Mazur ch12 — torque

- begin video preceding ws17 = mz12

Torque: the rotational analogue of force

Just as an unbalanced force causes linear acceleration

$$\vec{F} = m\vec{a}$$

an unbalanced torque causes rotational acceleration

$$\tau = I\alpha$$

Torque is **(lever arm)** $\times$ **(force)**

$$\tau = r_{\perp} F$$

where $r_{\perp}$ is the "perpendicular distance" from the rotation axis to the line-of-action of the force.

position

$$\vec{r} = (x, y)$$

velocity

$$\vec{v} = (v_x, v_y) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}$$

acceleration

$$\vec{a} = (a_x, a_y) = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

momentum

$$\vec{p} = m\vec{v}$$

force

$$\vec{F} = m\vec{a}$$

rotational coordinate

$$\vartheta = s/r$$

rotational velocity

$$\omega = \mathrm{d}\vartheta/\mathrm{d}t$$

rotational acceleration

$$\alpha = \mathrm{d}\omega/\mathrm{d}t$$

angular momentum

$$L = I\omega$$

$$L = r_\perp \ mv$$

torque

$$\tau = I\alpha$$

$$\tau = r_\perp \ F$$

I wind a string around a coffee can of radius $R = 0.05\ \mathrm{m}$. (That's 5 cm.) Friction prevents the string from slipping. I apply a tension $T = 20\ \mathrm{N}$ to the free end of the string. The free end of the string is tangent to the coffee can, so that the radial direction is perpendicular to the force direction. What is the magnitude of the torque exerted by the string on the coffee can?

(A) $1\ \mathrm{N \cdot m}$

(B) $2\ \mathrm{N \cdot m}$

(C) $5\ \mathrm{N \cdot m}$

(D) $10\ \mathrm{N \cdot m}$

(E) $20\ \mathrm{N \cdot m}$

Suppose that the angular acceleration of the can is $\alpha = 2\ \mathrm{s}^{-2}$ when the string exerts a torque of $1\ \mathrm{N} \cdot \mathrm{m}$ on the can. What would the angular acceleration of the can be if the string exerted a torque of $2\ \mathrm{N} \cdot \mathrm{m}$ instead?

(A) $\alpha = 0.5\ \mathrm{s}^{-2}$

(B) $\alpha = 1\ \mathrm{s}^{-2}$

(C) $\alpha = 2\ \mathrm{s}^{-2}$

(D) $\alpha = 4\ \mathrm{s}^{-2}$

(E) $\alpha = 5\ \mathrm{s}^{-2}$

(F) $\alpha = 10\ \mathrm{s}^{-2}$

I apply a force of 5.0 N at a perpendicular distance of 5 cm ($r_\perp = 0.05\ \mathrm{m}$) from this rotating wheel, and I observe some angular acceleration α. What force would I need to apply to this same wheel at $r_\perp = 0.10\ \mathrm{m}$ (that's 10 cm) to get the same angular acceleration α?

(A) $F = 1.0\ \mathrm{N}$

(B) $F = 2.5\ \mathrm{N}$

(C) $F = 5.0\ \mathrm{N}$

(D) $F = 10\ \mathrm{N}$

(E) $F = 20\ \mathrm{N}$

Suppose that I use the tension T in the string to apply a given torque $\tau = r_\perp T$ to this wheel, and it experiences a given angular acceleration α. Now I **increase** the rotational inertia I of the wheel and then apply the same torque. The new angular acceleration $\alpha_{\rm new}$ will be

(A) larger: $\alpha_{\rm new} > \alpha$

(B) the same: $\alpha_{\rm new} = \alpha$

(C) smaller: $\alpha_{\rm new} < \alpha$

I want to tighten a bolt to a torque of 1.0 newton-meter, but I don't have a torque wrench. I do have an ordinary wrench, a ruler, and a 1.0 kg mass tied to a string. How can I apply the correct torque to the bolt?

(A) Orient the wrench horizontally and hang the mass at a distance 0.1 m from the axis of the bolt

(B) Orient the wrench horizontally and hang the mass at a distance 1.0 m from the axis of the bolt

If the wrench is at 45° w.r.t. horizontal, will the 1.0 kg mass suspended at a distance 0.1 m along the wrench still exert a torque of 1.0 newton-meter on the bolt?

(A) Yes. The force of gravity has not changed, and the distance has not changed.

(B) No. The torque is now smaller — about 0.71 newton-meter — because the "perpendicular distance" is now smaller by a factor of $1/\sqrt{2}$.

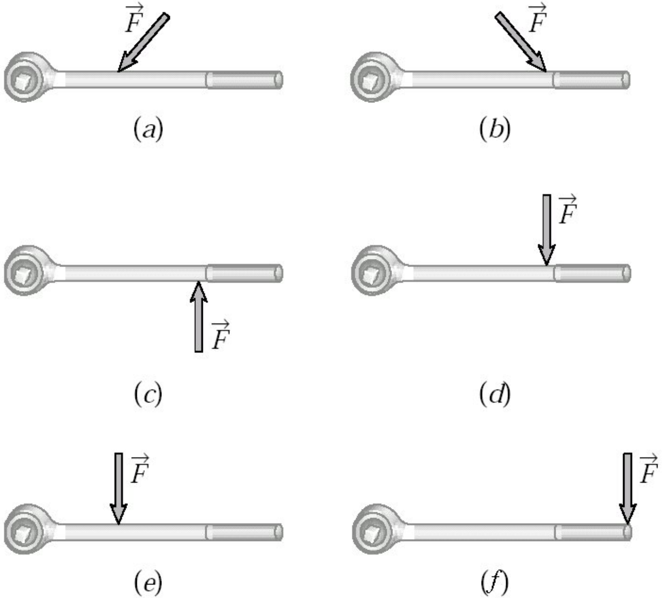
(C) No. The torque is now larger — about 1.4 newton-meter.

$$\tau \;=\; r_\perp F \;=\; r F_\perp \;=\; r F \sin\theta_{rF} \;=\; |\vec{r} \times \vec{F}|$$

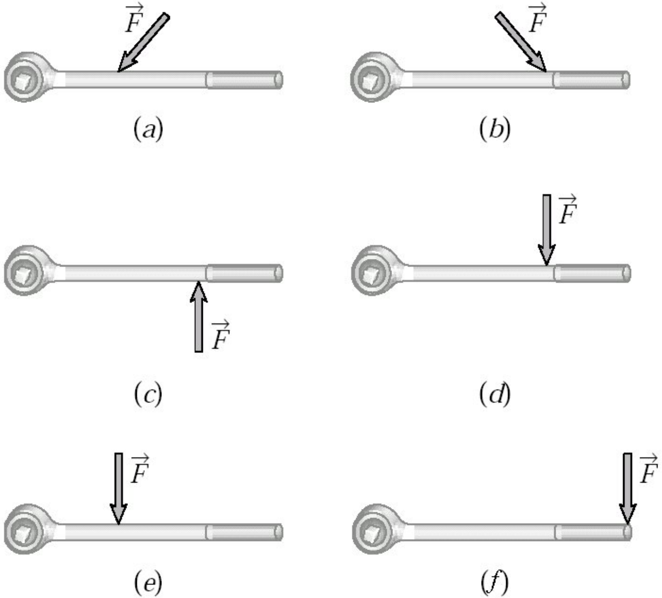
Four ways to get the magnitude of the torque

- (perpendicular component of distance) $\times$ (force)
- (distance) $\times$ (perpendicular component of force)
- (distance) (force) ($\sin\theta$ between $\vec{r}$ and $\vec{F}$)
- use magnitude of "vector product" $\vec{r} \times \vec{F}$ (a.k.a. "cross product")

To tighten a bolt, I apply a force of the same magnitude F at different positions and angles. Which torque is *largest*?



To tighten a bolt, I apply a force of magnitude F at different positions and angles. Which torque is *smallest*?



I want to apply to this meter stick two torques of the same magnitude and opposite sense, so that the stick has zero rotational acceleration. I apply one force of 5 N at a lever arm of 0.5 m. I want to apply an opposing force at a lever arm of 0.2 m, so that the second torque balances the first torque. How large must this second force be?

(A) 1.0 N
(B) 2.0 N
(C) 12.5 N
(D) 25 N

I want to apply to this meter stick two torques of the same magnitude and opposite sense, so that the stick has zero rotational acceleration. I apply one force of 10 N at a lever arm of 0.5 m. I tie a second string on the opposite end, 0.5 m from the pivot point. **The second force is applied at a 45° angle w.r.t. the vertical.** How large must this second force be?

(A) 5 N

(B) 7 N

(C) 10 N

(D) 14 N

(E) 20 N

If the rod doesn't accelerate (rotationally, about the pivot), what force does the scale read?

(A) 1.0 N

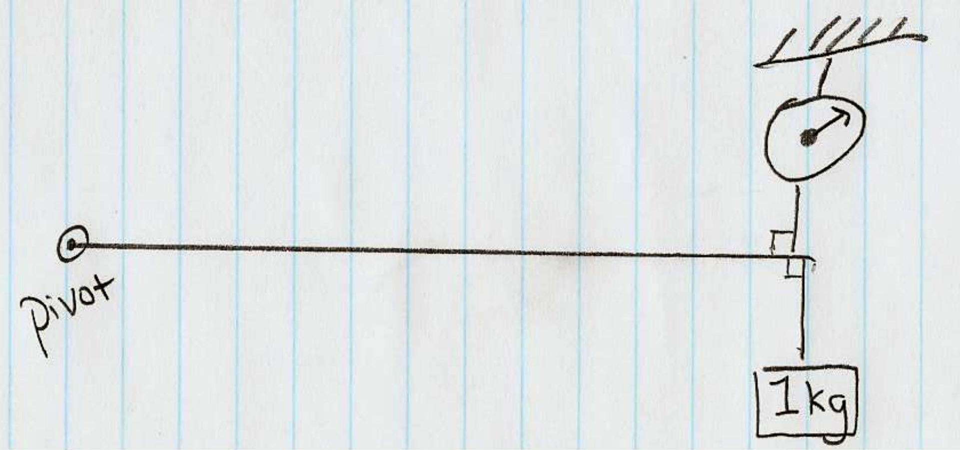
(B) 5.0 N

(C) 7.1 N

(D) 10 N

(E) 14 N

(F) 20 N



If the rod doesn't accelerate, what force does the scale read?

(A) 1.0 N

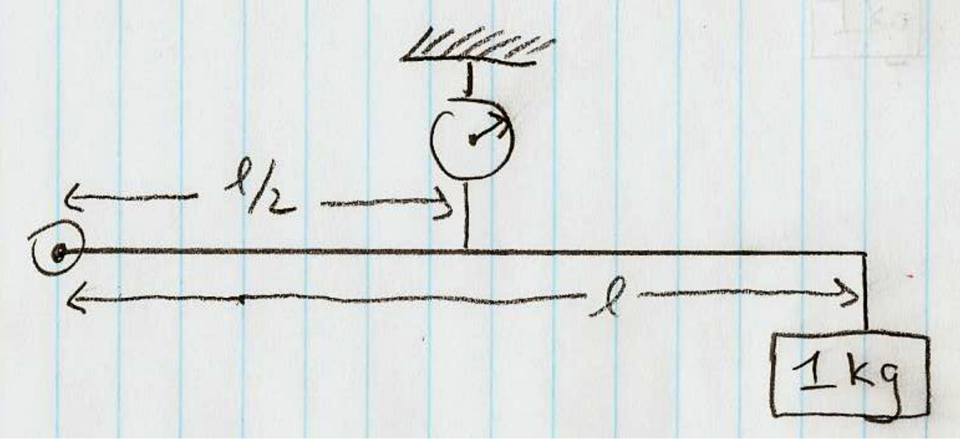
(B) 5.0 N

(C) 7.1 N

(D) 10 N

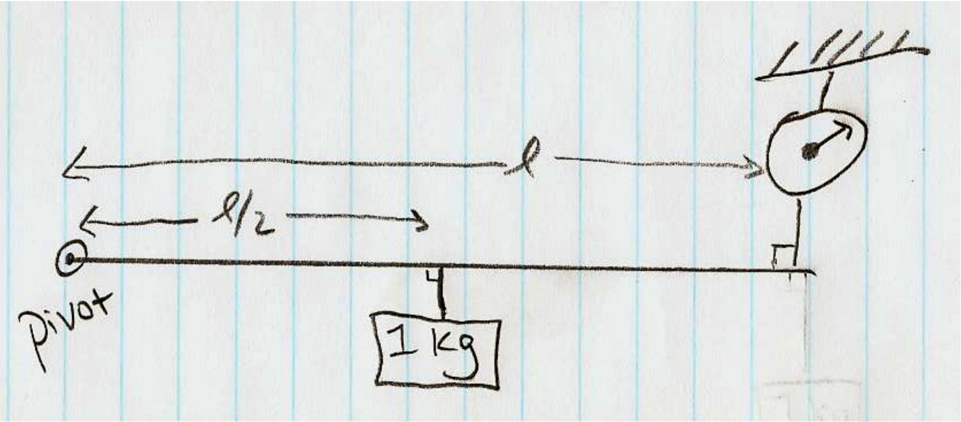
(E) 14 N

(F) 20 N



If the rod doesn't accelerate, what force does the scale read?

(A) 1.0 N
(B) 5.0 N
(C) 7.1 N
(D) 10 N
(E) 14 N
(F) 20 N



If the rod doesn't accelerate, what force does the scale read?

(A) 1.0 N

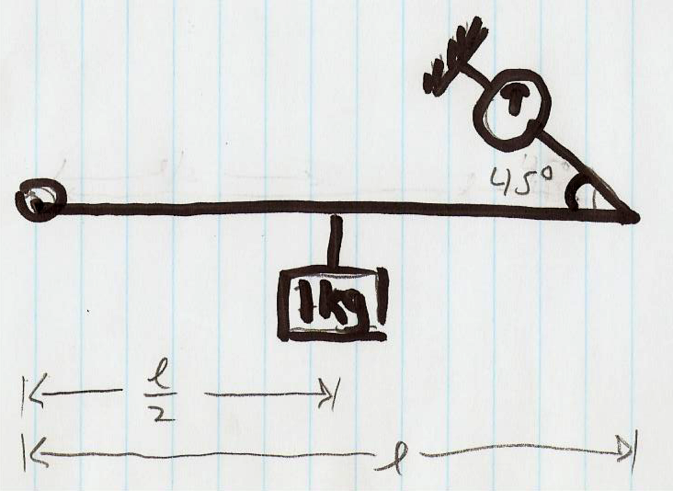
(B) 5.0 N

(C) 7.1 N

(D) 10 N

(E) 14 N

(F) 20 N



If the rod doesn't accelerate, what force does the scale read?

(A) 1.0 N

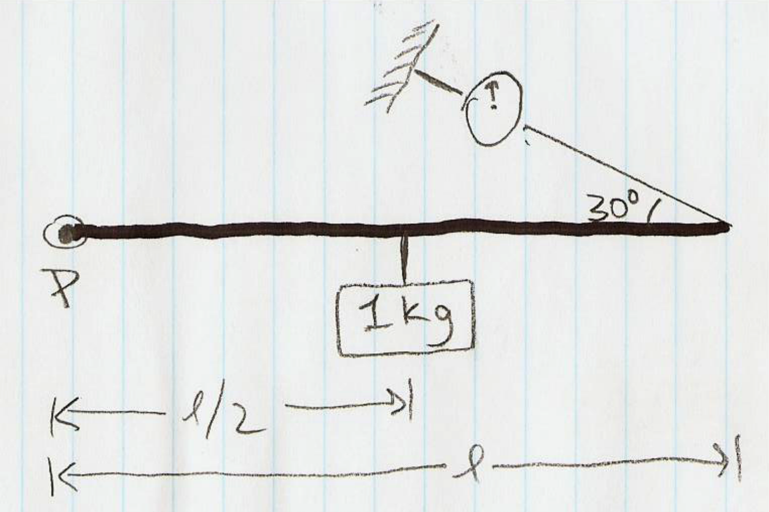
(B) 5.0 N

(C) 7.1 N

(D) 10 N

(E) 14 N

(F) 20 N



$$\tau = r_\perp F = rF_\perp = rF \sin\theta_{rF} = |\vec{r} \times \vec{F}|$$

Four ways to get the magnitude of the torque due to a force:

- (perpendicular component of distance) $\times$ (force)
- (distance) $\times$ (perpendicular component of force)
- (distance) (force) ($\sin\theta$ between $\vec{r}$ and $\vec{F}$)
- use magnitude of "vector product" $\vec{r} \times \vec{F}$ (a.k.a. "cross product")

To get the "direction" of a torque, use the right-hand rule.

Note right-hand rule for vector product $\vec{\tau} = \vec{r} \times \vec{F}$.

Note that most screws have “right-handed” threads.

Turn “right” (clockwise) to tighten, turn “left” (counterclockwise) to loosen.

If you look at the face of a clock, whose hands are moving clockwise, do the rotational velocity vectors of the clock's hands point toward you or toward the clock?

(A) Toward me

(B) Toward the clock

(C) Neither — when I curl the fingers of my right hand toward the clock, my thumb points to the left, in the 9 o'clock direction

Torques are important in architecture because they allow us to determine the conditions for a structure to stay put.

For an object (such as a structure or a part of a structure) to stay put, it must have zero acceleration, and it must have zero rotational acceleration.

So the vector sum of all forces must add to zero, and the sum of all torques (about any axis) must also be zero (to keep $\vec{a} = 0$ and $\alpha = 0$).

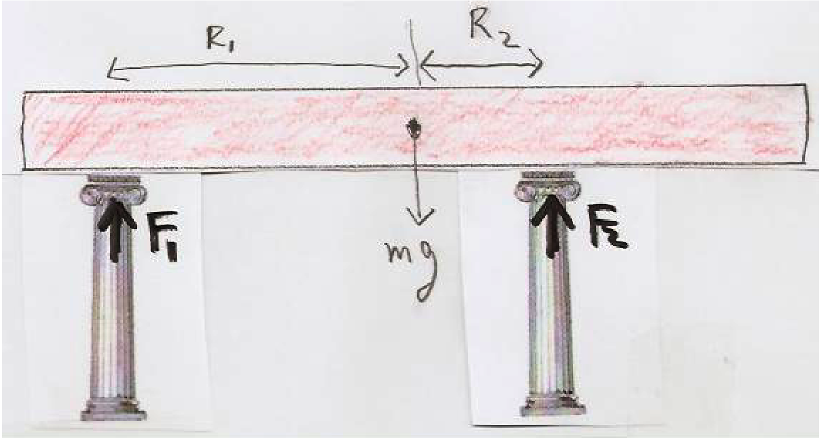
If these conditions are met, the object is in **equilibrium**: no unbalanced forces or torques.

Which column supports more of the beam's weight?

(A) Left column supports more than half of the beam's weight.

(B) Right column supports more than half of the beam's weight.

(C) Same. Each column supports half of the beam's weight.



Let's analyze this configuration, then demonstrate using two scales.

- How do I write $\sum F_y = 0$?
- What is a good choice of "rotation" axis here?
- How do I write $\sum \tau = 0$?
- What if I picked a different axis?

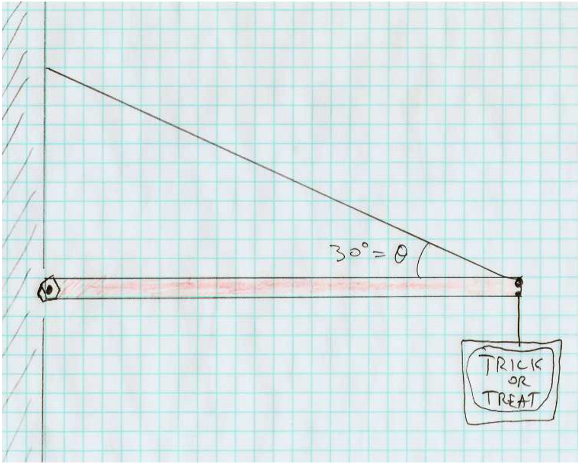
A beam of mass $M = 20$ kg and length $L = 2$ m is attached to a wall by a hinge. A sign of mass $m = 10$ kg hangs from the end of the beam. The end of the beam is supported by a cable (at $\theta = 30^\circ$ angle w.r.t. horizontal beam), which is anchored to the wall above the hinge.

What forces act on the beam? (Draw EFBD.)

Find the cable tension T.

Find the "reaction" forces F_x and F_y exerted by the hinge on the beam.

What 3 equations can we write for the beam? (Next few slides.)



Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of horizontal forces (on beam) = 0" ?

(A) $F_x + T\cos\theta = 0$

(B) $F_x + T\sin\theta = 0$

(C) $F_x - T\cos\theta = 0$

(D) $F_x - T\sin\theta = 0$

Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of vertical forces (on beam) = 0" ?

(A) $F_y + T\cos\theta + (M+m)g = 0$

(B) $F_y + T\cos\theta - (M+m)g = 0$

(C) $F_y + T\cos\theta - (M+m)g = 0$

(D) $F_y + T\sin\theta + (M+m)g = 0$

(E) $F_y + T\cos\theta - (M+m)g = 0$

(F) $F_y + T\sin\theta - (M+m)g = 0$

(G) $F_y - T\cos\theta - (M+m)g = 0$

(H) $F_y - T\sin\theta - (M+m)g = 0$

Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of torques (about hinge) = 0" ?

(A) $+\frac{L}{2}Mg + Lmg + LT\cos\theta = 0$

(B) $+\frac{L}{2}Mg + Lmg + LT\sin\theta = 0$

(C) $-\frac{L}{2}Mg + Lmg + LT\cos\theta = 0$

(D) $-\frac{L}{2}Mg + Lmg + LT\sin\theta = 0$

(E) $-\frac{L}{2}Mg - Lmg + LT\cos\theta = 0$

(F) $-\frac{L}{2}Mg - Lmg + LT\sin\theta = 0$

The 3 equations for static equilibrium in the xy plane

sum of horizontal forces = 0:

$$F_x - T\cos\theta = 0$$

sum of vertical forces = 0:

$$F_y + T\sin\theta - (M+m)g = 0$$

sum of torques (a.k.a. moments) about hinge = 0:

$$-\frac{L}{2}Mg - Lmg + LT\sin\theta = 0$$

Here's my solution: let's compare with the demonstration

$$F_x - T\cos\theta = 0$$

$$F_y - Mg - mg + T\sin\theta = 0$$

$$-\left(\frac{L}{2}\right)(Mg) - (L)(mg) + (L)(T\sin\theta) = 0$$

$$LT\sin\theta = Lmg + \frac{1}{2}LMg = L\left(m + \frac{M}{2}\right)g$$

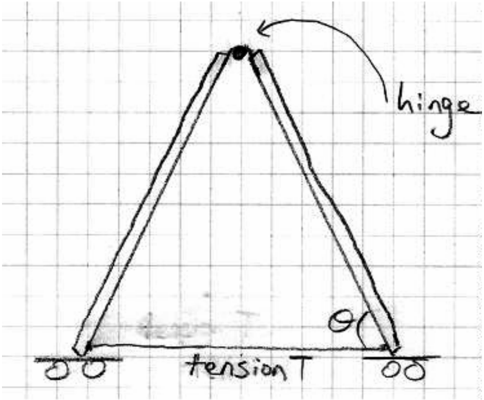
$$T = \frac{\left(m + \frac{M}{2}\right)g}{\sin\theta}$$

$$F_x = T\cos\theta = \frac{\left(m + \frac{M}{2}\right)g}{\tan\theta}$$

$$F_y = (M+m)g - T\sin\theta = (M+m)g - \left(m + \frac{M}{2}\right)g = \frac{Mg}{2}$$

Let's build & measure a simplified arch

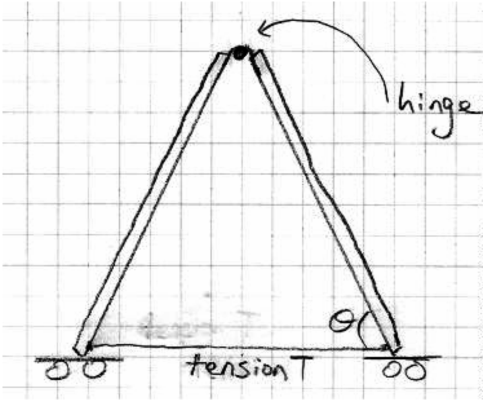
Often the essence of physics is to reduce a complicated problem to a similar problem that is easier to analyze.



(Does this make the function of a "roller support" more obvious?!)

(We'll emphasize function over form here . . .)

Often the essence of physics is to reduce a complicated problem to a similar problem that is easier to analyze. Use a cable to hold bottom together so that we can use scale to measure tension.

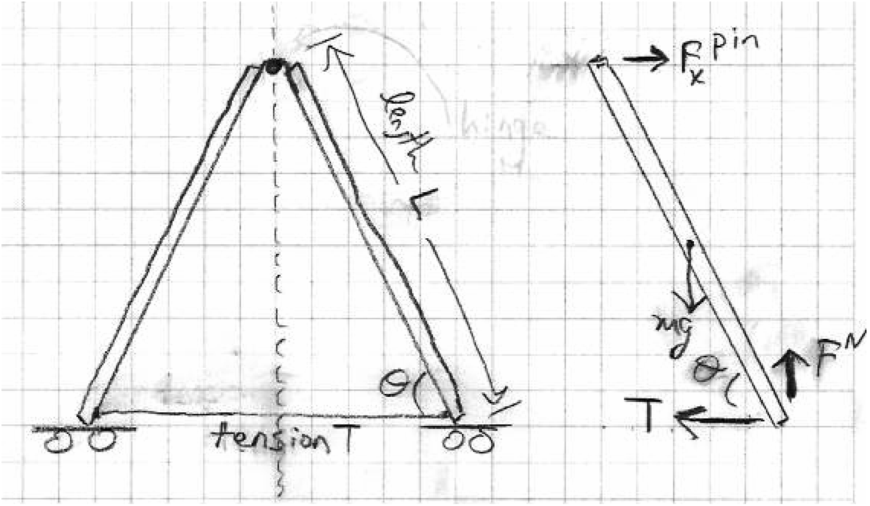


Weight (mg) of each side is 20 N.

We'll exploit mirror symmetry and analyze just one side of arch.

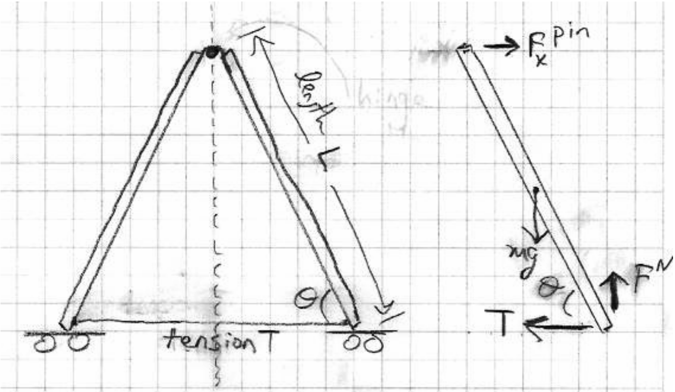
What forces act (and where) on the r.h.s. of the arch? **(Draw EFBD for the right-hand board.)**

Use a cable to hold bottom of "arch" together so that we can use scale to measure tension. Weight (mg) of each side is 20 N. We'll exploit mirror symmetry and analyze just one side of arch.



Right side shows EFBD for right-hand board.

How many unknown variables is it possible to determine using the equations for static equilibrium in a plane?



(A) one

(B) two

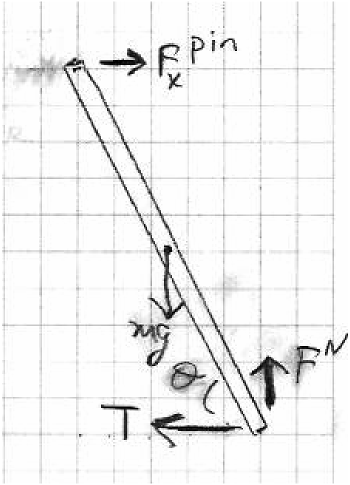
(C) three

(D) four

(E) five

Static equilibrium lets us write down three equations for a given object: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$. Let's first sum up the "moments" (a.k.a. torques) **about the top hinge.**

Which statement correctly expresses $\sum M_z = 0$ (a.k.a. $\sum \tau = 0$)? (Let the mass and length of each wooden board be L and m.)



(A) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(B) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(C) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(D) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

(E) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

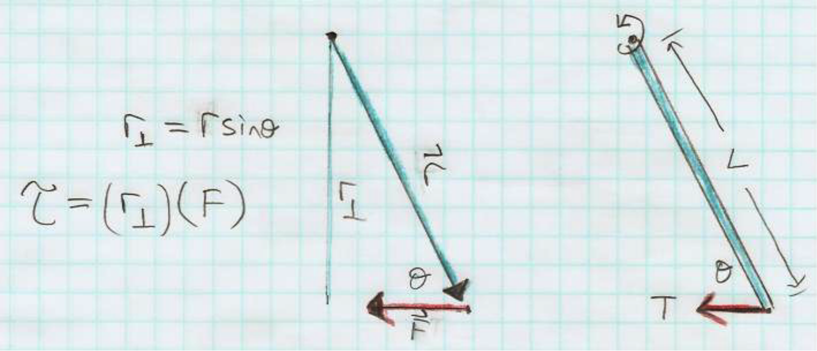
(F) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(G) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(H) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

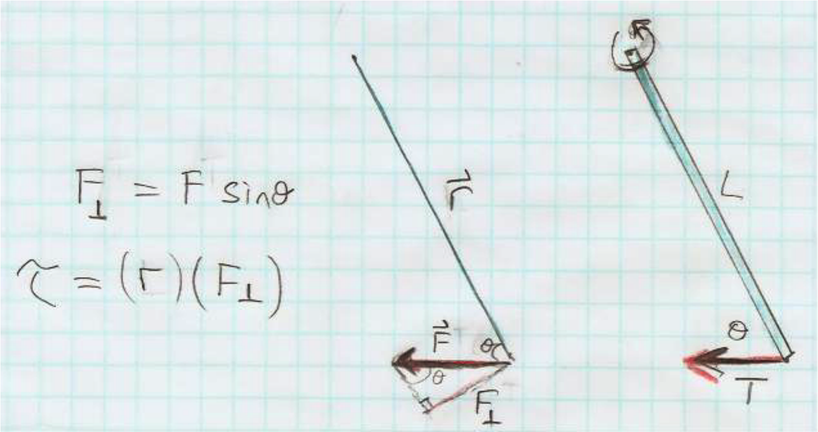
Let's start with torque (about top hinge) due to tension T.

- Usual convention: clockwise = negative, ccw = positive.
- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the "lever arm."
- Magnitude of torque is $|\tau| = (r_\perp)(F)$.



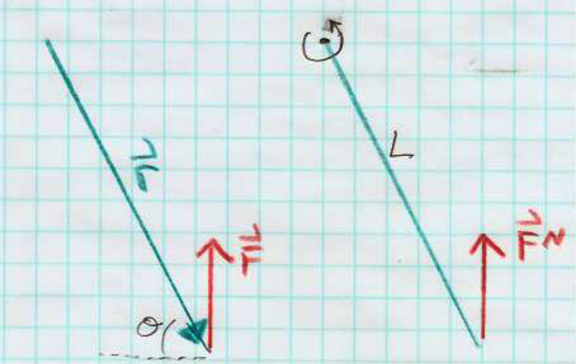
Alternative method: use $(r)(F_\perp)$ instead of $(r_\perp)(F)$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{F}$ to find component $F_\perp$ perpendicular to $\vec{r}$.
- Magnitude of torque is $|\tau| = (r)(F_\perp)$.



Now you try it for the normal force $\vec{F}^N$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the "lever arm."
- Magnitude of torque is $|\tau| = r_\perp F$.



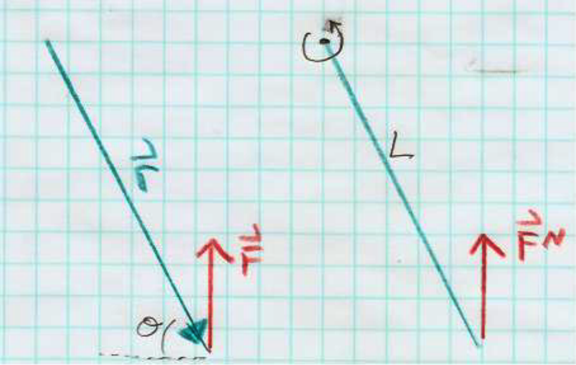
Which component of $\vec{r}$ is perpendicular to the normal force $\vec{F}^N$?

(A) horizontal component

(B) vertical component

Now you try it for the normal force $\vec{F}^N$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the "lever arm."
- Magnitude of torque is $|\tau| = r_\perp F$.



How long is the horizontal component of $\vec{r}$ (i.e. the $\vec{r}$ component which is perpendicular to $\vec{F}$) ?

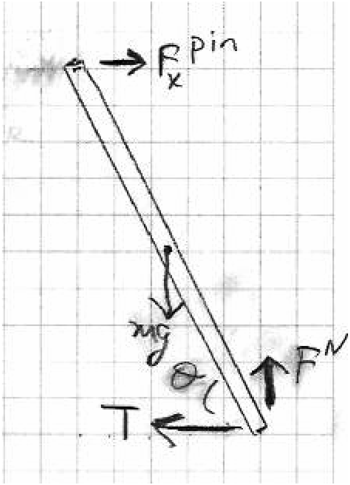
(A) $L\cos\theta$

(B) $L\sin\theta$

(C) $L\tan\theta$

OK, now back to the original question: Let's sum up the "moments" (a.k.a. torques) **about the top hinge.**

Which statement correctly expresses $\sum M_z = 0$ (a.k.a. $\sum \tau = 0$)? (Let the mass and length of each wooden board be L and m.)



(A) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(B) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(C) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(D) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

(E) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(F) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(G) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(H) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

Next: what about $\sum F_x = 0$ and $\sum F_y = 0$?

Next: what about $\sum F_x = 0$ and $\sum F_y = 0$?

$$-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$$

$$\sum F_y = 0 \Rightarrow -mg + F^N = 0$$

$$-mg\frac{L}{2}\cos\theta + mgL\cos\theta - TL\sin\theta = 0$$

$$mg\frac{L}{2}\cos\theta = TL\sin\theta$$

$$T = \frac{mg\cos\theta}{2\sin\theta} = \boxed{\frac{mg}{2\tan\theta} = T}$$

We said $mg = 20\ \mathrm{N}$, so we expect the string tension to be

$$T = \frac{10\ \mathrm{N}}{\tan\theta}$$

How would this change if we suspended a weight Mg from the hinge? (By symmetry, each side of arch carries **half** of this Mg.)

Let's use forces and torques to analyze the big red wheel that we first saw last Monday. The wheel has rotational inertia I.

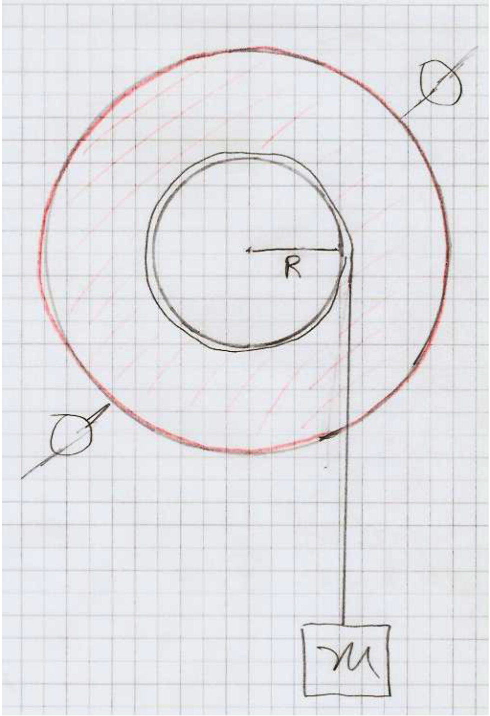
The string is wrapped at radius R, with an object of mass m dangling on the string.

For the dangling object, write

$$ma_y = \sum F_y$$

For the cylinder, write

$$I\alpha = \sum \tau$$



After some math, I get

$$\alpha = \frac{mgR}{I_{\text{wheel}} + mR^2} \approx \frac{mgR}{I_{\text{wheel}}}$$

(The approximation is for the limit where the object falls at $a \ll g$, so the string tension is $T = (mg - ma) \approx mg$.)

$$I\alpha = \tau = RT \Rightarrow T = \frac{I\alpha}{R}$$

$$ma = mg - T \Rightarrow m(\alpha R) = mg - \left(\frac{I\alpha}{R}\right)$$

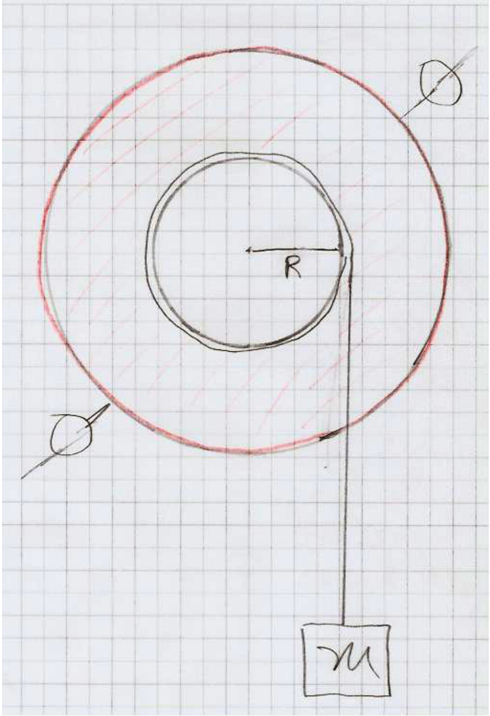
$$\alpha = \frac{g}{R} - \left(\frac{I}{mR^2}\right)\alpha \Rightarrow \alpha\left(1 + \frac{I}{mR^2}\right) = \frac{g}{R}$$

$$\alpha = \frac{g}{R\left(1 + \frac{I}{mR^2}\right)} = \frac{mgR}{I + mR^2}$$

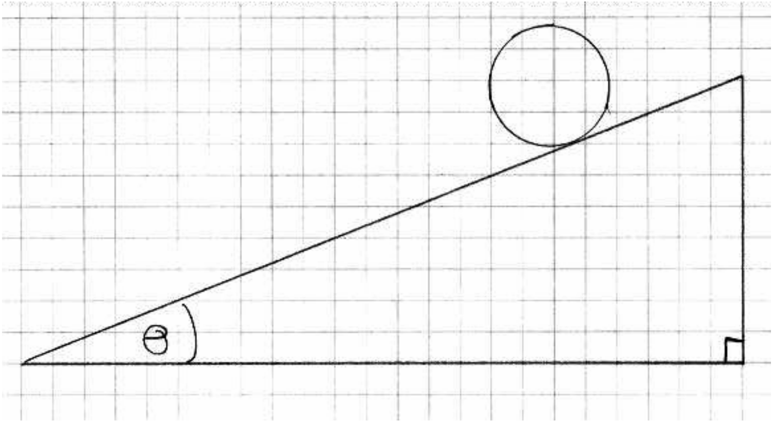
Why did increasing the dangling mass m **increase** the wheel's rotational acceleration α ?

Why did increasing the radius R from which the dangling mass was suspended **increase** the wheel's rotational acceleration?

Why did sliding the big rotating masses farther out on the extended "arms" **decrease** the wheel's rotational acceleration?



Let's go back and use torque to analyze another problem that last week we were only able to analyze using energy conservation: a cylinder rolling (without slipping) down an inclined plane.



What 3 forces act on the cylinder? What is the rotation axis? Draw FBD and extended FBD. What are the torque(s) about this axis? How are α and a related? Write $\vec{F} = m\vec{a}$ and $\tau = I\alpha$.

$$I\alpha = \sum \tau$$

$$I\left(\frac{a_x}{R}\right) = RF^s$$

$$F^s = \left(\frac{I}{R^2}\right) a_x$$

$$ma_x = mg\sin\theta - F^s$$

$$ma_x = mg\sin\theta - \left(\frac{I}{R^2}\right) a_x$$

$$\left(m + \frac{I}{R^2}\right) a_x = mg\sin\theta$$

$$a_x = \frac{mg\sin\theta}{m + \frac{I}{R^2}} = \frac{g\sin\theta}{1 + \left(\frac{I}{mR^2}\right)}$$

Remember that the object with the larger “shape factor” $I/(mR^2)$ rolls downhill more slowly.

While we're here, let's revisit the "center-of-mass chalkline" demonstration from a few weeks ago.

Now that we know about torque, we can see why the CoM always winds up directly beneath the pivot, once we understand that the line-of-action for gravity passes through the CoM.

Another equilibrium problem!

The top end of a ladder of inertia m rests against a smooth (i.e. slippery) wall, and the bottom end rests on the ground. The coefficient of static friction between the ground and the ladder is μ_s. What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Let's start by drawing an EFBD for the ladder.

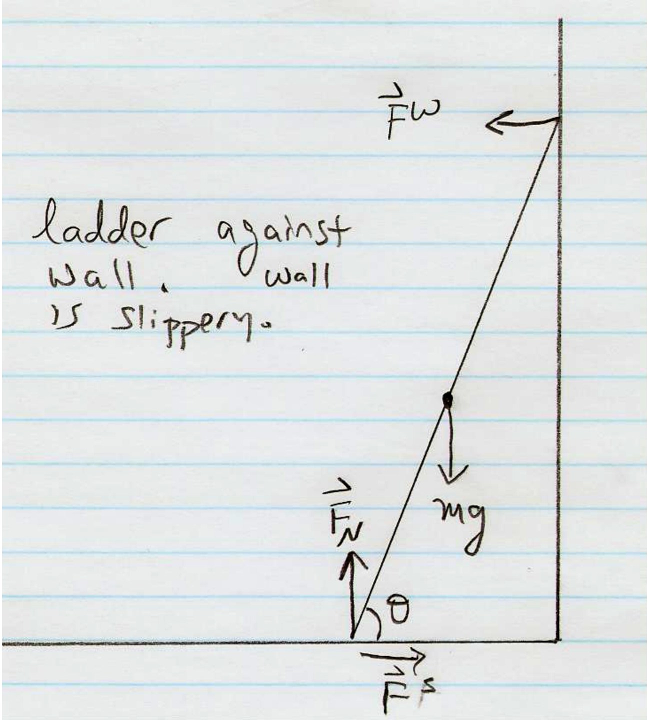
Why must we say the wall is slippery?

Is the slippery wall more like a pin or a roller support?

What plays the role here that string tension played in the previous problem?

Does the combination of two forces at the bottom act more like a pin or a roller support?

Which forces would an engineer call "reaction" forces?

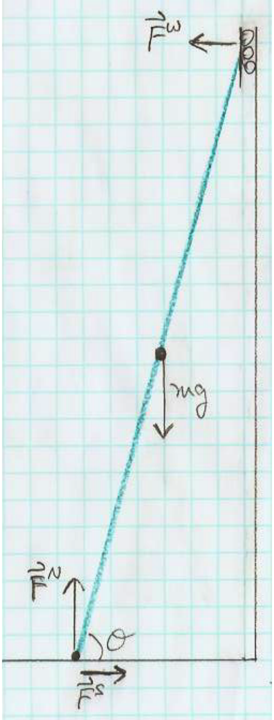


Which choice of pivot axis will give us the simplest equation for $\sum M_z = 0$? (We'll get an equation involving only two forces if we choose this axis.)

(A) Use bottom of ladder as pivot axis.

(B) Use center of ladder as pivot axis.

(C) Use top of ladder as pivot axis.



How would I write $\sum M_z = 0$ about the bottom end of the ladder? (Take length of ladder to be L.)

(A) $F^W L\cos\theta + mgL\sin\theta = 0$

(B) $F^W L\cos\theta + mg\frac{L}{2}\sin\theta = 0$

(C) $F^W L\cos\theta - mgL\sin\theta = 0$

(D) $F^W L\cos\theta - mg\frac{L}{2}\sin\theta = 0$

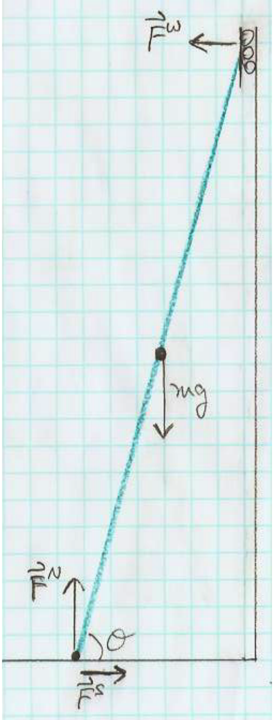
(E) $F^W L\sin\theta + mgL\cos\theta = 0$

(F) $F^W L\sin\theta + mg\frac{L}{2}\cos\theta = 0$

(G) $F^W L\sin\theta - mgL\cos\theta = 0$

(H) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?



What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

Let's answer the original question:

What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Suppose we add to this picture a woman of mass M who has climbed up a distance d along the length of the ladder. Now how do we write the moment equation $\sum M_z = 0$?

(A) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\cos\theta = 0$

(B) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\sin\theta = 0$

(C) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta + Mgd\cos\theta = 0$

(D) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta + Mgd\sin\theta = 0$

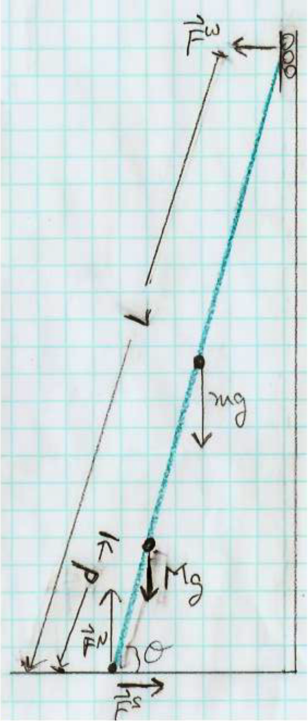
(E) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\cos\theta = 0$

(F) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\sin\theta = 0$

(G) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta - Mgd\cos\theta = 0$

(H) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta - Mgd\sin\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

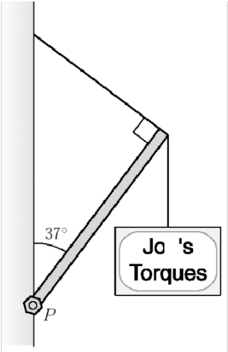


What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

For a given θ, how far up can she climb before the ladder slips?

Here's a trickier equilibrium problem:

4*. You want to hang a 22 kg sign (shown at right) that advertises your new business. To do this, you attach a 7.0 kg beam of length 1.0 m to a wall at its base by a pivot P. You then attach a thin cable to the beam and to the wall in such a way that the cable and beam are perpendicular to each other. The beam makes an angle of 37° with the vertical. You hang the sign from the end of the beam to which the cable is attached. (a) What must be the minimum tensile strength of the cable (the amount of tension it can sustain) if it is not to snap? (b) Determine the horizontal and vertical components of the force the pivot exerts on the beam.



What forces act on the beam?

What 3 equations can we write for the beam?

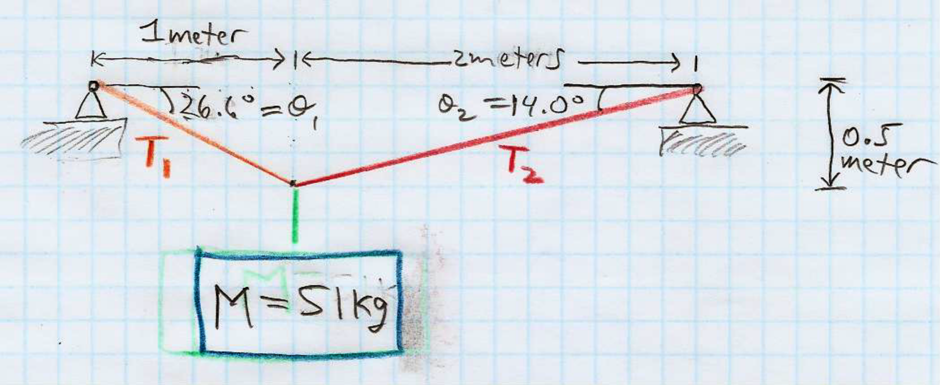
A tightly stretched "high wire" has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



How would you find the tensions T_1 and T_2?

Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?