent two identical sections of rope (which can be considered massless). Both cables are taut, and friction (if any) is the same for both blocks. What is the ratio of  $T_1$  to  $T_2$ ?



- (A) zero:  $T_1 = 0$  and  $T_2 \neq 0$ .
- (B)  $T_1 = \frac{1}{2}T_2$
- (C)  $T_1 = T_2$
- (D)  $T_1 = 2T_2$
- (E) infinite:  $T_2 = 0$  and  $T_1 \neq 0$ .

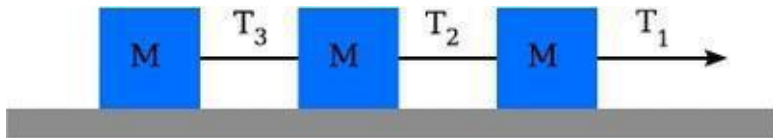
It's worth drawing an FBD first for the two-mass system, then for the left mass, then for the right mass.

Three blocks of equal mass are pulled to the right by a constant force. The blocks are connected by identical sections of rope (which can be considered massless). All cables are taut, and friction (if any) is the same for all blocks. What is the ratio of  $T_1$  to  $T_2$ ?



- (A)  $T_1 = \frac{1}{3} T_2$
- (B)  $T_1 = \frac{2}{3} T_2$
- (C)  $T_1 = T_2$
- (D)  $T_1 = \frac{3}{2} T_2$
- (E)  $T_1 = 2T_2$
- (F)  $T_1 = 3T_2$

Three blocks of equal mass are pulled to the right by a constant force. The blocks are connected by identical sections of rope (which can be considered massless). All cables are taut, and friction (if any) is the same for all blocks. What is the ratio of  $T_1$  to  $T_3$ ?



- (A)  $T_1 = \frac{1}{3} T_3$
- (B)  $T_1 = \frac{2}{3} T_3$
- (C)  $T_1 = T_3$
- (D)  $T_1 = \frac{3}{2} T_3$
- (E)  $T_1 = 2T_3$
- (F)  $T_1 = 3T_3$

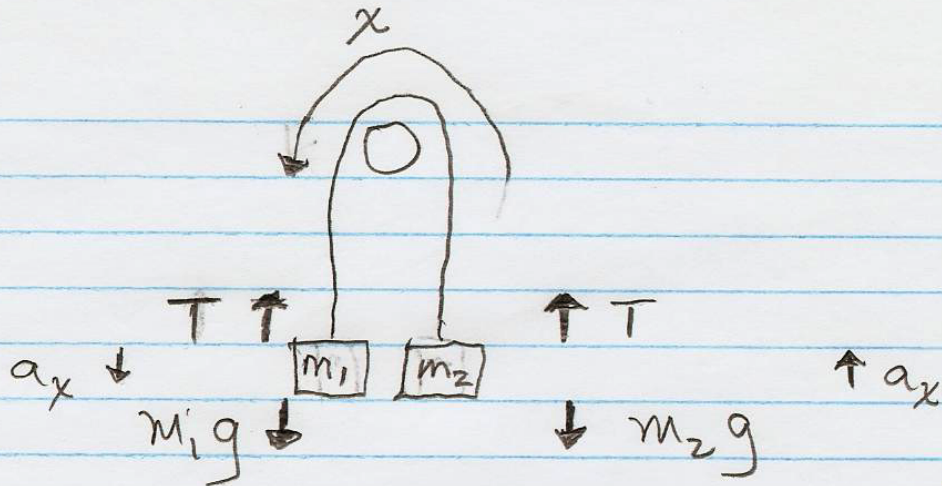
A contraption something like this Atwood machine appears in a worksheet (but with a spring added, to keep things interesting).

- ▶ Why aren't the two masses accelerating?
- ▶ What is the tension in the cable when the two masses are equal (both 5.0 kg) and stationary, as they are now?
- ▶ If I make one mass equal 5.0 kg and the other mass equal 5.1 kg, what will happen? Can you predict what the acceleration will be?
- ▶ If I make one mass equal 5.0 kg and the other mass equal 6.0 kg, will the acceleration be larger or smaller than in the previous case?
- ▶ Try drawing a free-body diagram for each of the two masses
- ▶ By how much do I change the gravitational potential energy of the machine+Earth system when I raise the 6 kg mass 1 m?

- ▶ Two more comments:
- ▶ This machine was originally invented as a mechanism for measuring  $g$  and for studying motion with constant acceleration.
- ▶ The same concept is used by the “counterweight” in an elevator for a building.

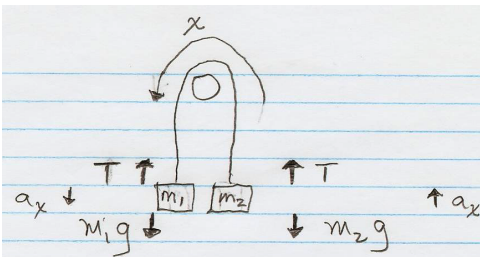


Atwood machine: take  $m_1 > m_2$



Pause here: how can we solve for  $a_x$ ? Try it before we go on.

## Atwood machine: write masses' equations of motion



$$m_1g - T = m_1a_x$$

$$T - m_2g = m_2a_x$$

Solve second equation for  $T$ ; plug  $T$  into first equation; solve for  $a_x$ :

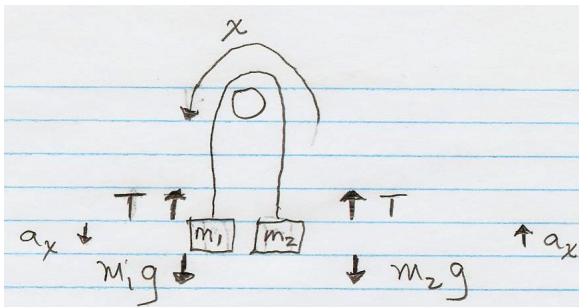
$$T = m_2a_x + m_2g \Rightarrow m_1g - (m_2a_x + m_2g) = m_1a_x \Rightarrow$$

$$(m_1 - m_2)g = (m_1 + m_2)a_x \Rightarrow \boxed{a_x = \frac{m_1 - m_2}{m_1 + m_2} g}$$

For  $m_2 = 0$ ,  $a_x = g$  (just like picking up  $m_1$  and dropping it)

For  $m_1 \approx m_2$ ,  $a_x \ll g$ : small difference divided by large sum.

$$a_x = \frac{m_1 - m_2}{m_1 + m_2} g$$



For example,  $m_1 = 4.03 \text{ kg}$ ,  $m_2 = 3.73 \text{ kg}$ :

$$a_x = \frac{m_1 - m_2}{m_1 + m_2} g = \left( \frac{0.30 \text{ kg}}{7.76 \text{ kg}} \right) (9.8 \text{ m/s}^2) = 0.38 \text{ m/s}^2$$

How long does it take  $m_1$  to fall 2 meters?

$$x = \frac{a_x t^2}{2} \Rightarrow t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{(2)(2 \text{ m})}{(0.38 \text{ m/s}^2)}} \approx 3.2 \text{ s}$$

You can also solve for  $T$  if you like (eliminate  $a_x$ ), to find the tension while the two masses are free to accelerate (no interaction with my hand or the floor).

Start from masses' equations of motion:

$$m_1g - T = m_1a_x, \quad T - m_2g = m_2a_x$$

Eliminate  $a_x$ :

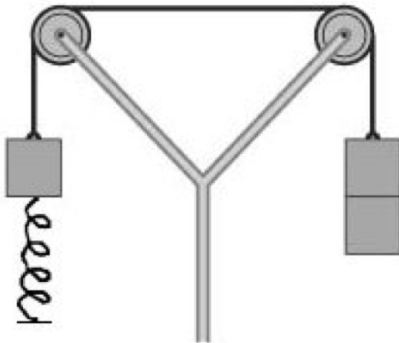
$$\frac{m_1g - T}{m_1} = \frac{T - m_2g}{m_2} \Rightarrow m_1m_2g - m_2T = m_1T - m_1m_2g$$

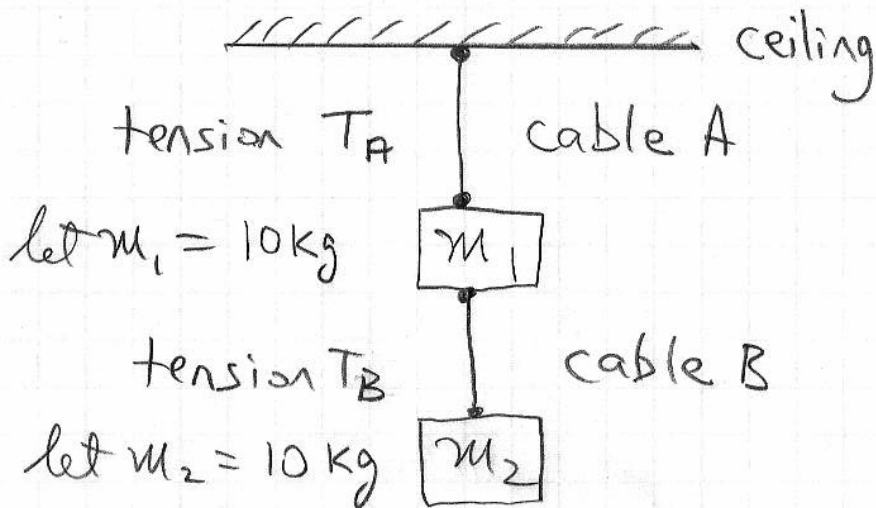
$$\Rightarrow 2m_1m_2g = (m_1 + m_2)T \Rightarrow T = \frac{2m_1m_2}{m_1 + m_2} g$$

consider extreme cases:  $m_2 = m_1$  vs.  $m_2 \ll m_1$ .

## worksheet problem: tricky!

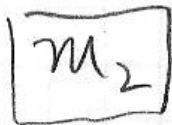
7\*. A modified Atwood machine is shown below. Each of the three blocks has the same inertia  $m$ . One end of the vertical spring, which has spring constant  $k$ , is attached to the single block, and the other end of the spring is fixed to the floor. The positions of the blocks are adjusted until the spring is at its **relaxed** length. The blocks are then released from rest. What is the acceleration of the two blocks on the right after they have fallen a distance  $D$ ?





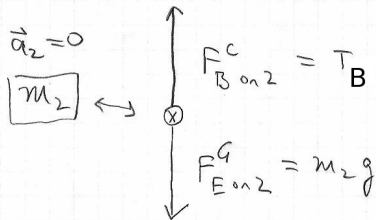
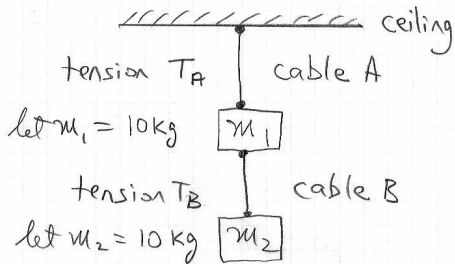
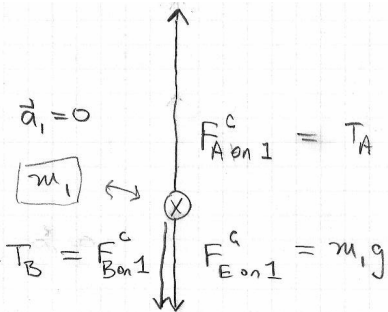
Draw a FBD for mass 2. Then draw a FBD for mass 1. Assume that  $\vec{a} = \vec{0}$  for both masses.

$$\vec{a}_2 = 0$$



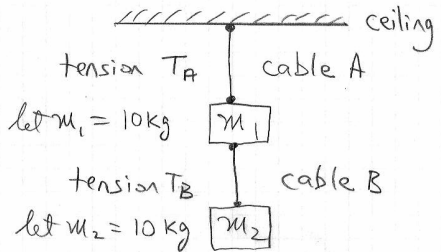
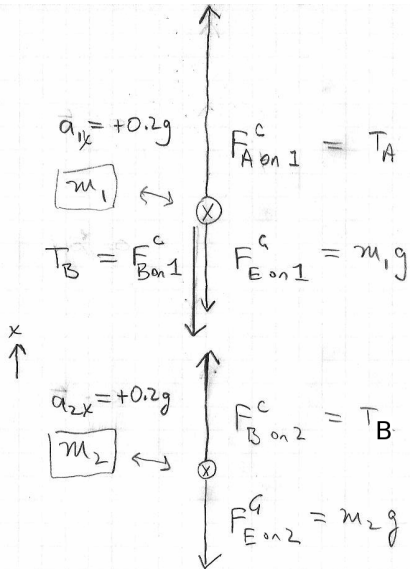
$$F_{B \text{ on } 2}^C = T_B$$

$$F_{E \text{ on } 2}^G = m_2 g$$

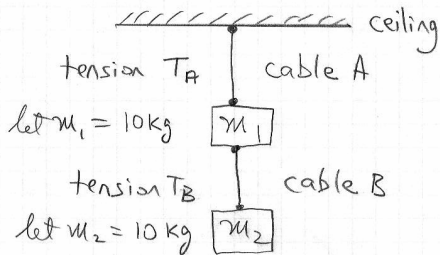
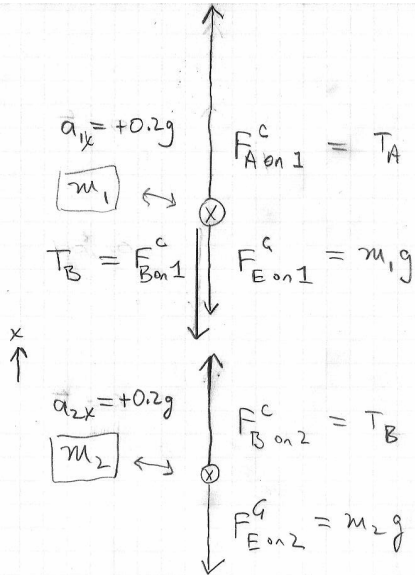


**Next:** How would these two diagrams change if we imagine that the ceiling is actually the ceiling of an elevator that is **accelerating upward** at  $a_x = +1.96 \text{ m/s}^2$  (that's  $0.2g$  — you can round off).





How do you use these two FBDs to write Newton's 2nd law for each of the two masses?



$$m_1 a_{1x} = T_A - m_1 g - T_B$$

$$m_2 a_{2x} = T_B - m_2 g$$

Note: because the length of an (**idealized**) taut cable doesn't change as its tension increases,  $a_{1x} = a_{2x}$ . Distance between blocks only changes if the cable goes slack (no longer in tension).

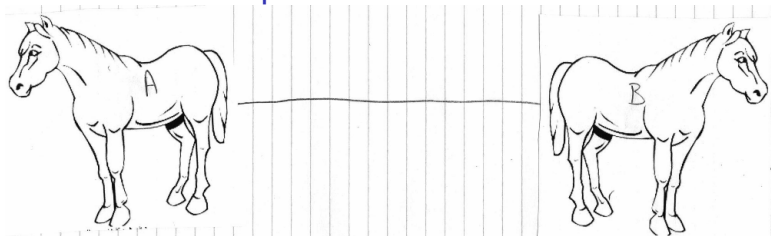
In the 17th century, Otto von Guericke, a physicist in Magdeburg, fitted two hollow bronze hemispheres together and removed the air from the resulting sphere with a pump. Two eight-horse teams could not pull the halves apart even though the hemispheres fell apart when air was readmitted. Suppose von Guericke had tied both teams of horses to one side and bolted the other side to a giant tree trunk. In this case, the tension on the hemispheres would be

- (A) twice
- (B) exactly the same as
- (C) half

what it was before.

(To avoid confusion, you can replace the phrase “the hemispheres” with the phrase “the cable” if you like. The original experiment was a demonstraton of air pressure, but we are interested in tension.)

Suppose a horse can pull 1000 N



$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

$$|\vec{F}_{A \text{ on } B}| = |\vec{F}_{B \text{ on } A}| = 1000 \text{ N}$$

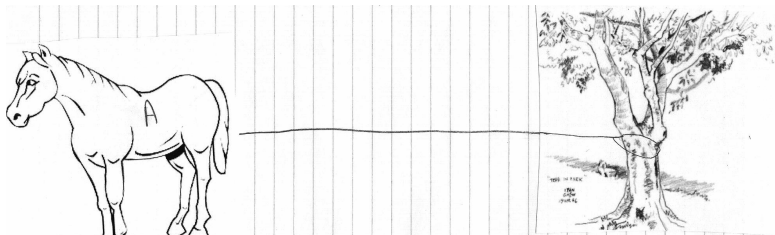
$$T = 1000 \text{ N}$$

$$\vec{a}_A = \vec{0}$$

$$\vec{a}_B = \vec{0}$$

The acceleration of each horse is zero. What are the two horizontal forces acting on horse A? What are the two horizontal forces acting on horse B?

Suppose tree stays put, no matter how hard horse pulls



$$\vec{F}_{A \text{ on tree}} = -\vec{F}_{\text{tree on } A}$$

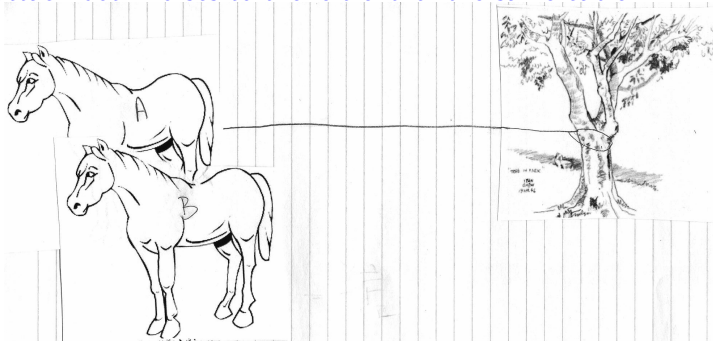
$$|\vec{F}_{A \text{ on tree}}| = |\vec{F}_{\text{tree on } A}| = 1000 \text{ N}$$

$$T = 1000 \text{ N}$$

$$\vec{a}_A = \vec{0}$$

What are the two horizontal forces acting on horse A?

Suppose tree stays put, no matter how hard horses pull. Somehow we attach both horses to the left end of the same cable.



$$\vec{F}_{A+B \text{ on tree}} = -\vec{F}_{\text{tree on } A+B}$$

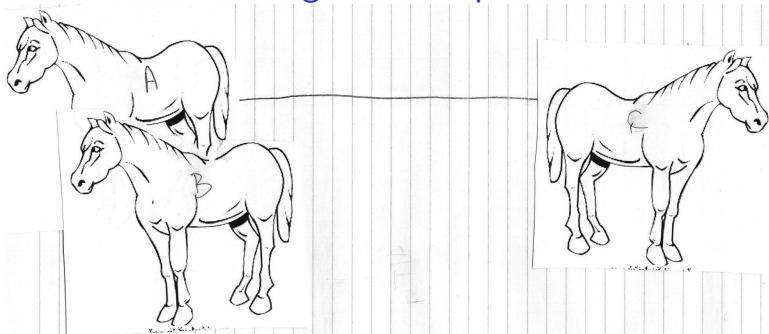
$$|\vec{F}_{A+B \text{ on tree}}| = |\vec{F}_{\text{tree on } A+B}| = 2000 \text{ N}$$

$$T = 2000 \text{ N}$$

$$\vec{a}_{\text{horses } A+B} = \vec{0}$$

What are the external forces acting on the two-horse system (system = horse A + horse B)?

Horse C loses his footing when he pulls  $> 1000 \text{ N}$



$$|\vec{F}_{A+B \text{ on } C}| = |\vec{F}_{C \text{ on } A+B}| = 2000 \text{ N}$$

$$T = 2000 \text{ N}$$

Force of ground on C is 1000 N to the right. Tension pulls on C 2000 N to the left. C accelerates to the left.

$$|\vec{a}_C| = (2000 \text{ N} - 1000 \text{ N})/m_C$$

Today, while we happen to have this rope attached to the ceiling, I want to re-visit something (related to forces) that I demonstrated on the first day of class. Believe it or not, this relates pretty directly to architecture.

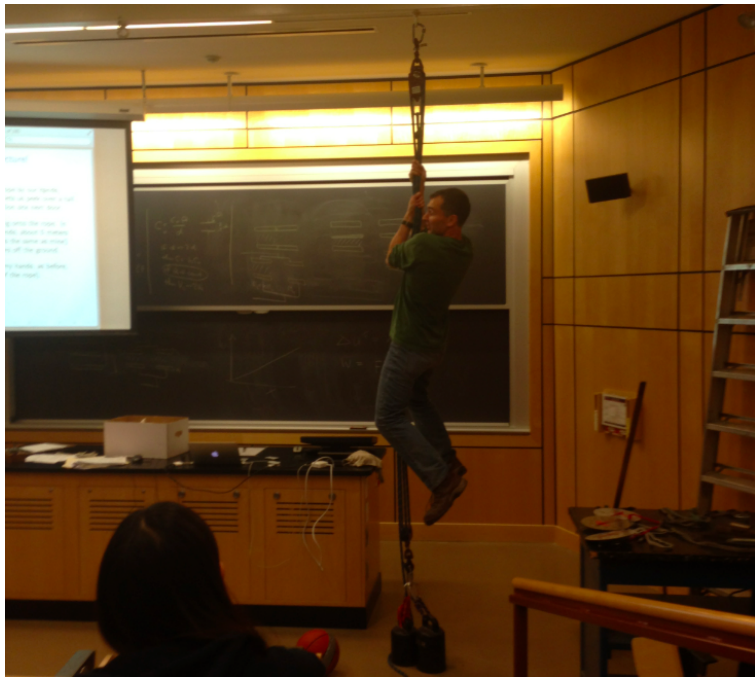
My friend and I both want to hang on to a rope by our hands, perhaps because being up above the ground lets us peek over a tall fence and see into an amazing new construction site next door.

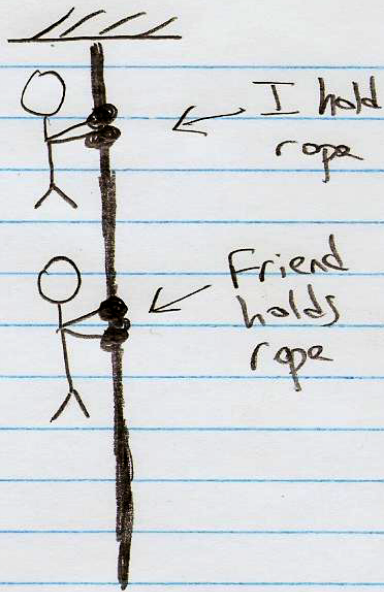
We consider two different methods of hanging onto the rope. In the first method, I hold the rope with my hands, about 5 meters off the ground, and my friend (whose mass is the same as mine) holds the rope with his hands, about 3 meters off the ground.

In the second method, I hold the rope with my hands, as before, and my friend holds onto my feet (instead of the rope).

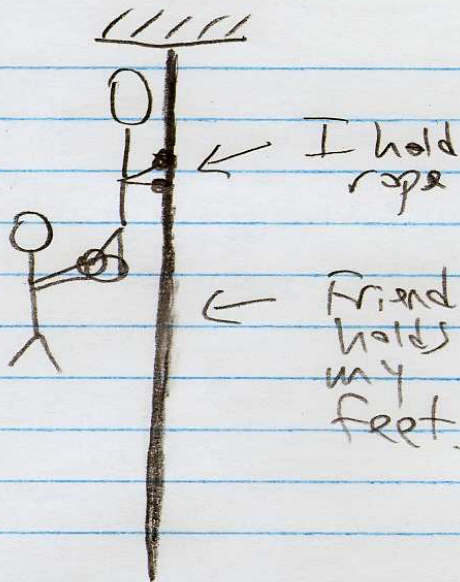
Let's draw a picture, to make it more clear.





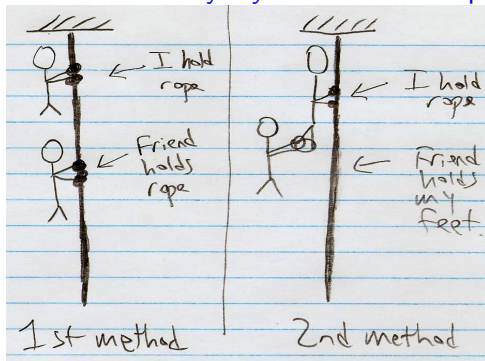


1st method



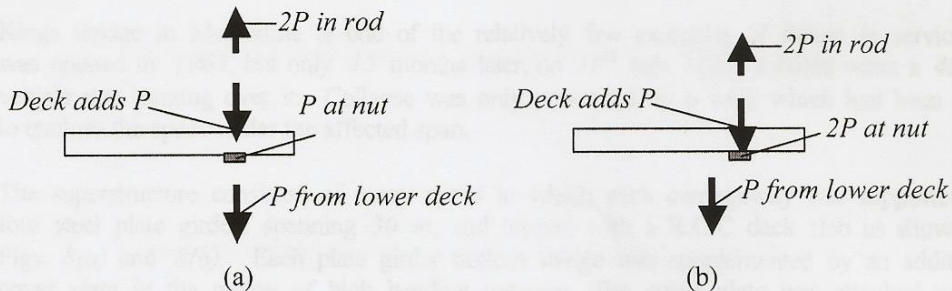
2nd method

The downward force exerted by my hands on the rope is ...



- (A) The same for both methods: equal to  $mg$  ( $m = \text{my mass}$ )
- (B) The same for both methods: equal to  $2mg$
- (C) Twice as much for 1st method ( $2mg$  vs.  $mg$ )
- (D) Twice as much for 2nd method ( $2mg$  vs.  $mg$ )

A real-world use for free-body diagrams! But these diagrams aren't careful to single out one object, to indicate clearly what that object is, and to draw only the forces acting ON that object. (Alas.)



**Fig. 6: Free-Body Diagram (a) As Designed (b) As Built**

The author uses the symbol  $P$  for a “point” force (or point load, or a “concentrated load”), as is the custom in engineering and architecture. When you see “ $P$ ” here, pretend it says “ $F$ ” or “ $mg$ ” instead.

## worksheet problem: slightly modified (skip?)

9\*. A tugboat pulls two barges (connected in series, like a train, with taut ropes as couplings) down a river. The barge connected to the tugboat, carrying coal, has inertia  $m_1$ . The other barge, carrying pig iron, has inertia  $m_2$ . The frictional force exerted by the water on the coal barge is  $F_{w1}^f$ , and that exerted by the water on the pig-iron barge is  $F_{w2}^f$ . The common acceleration of all three boats is  $a_x$ . Even though the ropes are huge, the gravitational force exerted on them is negligible, as are the ropes' inertias. How can you solve for the tension in each rope?

## worksheet problem (modified): (skip?)

10\*. A red cart of mass  $m_{\text{red}}$  is connected to a green cart of mass  $m_{\text{green}}$  by a **relaxed** spring of spring constant  $k$ . The green cart is resting against a blue cart of mass  $m_{\text{blue}}$ . All are on a low-friction track. You push the red cart to the right, in the direction of the green cart, with a constant force  $F_{\text{you,green}}^c$ . (a) What is the acceleration of the center-of-mass of the three-cart system? (b) What is the acceleration of each cart **the instant you begin to push**? (c) What is the acceleration of each cart the instant when the spring is compressed a distance  $D$  with respect to its relaxed length?

(skip?)

Estimate the spring constant of your car springs. (Experiment: sit on one fender.)

(What do you think?)

(skip)

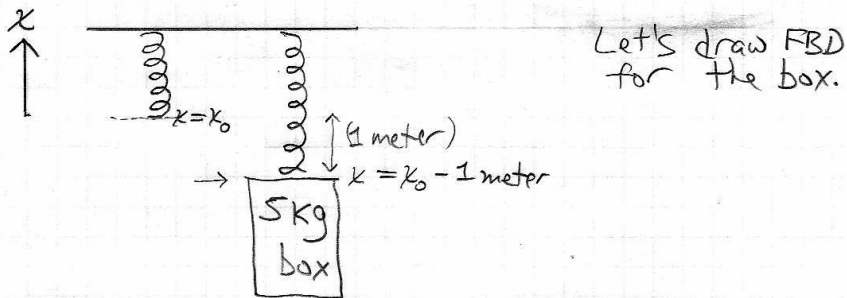
When a 5.0 kg box is suspended from a spring, the spring stretches to 1.0 m beyond its equilibrium length. In an elevator accelerating upward at  $0.98 \text{ m/s}^2$  (that's "0.1 g"), how far will the spring stretch with the same box attached?

- (A) 0.50 m
- (B) 0.90 m
- (C) 1.0 m
- (D) 1.1 m
- (E) 1.2 m
- (F) 1.9 m
- (G) 2.0 m

(By the way: When a tall building sways back and forth in the wind, the uncomfortable acceleration experienced by the occupants is often measured as a fraction of "g.")

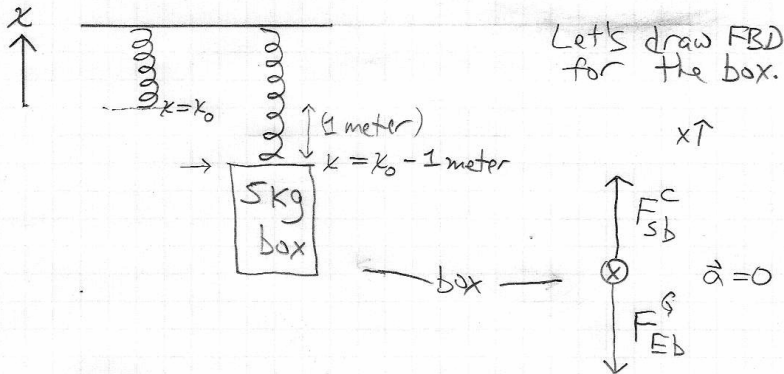


(skip)



Let's start by drawing a FBD for the box when the elevator is **not** accelerating.

(skip)



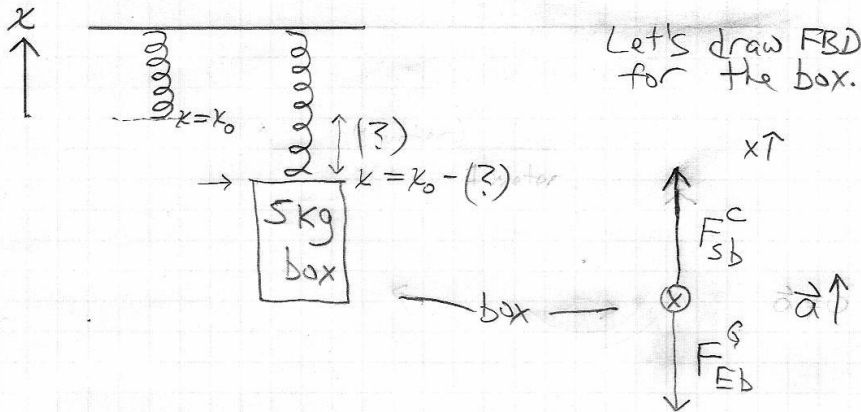
$$F_{sb,x}^c + F_{Eb,x}^g = ma_x = 0$$

$$F_{sb,x}^c = -k(x - x_0) = -k(-1 \text{ meter}) \quad F_{Eb,x}^g = -mg$$

$$+k(1 \text{ meter}) - mg = ma_x = 0$$

Next, what happens if elevator is accelerating upward at  $1 \text{ m/s}^2$ ?

(skip)



$$F_{sb,x}^c + F_{Eb,x}^g = ma_x = +1 \text{ m/s}^2$$

$$F_{sb,x}^c = -k(x - x_0) \quad F_{Eb,x}^g = -mg$$

$$-k(x - x_0) - mg = ma_x = +0.1mg$$

combine with  $+k(1 \text{ meter}) - mg = 0$  from last page

(skip)

$$-k(x - x_0) - mg = ma_x = +0.1g \Rightarrow \boxed{-k(x - x_0) = +1.1mg}$$

$$\text{combine with } +k(1 \text{ meter}) - mg = 0 \Rightarrow \boxed{+k(1 \text{ meter}) = mg}$$

Divide two boxed equations: get  $x - x_0 = -1.1 \text{ meters}$

So the spring is now stretching 1.1 meters beyond its relaxed length (vs. 1.0 meters when  $a_x = 0$ ).

The upward force exerted by the spring on the box is  $m(g + a_x)$ .

(skip)

When a 5.0 kg box is suspended from a spring, the spring stretches to 1.0 m beyond its equilibrium length. In an elevator accelerating upward at  $0.98 \text{ m/s}^2$ , how far will the spring stretch with the same box attached?

- (A) 0.50 m
- (B) 0.90 m
- (C) 1.0 m
- (D) 1.1 m
- (E) 1.2 m
- (F) 1.9 m
- (G) 2.0 m