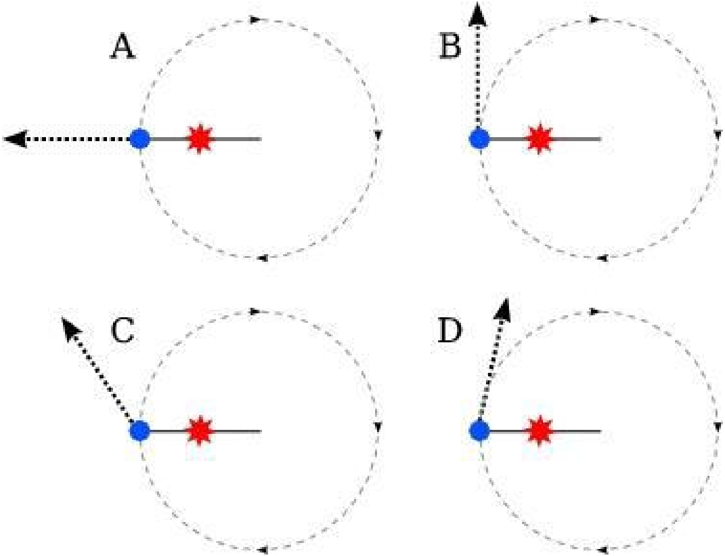
Mazur ch11 — motion in a circle

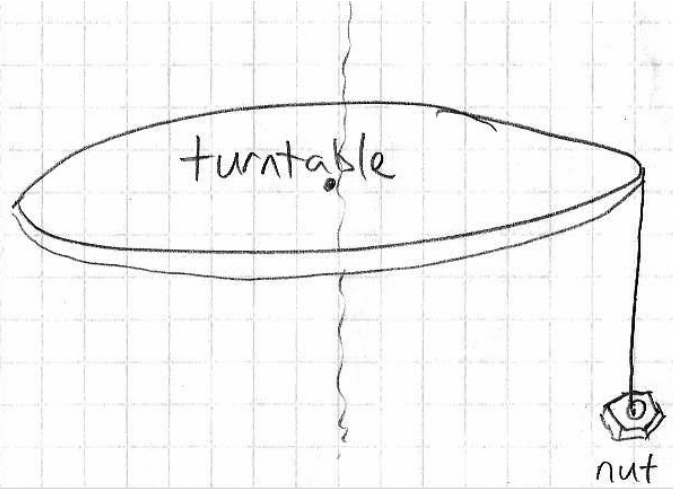
- If you go around in a circle at constant speed, your velocity vector is always changing direction.
- A change in velocity (whether magnitude, direction, or both) requires acceleration.
- For motion in a circle of radius R at constant speed v

$$a = \frac{v^2}{R}$$

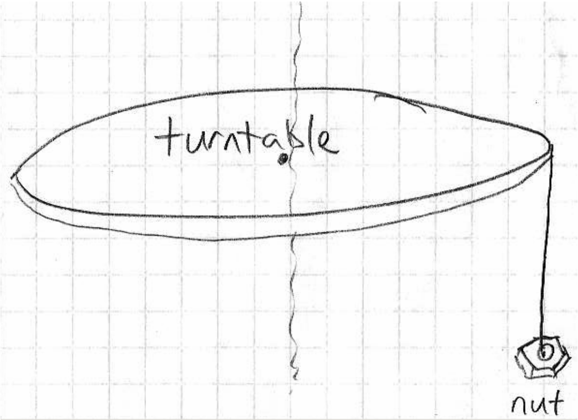
- This is called **centripetal acceleration**, and points toward the center of the circle.
- In the absence of a force (i.e. if vector sum of forces (if any) is zero), there is no acceleration, hence no change in velocity.

You are looking down (plan view) as I spin a (blue) ball on a string above my head in a circle at constant speed. The string breaks at the instant shown below. Which picture depicts the subsequent motion of the ball?





If the nut has mass m and the turntable is sitting idle, what is the tension in the string?

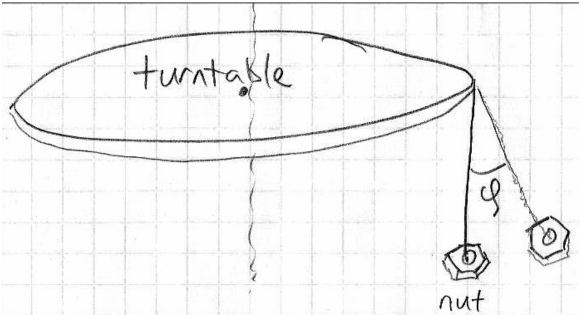


What will happen when I start the turntable spinning?

(A) The nut will continue to hang down vertically.

(B) The nut will move inward somewhat, making some angle φ w.r.t. the vertical axis.

(C) The nut will move outward somewhat, making some angle φ w.r.t. the vertical axis.



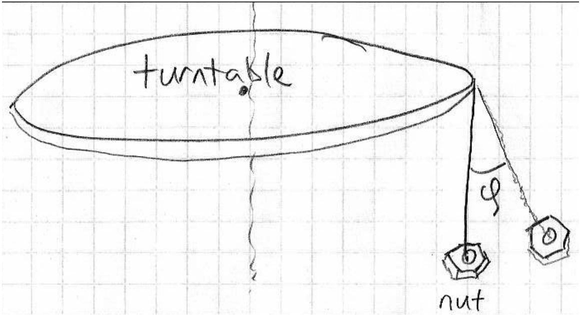
What will happen if I spin the turntable faster? Let T be the tension in the string.

(A) The nut will move farther outward. $T \sin\varphi$ provides the centripetal force mv/R^2, while $T \cos\varphi$ balances gravity mg.

(B) The nut will move farther outward. $T \cos\varphi$ provides the centripetal force mv/R^2, while $T \sin\varphi$ balances gravity mg.

(C) The nut will move farther outward. $T \sin\varphi$ provides the centripetal force mv^2/R, while $T \cos\varphi$ balances gravity mg.

(D) The nut will move farther outward. $T \cos\varphi$ provides the centripetal force mv^2/R, while $T \sin\varphi$ balances gravity mg.



$$m\vec{a}_{\text{nut}} = \vec{F}^{\text{tension}}_{s,\text{nut}} + \vec{F}^{G}_{E,\text{nut}}$$

$$0 = ma_y = T\cos\varphi - mg$$

$$T\cos\varphi = mg$$

$$-\frac{mv^2}{R} = ma_x = -T\sin\varphi$$

$$\frac{T\sin\varphi}{T\cos\varphi} = \tan\varphi = \frac{mv^2/R}{mg} = \frac{v^2}{gR}$$

Now suppose that friction provides the centripetal force

Suppose that a highway offramp that I often use bends with a radius of 20 meters. I notice that my car tires allow me (in good weather) to take this offramp at 15 m/s without slipping. How large does the offramp's bending radius need to be for me to be able to make the turn at 30 m/s instead?

(Assume that the frictional force between the road and my tires is the same in both cases and that the offramp is level (horizontal), i.e. not "banked.")

(A) 5 meters
(B) 10 meters
(C) 20 meters
(D) 30 meters
(E) 40 meters
(F) 80 meters

Now suppose that friction provides the centripetal force

- If velocity gets too large, penny flies off of turntable, as friction is no longer large enough to hold it in place.
- What if there are several pennies placed on the turntable at several different radii?
- As I slowly increase the speed at which the turntable rotates, do all of the pennies fly off at the same time?!

If I put several pennies on the turntable at several different radii and turn the turntable slowly enough that all pennies stay put, which of the following statements is true?

(A) All pennies have the same velocity.

(B) All pennies make the same number of revolutions per second.

(C) All pennies have the same "angular velocity" $\omega = v/r$, but r will vary from penny to penny, so v will also vary.

(D) B and C are both true.

(E) A, B, and C are all true.

How can I best express the centripetal acceleration for each penny on the turntable?

(A) $a = v^2/r$

(B) $a = v^2/r = (r\omega)^2/r = \omega^2 r$

(C) a is the same for all pennies on the turntable

(D) (A) and (B) are both true, but (B) is a more useful way to describe what is happening on the turntable, because v varies from penny to penny, while ω is the same for all pennies.

(E) (A), (B), and (C) are all true

(F) (A), (B), and (C) are all false

Now we know that the centripetal acceleration can be written $a = \omega^2 r$ and varies with the radius of each penny. We also know that static friction must provide the force $m\omega^2 r$ to keep each penny going in a circle. Can we predict which pennies will slide off of the turntable first as I gradually increase the rotational velocity of the turntable?

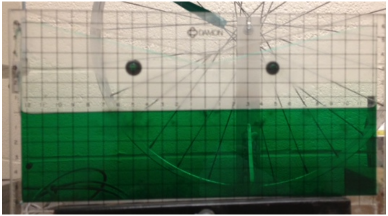
(A) The inside pennies will fly off first. This makes sense, because (for a given speed v) your tires screech more when you go around a turn with small radius than when you go around a turn with large radius.

(B) The outside pennies will fly off first. This makes sense because ω is the same for all pennies (v is not the same for all pennies), but $m\omega^2 r$ is largest for the outermost pennies.

(C) They all slide off at the same time.

(D) No way to predict.

What happens to the surface of this liquid if I center the tank atop the turntable and spin the turntable? (You'll have to make a leap of intuition, by analogy with the spinning nut-on-string, as we haven't studied fluids in this course.)



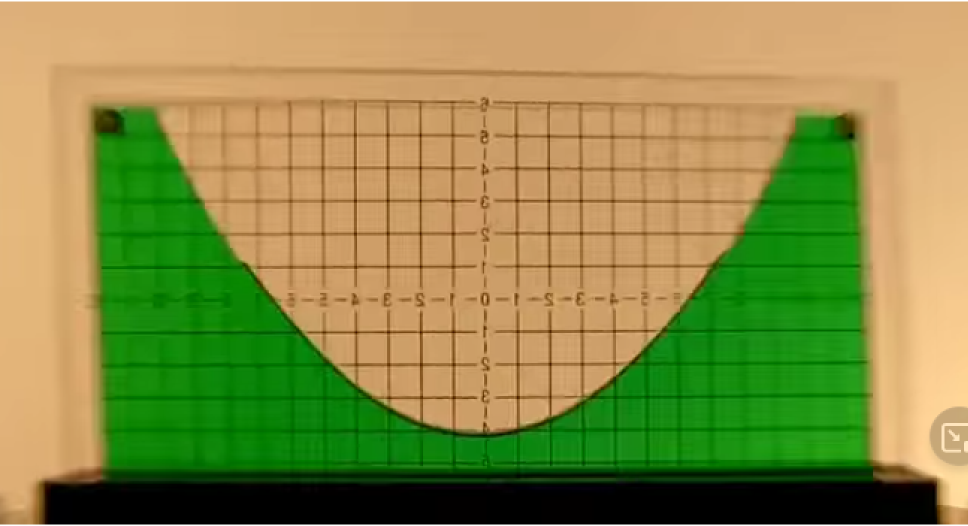
(A) The water surface will stay horizontal.

(B) The water surface is (when not spinning) perpendicular to the vector $(0, -g)$, i.e. it is horizontal. When spinning, the water surface will be perpendicular to the vector $(\omega^2 r, -g)$, with slope $= \omega^2 r/g$. So the surface will be triangular.

(C) The water surface is (when not spinning) perpendicular to the vector $(0, -mg)$, i.e. it is horizontal. When spinning, the water surface will be perpendicular to the vector $(\omega^2 r, -g)$, with slope $= \omega^2 r/g$. So the surface will be a parabola.

(D) All of the water will be stuck against the outer walls, as if trying to escape from a salad spinner.

Our version of this demo is currently broken, but you can see the effect in this video:
`https://www.youtube.com/watch?v=1F5yPSalXb8`



Why does a salad spinner work?

(A) The outer wall of the spinner provides the centripetal force that pushes the lettuce toward the center of rotation, but the water feels no such force, because it can flow through the holes in the outer wall, thus separating water from lettuce.

(B) I really want to say "centrifugal force," even though my high-school teachers told me that there is really no such thing as "centrifugal force" — it's just a pseudo-force that one perceives when observing from the confusing perspective of a non-inertial reference frame.

(C) I guess you could say (A) or (B), but (A) is the way we've learned to analyze the situation methodically from Earth's reference frame. We haven't learned how to do calculations in non-inertial reference frames.

(D) While the obvious answer is (A), I am so fascinated by the pseudo-forces that appear in non-inertial reference frames that I went and read the Wikipedia article on the Coriolis effect!

Here is a good answer to the salad-spinner question: "The explanation for the physics going on as the spinner does its job is centripetal acceleration. The centripetal acceleration of an object in circular motion at constant speed tells us that the vector sum of the forces exerted on the object must be directed toward the center of the circle, continuously adjusting the object's direction. Without this inward pointing vector sum of forces, the object would move in a straight line. Centripetal force between the lettuce and the inside of the spinner pushes the lettuce around in a circle. On the other hand, the water can slip through the drain holes, so there's nothing to give it the same kind of push (and consequently there's no centripetal force to make it go in a circle). Thus, the lettuce experiences centripetal force while the water doesn't. In this way, the spinner manages to separate the two as the lettuce goes round in a circle and the water in a straight line through the holes."

People often point out that we expect the water to shoot out *tangentially* from the spinner, since the water, once it loses contact with the lettuce, should travel in a straight line in the absence of a centripetal force. Need transparent salad spinner to verify!

Suppose I try to spin a pail of water in a vertical circle at constant rotational speed ω, with the water a distance R from the pivot point at my shoulder. So the water is moving at speed $v = \omega R$. (I'll demonstrate first with an empty pail.) Will the water fall out of the pail?

(A) The water will fall out while the pail is upside down, no matter how fast you spin it around.

(B) The water will stay in the pail, no matter how slowly you spin it around.

(C) The water will stay in the pail as long as you spin it fast enough. "Fast enough" means $v/R^2 > g$ (or equivalently $\omega^2/R > g$) when the bucket is upside-down.

(D) The water will stay in the pail as long as you spin it fast enough. "Fast enough" means $v^2/R > g$ (or equivalently $\omega^2 R > g$) when the bucket is upside-down.

The way to think about the water-in-bucket problem is

(A) The bottom surface of the bucket can both push and pull on the water, as if the water and bucket were glued together.

(B) The bottom surface of the bucket can push on the water (compressive force) but cannot pull on the water (no tensile force). If the required centripetal acceleration is large enough that the bucket must push on the water to keep it moving in a circle (even when Earth's gravity is pulling down on the water), then the water will stay in the bucket.

(C) When the bucket is upside down, the bottom surface of the bucket must "pull up" on the water to keep it inside the bucket, or else the water will spill out.

(D) The water stays in the upside-down bucket if the outward "centrifugal pseudo-force" (magnitude mv^2/R or $m\omega^2 R$) is at least as large as the downward force of gravity.

(E) I think you could say (B) or (D), but we haven't learned in this course how to analyze the "pseudo-forces" that one perceives when working in a non-inertial reference frame. So I prefer (B), which uses the Earth reference frame.

How does this thing work?

`http://www.youtube.com/watch?v=oh9sn5gn2fk`

Can you tell me what movie this is from?
(Hints: directed by Stanley Kubrick, story by A.C. Clarke.)

An ice cube and a rubber ball are both placed at one end of a warm cookie sheet, and the sheet is then tipped up. The ice cube slides down with virtually no friction (unrealistic, but let's suspend our disbelief), and the ball rolls down without slipping. Which one makes it to the bottom first?

(A) They reach the bottom at the same time.

(B) The ball gets there slightly faster, because the ice cube's friction (while very small) is kinetic and dissipates some energy, while the rolling ball's friction is static and does not dissipate energy.

(C) The ice cube gets there substantially faster, because the ball's initial potential energy mgh gets shared between $\frac{1}{2}mv^2$ (translational) and $\frac{1}{2}I\omega^2$ (rotational), while essentially all of the ice cube's initial mgh goes into $\frac{1}{2}mv^2$ (translational).

(D) The ice cube gets there faster because the ice cube's friction is negligible, while the frictional force between the ball and the cookie sheet dissipates the ball's kinetic energy into heat.

A hollow cylinder and a solid cylinder both roll down an inclined plane without slipping. Does friction play an important role in the cylinders' motion?

(A) No, friction plays a negligible role.

(B) Yes, (kinetic) friction dissipates a substantial amount of energy as the objects roll down the ramp.

(C) Yes, (static) friction is what causes the objects to roll rather than to slide. Without static friction, they would just slide down, so there would be no rotational motion (if you just let go of each cylinder from rest at the top of the ramp).

Why are people who write physics problems (e.g. about cylinders rolling down inclined planes) so fond of the phrase “rolls without slipping?”

(A) Because Nature abhors the frictional dissipation of energy.

(B) Because “rolls without slipping” implies that $\boxed{v = \omega R}$, where v is the cylinder’s (translational) speed down the ramp. This lets you directly relate the rotational and translational parts of the motion.

(C) No good reason. You could analyze the problem just as easily if the cylinders were slipping somewhat while they roll.

How do I write the total kinetic energy of an object that has both translational motion at speed v and rotational motion at speed ω?

(Note that the symbol I is a capital I (for rotational "inertia") in the sans-serif font that I use to make my slides. Sorry!)

(A) $K = \frac{1}{2}mv^2$
(B) $K = \frac{1}{2}I\omega^2$
(C) $K = \frac{1}{2}I^2\omega$
(D) $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
(E) $K = \frac{1}{2}mv^2 + \frac{1}{2}I^2\omega$
(F) $K = \frac{1}{2}m\omega^2 + \frac{1}{2}Iv^2$

While you ponder, I'll throw a familiar object across the room, for you to look at now from the perspective of Chapter 11 (and 12).

Sliding vs rolling downhill:

For translational motion with no friction, $v_f = \sqrt{2gh}$ because

$$mgh_i = \frac{1}{2}mv_f^2$$

For rolling without slipping, we can write $\omega_f = v_f/R$:

$$mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\left(\frac{v_f}{R}\right)^2$$

$$mgh_i = \frac{1}{2}mv_f^2\left(1 + \frac{I}{mR^2}\right)$$

So the final velocity is slower (as are all intermediate velocities):

$$v_f = \sqrt{\frac{2gh}{1 + \frac{I}{mR^2}}}$$

A hollow cylinder and a solid cylinder both roll down an inclined plane without slipping. Assuming that the two cylinders have the same mass and same outer radius, which one has the larger rotational inertia?

(A) The hollow cylinder has the larger rotational inertia, because the material is concentrated at larger radius.

(B) The solid cylinder has the larger rotational inertia, because the material is distributed over more area.

(C) The rotational inertias are the same, because the masses and radii are the same.

The rolling object's downhill acceleration is smaller by a factor

$$\left(\frac{1}{1+\frac{I}{mR^2}}\right)$$

$I = mR^2$ for hollow cylinder. $\frac{1}{1+1} = 0.5$

$I = \frac{2}{3}mR^2$ for hollow sphere. $\frac{1}{1+(2/3)} = 0.60$

$I = \frac{1}{2}mR^2$ for solid cylinder. $\frac{1}{1+(1/2)} = 0.67$

$I = \frac{2}{5}mR^2$ for solid sphere. $\frac{1}{1+(2/5)} = 0.71$

Using Chapter 11 ideas, we know how to analyze the rolling objects' motion using energy arguments. (With Chapter 12 ideas, we will look again at the same problem using torque arguments, and directly find each object's downhill acceleration.)

Rotational inertia

For an extended object composed of several particles, with particle j having mass m_j and distance r_j from the rotation axis,

$$I = \sum_{j \in \text{particles}} m_j r_j^2$$

For a continuous object like a sphere or a solid cylinder, you have to integrate (or more often just look up the answer):

$$I = \int r^2 \, \mathrm{d}m$$

If you rearrange the same total mass to put it at larger distance from the axis of rotation, you get a larger rotational inertia.

(In which configuration does this adjustable cylinder-like object have the larger rotational inertia?)

inertia

$$m$$

translational velocity

$$v$$

translational K.E.

$$K = \frac{1}{2}mv^2$$

momentum

$$p = mv$$

rotational inertia

$$I = \sum mr^2$$

rotational velocity

$$\omega$$

rotational K.E.

$$K = \frac{1}{2}I\omega^2$$

angular momentum

$$L = I\omega$$

We learned earlier that momentum can be transferred from one object to another, but cannot be created or destroyed.

Consequently, a system on which no external forces are exerted (an "isolated system") has a constant momentum ($\vec{p} = m\vec{v}$):

$$\Delta\vec{p} = 0$$

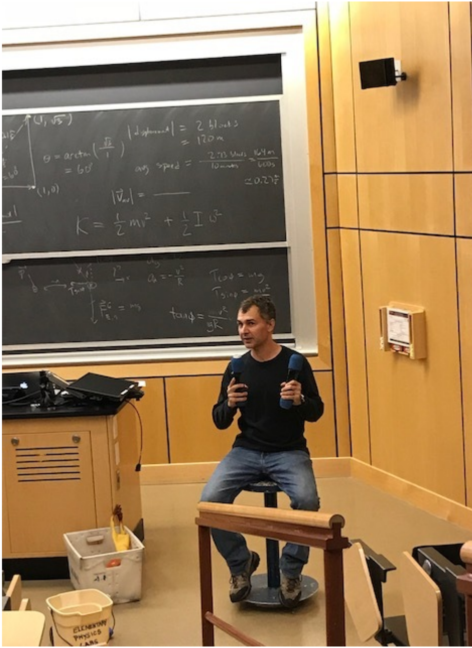
We now also know that angular momentum can be transferred from one object to another, but cannot be created or destroyed.

So a system on which no external torques are exerted has a constant angular momentum ($L = I\omega$):

$$\Delta\vec{L} = 0$$

If I spin around while sitting on a turntable (so that I am rotationally "isolated") and suddenly decrease my own rotational inertia, what happens to my rotational velocity?

In which photo is this character spinning faster (larger ω)?



position

$$\vec{r} = (x, y)$$

velocity

$$\vec{v} = (v_x, v_y) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}$$

acceleration

$$\vec{a} = (a_x, a_y) = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

if a_x is constant then:

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$v_{x,f}^2 = v_{x,i}^2 + 2 a_x \Delta x$$

rotational coordinate

$$\vartheta = s/r$$

rotational velocity

$$\omega = \frac{\mathrm{d}\vartheta}{\mathrm{d}t}$$

rotational acceleration

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$$

if α is constant then:

$$\omega_f = \omega_i + \alpha t$$

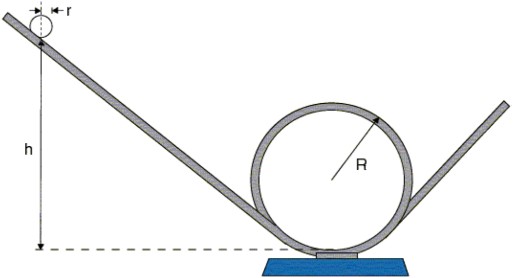
$$\vartheta_f = \vartheta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2 \alpha \Delta\vartheta$$

4. An automobile accelerates from rest starting at t = 0 such that its tires undergo a constant rotational acceleration $\alpha = 5.9\ \mathrm{s}^{-2}$. The radius of each tire is 0.29 m. At t = 11 s after the acceleration begins, find (a) the instantaneous rotational speed ω of the tires, (b) the total rotational displacement $\Delta\vartheta$ of each tire, (c) the linear speed v of the automobile (assuming the tires stay perfectly round) and (d) the total distance the car travels in the 11 s.

(Let's not spend time solving this today. But think about which equations from the previous slide would be useful.)

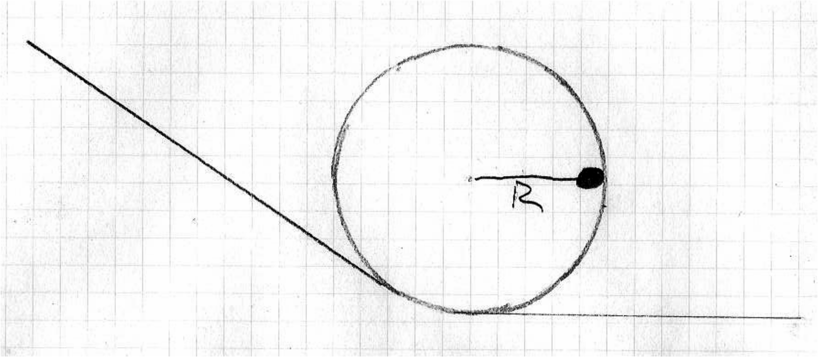
Let R be the **radius** of the circle in this loop-the-loop demo. I want the ball to make it all the way around the loop without falling off. What is the lowest height h at which I can start the ball (from rest)?



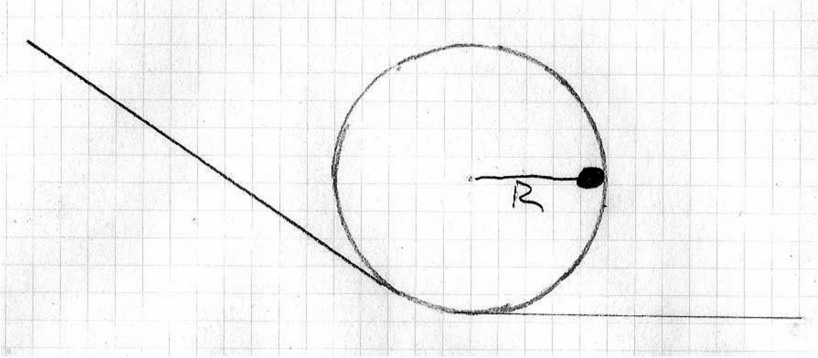
(A) The ball will make it all the way around if $h \geq R$.

(B) The ball will make it all the way around if $h \geq 2R$.

(C) If $h = 2R$, the ball will just make it to the top and will then fall down (assuming, for the moment, that it slides frictionlessly along the track). When the ball is at the top of the circle, its velocity must still be large enough to require a downward normal force exerted by the track on the ball. So the minimum h is even larger than $2R$. My neighbor and I are discussing now just how much higher that should be.

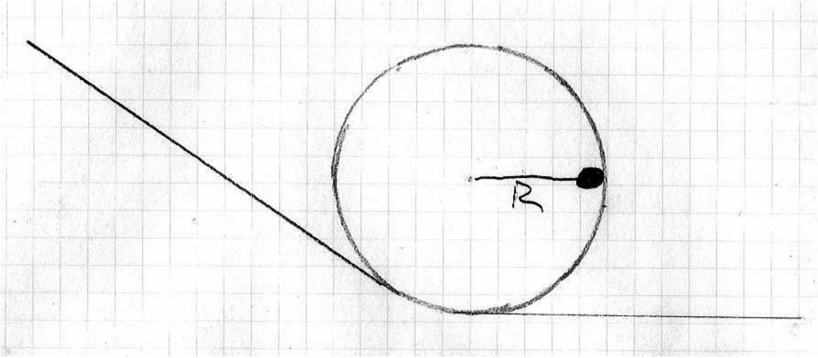


The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), in what direction does its velocity vector point?



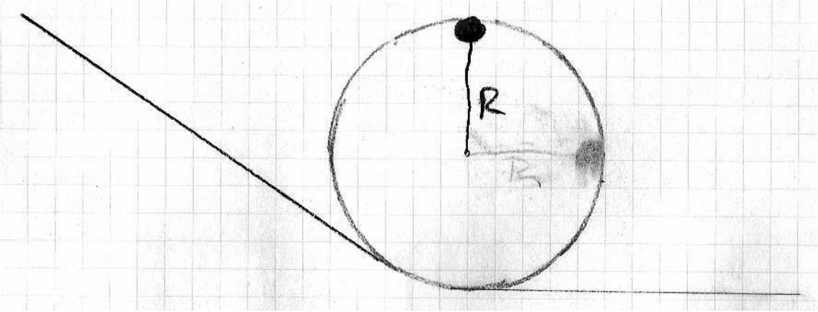
The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), what do we know about the vertical (y axis points up) component, a_y, of the ball's acceleration vector?

If $a_y \neq 0$, what vertical force(s) F_y is/are responsible?



The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), what do we know about the horizontal (x axis points right) component, a_x, of the ball's acceleration vector?

If $a_x \neq 0$, what horizontal force(s) F_x is/are responsible?



Suppose the ball makes it all the way around the circle without falling off. At the instant when the ball is at the position shown (at top of circle), what do we know about the vertical component, a_y, of the ball's acceleration vector?

If $a_y \neq 0$, what vertical force(s) F_y is/are responsible?

Suppose the ball makes it all the way around the loop-the-loop with much more than sufficient speed to stay on the circular track. Let the y-axis point upward, and let v_{top} be the ball's speed when it reaches the top of the loop. What is the y component, a_y, of the ball's acceleration when it is at the very top of the loop?

(A) $a_y = -g$

(B) $a_y = +g$

(C) $a_y = +v_{\mathrm{top}}^2/R$

(D) $a_y = -v_{\mathrm{top}}^2/R$

(E) $a_y = +g + v_{\mathrm{top}}^2/R$

(F) $a_y = -g - v_{\mathrm{top}}^2/R$

(G) $a_y = +g + v_{\mathrm{top}}/R^2$

(H) $a_y = -g - v_{\mathrm{top}}/R^2$

The track can push on the ball, but it can't pull on the ball! How do I express the fact that the track is still pushing on the ball even at the very top of the loop?

(A) Write the equation of motion for the ball: $m\vec{a} = \sum \vec{F}_{\mathrm{on\ ball}}$, and require the normal force exerted by the track on the ball to point inward, even at the very top. (At the very top, "inward" is "downward.") If the equation $m\vec{a} = \sum \vec{F}_{\mathrm{on\ ball}}$ gave us an outward-pointing normal force (exerted by track on ball), that would be inconsistent with the ball's staying in contact with the track.

(B) Use conservation of angular momentum.

(C) Draw a free-body diagram for the ball, and require that gravity and the normal force point in opposite directions.

(D) Draw a free-body diagram for the ball, and require that the magnitude of the normal force be at least as large as the force of Earth's gravity on the ball.

For the ball to stay in contact with the track when it is at the top of the loop, there must still be an inward-pointing normal force exerted by the track on the ball, even at the very top. How can I express this fact using $\boxed{ma_y = \sum F_y}$? Let $v_{\rm top}$ be the ball's speed at the top of the loop.

(A) $+mv_{\rm top}^2/R = +mg + F^N_{tb}$ with $F^N_{tb} > 0$

(B) $+mv_{\rm top}^2/R = +mg - F^N_{tb}$ with $F^N_{tb} > 0$

(C) $+mv_{\rm top}^2/R = -mg + F^N_{tb}$ with $F^N_{tb} > 0$

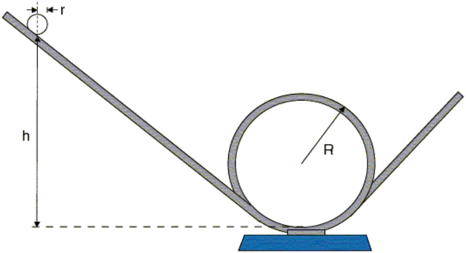
(D) $+mv_{\rm top}^2/R = -mg - F^N_{tb}$ with $F^N_{tb} > 0$

(E) $-mv_{\rm top}^2/R = +mg + F^N_{tb}$ with $F^N_{tb} > 0$

(F) $-mv_{\rm top}^2/R = +mg - F^N_{tb}$ with $F^N_{tb} > 0$

(G) $-mv_{\rm top}^2/R = -mg + F^N_{tb}$ with $F^N_{tb} > 0$

(H) $-mv_{\rm top}^2/R = -mg - F^N_{tb}$ with $F^N_{tb} > 0$



How do I decide the minimum height h from which the ball will make it all the way around the loop without losing contact with the track? For simplicity, assume that the track is very slippery, so that you can neglect the ball's rotational kinetic energy.

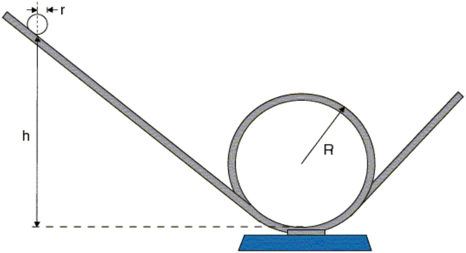
(A) $2mgR = \frac{1}{2}mv_{\rm top}^2 + mgh$ with $v_{\rm top} = \sqrt{gR}$

(B) $mgR = \frac{1}{2}mv_{\rm top}^2 + mgh$ with $v_{\rm top} = \sqrt{gR}$

(C) $mgh = \frac{1}{2}mv_{\rm top}^2 + 2mgR$ with $v_{\rm top} = \sqrt{gR}$

(D) $mgh = \frac{1}{2}mv_{\rm top}^2 + mgR$ with $v_{\rm top} = \sqrt{gR}$

(By the way, how would the answer change if I said instead that the (solid) ball rolls without slipping on the track?)



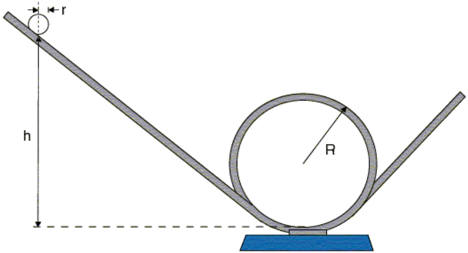
How do I decide the minimum height h from which the ball will make it all the way around the loop without losing contact with the track? Let's now be realistic: the ball is a solid sphere that rolls without slipping on the track.

(A) $mgh = \frac{1}{2}mv_{\rm top}^2 + 2mgR$

(B) $mgh = \frac{1}{2}mv_{\rm top}^2 + \frac{1}{2}I\omega_{\rm top}^2 + 2mgR$

with $v_{\rm top} = \sqrt{gR}$ and $\omega_{\rm top} = v_{\rm top}/r_{\rm ball}$

(Little "$r_{\rm ball}$" is the radius of the ball. Big "R" is the radius of the loop-the-loop.)

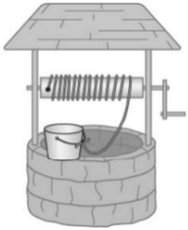


$$mgh = \frac{1}{2} m v_{\rm top}^2 + \frac{1}{2} I \omega_{\rm top}^2 + 2mgR$$

with $v_{\rm top} = \sqrt{gR}$ and $\omega_{\rm top} = v_{\rm top}/r_{\rm ball}$ and $I = \frac{2}{5} m r_{\rm ball}^2$.

$$mgh = \frac{1}{2} m(gR) + \frac{\frac{2}{5} m r_{\rm ball}^2}{2 r_{\rm ball}^2}(gR) + 2mgR = 2.7 mgR$$

6*. You accidentally knock a full bucket of water off the side of the well shown in the figure at right. The bucket plunges 18 m to the bottom of the well. Attached to the bucket is a light rope that is wrapped around the crank cylinder. How fast is the handle turning (rotational speed) when the bucket hits bottom? The inertia of the bucket plus water is 13 kg. The crank cylinder is a solid cylinder of radius 0.65 m and inertia 5.0 kg. (Assume the small handle's inertia is negligible in comparison with the crank cylinder.)



How would you approach this problem? Discuss with your neighbor while I set up a demonstration along the same lines ...

(A) initial angular momentum of bucket equals final angular momentum of cylinder + bucket

(B) initial G.P.E. equals final K.E. (translational for bucket + rotational for cylinder)

(C) initial G.P.E. equals final K.E. of bucket

(D) initial G.P.E. equals final K.E. of cylinder

(E) initial K.E. of bucket equals final G.P.E.

(F) use torque = mgR to find constant angular acceleration

- What is the rotational inertia for a solid cylinder?
- How do you relate v of the bucket with ω of the cylinder? Why is this true?
- What is the expression for the total kinetic energy?
- Why is angular momentum not the same for the initial and final states?
- What are the two expressions for angular momentum used in Chapter 11?
- Does anyone know (though this is in Chapter 12 and is tricky) why using $\tau = mgR$ would not give the correct angular acceleration? What if you used $\tau = TR$, where T is the tension in the rope?

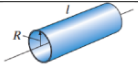
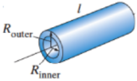
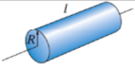
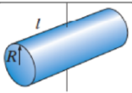
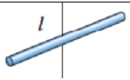
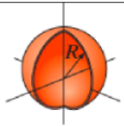
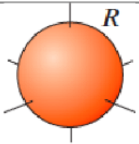
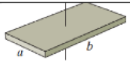
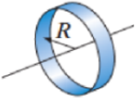
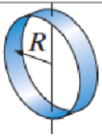
configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder $\perp$ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod $\perp$ to axis		$(1/12)m\ell^2$

Table 11.3. Also in "equation sheet"
`http://positron.hep.upenn.edu/p8/files/equations.pdf`

hollow sphere		$(2/3)mR^2$
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2+b^2)$
thin hoop about its axis		mR^2
thin hoop ⊥ to axis		$(1/2)mR^2$

5. A long, thin rod is pivoted from a hinge such that it hangs vertically from one end. (The hinge is at the top.) The length of the rod is 1.23 m. You want to hit the lower end of the rod just hard enough to get the rod to swing all the way up and over the pivot (i.e. to swing more than 180°). How fast do you have to make the end go?

How would you approach this problem? Discuss with neighbors!
Which (if any) of these statements is **false** ?

(A) I know the change in G.P.E from the initial to the desired final states. So the initial K.E. (which is rotational) of the rod needs to be at least this large.

(B) The book (or equation sheet) gives rotational inertia I for a long, thin rod about its center. So I can use the parallel-axis theorem to get I for the rod about one end.

(C) The angular momentum, $L = I\omega$, is the same for the initial and final states.

(D) Because the rod pivots about one end, the speed of the other end is $v = \omega\ell$ (where ℓ is length of rod)

(E) None. (All of the above statements are true.)

The rotational inertia for a long, thin rod of length ℓ about a perpendicular axis through its center is

$$I = \frac{1}{12} m\ell^2$$

What is its rotational inertia about one end?

(A) $\frac{1}{12} m\ell^2$
(B) $\frac{1}{24} m\ell^2$
(C) $\frac{1}{2} m\ell^2$
(D) $\frac{1}{3} m\ell^2$
(E) $\frac{1}{4} m\ell^2$
(F) $\frac{1}{6} m\ell^2$

(We'll repeat this question after some explanation.)

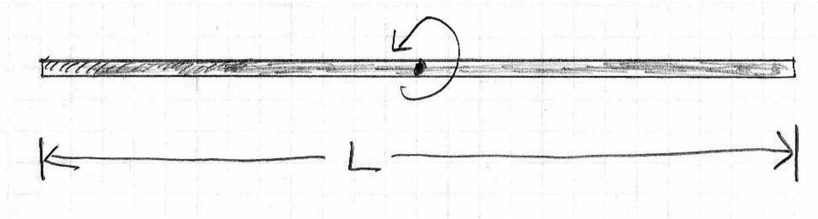
If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance $d_\perp$ from CoM), the rotational inertia is larger, because the object's CoM revolves around a circle of radius $d_\perp$ and in addition the object rotates about its own CoM.

This larger rotational inertia is given by the *parallel axis theorem*:

$$I = I_{\rm cm} + Md_\perp^2$$

where $I_{\rm cm}$ is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's CoM.

(We'll go over the parallel-axis theorem again next time. First I want to make sure you know what you need for this week's HW.)



The rotational inertia of a long, thin rod (whose thickness is negligible compared with its length) of mass M and length L, for rotation about its CoM, is

$$I = \frac{1}{12} ML^2$$

Using the parallel axis theorem, what is the rod's rotational inertia for rotation about one end? (Click next page.)

The rotational inertia for a long, thin rod of length ℓ about a perpendicular axis through its center is

$$I = \frac{1}{12} m\ell^2$$

What is its rotational inertia about one end?

(A) $\frac{1}{12} m\ell^2$

(B) $\frac{1}{24} m\ell^2$

(C) $\frac{1}{2} m\ell^2$

(D) $\frac{1}{3} m\ell^2$

(E) $\frac{1}{4} m\ell^2$

(F) $\frac{1}{6} m\ell^2$

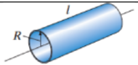
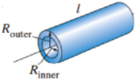
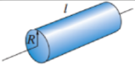
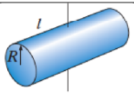
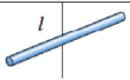
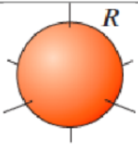
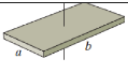
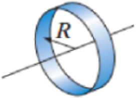
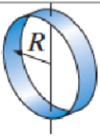
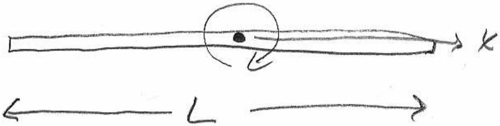
configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder $\perp$ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod $\perp$ to axis		$(1/12)m\ell^2$

Table 11.3. Also in “equation sheet”

`http://positron.hep.upenn.edu/p8/files/equations.pdf`

hollow sphere		$(2/3)mR^2$
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2 + b^2)$
thin hoop about its axis		mR^2
thin hoop ⊥ to axis		$(1/2)mR^2$

(In case you're curious where that $I = ML^2/12$ comes from.)



$$I = \sum m r^2 \rightarrow \int r^2 \, dm$$

$$dm = \frac{M}{L} dx \qquad r = |x|$$

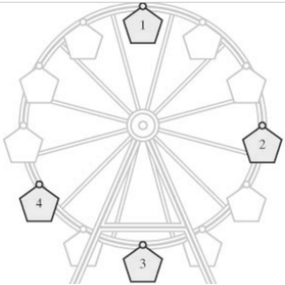
$$I = \int_{-L/2}^{L/2} x^2 \left(\frac{M}{L}\right) dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{x=-\frac{L}{2}}^{x=+\frac{L}{2}}$$

$$I = \frac{M}{L}\left[\frac{(L/2)^3}{3} - \frac{(-L/2)^3}{3}\right] = \frac{ML^2}{12}$$

3*. You have a weekend job selecting speed-limit signs to put at road curves. The speed limit is determined by the radius of the curve and the bank angle of the road w.r.t. horizontal. Your first assignment today is a turn of radius 250 m at a bank angle of 4.8°. (a) What speed limit sign should you choose for that curve such that a car traveling at the speed limit negotiates the turn successfully even when the road is wet and slick? (So at this speed, it stays on the road even when friction is negligible.) (b) Draw a free-body diagram showing all of the forces acting on the car when it is moving at this maximum speed. (Your diagram should also indicate the direction of the car's acceleration vector.)

Let's start by drawing an FBD (for the car) for the case where the car's speed is at exactly the value for which no friction at all is needed to keep the car moving in its circular path. In that case, what are the forces acting on the car?

Alongside the FBD, let's draw (elevation view) the car on the banked road. Let's assume that the road curves to the left.



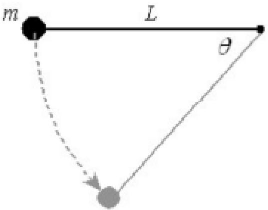
A Ferris wheel rotates at constant speed. Draw an FBD for each numbered carriage. For scale, draw $m\vec{a}$ on each FBD instead of the usual $\vec{a}$. Then the two force vectors must sum to $m\vec{a}$. For a realistic Ferris wheel, ma would be much smaller than mg, but to make drawings that bring out the physics, let's make ma be half the size of mg. In other words, for the sake of illustration, we'll spin the Ferris wheel fast enough that $v^2/R = g/2$.

I'll try to use these values for some ad-hoc questions:

(A) E (B) NE (C) N (D) NW (E) W (F) SW (G) S (H) SE

(A) 0.5 (B) 1.0 (C) $\sqrt{\frac{5}{4}} = 1.118$ (D) 1.32 (E) 1.5 (F) 2.0

2. You attach one end of a string of length L to a small ball of inertia m. You attach the string's other end to a pivot that allows free revolution. You hold the ball out to the side with the string taut along a horizontal line, as the in figure (below, left). (a) If you release the ball from rest, what is the tension in the string as a function of angle ϑ swept through? (b) What should the tensile strength of the string be (the maximum tension it can sustain without breaking) if you want it not to break through the ball's entire motion (from $\vartheta = 0$ to $\vartheta = \pi$)?

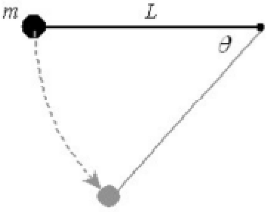


- Since the string's length L stays constant, what shape does the ball's path trace out as it moves?
- Does the ball's acceleration have a component that points along the axis of the string? If so, does its magnitude depend on the ball's speed?
- What two forces are acting on the ball?
- Assuming that no energy is dissipated, how can we relate the ball's speed v to its height y ?
- Can you write $m\vec{a} = \sum \vec{F}$ for the component of $\vec{a}$ and $\sum \vec{F}$ that points along the string?

("small" ball $\Rightarrow$ neglect the ball's rotation about its own CoM)

2. You attach one end of a string of length L to a small ball of inertia m. You attach the string's other end to a pivot that allows free revolution. You hold the ball out to the side with the string taut along a horizontal line, as the in figure (below, left). (a) If you release the ball from rest, what is the tension in the string as a function of angle ϑ swept through? (b) What should the tensile strength of the string be (the maximum tension it can sustain without breaking) if you want it not to break through the ball's entire motion (from $\vartheta = 0$ to $\vartheta = \pi$)?

How do I relate angle θ to speed v? $E_i = mgL \quad \rightarrow \quad E = \frac{1}{2}mv^2 + mgy$



(A) $\frac{1}{2}mv^2 = mg(L - y) = mgL(1 - \cos\theta)$

(B) $\frac{1}{2}mv^2 = mg(L - y) = mgL(1 - \sin\theta)$

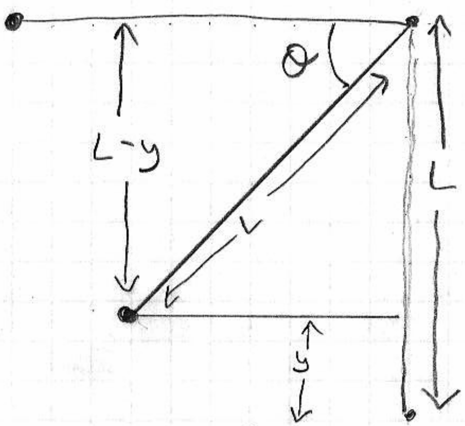
(C) $\frac{1}{2}mv^2 = mg(L - y) = mgL\cos\theta$

(D) $\frac{1}{2}mv^2 = mg(L - y) = mgL\sin\theta$

Hint: draw on the figure a vertical line of length $L - y = y_i - y$

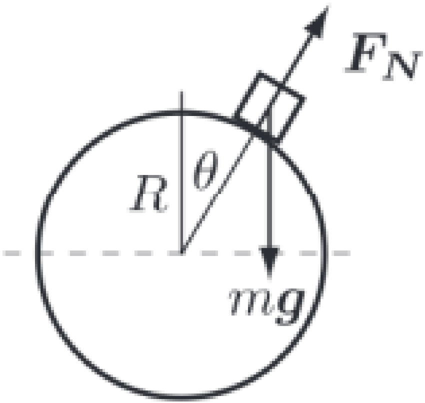
Next: write "radial" component of $m\vec{a} = \sum \vec{F}$ to find T

$$\sin\theta = \frac{L-y}{L}$$

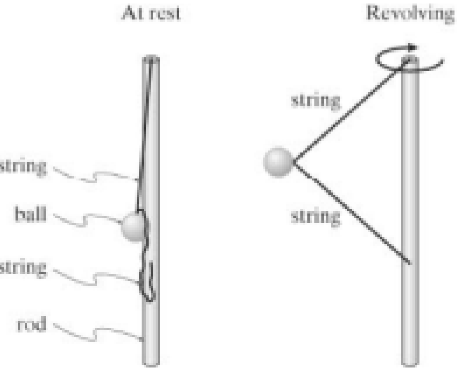


When does the block lose contact with the sphere?

A small block of mass m slides down a sphere of radius R, starting from rest at the top. The sphere is immobile, and friction between the block and the sphere is negligible. In terms of m, g, R, and θ, determine:
(a) the K.E. of the block;
(b) the centripetal acceleration of the block;
(c) the normal force exerted by the sphere on the block.
(d) At what value of θ does the block lose contact with the sphere?

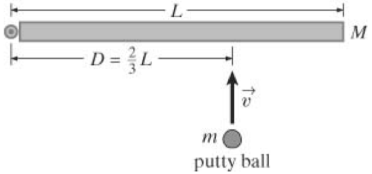


8*. A ball is attached to a vertical rod by two strings of equal strength and equal length. (See figure, below left.) The strings are very light and do not stretch. The rod begins to rotate under the influence of a constant rotational acceleration. (a) Which string breaks first? (b) Draw a free-body diagram for the ball, indicating all forces and their relative magnitudes, to justify your answer to (a).



- Are the angles of the two strings w.r.t. horizontal equal?
- Are the tensions in the two strings equal? How do you know?
- What three forces act on the ball?
- Is the ball accelerating vertically? Horizontally?
- Draw a FBD for the ball, showing both horizontal (radial) and vertical component of each force.

Notice that the ball's speed v increases with time, until finally one string breaks. Which one? (Which string's tension is larger?)



(plan view — from above)

9*. An open door of inertia M and width L is at rest when it is struck by a thrown putty ball of inertia m that is moving at linear speed v at the instant it strikes the door. (See figure, above right.) The impact point is a distance $D = \frac{2}{3}L$ from the rotation axis through the hinges. The putty ball strikes at a right angle to the door face and sticks after it hits. What is the rotational speed of the door and putty? Do not ignore the inertia m.

How would you approach this problem? Discuss with neighbors!

(A) The final K.E. (rotational+translational) equals the initial K.E. of the ball.

(B) The initial momentum $m\vec{v}$ of the ball equals the final momentum $(m+M)\vec{v}$ of the door+ball.

(C) The initial angular momentum $L = r_\perp mv$ of the ball w.r.t. the hinge axis equals the final angular momentum $L = I\omega$ of the door+ball.

9*. An open door of inertia M and width L is at rest when it is struck by a thrown putty ball of inertia m that is moving at linear speed v at the instant it strikes the door. (See figure, above right.) The impact point is a distance $D = \frac{2}{3}L$ from the rotation axis through the hinges. The putty ball strikes at a right angle to the door face and sticks after it hits. What is the rotational speed of the door and putty? Do not ignore the inertia m.

I know that the rotational inertia of a thin rod of length L about a perpendicular axis through its center is $I = \frac{1}{12}mL^2$. The rotational inertia I to use for the final state here is

(A) $I = ML^2 + mL^2$

(B) $I = \frac{1}{12}ML^2 + M(\frac{L}{2})^2 + m(\frac{2}{3}L)^2$

(C) $I = \frac{1}{12}ML^2 + \frac{2}{3}mL^2$

(D) $I = \frac{1}{12}ML^2 + m(\frac{2}{3}L)^2$

(E) $I = \frac{1}{12}ML^2 + mL^2$

(Challenge: Also think how the answer would change if the radius of the putty ball were non-negligible. What if the thickness of the door were non-negligible? Does the height of the door matter?)

Three different expressions for angular momentum:

$$L = I\omega$$

$$L = r_{\perp}\ mv$$

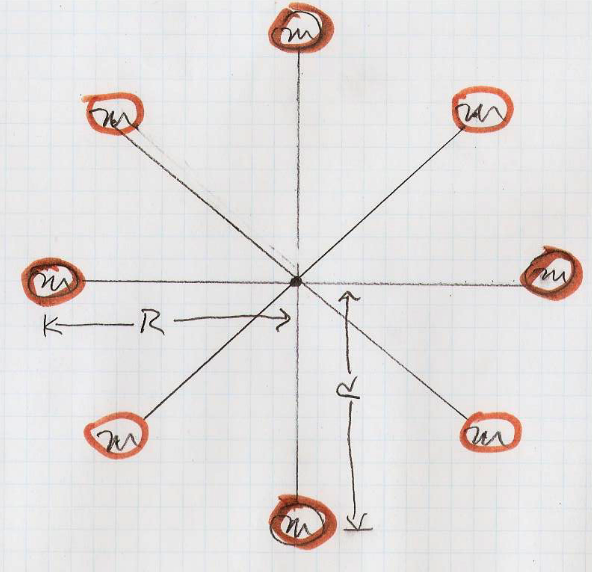
$$L = r\ mv_{\perp}$$

The second expression is telling you that momentum times lever arm (w.r.t. the relevant pivot axis) equals angular momentum.

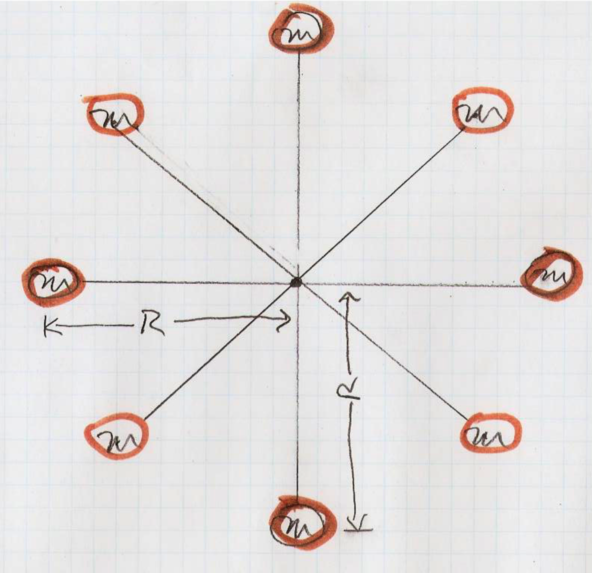
The second and third expressions are both simplified ways of writing the more general (but more difficult) expression

$$\vec{L} = \vec{r} \times \vec{p}$$

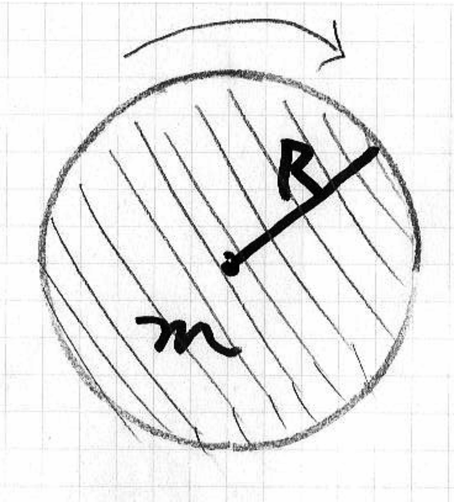
10*. Two skaters skate toward each other, each moving at 3.1 m/s. Their lines of motion are separated by a perpendicular distance of 1.8 m. Just as they pass each other (still 1.8 m apart), they link hands and spin about their common center of mass. What is the rotational speed of the couple about the center of mass? Treat each skater as a point particle, each with an inertia of 52 kg.



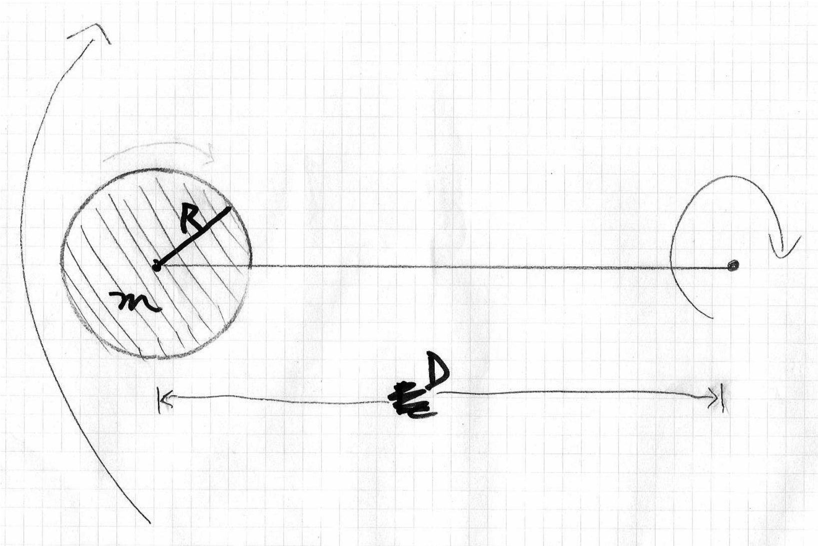
Where is the center of mass of this pinwheel-like object?



What is this object's rotational inertia, for rotation about its center of mass? Assume that all of the mass is concentrated in the orange blobs, and assume that the orange blobs are "point masses," i.e. that their size is much smaller than R.



Suppose I have a solid disk of radius R and mass m. I rotate it about its CoM, about an axis $\perp$ to the plane of the page. What is its rotational inertia? (If you don't happen to remember — is it bigger than, smaller than, or equal to mR^2 ?)



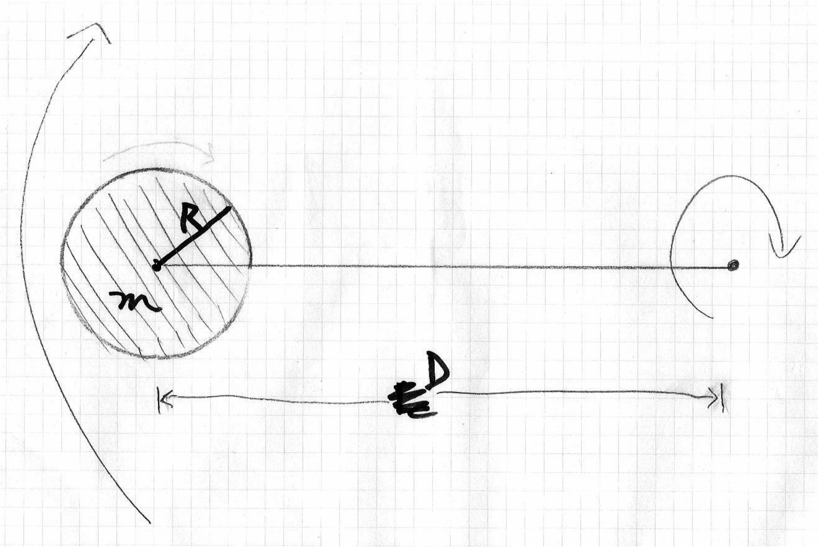
Now I take the same disk, attach it to a string or a lightweight stick of length D, and make the disk's CoM go around in circles of radius D. Is the mass now farther than or closer to the rotation axis than in the original rotation (about CoM)? What happens to I ?

If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance D from CoM), the rotational inertia is larger, because the object's CoM revolves around a circle of radius D and in addition the object rotates about its own CoM.

This larger rotational inertia is given by the *parallel axis theorem*:

$$I = I_{\rm cm} + MD^2$$

where $I_{\rm cm}$ is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's CoM.



Using the parallel axis theorem, what is the disk's rotational inertia about the displaced axis (the axis that is distance D away from the CoM)?