(begin video covering Mazur ch10)

Things to understand before studying architectural structures:

- forces ✓ (but we will continue to use, all term!)
- vectors — (now)
- torques — (chapter 12)

- The next few chapters (10,11,12) are the most difficult material in the course. We will slow down for them. After that, the fun begins: we can apply our knowledge of forces (ch8), vectors (ch10), and torque (ch12) to structures.

I have an astonishing 86 slides of material for ch10, so I expect we'll split ch10 into two parts.

A Chapter 10 reading question:

Can an object be accelerated without changing its kinetic energy?

Answer: Yes. You can change an object's direction without changing its speed. So its velocity can change without changing its kinetic energy.

Over a finite time interval, this is easy to arrange.

Over an infinitessimal time interval, if the acceleration vector is perpendicular to the velocity vector, then direction changes, but speed does not. This will be important in Chapter 11!

Let's start with the familiar "ball-popper" cart

New (ch10): use two coordinate axes. In most cases, make y-axis point upward (vertical), and x-axis point to the right (horizontal).

Vertical equation of motion ($a_y = -g$ is constant):

$$y = y_i + v_{i,y}t - \frac{1}{2}gt^2$$

$$v_y = v_{i,y} - gt$$

Horizontal equation of motion ($v_x = v_{i,x}$ is constant):

$$x = x_i + v_{i,x}t$$

If you let $x_i = 0$ (simpler) and solve horizontal eqn. for t, you get

$$t = \frac{x}{v_{i,x}}$$

Now plug this into the equation for y ...

$$y = y_i + v_{i,y} t - \frac{1}{2} g t^2$$

Now plug $t = \frac{x}{v_{i,x}}$ into the equation for y:

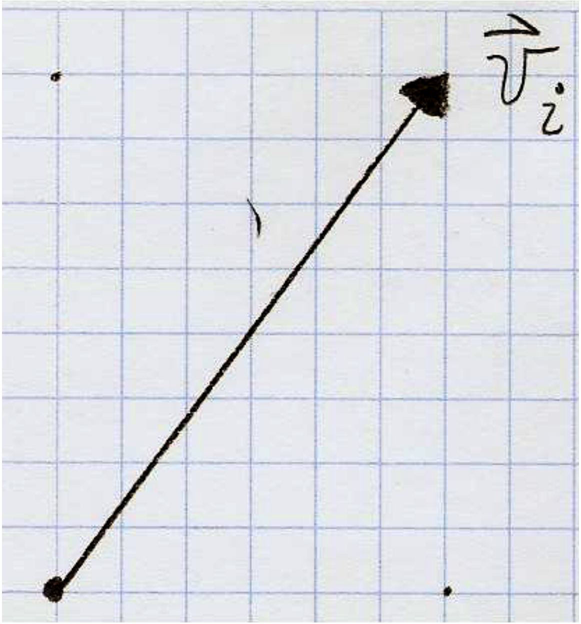
$$y = y_i + v_{i,y} \left(\frac{x}{v_{i,x}} \right) - \frac{1}{2} g \left(\frac{x}{v_{i,x}} \right)^2$$

Separate out the constants to see that $y(x)$ is a parabola:

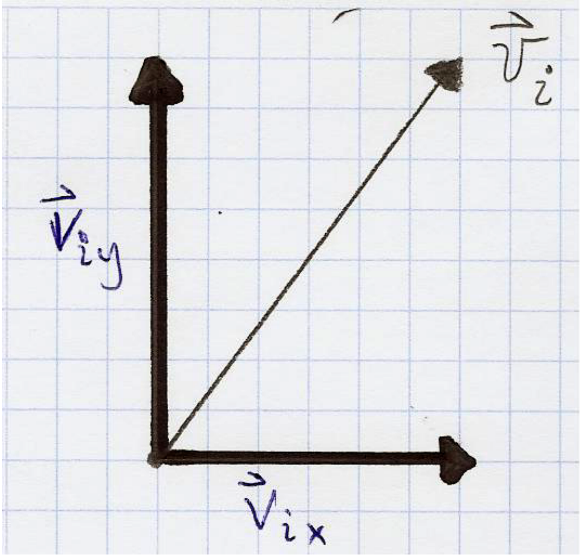
$$y = y_i + \left(\frac{v_{i,y}}{v_{i,x}} \right) x - \left(\frac{g}{2 v_{i,x}^2} \right) x^2$$

(You can "see" this either by drawing a graph or by happening to remember from math that $y = Ax^2 + Bx + C$ is a parabola.)

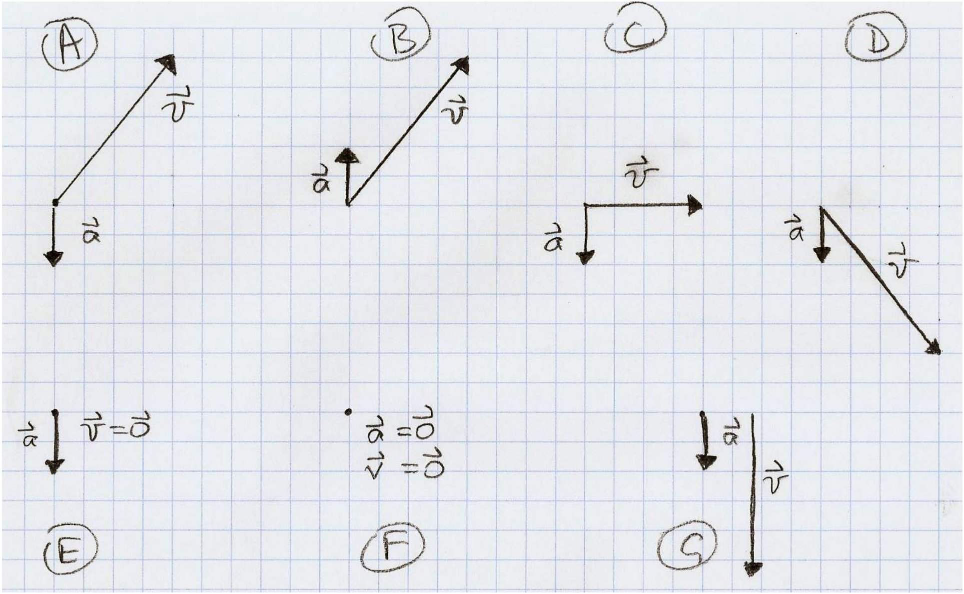
Let's draw and "decompose" the velocity vector at the moment the ball is launched from the cart.



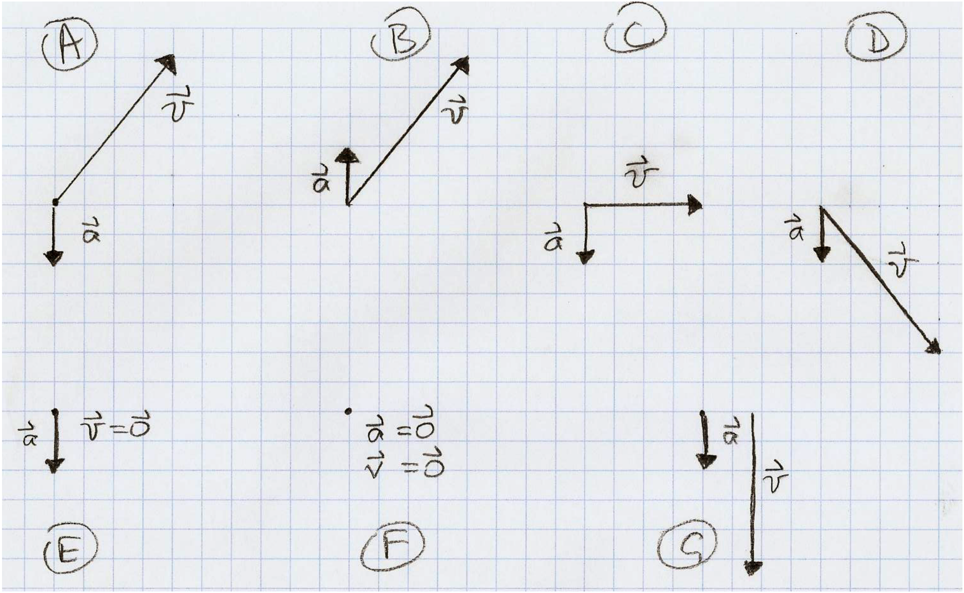
Now decompose into x and y components ...



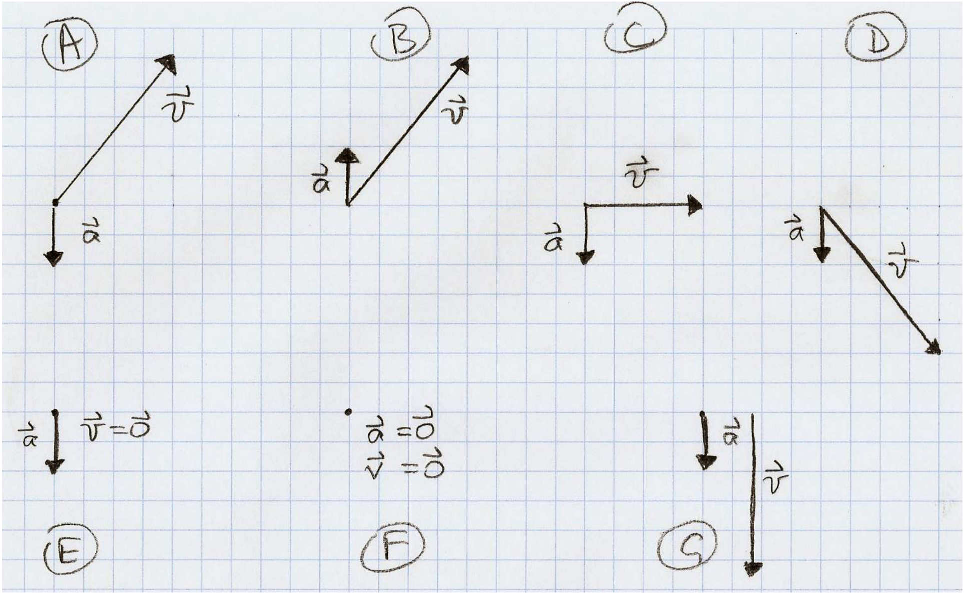
Notice (blackboard) that adding the two components together gives back the original vector.



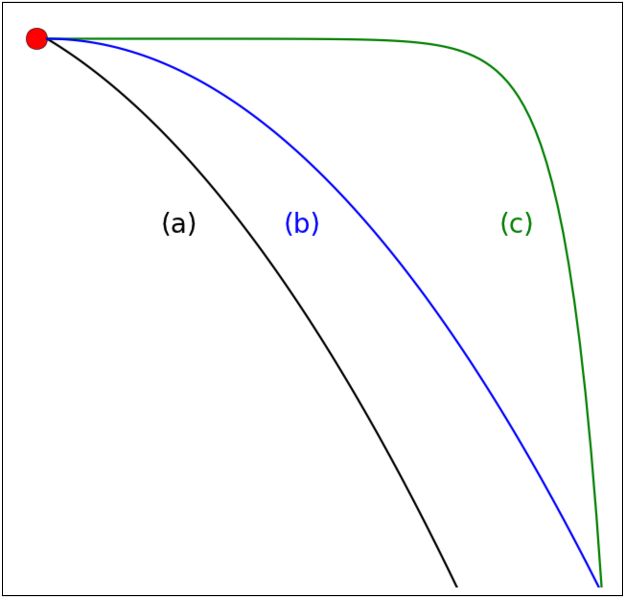
Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ the instant **after** the ball is launched from the cart?



Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ at the top of the ball's trajectory?

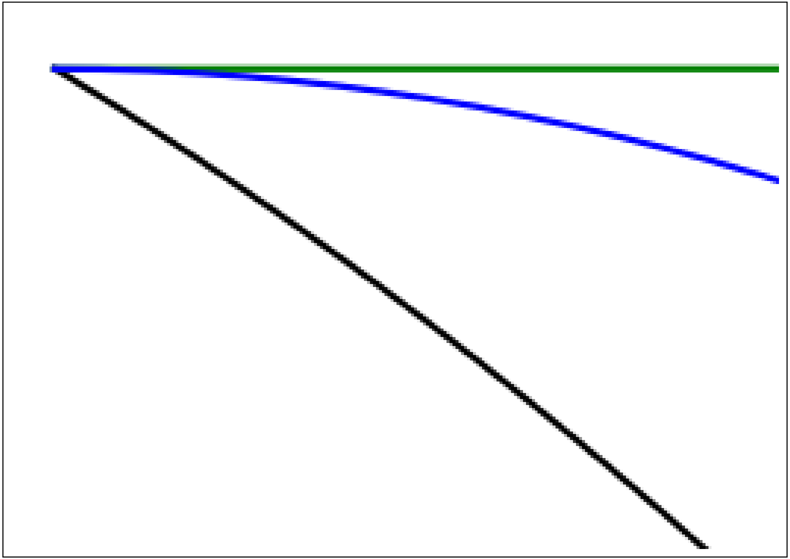


Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ the instant before the ball lands in the cart?



Which path best represents the trajectory of a cantaloupe *thrown horizontally* off a bridge? (What's wrong with the other two?)
(Next slide zooms in on corner.)

zoom in on top-left corner (launch position)



Which path best represents the trajectory of a cantaloupe *thrown horizontally* off a bridge? (What's wrong with the other two?)

Two steel balls are released simultaneously from the same height above the ground.

One ball is simply dropped (zero initial velocity).

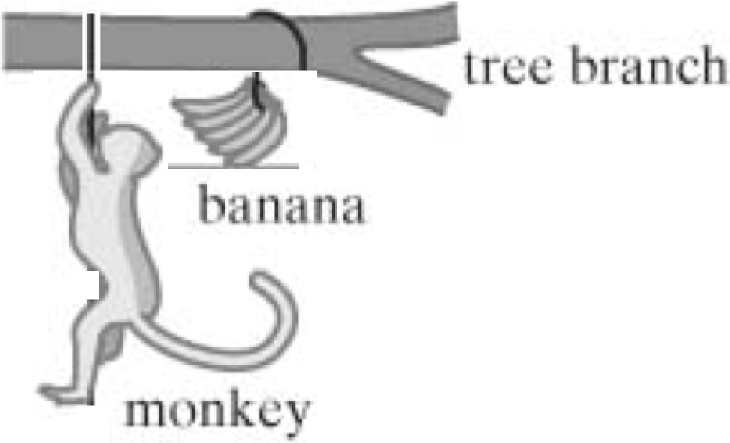
The other ball is thrown horizontally (initial velocity is nonzero, but is purely horizontal).

Which ball will hit the ground first?

(A) The ball thrown horizontally will hit the ground first.
(B) The ball released from rest will hit the ground first.
(C) Both balls will hit the ground at the same time.

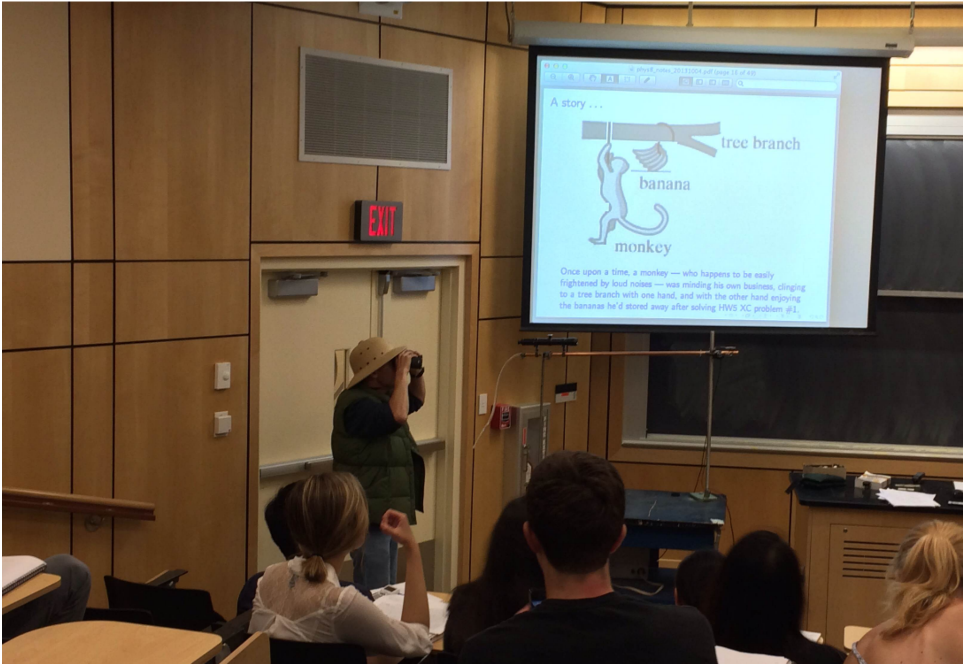
(I should draw a picture of both trajectories on the board.)

A story . . .



Once upon a time, a monkey — who happened to be easily frightened by loud noises — was minding his own business, clinging to a tree branch with one hand, and with the other hand enjoying the bananas he'd stored away after solving XC physics problems.

Look out . . .

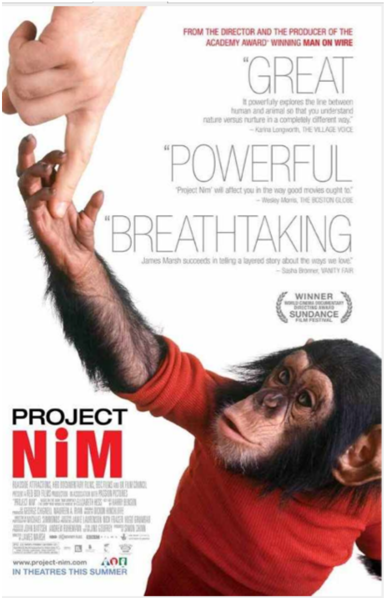

EXIT
A story . . .
tree branch
banana
monkey
Once upon a time, a monkey — who happens to be easily frightened by loud noises — was minding his own business, clinging to a tree branch with one hand, and with the other hand enjoying the bananas he'd stored away after solving HW5 XC problem #1.

Now let's move on to two questions of much more practical significance:

1. Should the "ecologist" shoot the "tranquilizer dart" at Nim Chimpsky, or at Mr. Bill? (She needs to collect a harmless DNA sample from one of these two characters for the Primate Genome Project.)

(A) Tranquilize Nim Chimpsky! (His DNA sample may explain why he was smart enough to learn all those words of American Sign Language.)

(B) Tranquilize Mr. Bill! (If you manage to find any real DNA in his sample, the result will definitely be a publishable paper, if not a Nobel Prize.)

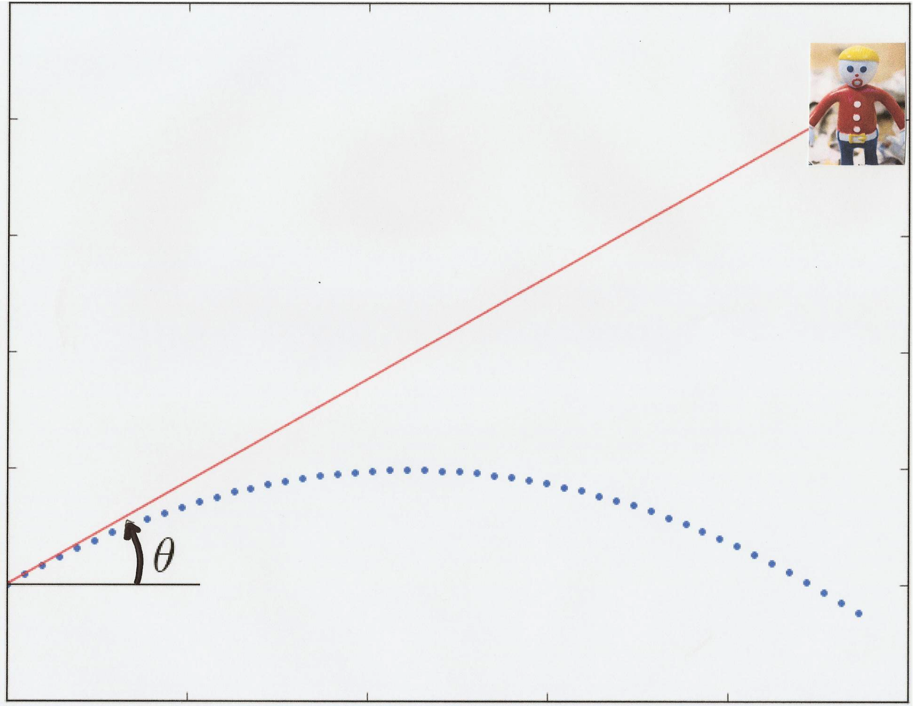


(A) study Nim Chimpsky.
(B) study Mr. Bill.

Now let's move on to two questions of much more practical significance:

1. Should the "ecologist" shoot the "tranquilizer pellet" at Nim Chimpsky, or at Mr. Bill?
2. It takes the pellet some time to travel across the width of the room.
   - In that time interval, gravity will cause Nim/Bill to fall.
   - So where should I aim the pea-shooter so that the pellet hits Nim/Bill as he drops?

Before you answer, let's explain in detail how this game works, why Nim/Bill lets go of the tree, what each trajectory will look like, etc.


θ

What shall I aim for?

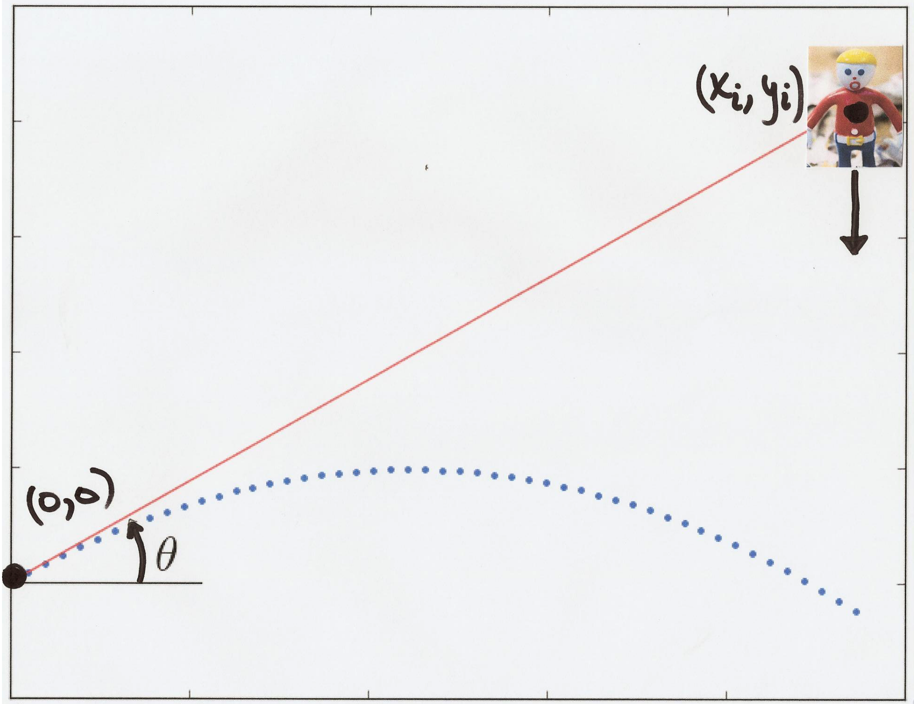
(A) Aim high, because the steel pellet is so much heavier than Mr. Bill, and will be pulled down more by gravity.

(B) Aim low, because Mr. Bill will be falling while the pellet travels.

(C) Aim directly for Mr. Bill. This is clearly what you would do if gravity were absent. The presence of gravity will affect Mr. Bill and the pellet in the same way (they experience the same downward gravitational acceleration), so aiming directly for Mr. Bill will result in a direct hit.

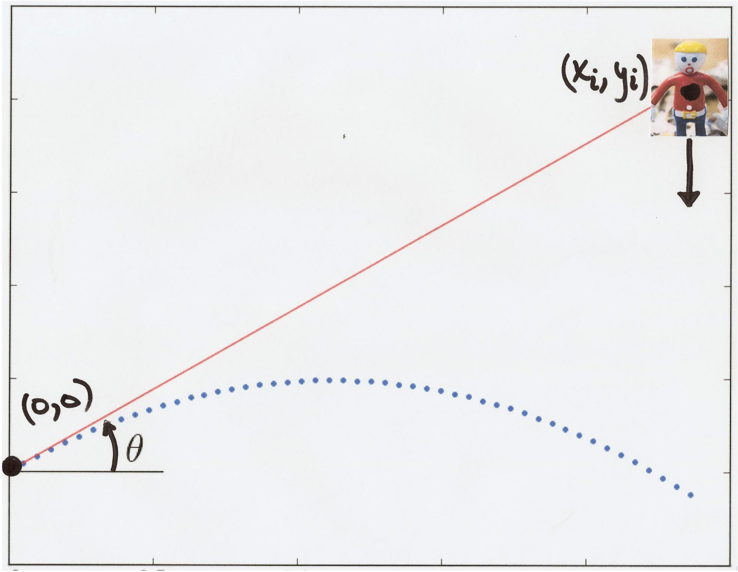
(D) How much below Mr. Bill you need to aim depends on the speed with which you fire the pellet, because the time that it takes the pellet to reach Mr. Bill will depend on how fast the pellet is shot.

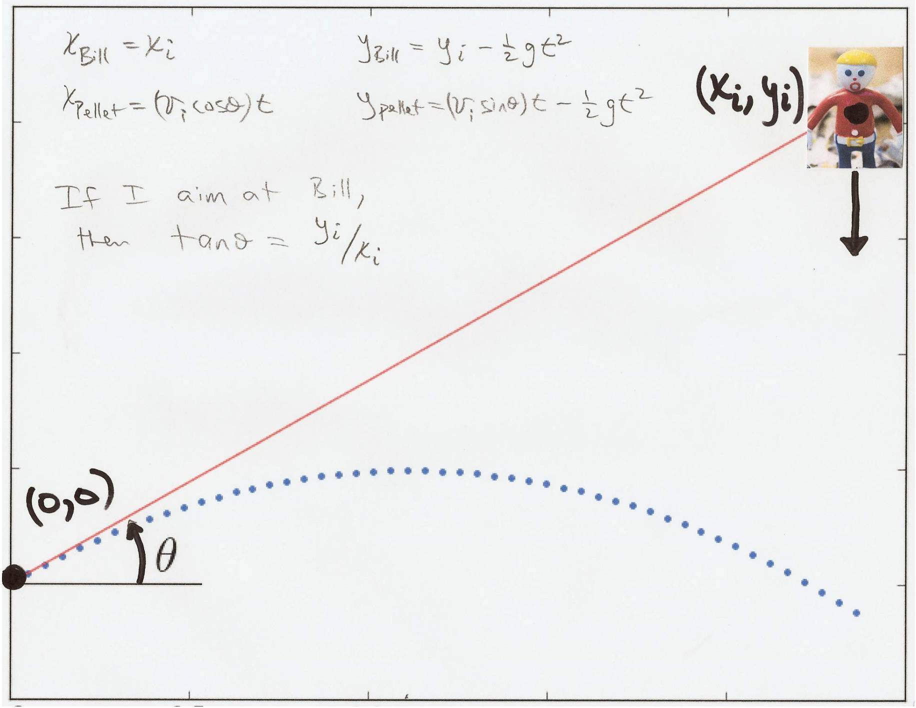
(I'm not going to give away my own answer yet!)



(x_i, y_i)
$(0,0)$
θ

Try writing equations for $\boxed{x_{\mathrm{Bill}}(t),\ y_{\mathrm{Bill}}(t),\ x_{\mathrm{pellet}}(t),\ y_{\mathrm{pellet}}(t)}$, in terms of x_i, y_i, θ (shown on diagram) and initial pellet speed v_i.





x_Bill = x_i
y_Bill = y_i − ½gt²
x_pellet = (v_i cosθ)t
y_pellet = (v_i sinθ)t − ½gt²
(x_i, y_i)
If I aim at Bill,
then tanθ = y_i/x_i
(0,0)
θ

Anybody want to change your vote?

(A) Aim high, because the steel pellet is so much heavier than Mr. Bill, and will be pulled down more by gravity.

(B) Aim low, because Mr. Bill will be falling while the pellet travels.

(C) Aim directly for Mr. Bill. **This is clearly what you would do if gravity were absent.** The presence of gravity will affect Mr. Bill and the pellet in the same way, so aiming directly for Mr. Bill will result in a direct hit.

(D) How much below Mr. Bill you need to aim depends on the speed with which you fire the pellet, because the time that it takes the pellet to reach Mr. Bill will depend on how fast the pellet is shot.

Mr. Bill starts from rest at (x_i, y_i). Pellet starts at $(0, 0)$ with initial velocity $(v_i \cos\theta, v_i \sin\theta)$. Equations of motion:

$$x_{\mathrm{bill}} = x_i$$

$$y_{\mathrm{bill}} = y_i - \frac{1}{2} g t^2$$

$$x_{\mathrm{pellet}} = v_i \cos\theta \ t$$

$$y_{\mathrm{pellet}} = v_i \sin\theta \ t - \frac{1}{2} g t^2$$

When does pellet cross Mr. Bill's downward path?

$$x_{\mathrm{pellet}} = x_{\mathrm{bill}} \quad \Rightarrow \quad v_i \cos\theta \ t = x_i$$

$$t = \frac{x_i}{v_i \cos\theta}$$

Plugging in $t = \left(\frac{x_i}{v_i \cos\theta}\right)$:

$$x_{\text{bill}} = x_i$$

$$x_{\text{pellet}} = v_i \cos\theta \left(\frac{x_i}{v_i \cos\theta}\right) = x_i$$

$$y_{\text{bill}} = y_i - \frac{1}{2} g \left(\frac{x_i}{v_i \cos\theta}\right)^2$$

$$y_{\text{pellet}} = v_i \sin\theta \left(\frac{x_i}{v_i \cos\theta}\right) - \frac{1}{2} g \left(\frac{x_i}{v_i \cos\theta}\right)^2$$

What is vertical separation between Mr. Bill and the pellet at the instant when $x_{\text{pellet}} = x_{\text{bill}} = x_i$?

$$y_{\text{bill}} - y_{\text{pellet}} = y_i - v_i \sin\theta \left(\frac{x_i}{v_i \cos\theta}\right)$$

$$y_{\text{bill}} - y_{\text{pellet}} = y_i - x_i \tan\theta = y_i - y_i = 0$$

Anybody want to change your vote?

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Oh noooo . . .

https://en.wikipedia.org/wiki/Mr._Bill

From a height h above the ground, I throw a ball with an initial velocity that is nonzero only in the horizontal direction: $v_{xi} > 0$, $v_{yi} = 0$. How do I determine how long it takes to reach the ground?

(A) $h + v_{xi} t = 0$

(B) $h + v_{xi} t - \frac{1}{2} g t^2 = 0$

(C) $h + v_{yi} t = 0$

(D) $h - \frac{1}{2} g t^2 = 0$

From a height h above the ground, I throw a ball with an initial velocity that is nonzero only in the horizontal direction: $v_{xi} > 0$, $v_{yi} = 0$. If the ball's initial x coordinate is $x_i = 0$, how do I determine the x coordinate where the ball hits the ground?

(A) $x_f = h + v_{xi}t - \frac{1}{2}gt^2$, with t given on the previous page

(B) $x_f = h + v_{xi}t - \frac{1}{2}gt^2$, with $t = 0$

(C) $x_f = x_i + v_{xi}t$, with t given on the previous page

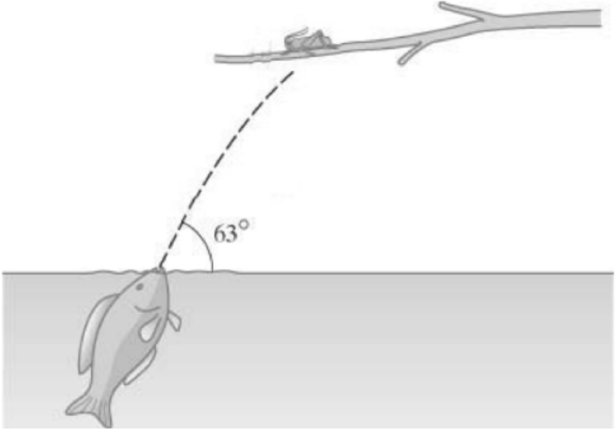
(D) $x_f = x_i + v_{xi}t$, with $t = 0$

(E) $y_f = y_i + v_{xi}t$, with t given on the previous page

(F) $y_f = y_i + v_{xi}t$, with $t = 0$

From a height h above the ground, I throw a ball with an initial velocity that is nonzero only in the horizontal direction: $v_{xi} > 0$, $v_{yi} = 0$. How do I determine the x and y components of the ball's velocity, v_x and v_y, at the instant before the ball hits the ground?

6. The archer fish shown in the figure, peering from just below the surface of the water, spits a drop of water at the grasshopper and knocks it into the water. The grasshopper's initial position is 0.45 m above the water surface and 0.25 m horizontally away from the fish's mouth. If the launch angle of the drop of water is 63° with respect to the horizontal water surface, how fast is the drop moving when it leaves the fish's mouth?



Let's quickly revisit free-body diagrams in 1D

You push on a crate, and it starts to move but you don't. Draw a free-body diagram for you and one for the crate. Then use the diagrams and Newton's third law of motion to explain why the crate moves but you don't.

(A) The force I exert on the crate is larger than the force the crate exerts on me.

(B) The crate's force on me is equal and opposite to my force on the crate. The frictional force between my shoes and the floor is equal in magnitude to the crate's push on me, while the frictional force between the crate and the floor is smaller than my push on the crate.

(C) The crate and I exert equal and opposite forces on each other, but I don't move because I am much more massive than the crate.

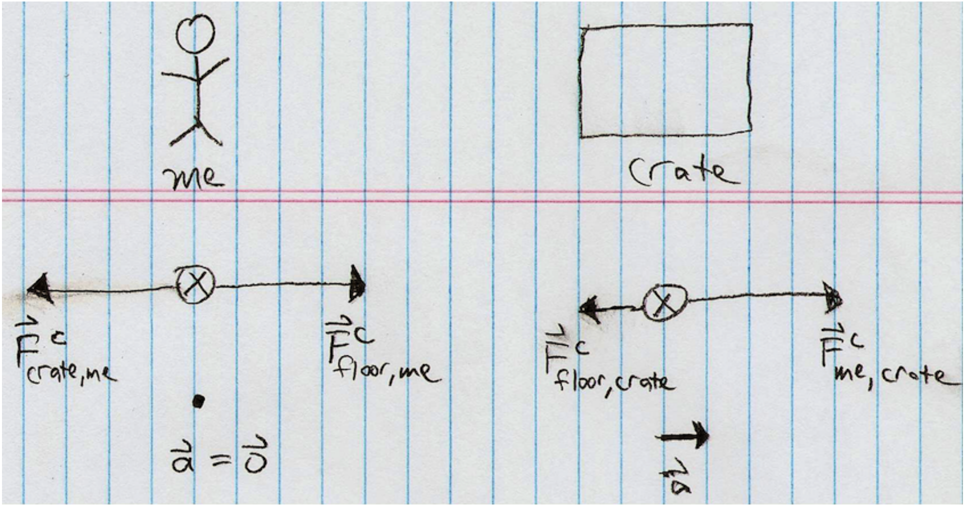
(free-body diagrams in one dimension)

If the crate and I were both standing on an ice rink, then it seems clear that we would both start to move. If the crate and I were both bolted to the floor, then it seems clear that neither one of us would start to move. So the grip of the floor's friction on my feet must be greater in magnitude than the grip of the floor's friction on the crate.

Let's say that I push to the right on the crate with a force $\vec{F}_{\mathrm{me,crate}}$, so the crate pushes to the left on me with a force $\vec{F}_{\mathrm{crate,me}} = -\vec{F}_{\mathrm{me,crate}}$. Meanwhile, the floor pushes to the right on me with a force $\vec{F}_{\mathrm{floor,me}}$, and the floor pushes (by a smaller amount) to the left on the crate with a force $\vec{F}_{\mathrm{floor,crate}}$.

It is reasonable that $|\vec{F}_{\mathrm{floor,crate}}| < |\vec{F}_{\mathrm{floor,me}}|$, because the bottom of the crate is wood, while the soles of my shoes are rubber.

(free-body diagrams in one dimension)



If I gently step on my car's accelerator pedal, and the car starts to move faster (without any screeching sounds), the frictional force between the road and the rubber tire surface that causes my car to accelerate is

(A) static friction.

(B) kinetic friction.

(C) normal force.

(D) gravitational force.

(E) there is no frictional force between road and tire.

If I **slam down** on my car's accelerator pedal, and the car **screeches** forward noisily like a drag-race car, the frictional force between the road and the rubber tire surface that causes my car to accelerate is

(A) static friction.

(B) kinetic friction.

(C) normal force.

(D) gravitational force.

(E) there is no frictional force between road and tire.

Why do modern cars have anti-lock brakes?

(A) because the pumping action of the anti-lock brake mechanism keeps the brake pads from getting too hot.

(B) because pulsing the brakes on and off induces kinetic friction, which is preferable to static friction.

(C) because the cofficient of static friction is larger than the coefficient of kinetic friction, so you stop faster if your wheels roll on the ground than you would if your wheels were skidding on the ground.

(D) because the weird pulsating sensation you feel when the anti-lock brakes engage is fun and surprising!

Anti-Lock Brakes

(photo credit: Bill Berner)

Static friction and kinetic (sometimes confusingly called "sliding") friction:

$$F^{\mathrm{Static}} \leq \mu_S \, F^{\mathrm{Normal}}$$

$$F^{\mathrm{Kinetic}} = \mu_K \, F^{\mathrm{Normal}}$$

"normal" & "tangential" components are $\perp$ to and $\parallel$ to surface

Static friction is an example of what physicists call a "force of constraint" and engineers call a "reaction force." In most cases, you don't know its magnitude until you solve for the other forces in the problem and impose the condition that $\vec{a} = \vec{0}$. (An exception is if we're told that static friction "just barely holds on / just barely lets go," i.e. has its maximum possible value.)

TABLE 10.1
Coefficients of friction

Material1	Material 2	μ_s	μ_k
aluminum	aluminum	1.1–1.4	1.4
aluminum	steel	0.6	0.5
glass	glass	0.9–1.0	0.4
glass	nickel	0.8	0.6
ice	ice	0.1	0.03
oak	oak	0.6	0.5
rubber	concrete	1.0–4.0	0.8
steel	steel	0.8	0.4
steel	brass	0.5	0.4
steel	copper	0.5	0.4
steel	lead	0.95	0.95
wood	wood	0.25–0.5	0.2

The values given are for clean, dry, smooth surfaces.

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oak	oak	0.6	0.5
rubber	concrete	1.0–4.0	0.8
steel	steel	0.8	0.4
steel	brass	0.5	0.4
steel	copper	0.5	0.4
steel	lead	0.95	0.95
wood	wood	0.25–0.5	0.2

The values given are for clean, dry, smooth surfaces.

- Steel on steel μ_K is about half that of rubber on concrete, and much less than that of μ_S for rubber on concrete.
- So a train can take a while to skid to a stop!
- Even more so if the tracks are wet: $\mu_K \approx 0.1$
- At $\mu = 0.1$ on level ground: 360 m to stop from 60 mph.
- At $\mu = 0.1$ on 6° slope: not possible to stop.

A car of mass 1000 kg travels at constant speed 20 m/s on dry, level pavement. The friction coeffs are $\mu_k = 0.8$ and $\mu_s = 1.2$. What is the **normal force** exerted by the road on the car?

(A) 1000 N downward

(B) 1000 N upward

(C) 1000 N forward

(D) 1000 N backward

(E) 9800 N downward

(F) 9800 N upward

(G) 11800 N downward

(H) 11800 N upward

A car of mass 1000 kg is traveling (in a straight line) at a constant speed of 20 m/s on dry, level pavement, with the cruise control engaged to maintain this speed. The friction coefficients are $\mu_k = 0.8$ and $\mu_s = 1.2$. The tires roll on the pavement without slipping. What is the frictional force exerted by the road on the car? (Let's use $g \approx 10\ \mathrm{m/s^2}$ for simplicity here.)

(A) 8000 N backward

(B) 8000 N forward

(C) 8000 N upward

(D) 10000 N backward

(E) 10000 N forward

(F) 12000 N backward

(G) 12000 N forward

(H) It points forward, must have magnitude $\leq 12000\ \mathrm{N}$, and has whatever value is needed to counteract air resistance.

A car of mass 1000 kg is initially traveling (in a straight line) at 20 m/s on dry, level pavement, when suddenly the driver jams on the (**non**-anti-lock) brakes, and the car skids to a stop with its wheels locked. The friction coefficients are $\mu_k = 0.8$ and $\mu_s = 1.2$. What is the frictional force exerted by the road on the car? (Let's use $g \approx 10\ \mathrm{m/s^2}$ for simplicity here.)

(A) 8000 N backward

(B) 8000 N forward

(C) 8000 N upward

(D) 10000 N backward

(E) 10000 N forward

(F) 12000 N backward

(G) 12000 N forward

(H) It points forward, must have magnitude $\leq 12000\ \mathrm{N}$, and has whatever value is needed to counteract air resistance.

Suppose that for rubber on dry concrete, $\mu_k = 0.8$ and $\mu_s = 1.2$. If a car of mass m traveling at initial speed v_i on a level road jams on its brakes and skids to a stop with its wheels locked, how do I solve for the length L of the skid marks? (Let's use $g \approx 10\ \mathrm{m/s^2}$ for simplicity here.)

(A) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -2.0\ \mathrm{m/s^2}$

(B) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -4.0\ \mathrm{m/s^2}$

(C) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -6.0\ \mathrm{m/s^2}$

(D) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -8.0\ \mathrm{m/s^2}$

(E) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -10.0\ \mathrm{m/s^2}$

(F) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -12.0\ \mathrm{m/s^2}$

(G) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -14.0\ \mathrm{m/s^2}$

Suppose that for rubber tires on dry, level pavement, the friction coefficients are $\mu_k = 0.8$ and $\mu_s = 1.2$. If you assume that the forces between the ground and the tires are the same for all four tires (4-wheel drive, etc.), what is a car's maximum possible acceleration for this combination of tires and pavement? (Let's use $g \approx 10\ \mathrm{m/s^2}$ for simplicity here.)

(A) $1.0\ \mathrm{m/s^2}$

(B) $5.0\ \mathrm{m/s^2}$

(C) $8.0\ \mathrm{m/s^2}$

(D) $10.0\ \mathrm{m/s^2}$

(E) $12.0\ \mathrm{m/s^2}$

Easier example (quickly, or skip)

How hard do you have to push a 1000 kg car (with brakes on, all wheels, on level ground) to get it to start to slide? Let's take $\mu_S \approx 1.2$ for rubber on dry pavement.

$$F^{\text{Normal}} = mg = 9800 \text{ N}$$

$$F^{\text{Static}} \leq \mu_S F^N = (1.2)(9800 \text{ N}) \approx 12000 \text{ N}$$

So the static friction gives out (hence car starts to slide) when your push exceeds 12000 N.

How hard do you then have to push to keep the car sliding at constant speed? Let's take $\mu_K \approx 0.8$ for rubber on dry pavement.

$$F^{\text{Kinetic}} = \mu_K F^N = (0.8)(9800 \text{ N}) \approx 8000 \text{ N}$$

How far does your car slide on dry, level pavement if you jam on the brakes, from 60 mph (27 m/s)?

$$F^N = mg, \quad F^K = \mu_K mg$$

$$a = ? \qquad \Delta x = ?$$

(The math is worked out on the next slides, but we won't go through them in detail. It's there for you to look at later.)

How far does your car slide on dry, level pavement if you jam on the brakes, from 60 mph (27 m/s)?

$$F^N = mg, \quad F^K = \mu_K mg$$

$$a = -F^K/m = -\mu_K g = -(0.8)(9.8\ \mathrm{m/s^2}) \approx -8\ \mathrm{m/s^2}$$

Constant force $\rightarrow$ constant acceleration from 27 m/s down to zero:

$$v_f^2 = v_i^2 + 2ax$$

$$x = \frac{v_i^2}{-2a} = \frac{(27\ \mathrm{m/s})^2}{2 \times (8\ \mathrm{m/s^2})} \approx 45\ \mathrm{m}$$

How much time elapses before you stop?

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{27\ \mathrm{m/s}}{8\ \mathrm{m/s^2}} = 3.4\ \mathrm{s}$$

How does this change if you have anti-lock brakes (or good reflexes) so that the tires never skid? Remember $\mu_S > \mu_K$. For rubber on dry pavement, $\mu_S \approx 1.2$ (though there's a wide range) and $\mu_K \approx 0.8$. The best you can do is *maximum* static friction:

$$F^S \leq \mu_S mg$$

$$a = -F^S/m = -\mu_S g = -(1.2)(9.8\ \mathrm{m/s^2}) \approx -12\ \mathrm{m/s^2}$$

Constant force → constant acceleration from 27 m/s down to zero:

$$v_f^2 = v_i^2 + 2ax$$

$$x = \frac{v_i^2}{-2a} = \frac{(27\ \mathrm{m/s})^2}{2 \times (12\ \mathrm{m/s^2})} \approx 30\ \mathrm{m}$$

How much time elapses before you stop?

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{27\ \mathrm{m/s}}{12\ \mathrm{m/s^2}} = 2.2\ \mathrm{s}$$

So you can stop in about 2/3 the time (and 2/3 the distance) if you don't let your tires skid. Or whatever μ_K/μ_S ratio is.

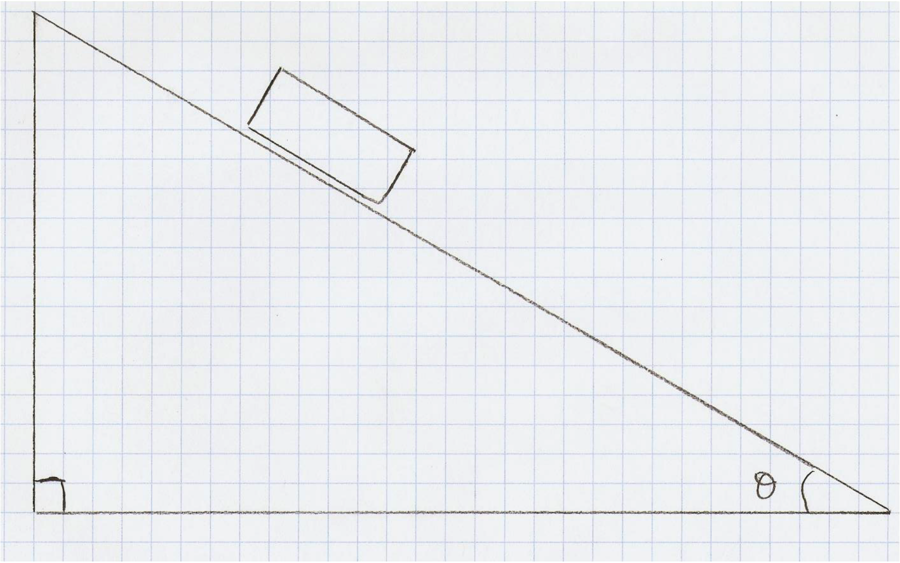
A Ch10 problem that may not fit

Calculate $\vec{C} \cdot (\vec{B} - \vec{A})$ if $\vec{A} = 3.0\hat{i} + 2.0\hat{j}$, $\vec{B} = 1.0\hat{i} - 1.0\hat{j}$, and $\vec{C} = 2.0\hat{i} + 2.0\hat{j}$. Remember that there are two ways to compute a dot product—choose the easier method in a given situation: one way is $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}| \cos\varphi$, where φ is the angle between vectors $\vec{P}$ and $\vec{Q}$, and the other way is $\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y$.

A Ch10 problem that may not fit

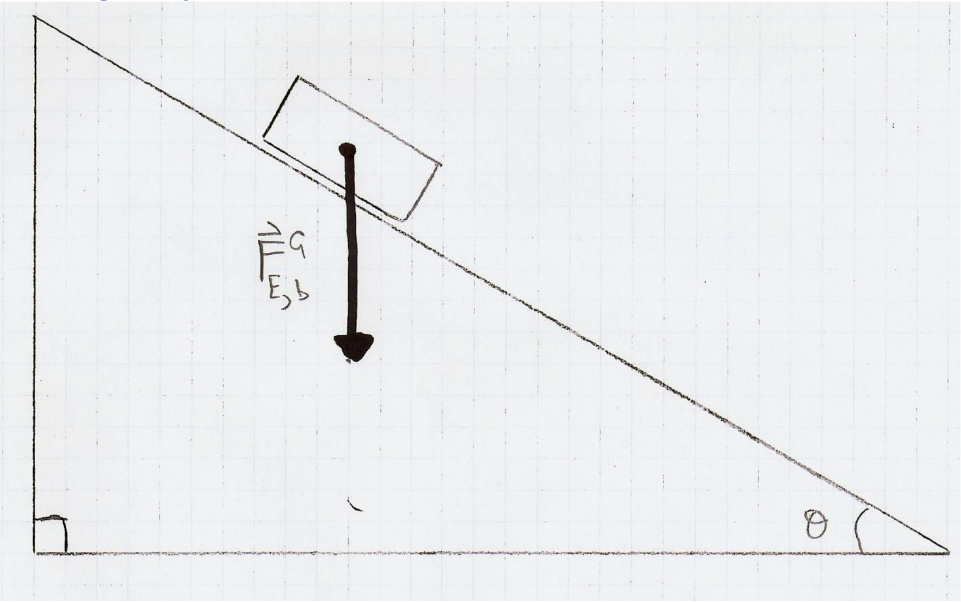
A child rides her bike 1.0 block east and then $\sqrt{3} \approx 1.73$ blocks north to visit a friend. It takes her 10 minutes, and each block is 60 m long. What are (a) the magnitude of her displacement, (b) her average velocity (magnitude and direction), and (c) her average speed?

Block sliding down inclined plane: try drawing free-body diagram. Suppose some kinetic friction is present, but block still accelerates downhill. Try drawing this, one step ahead of me.



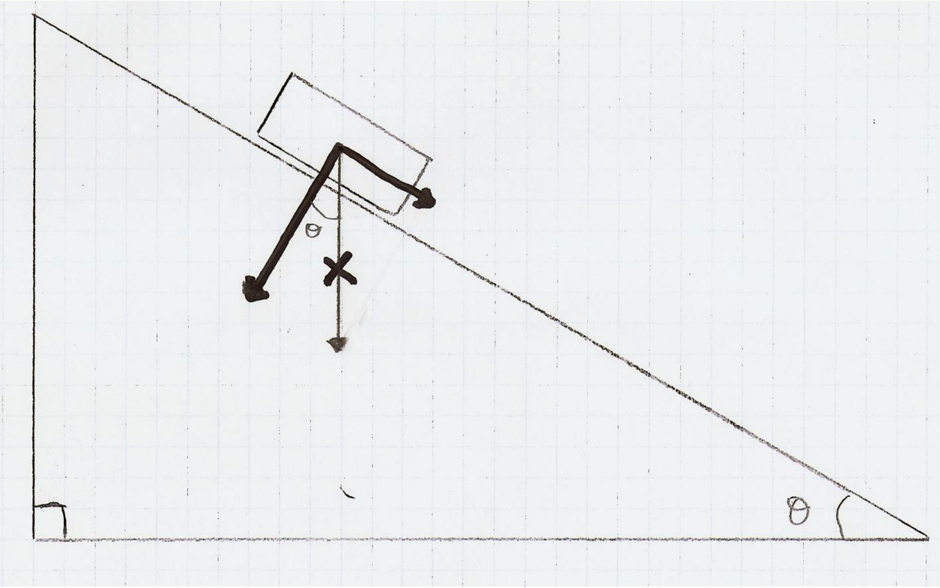
First: let's draw $\vec{F}^G_{E,b}$ for gravity.

Add gravity vector



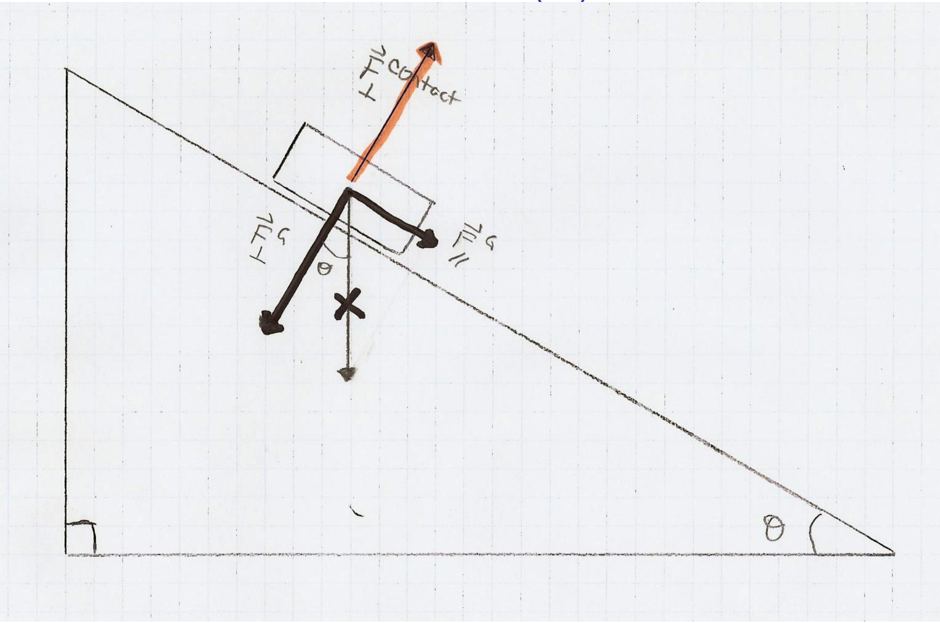
Next decompose $\vec{F}^G_{E,b}$ into components $\parallel$ and $\perp$ to surface.

Decompose gravity vector: $\parallel$ and $\perp$ to surface



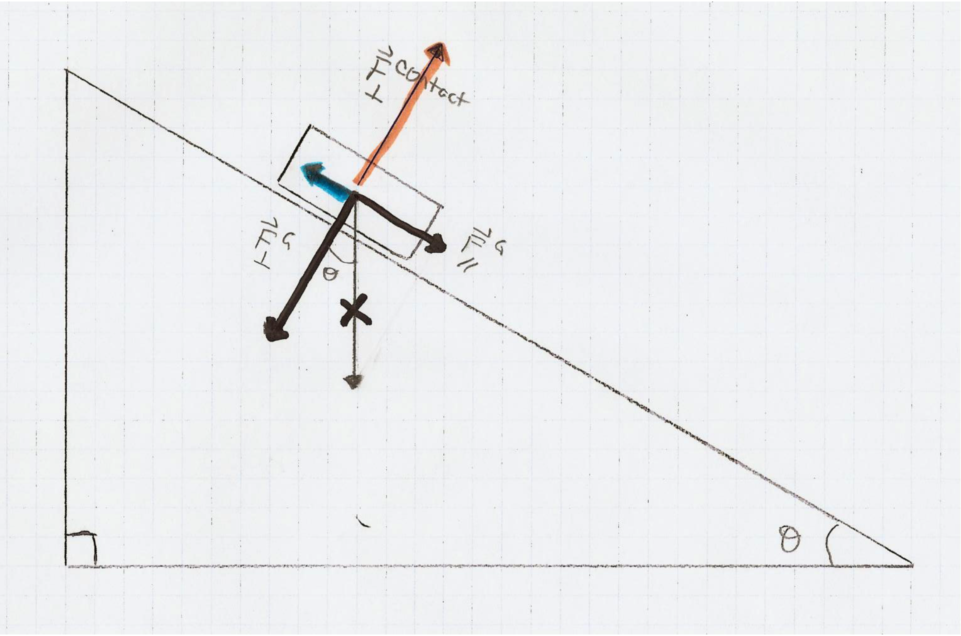
Next: add contact force "normal" ($\perp$) to surface.

Now add contact force "normal" (⊥) to surface



Next: add friction.

Now add friction (∥ to surface, opposing *relative* motion)



The block shown in this free-body diagram is

(A) at rest.

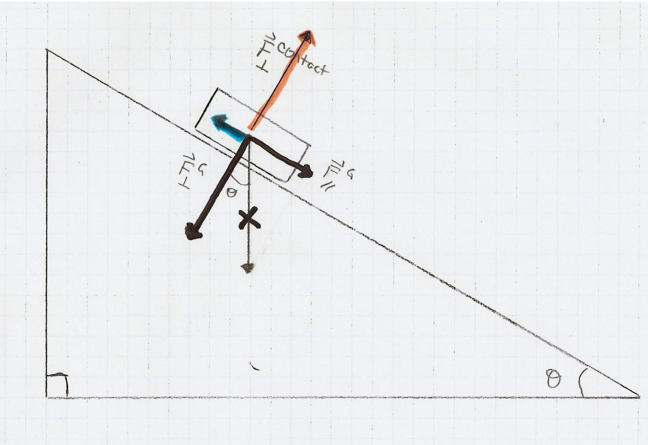
(B) sliding downhill at constant speed.

(C) sliding downhill and speeding up.

(D) sliding downhill and slowing down.

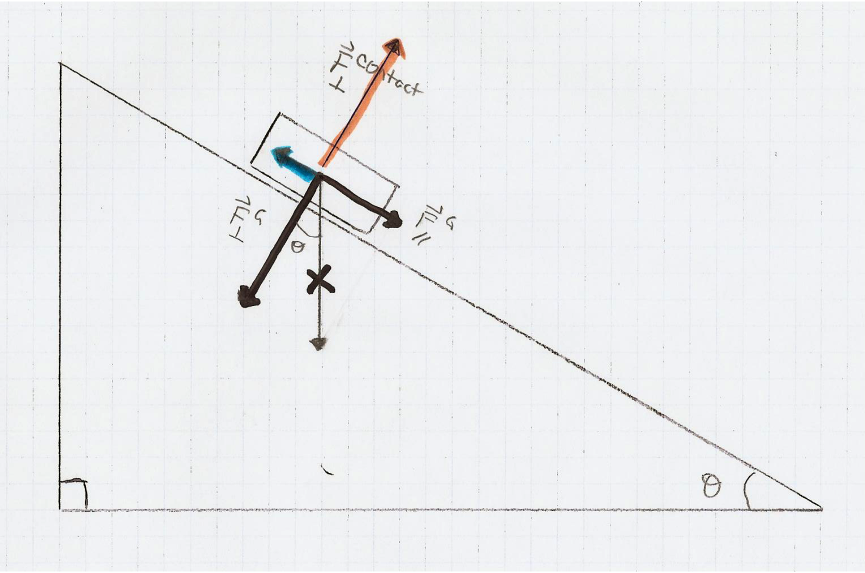
(E) sliding uphill and speeding up.

(F) sliding uphill and slowing down.



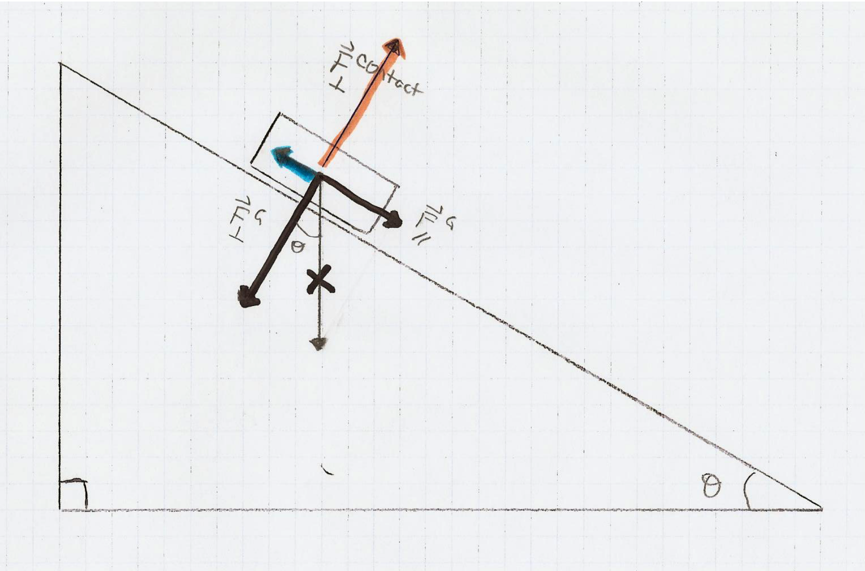
How would I change this free-body diagram . . .

if the block were at rest?



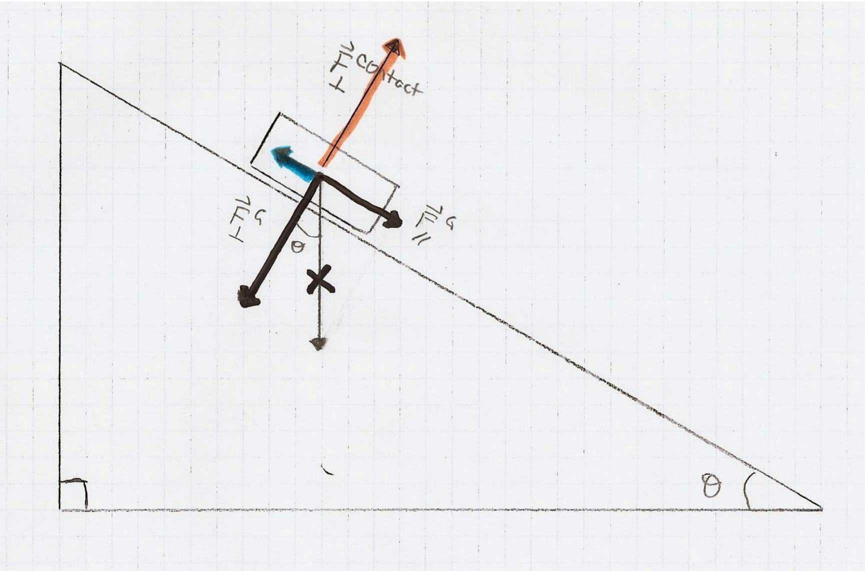
How would I change this free-body diagram . . .

if the block were sliding downhill at constant speed?



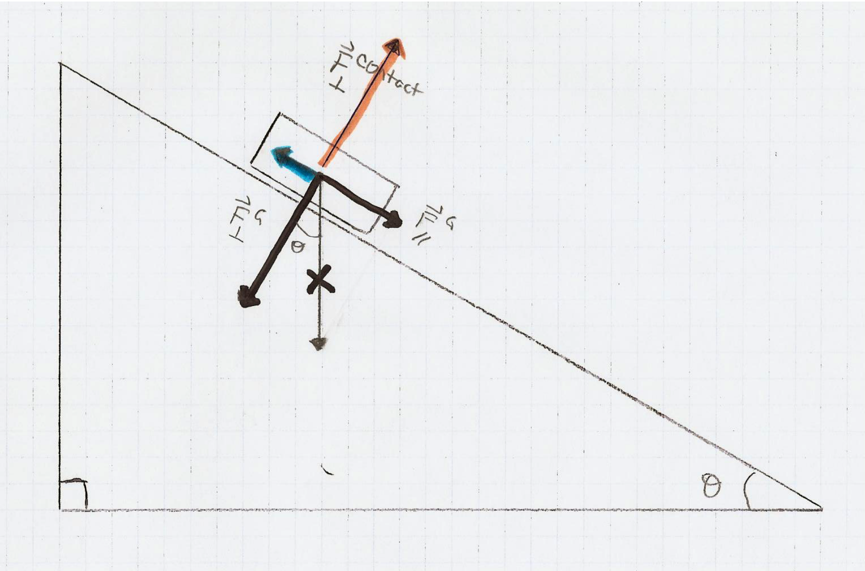
How would I change this free-body diagram . . .

if the block were sliding downhill and slowing down?



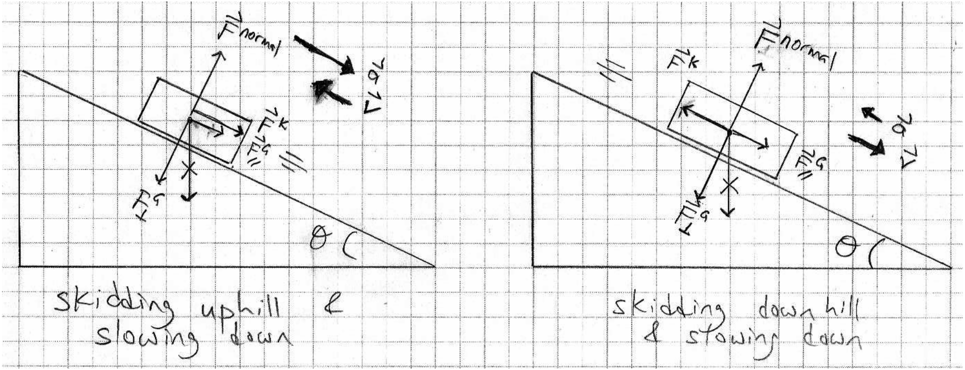
How would I change this free-body diagram . . .

if the block were sliding uphill and slowing down?



Another Chapter 10 reading question:

You've slammed on the brakes, and your car is skidding to a stop on a steep and slippery winter road. Other things being equal, will the car come to rest more quickly if it is traveling uphill or if it is traveling downhill? Why? (Consider FBD for each case.)



An object "O" of mass m slides down an inclined surface "S" at constant velocity. What is the magnitude of the (kinetic) **frictional force** F^k_{so} exerted by the surface on the object?

(A) $F^k_{so} = mg$

(B) $F^k_{so} = mg \sin\theta$

(C) $F^k_{so} = mg \cos\theta$

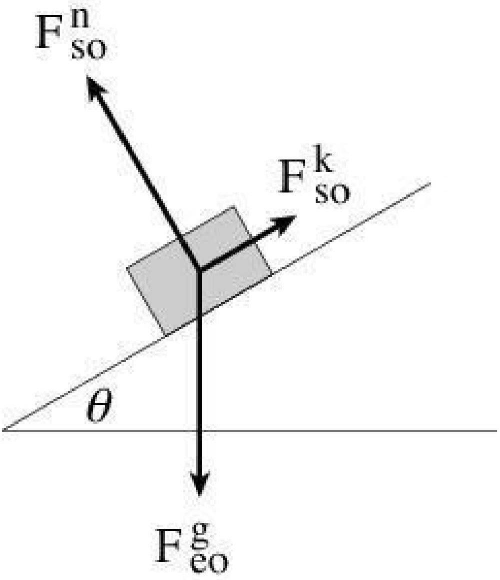
(D) $F^k_{so} = mg \tan\theta$

(E) $F^k_{so} = \mu_k mg$

(F) $F^k_{so} = \mu_k mg \sin\theta$

(G) $F^k_{so} = \mu_k mg \cos\theta$

(H) $F^k_{so} = \mu_k mg \tan\theta$



An object "O" of mass m slides down an inclined surface "S" at constant velocity. What is the magnitude of the **gravitational force** F_{eo}^{g} exerted by Earth on the object?

(A) $F_{eo}^{g} = mg$

(B) $F_{eo}^{g} = mg \sin\theta$

(C) $F_{eo}^{g} = mg \cos\theta$

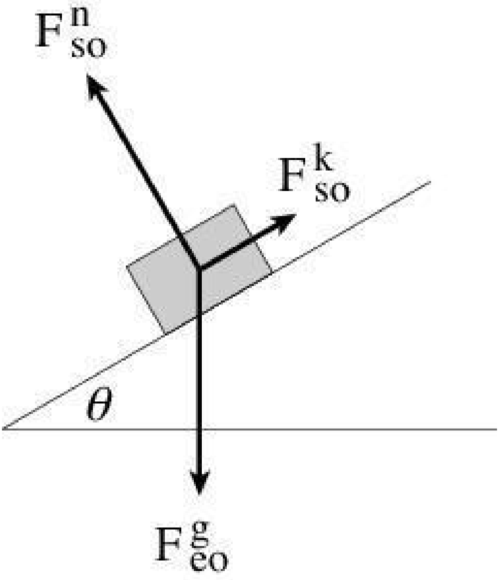
(D) $F_{eo}^{g} = mg \tan\theta$

(E) $F_{eo}^{g} = \mu_k mg$

(F) $F_{eo}^{g} = \mu_k mg \sin\theta$

(G) $F_{eo}^{g} = \mu_k mg \cos\theta$

(H) $F_{eo}^{g} = \mu_k mg \tan\theta$



An object "O" of mass m slides down an inclined surface "S" at constant velocity. Let the x-axis point downhill. What is the magnitude of the **downhill (tangential) component** $F^g_{eo,x}$ of the gravitational force exerted by Earth on the object?

(A) $F^g_{eo,x} = mg$

(B) $F^g_{eo,x} = mg \sin\theta$

(C) $F^g_{eo,x} = mg \cos\theta$

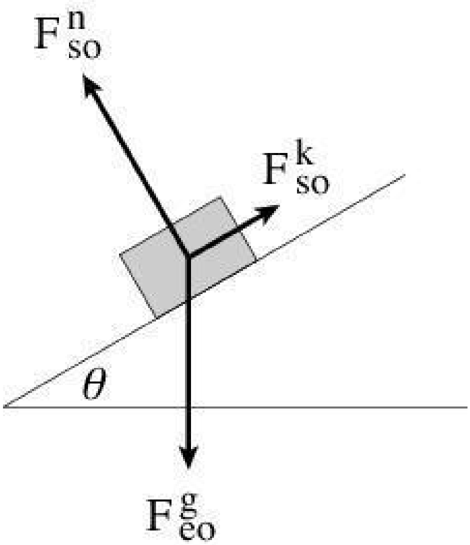
(D) $F^g_{eo,x} = mg \tan\theta$

(E) $F^g_{eo,x} = \mu_k mg$

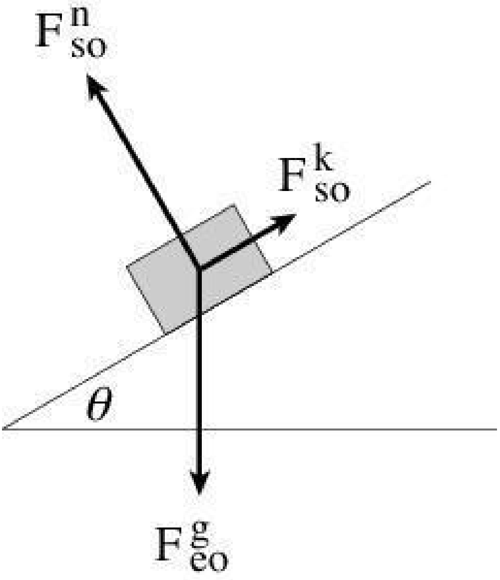
(F) $F^g_{eo,x} = \mu_k mg \sin\theta$

(G) $F^g_{eo,x} = \mu_k mg \cos\theta$

(H) $F^g_{eo,x} = \mu_k mg \tan\theta$



An object “O” of mass m slides down an inclined surface “S” at constant velocity. What is the magnitude of the **normal force** F_{so}^{n} exerted by the surface on the object?



(A) $F_{so}^{n} = mg$

(B) $F_{so}^{n} = mg\sin\theta$

(C) $F_{so}^{n} = mg\cos\theta$

(D) $F_{so}^{n} = mg\tan\theta$

(E) $F_{so}^{n} = \mu_k mg$

(F) $F_{so}^{n} = \mu_k mg\sin\theta$

(G) $F_{so}^{n} = \mu_k mg\cos\theta$

(H) $F_{so}^{n} = \mu_k mg\tan\theta$

Since object "O" slides down surface "S" at constant velocity, the forces on O must sum vectorially to zero. How do I express this fact for the forces acting along the downhill (tangential) axis?

(A) $\mu_k mg = mg\cos\theta$

(B) $\mu_k mg = mg\sin\theta$

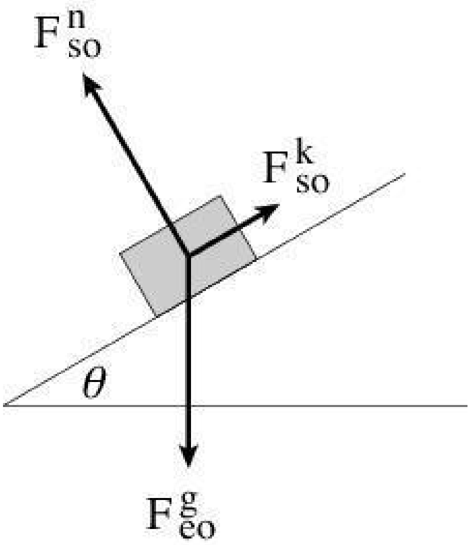
(C) $\mu_k mg\cos\theta = mg$

(D) $\mu_k mg\sin\theta = mg$

(E) $\mu_k mg\cos\theta = mg\sin\theta$

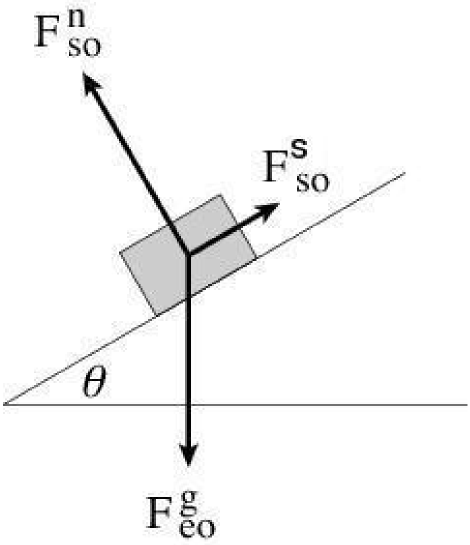
(F) $\mu_k mg\sin\theta = mg\cos\theta$

(G) $mg\sin\theta = mg\cos\theta$

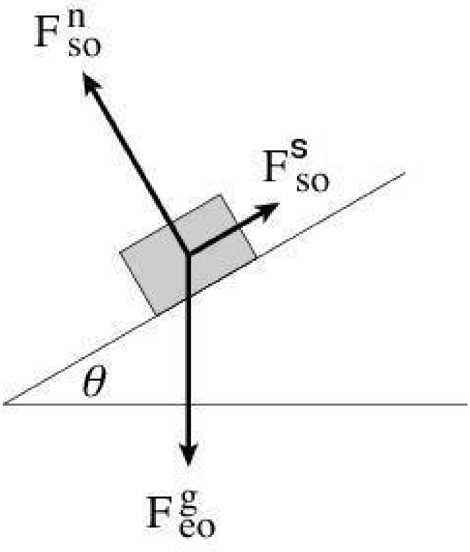


Suppose friction holds object "O" at rest on surface "S." Which statement is true?

(A) $mg \sin\theta = F^s_{so} = \mu_k mg \cos\theta$
(B) $mg \sin\theta = F^s_{so} = \mu_s mg \cos\theta$
(C) $mg \sin\theta = F^s_{so} \leq \mu_k mg \cos\theta$
(D) $mg \sin\theta = F^s_{so} \leq \mu_s mg \cos\theta$
(E) $mg \cos\theta = F^s_{so} = \mu_k mg \sin\theta$
(F) $mg \cos\theta = F^s_{so} = \mu_s mg \sin\theta$
(G) $mg \cos\theta = F^s_{so} \leq \mu_k mg \sin\theta$
(H) $mg \cos\theta = F^s_{so} \leq \mu_s mg \sin\theta$



Suppose friction holds object "O" at rest on surface "S." Then I gradually increase θ until the block just begins to slip. Which statement is true at the instant when the block starts slipping?



(A) $mg \sin\theta = F^s_{so} = \mu_k mg \cos\theta$

(B) $mg \sin\theta = F^s_{so} = \mu_s mg \cos\theta$

(C) $mg \sin\theta = F^s_{so} \leq \mu_k mg \cos\theta$

(D) $mg \sin\theta = F^s_{so} \leq \mu_s mg \cos\theta$

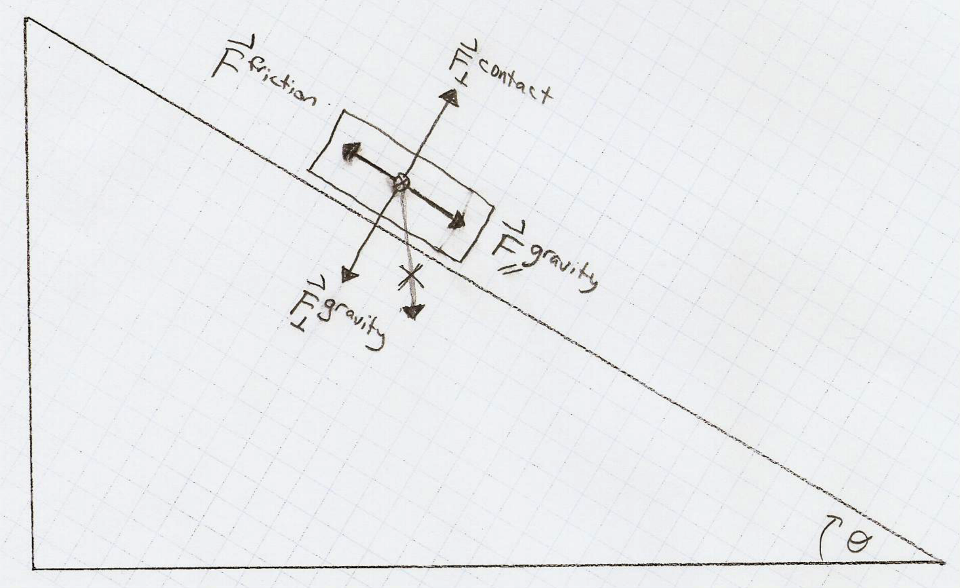
(E) $mg \cos\theta = F^s_{so} = \mu_k mg \sin\theta$

(F) $mg \cos\theta = F^s_{so} = \mu_s mg \sin\theta$

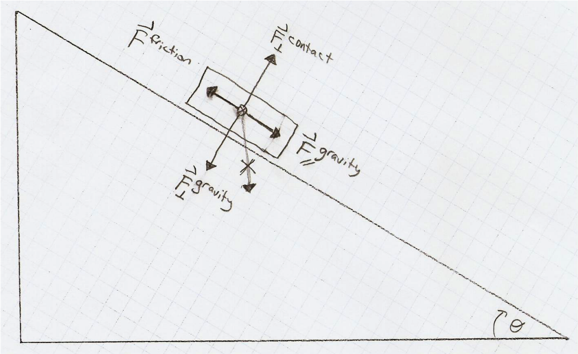
(G) $mg \cos\theta = F^s_{so} \leq \mu_k mg \sin\theta$

(H) $mg \cos\theta = F^s_{so} \leq \mu_s mg \sin\theta$

Friction on inclined plane



Why do I "cross off" the downward gravity arrow?



Take x-axis to be downhill, y-axis to be upward $\perp$ from surface.

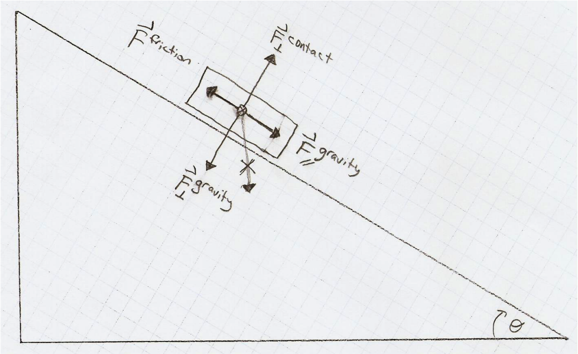
$$\vec{F}^G_\perp = -mg\cos\theta\;\hat{j}, \qquad \vec{F}^N = +mg\cos\theta\;\hat{j}$$

$$\vec{F}^G_\parallel = +mg\sin\theta\;\hat{i}$$

If block is not sliding then friction balances downhill gravity:

$$\vec{F}^S = -mg\sin\theta\;\hat{i}$$

(I'll skip this slide, but it's here for reference.)



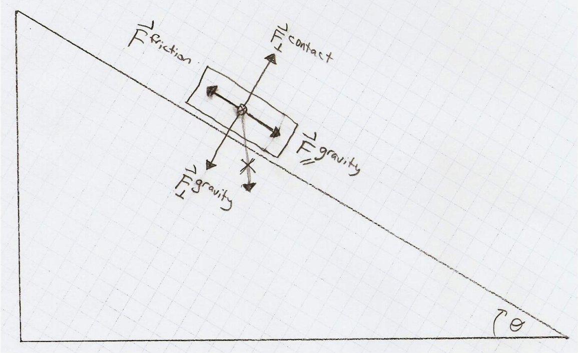
Magnitude of "normal" force ("normal" is a synonym for "perpendicular") between surfaces is

$$F^N = mg\cos\theta$$

Magnitude of static friction must be less than maximum:

$$F^S \ \leq \ \mu_S F^N = \mu_S \ mg\cos\theta$$

Block begins sliding when downhill component of gravity equals maximum magnitude of static friction ...



Block begins sliding when downhill component of gravity equals maximum magnitude of static friction:

$$\mu_S \ mg \cos\theta = mg \sin\theta$$

$$\mu_S = \frac{mg \sin\theta}{mg \cos\theta}$$

$$\mu_S = \tan\theta$$

A Ch10 problem that may not fit

The coefficient of static friction of tires on ice is about 0.10. (a) What is the steepest driveway on which you could park under those circumstances? (b) Draw a free-body diagram for the car when it is parked (successfully) on an icy driveway that is just a tiny bit less steep than this maximum steepness. [We might want to do (b) before we do (a).]

Answering part (a) starts by expressing (in math) which statement:

(A) (total gravitational force on car) equals (kinetic friction)
(B) (total gravitational force on car) equals (largest possible value of static friction)
(C) (downhill component of gravity) equals (kinetic friction)
(D) (downhill component of gravity) equals (largest possible value of static friction)

A Ch10 problem that may not fit

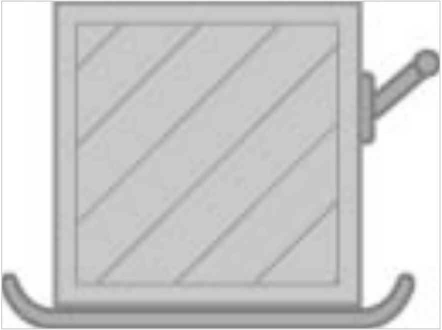
A fried egg of inertia m slides (at constant speed) down a Teflon frying pan tipped at an angle θ above the horizontal. [This only works if the angle θ is just right.] (a) Draw the free-body diagram for the egg. Be sure to include friction. (b) What is the “net force” (i.e. the vector sum of forces) acting on the egg? (c) How do these answers change if the egg is instead speeding up as it slides?

A heavy crate has plastic skid plates beneath it and a tilted handle attached to one side. Which requires a smaller force (directed along the diagonal rod of the handle) to move the box? Why?

(A) Pushing the crate is easier than pulling.

(B) Pulling the crate is easier than pushing.

(C) There is no difference.



Example (tricky!) problem

A woman applies a constant force to pull a 50 kg box across a floor **at constant speed**. She applies this force by pulling on a rope that makes an angle of 37° above the horizontal. The friction coefficient between the box and the floor is $\mu_k = 0.10$.

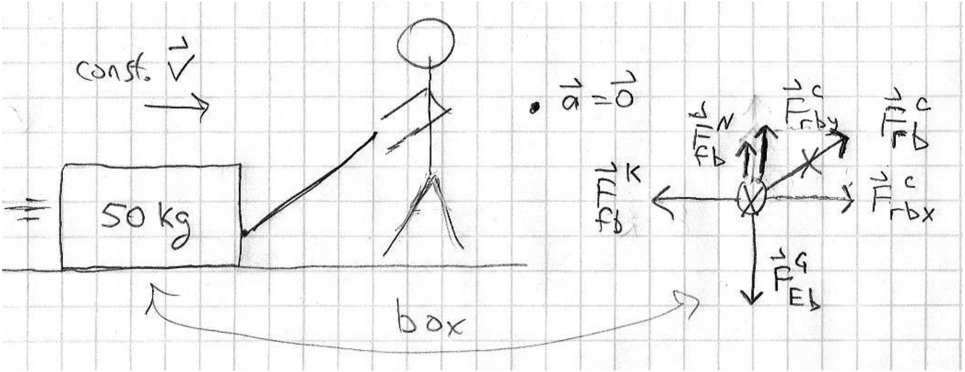
(a) Find the tension in the rope.

(b) How much work does the woman do in moving the box 10 m?

What are all of the forces acting on the box? Try drawing your own FBD for the box. It's tricky!

free-body diagram for box

What are all of the forces acting on the box? Try drawing your own FBD for the box. It's tricky!



(I should redraw the RHS of this diagram on the board.)

find tension in rope

Step one: If T is the tension in the rope, then what is the normal force (by floor on box)?

(A) $F^N = mg$
(B) $F^N = mg + T\cos\theta$
(C) $F^N = mg + T\sin\theta$
(D) $F^N = mg - T\cos\theta$
(E) $F^N = mg - T\sin\theta$

find tension in rope

Step two: what is the frictional force exerted by the floor on the box (which is sliding across the floor at constant speed)?

(A) $F^K = \mu_K(mg - T\sin\theta)$
(B) $F^K = \mu_K(mg - T\cos\theta)$
(C) $F^K = \mu_S(mg - T\sin\theta)$
(D) $F^K = \mu_S(mg - T\cos\theta)$
(E) $F^K = (mg - T\sin\theta)$
(F) $F^K = (mg - T\cos\theta)$

find tension in rope

Step three: how do I use the fact that the box is moving at constant velocity (and hence is not accelerating)?

(A) $T = F^K = \mu_K(mg - T\sin\theta)$

(B) $T\cos\theta = F^K = \mu_K(mg - T\sin\theta)$

(C) $T\sin\theta = F^K = \mu_K(mg - T\sin\theta)$

solution (part a): find tension in rope

Force by rope on box has upward vertical component $T \sin\theta$. So the normal force (by floor on box) is $\boxed{F^N = mg - T\sin\theta}$.

Force of friction is $\boxed{F^K = \mu_K(mg - T\sin\theta)}$. To keep box sliding at constant velocity, horizontal force by rope on box must balance F^K.

$$\boxed{T\cos\theta = F^K = \mu_K(mg - T\sin\theta)} \quad \Rightarrow \quad T = \frac{\mu_K mg}{\cos\theta + \mu_K \sin\theta}$$

This reduces to familiar $T = \mu_K mg$ if $\theta = 0^\circ$ (pulling horizontally) and even reduces to a sensible $T = mg$ if $\theta = 90^\circ$ (pulling vertically).

Plugging in $\theta = 37^\circ$, so $\cos\theta = 4/5 = 0.80$, $\sin\theta = 3/5 = 0.60$,

$$T = \frac{(0.10)(50\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{(0.80) + (0.10)(0.60)} = 57\ \mathrm{N}$$

solution (part b): work done by pulling for 10 meters

In part (a) we found tension in rope is $T = 57\ \mathrm{N}$ and is oriented at an angle $\theta = 36.9^\circ$ above the horizontal.

In 2D, work is displacement times **component of force along direction of displacement** (which is horizontal in this case). So the work done by the rope on the box is

$$W = \vec{F}_{rb} \cdot \Delta\vec{r}_b$$

This is the dot product (or "scalar product") of the force $\vec{F}_{rb}$ (by rope on box) with the displacement $\Delta\vec{r}_b$ of the point of application of the force.

In part (a) we found tension in rope is $T = 57\ \mathrm{N}$ and is oriented at an angle $\theta = 36.9^\circ$ above the horizontal.

What is the work done by the rope on the box by pulling the box across the floor for 10 meters? (Assume my arithmetic is correct.)

(In two dimensions, work is the dot product of the force $\vec{F}_{rb}$ with the displacement $\Delta\vec{r}_b$ of the point of application of the force.)

(A) $W = (10\ \mathrm{m})(T) = (10\ \mathrm{m})(57\ \mathrm{N}) = 570\ \mathrm{J}$

(B) $W = (10\ \mathrm{m})(T\cos\theta) = (10\ \mathrm{m})(57\ \mathrm{N})(0.80) = 456\ \mathrm{J}$

(C) $W = (10\ \mathrm{m})(T\sin\theta) = (10\ \mathrm{m})(57\ \mathrm{N})(0.60) = 342\ \mathrm{J}$

(D) $W = (8.0\ \mathrm{m})(T\cos\theta) = (8.0\ \mathrm{m})(57\ \mathrm{N})(0.80) = 365\ \mathrm{J}$

(E) $W = (8.0\ \mathrm{m})(T\sin\theta) = (8.0\ \mathrm{m})(57\ \mathrm{N})(0.60) = 274\ \mathrm{J}$

Repeat, now that we've analyzed this quantitatively

A heavy crate has plastic skid plates beneath it and a tilted handle attached to one side. Which requires a smaller force (directed along the diagonal rod of the handle) to move the box? Why?

(A) Pushing the crate is easier than pulling.

(B) Pulling the crate is easier than pushing.

(C) There is no difference.