phys8_slides_13.pdf

- This file has **nothing** to do with Mazur ch13, which we will not cover in this course. (The next planned Mazur chapter is ch15 in the last week of the term.)
- These slides draw from:
- Giancoli ch09 (statics, etc)
- Onouye/Kane ch02 (statics)
- Onouye/Kane ch03 (analysis of trusses, etc)
- pages 12–16 of positron.hep.upenn.edu/p8/files/equations.pdf (which I'll paste into these slides for your convenience)

Chapter G9: static equilibrium, etc.

Static equilibrium: all forces/torques acting ON the object sum to zero

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum \tau = 0$$

Young's modulus: $\frac{\Delta L}{L_0} = \frac{1}{E} \left(\frac{\text{force}}{\text{area}}\right)$

Onouye/Kane ch1: introduction

static loads: gravitational forces, due to the weight of the structure or its contents. Includes *dead loads* due to the weight of the building and permanently attached components thereof, and *live loads* that come and go, such as furniture and people.

dynamic loads: inertial forces, due to resisting the motion of mass. For example: wind, vibration, earthquakes, falling objects.

Onouye/Kane ch2: statics

A force is characterized by *point of application*, magnitude, and direction. The force's *line of action* passes through the point of application of the force and is in the same direction as the force.

When we idealize an object as a *rigid body*, we assume that it undergoes negligible deformation in response to applied forces. If you think of a body as a huge number of constituent particles, the body is rigid if the relative distance between every pair of consituent particles is fixed. The rules of *statics* (i.e. static equilibrium) apply to rigid bodies. Statics in a plane gives us 3 equations (see below), allowing us to solve for 3 unknown forces (or 3 unknown force components, if directions are unknown). If there are more than 3 unknown force components, then we need additional information about how the body deforms in response to applied forces (i.e. the rigid-body idealization is no longer sufficient); that goes beyond the scope of statics. [But occasionally one can use statics to determine more than 3 unknown forces by using e.g. mirror symmetry to eliminate all but 3 unknowns.]

(O/K ch2) Usually a *load* is a specified external force that a structure must be designed to bear, such as the weight of snow on the roof or the weight of the building itself. Usually a *reaction* is an unknown external force whose value is calculated by imposing the conditions of static equilibrium on an object. If you and I sit on a see-saw, our weights and the weight of the wooden plank are loads; the upward contact force exerted by the pin on the center of the plank is a reaction. A free-body diagram for the plank includes the specified loads and the to-be-determined reaction.

Principle of transmissibility (applies to rigid bodies only): the acceleration and angular acceleration of a rigid body are unchanged by replacing a given force F_1 acting at point A with a new force F_2 acting at point B as long as forces A and B have the same line of action, same magnitude, and point in the same direction.

Concurrent forces have lines of action that intersect at a common point. The effects on a rigid body are unchanged by replacing several concurrent forces with a single resultant force. The *resultant* of several forces is the vector sum of those forces.

(O/K ch2) The *moment* of a force is engineers' term for what physicists call torque. It is force multiplied by perpendicular lever arm, with a sign given by the convention that counterclockwise is positive and clockwise is negative. In 3 dimensions, a torque (or moment) is given by the vector ("cross") product $\vec{\tau} = \vec{r} \times \vec{F}$ and the right-hand rule. You can't define a moment (torque) without first defining a reference point, also known as a pivot, or an axis, or an origin for a coordinate system. The vector $\vec{r}$ in the expression $\vec{r} \times \vec{F}$ is measured with respect to that pivot point, i.e. the tail of $\vec{r}$ is at the pivot.

Varignon's theorem: to compute the moment of a force, you can decompose the force into components (having the same point of application) and sum (algebraically, i.e. with proper signs) the moments of the components.

A *couple* is two forces that sum to zero ($\vec{F}$ and $-\vec{F}$) and have parallel (you might say antiparallel) lines of action separated by a distance d. A couple will tend to cause rotational acceleration but will not cause linear acceleration of a body. The moment of a couple has magnitude Fd.

(O/K ch2) A force $\vec{F}$ acting on a rigid body can be moved to any given point of application A (with a parallel line of action) provided that a couple $\vec{M}$ is added. The moment M of the couple equals $Fd_\perp$, where $d_\perp$ is the perpendicular distance between the original line of action and the new location A.

In the 2D plane, the three equations of statics are: $\sum F_x = 0$, $\sum F_y = 0$, and $\sum_{\circlearrowleft P} M = 0$, where P is a chosen pivot point for evaluating moments.

When engineers and architects say *Free Body Diagram*, they are referring to what Mazur calls an *Extended Free Body Diagram*. An EFBD starts with a cartoon-like sketch of the body in question and indicates with an arrow each external force acting on the body, carefully indicating the direction and the point of application of the force. Often unknown reaction forces are drawn with a single slash through the arrow. External moments (illustrated via types of connections) are indicated using curved arrows. An unknown moment reaction is indicated using a single slash through a curved arrow.

(O/K ch2) Support forces are often drawn as stereotyped *pin* (or hinge) supports and *roller* supports. A pin can exert both horizontal and vertical support (reaction) forces but cannot exert any moment (torque) about the pin axis. A roller can only exert a force normal to the surface on which it rolls and cannot exert a moment. So a pin (or hinge) support contributes two unknown reaction force components, while a roller support contributes only one unknown reaction force component. A body that has a pin support beneath one end and a roller support benath the opposite end is *simply supported*. One pin and one roller support constitute 3 total unknown forces, which is exactly the number of unknowns that the laws of statics in a 2D plane can determine.

Another type of connection, not illustrated in chapter 2, is a "built-in" connection, which (in the 2D plane) can exert two forces and a moment. For an example, think of how a lamppost is attached to the sidewalk. It resists motion along its axis, resists motion parallel to the sidewalk, and also resists the toppling over of the lamppost, i.e. it resists rotation about the point of connection.

(O/K ch2) A body on which more than three unknown forces are exerted is called *statically indeterminate.* To solve a statically indeterminate system, you need to know how the body deforms under the applied load.

Onouye/Kane ch3: selected determinate systems

The resultant force exerted on the end of a cable must be tangent to the end of the cable.

(O/K ch3) A *concentrated load* has a point of application that can be represented as a single point. For example, the weight of a single lead brick placed at the center of a long beam. A *distributed load* is spread out over a wide area (usually indicated as force per unit length on a 2D sketch). The most common symbol used for concentrated (or "point") loads is P. The most common symbol used for distributed loads is w, though some books use ω.

For statics calculations of rigid bodies (but not for elastic calculations such as the deflection of beams!) a distributed load w can be replaced by the equivalent concentrated load P. The point of application of P is the centroid of w, and the magnitude of P is the integral of w, i.e. the area under the $w(x)$ curve, $\int w(x)\,\mathrm{d}x$. Usually this "integral" can be simply calculated using formulas for the area of a rectangle, a triangle, a trapezoid, etc.

(O/K ch3) There are two ways to analyze a *truss*: one is the *method of joints* and the other is the *method of sections*. Analysis of a truss assumes: (a) members (*bars*) are straight line segments and can support only axial forces, i.e. forces parallel to the axis of the bar; (b) all *joints* are pin connections, i.e. connections that can exert horizontal and vertical forces but not moments about the pin; (c) the weight of the truss bars themselves is usually neglected; (d) loads are applied to the truss at the pinned joints only. A given bar is either in *compression* (the forces exerted on the ends of the bar are trying to squish the bar along its axis) or in *tension* (the forces exerted on the ends of the bar are trying to stretch the bar along its axis).

A necessary condition for a planar truss having J joints and B bars to be solvable using the methods of statics is $B = 2J - 3$. Solving the truss involves finding B unknown bar tensions/compressions plus 3 unknown support reactions (e.g. one pin and one roller support). The method of joints will give us 2 equations per joint. So we have $2J$ equations to determine $B + 3$ unknowns. Thus $2J = B + 3$.

(O/K ch3) The *method of joints* is conceptually simple, but can be tedious. At each joint, you apply the two force equations for static equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$ (consider forces acting *on the joint* itself). There is no moment equation because all forces at the joint have lines of action passing through the joint. I usually label the support "reaction" forces e.g. R_{Ax}, R_{Ay}, R_{Cy} for reaction forces at joints A and C, and then label the tension/compression of each bar as if every bar were in tension: T_{AB}, T_{BC}, T_{AC} for bars AB, BC, AC connecting joints A, B, C. In the end, you will find $T_{AB} > 0$ if bar AB is in tension and you will find $T_{AB} < 0$ if bar AB is in compression. To eliminate the need to solve large systems of simultaneous equations, always start from a joint having at most two unknown forces; if you find a joint having only one unknown force, so much the better.

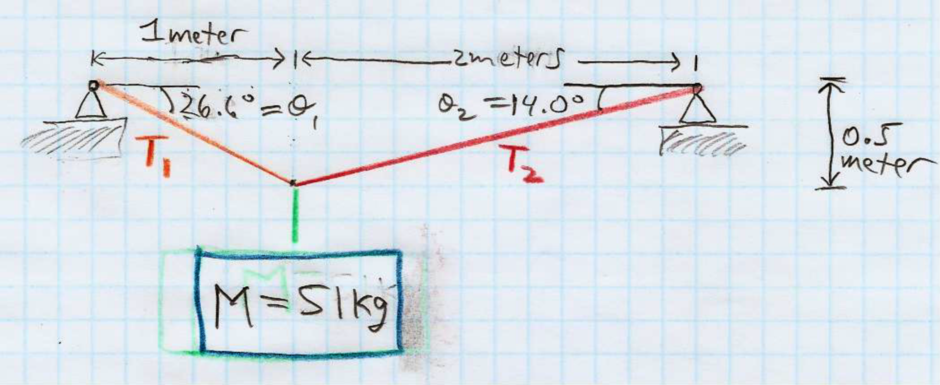
(O/K ch3) In the *method of sections* you often (but not always) start by drawing an EFBD for the truss as a whole and solving for the unknown support reactions; sometimes this step is unnecessary. Then you draw a hypothetical line (or curve) that divides the truss into two pieces; the line should pass through bars, not joints, and should cut through no more than three bars whose forces are unknown. If there is a particular bar whose tension/compression you want to find, be sure that your cut line passes through that bar. You then draw an EFBD for *either* the right side or the left side of the truss, including the forces exerted (by the invisible side of the truss) on the bars cut by the section line. Be careful with the directions: if the cut bar (let's say it's bar AB) is assumed to be in tension, then the EFBD for the right side of the cut includes T_{AB} pointing (in general diagonally) to the left; alternatively the EFBD for the left side of the cut would include T_{AB} pointing (in general diagonally) to the right. You want to draw the external forces exerted **on** the part of the truss whose EFBD you have drawn. . . .

(O/K ch3) *(method of sections)* ... You then use the three equations for static equilibrium in a plane: $\sum F_x = 0$, $\sum F_y = 0$, and $\sum_{\circlearrowleft P} M = 0$, where the pivot point P is strategically chosen so that the moment equation omits any forces that you do not care about. (Forces whose lines of action pass through the pivot P will have zero lever arm and will thus not appear in the moment equation.) You are summing forces and moments acting on the visible (i.e. left or right) portion of the truss as a whole. Whereas the method of joints found the conditions for each joint to be in equilibrium, the method of sections finds the conditions for the visible half of the truss as a whole to be in equilibrium. If you are only interested in finding a single bar force, and if you choose just the right section, and if you choose just the right pivot point, you can often find the desired force by solving only the moment equation. The method of joints is a brute-force method that you can imagine programming a computer to do for you; the method of sections requires some finesse.

Pinned frames, multiforce members, and retaining walls are outside the scope of this course.

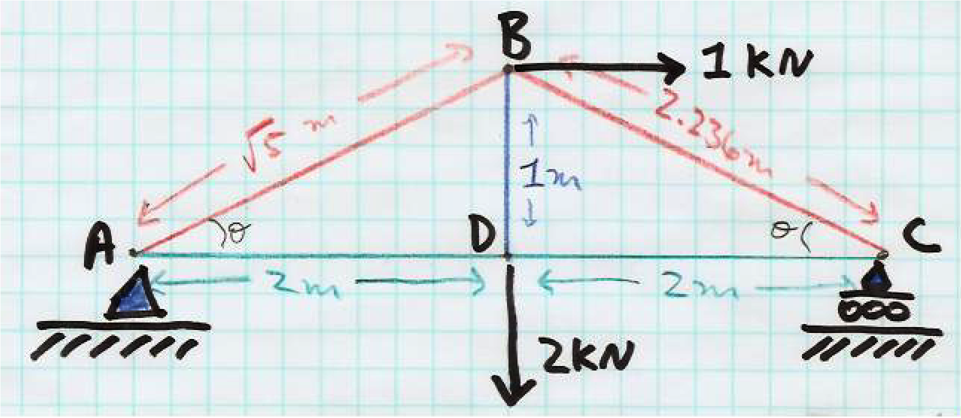
Here is where we left off last time:

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



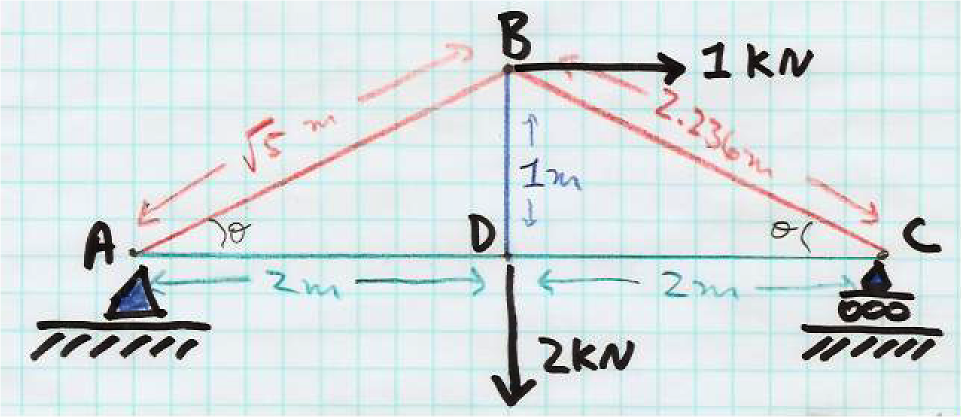
How would you find the tensions T_1 and T_2?

Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?



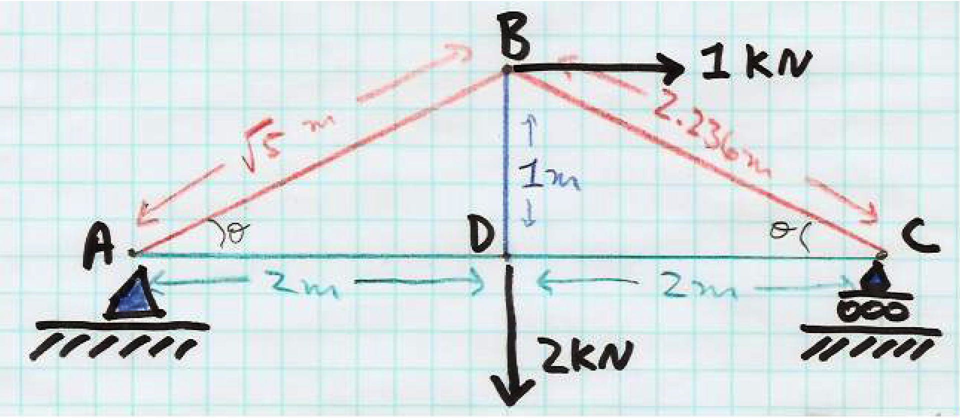
How many equations does the "method of joints" allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15



How many unknown internal forces (tensions or compressions) do we need to determine when we “solve” this truss?

(A) 4 (B) 5 (C) 6 (D) 7



This is a "simply supported" truss. How many independent "reaction forces" do the two supports exert on the truss? (If there are independent horizontal and vertical components, count them as separate forces.)

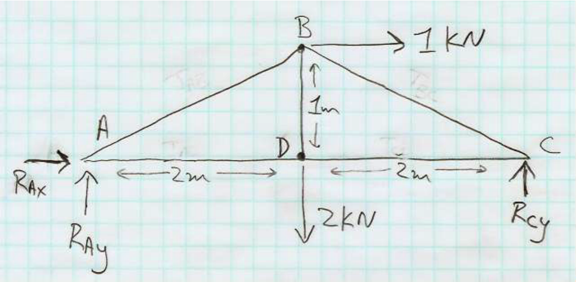
(A) 2 (B) 3 (C) 4 (D) 6

Notice that $8 = 5 + 3$.

For a planar truss that is stable and that you can solve using the equations of static equilibrium,

$$2N_{\mathrm{joints}} \;=\; N_{\mathrm{bars}} + 3$$

You get two force equations per joint. You need to solve for one unknown tension/compression per bar plus three support "reaction" forces.



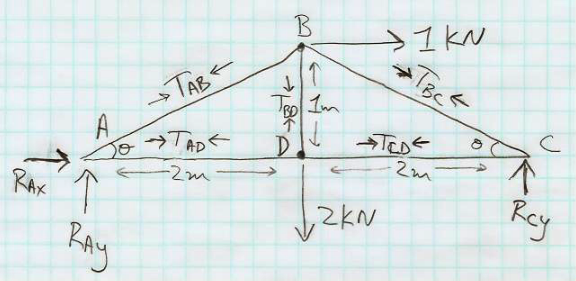
What do we learn by writing $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$ for the truss as a whole? (Use joint **A** as pivot.)

(I write R_{Ax}, R_{Ay}, R_{Cy} for the 3 "reaction forces" exerted by the supports on the truss.)

(A) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(2\ \mathrm{m}) = 0$

(B) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(1\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$

(C) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0$,
$R_{Ax} + 1\ \mathrm{kN} = 0$,
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$



What two equations does the "method of joints" let us write for joint **C** ?

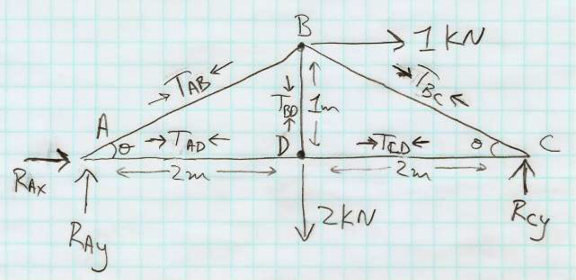
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $T_{CD} - T_{BC} \cos\theta = 0$
$R_{Cy} - T_{BC} \sin\theta = 0$

(B) $T_{CD} - T_{BC} \sin\theta = 0$
$R_{Cy} - T_{BC} \cos\theta = 0$

(C) $T_{CD} + T_{BC} \cos\theta = 0$
$R_{Cy} + T_{BC} \sin\theta = 0$

(D) $T_{CD} + T_{BC} \sin\theta = 0$
$R_{Cy} + T_{BC} \cos\theta = 0$



What two equations does the "method of joints" let us write for joint **A** ?

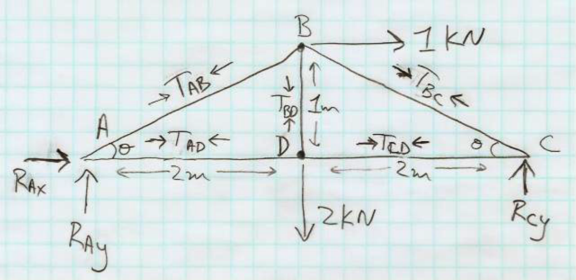
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $R_{Ax} - T_{AD} - T_{AB} \cos\theta = 0$
$R_{Ay} - T_{AB} \sin\theta = 0$

(B) $R_{Ax} - T_{AD} - T_{AB} \sin\theta = 0$
$R_{Ay} - T_{AB} \cos\theta = 0$

(C) $R_{Ax} + T_{AD} + T_{AB} \cos\theta = 0$
$R_{Ay} + T_{AB} \sin\theta = 0$

(D) $R_{Ax} + T_{AD} + T_{AB} \sin\theta = 0$
$R_{Ay} + T_{AB} \cos\theta = 0$



What two equations does the "method of joints" let us write for joint **D** ?

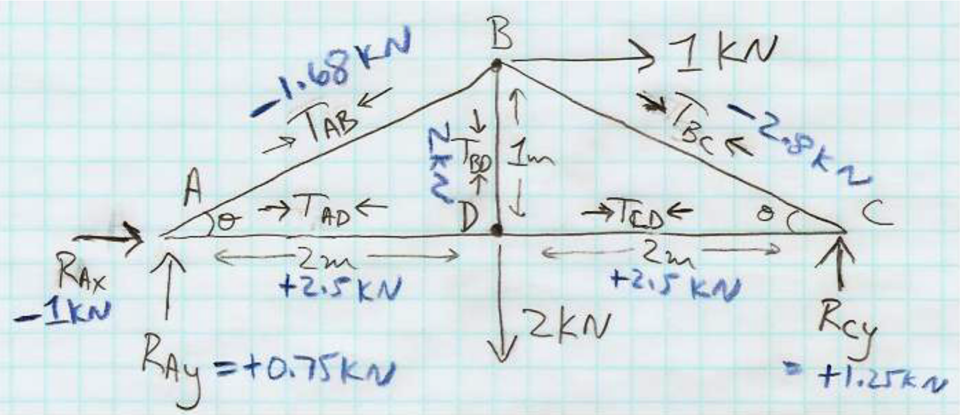
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

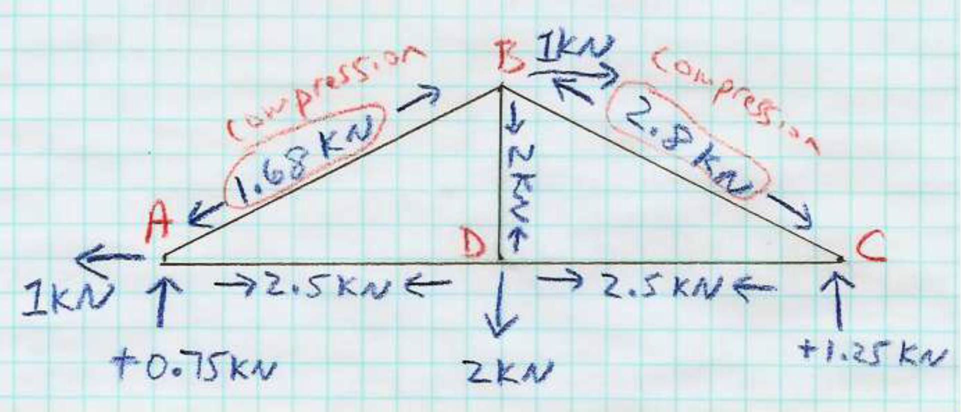
(B) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$

(C) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

(D) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$



I named each member force T_{ij} (for "tension") and let $T_{ij} > 0$ mean tension and $T_{ij} < 0$ mean compression. Once you've solved the truss, it's best to draw the arrows with the correct signs for clarity. (Next page.)

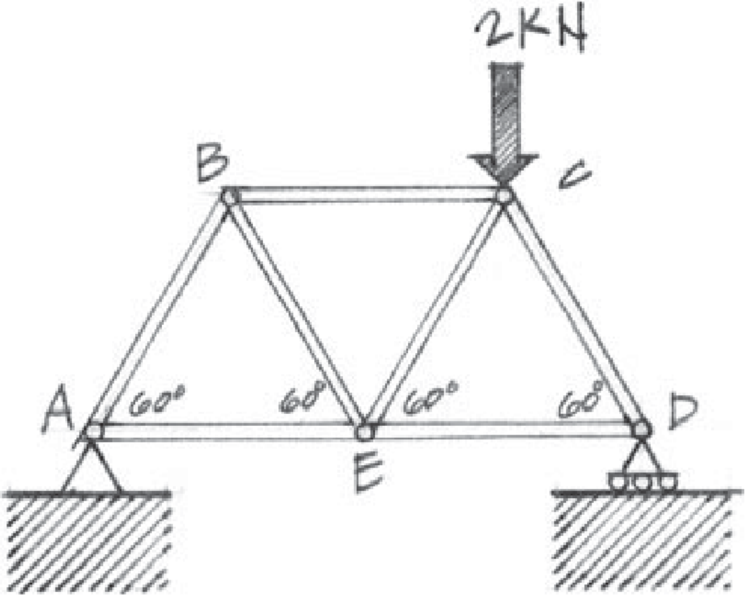


Forces redrawn with arrows in correct directions, now that we know the sign of each force. Members **AB** and **BC** are in compression. All other members are in tension.

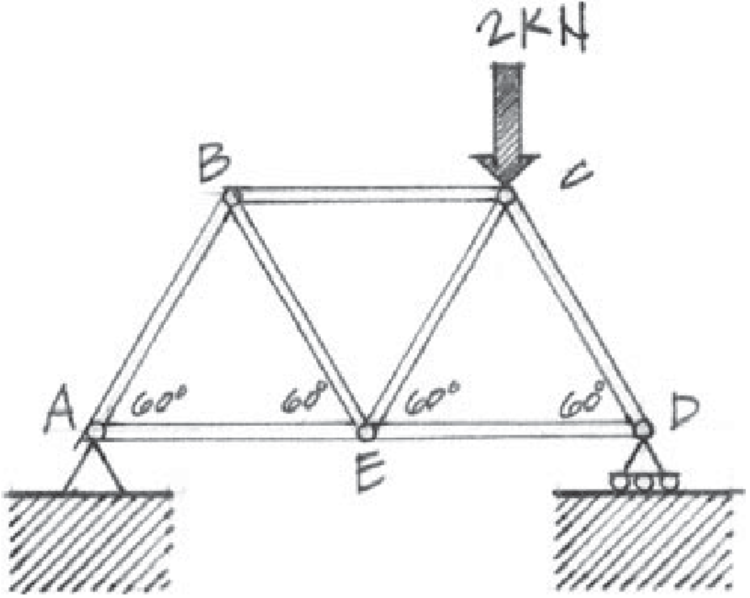
Another option is to write down all $2J$ equations at once and to type them into **Mathematica**, Maple, Wolfram Alpha, etc.

```
In[92] eq := {

RAx + TAB*cos + TAD == 0,
RAy + TAB*sin == 0,
-TAB*cos+TBC*cos+1 == 0,
-TBD-TAB*sin-TBC*sin == 0,
-TAD+TCD == 0,
-2 + TBD == 0,
-TCD - TBC*cos == 0,
RCy + TBC*sin == 0,

sin==1.0/Sqrt[5.0],
cos==2.0/Sqrt[5.0]

}
```

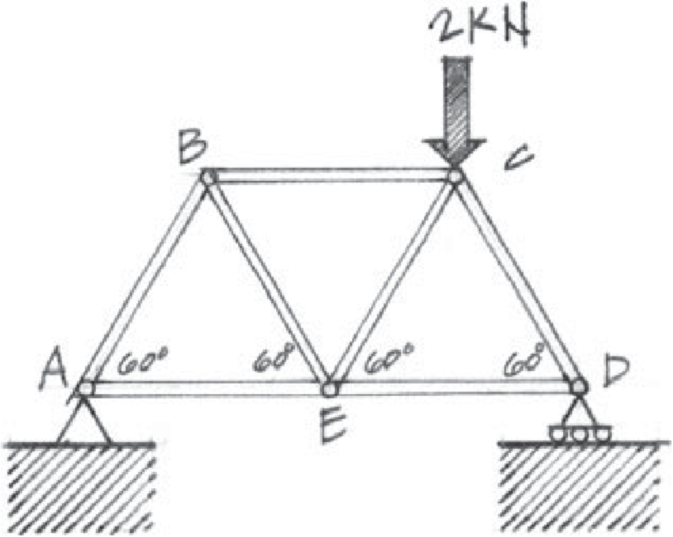
```
In[93] Solve[eq]

Out[93] {

RAx → -1.,
RAy → 0.75,
RCy → 1.25,
TAB → -1.67705,
TAD → 2.5,
TBC → -2.79508,
TBD → 2.,
TCD → 2.5,

cos → 0.894427,
sin → 0.447214

}
```



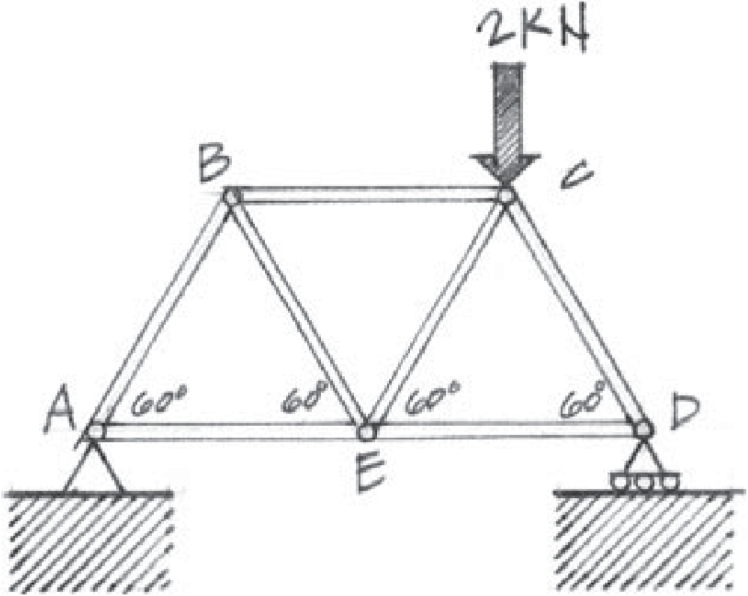
How many “reaction forces” are exerted by the supports (i.e. exerted on the truss by the supports)?



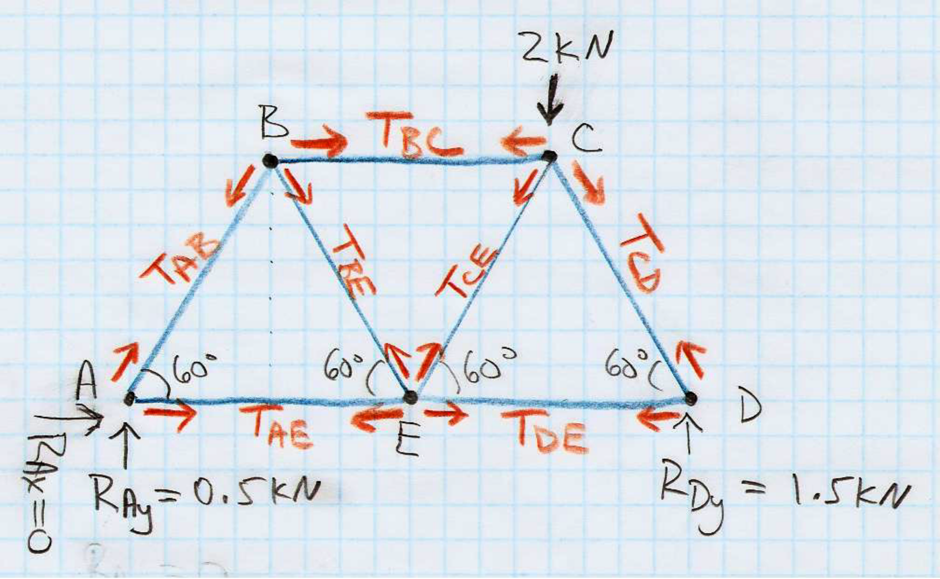
How many internal forces (tensions or compressions in the members) do we need to solve for to “solve” this truss?



Do you see any joint at which there are ≤ 2 unknown forces? If so, we can start there. If not, we need to start with an EFBD for the truss as a whole.



Try to guess $R_{A,x}$, $R_{A,y}$, and $R_{D,y}$ by inspection. Then let's check with the usual equations.



Now start from a joint having ≤ 2 unknown forces. In this case, I just went through the joints alphabetically. You can make your life easier by seeking out equations having just 1 unknown.

$\cos 60° = 0.5, \quad \sin 60° = 0.866$

Joint A: $0 + T_{AE} + T_{AB}\cos 60° = 0$

$0.5\text{kN} + T_{AB}\sin 60° = 0 \Rightarrow \boxed{T_{AB} = -0.577\text{kN}}$

$\boxed{T_{AE} = +0.289\text{kN}}$

Joint B: $-T_{AB}\cos 60° + T_{BE}\cos 60° + T_{BC} = 0$

$-T_{AB}\sin 60° - T_{BE}\sin 60° = 0 \Rightarrow \boxed{T_{BE} = +0.577\text{kN}}$

$\boxed{T_{BC} = -0.577\text{kN}}$

Joint C: $-T_{BC} - T_{CE}\cos 60° + T_{CD}\cos 60° = 0 \Rightarrow T_{CD} = T_{CE} - 1.155\text{kN}$

(could do CD, or E at this point)

$-2\text{kN} - T_{CE}\sin 60° - T_{CD}\sin 60° = 0$

$-2\text{kN} - T_{CE}\sin 60° - (T_{CE} - 1.155\text{kN})\sin 60° = 0$

$2T_{CE}\sin 60° = -2\text{kN} + 1.0\text{kN} = -1\text{kN}$

$\boxed{T_{CE} = -0.577\text{kN}}$

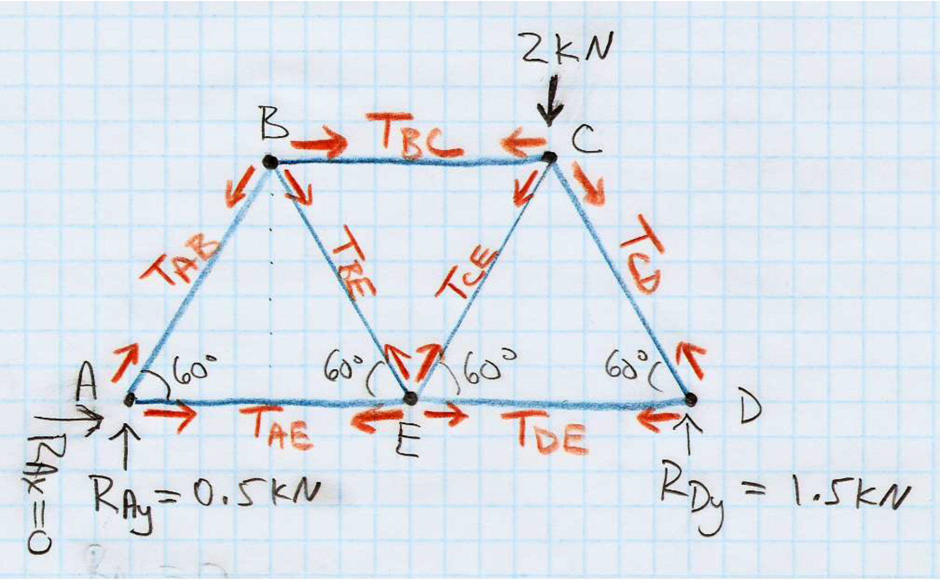
$T_{CD} = T_{CE} - 1.155\text{kN} = \boxed{-1.732\text{kN} = T_{CD}}$

Joint D: $-T_{DE} - T_{CD}\cos 60° = 0 \Rightarrow \boxed{T_{DE} = +0.866\text{kN}}$

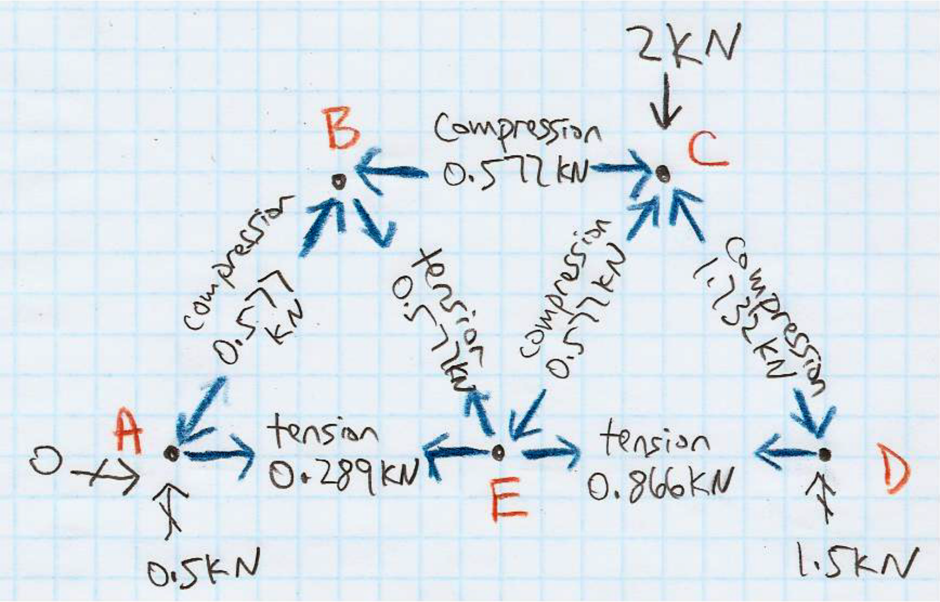
(check): $1.5\text{kN} + T_{CD}\sin 60° = 0$ ✓

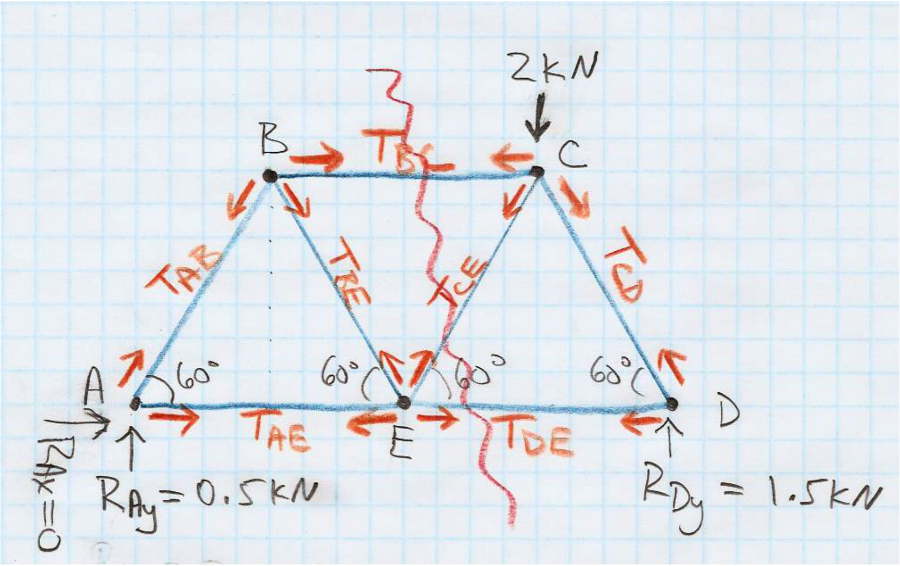
(check Joint E): $-T_{AE} + T_{DE} - T_{BE}\cos 60° + T_{CE}\cos 60° = 0$ ✓

$+T_{BE}\sin 60° + T_{CE}\sin 60° = 0$ ✓

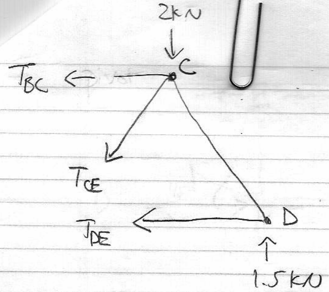


$T_{AB} = -0.577$ kN, $T_{AE} = +0.289$ kN, $T_{BE} = +0.577$ kN, $T_{BC} = -0.577$ kN, $T_{CE} = -0.577$ kN, $T_{CD} = -1.732$ kN, $T_{DE} = +0.866$ kN. My notation: tension > 0, compression < 0.


2KN
B
C
A
D
E
compression 0.577 kN
compression 0.577 KN
tension 0.577 kN
compression 0.577 kN
compression 1.732 kN
tension 0.289 kN
tension 0.866 kN
0
0.5 kN
1.5 kN



Let's try drawing an EFBD for the **right** side of the cut ("section").



$$\sum F_x = 0 \Rightarrow -T_{BC} - T_{CE}\cos 60^\circ - T_{DE} = 0$$

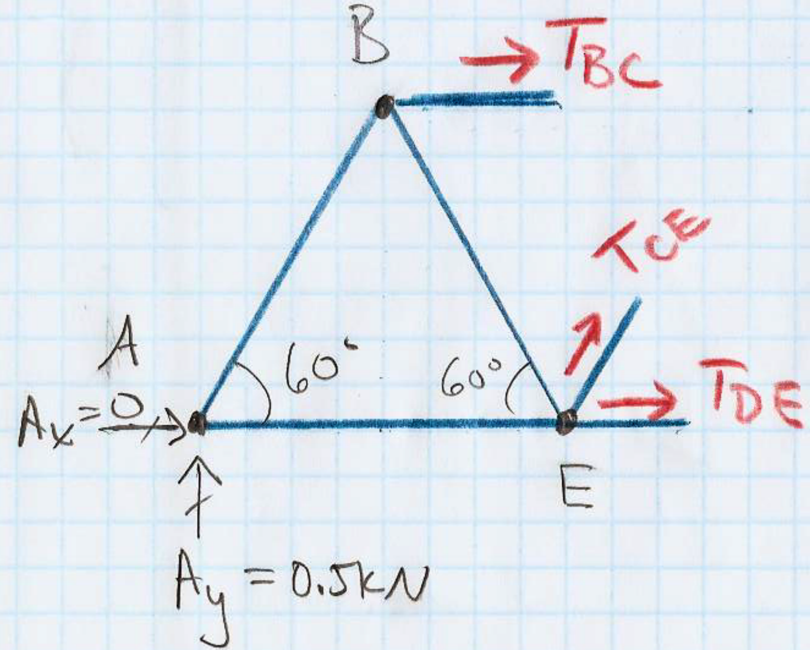
$$\sum F_y = 0 \Rightarrow -2kN + 1.5kN - T_{CE}\sin 60^\circ = 0$$

$$\sum M_C = 0 \Rightarrow +(1.5kN)\left(\frac{L}{2}\right) - T_{DE}\, L\sin 60^\circ = 0$$

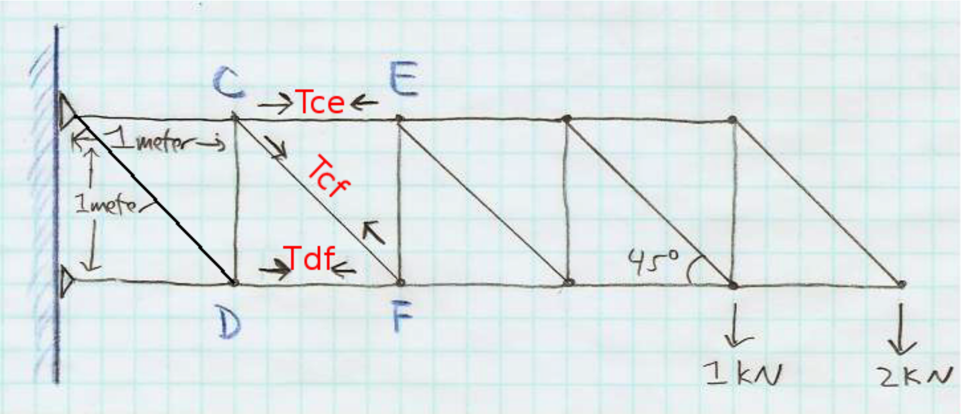
$$T_{DE} = \frac{1.5kN}{2\sin 60^\circ} = +0.866kN$$

$$T_{CE} = \frac{-0.5kN}{\sin 60^\circ} = -0.577kN$$

$$T_{BC} = -(T_{CE}\cos 60^\circ + T_{DE}) = -0.577$$

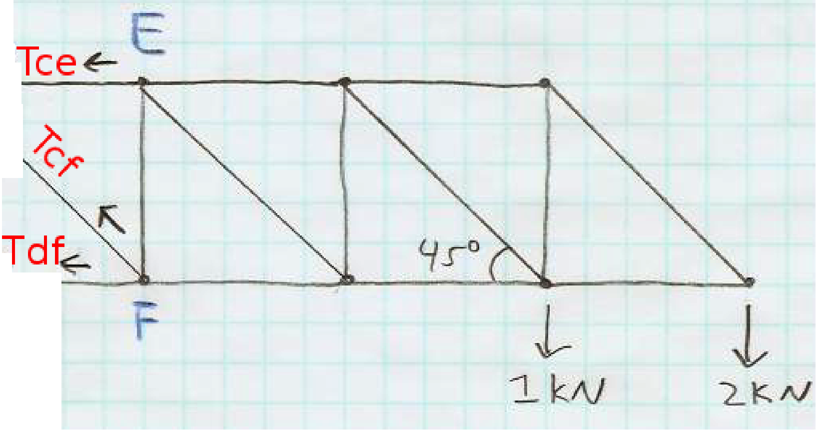


B
TBC
TCE
A
60°
60°
TDE
Ax=0
E
Ay=0.5kN

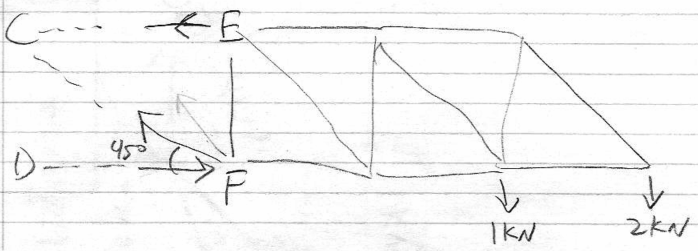


Here's another truss problem that you can solve using the Method of Sections. Find forces in members **CE**, **CF**, and **DF**, with assumed force directions as shown.

- What happens if an assumed force direction is backwards?
- Where should we "section" the truss?
- Then what do we do next?



If all goes well, we should get
$T_{CF} = +3\sqrt{2}$ kN, $T_{CE} = +8$ kN, $T_{DF} = -11$ kN.



$\Sigma F_x = 0 = -CE - CF\cos 45° + DF = 0$

$\Sigma F_y = 0 = CF\sin 45° = 3\text{kN} \Rightarrow \boxed{CF = 3\text{kN}\sqrt{2}}$

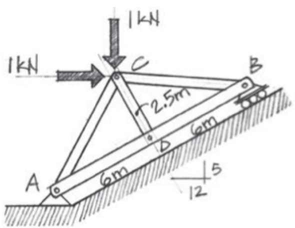
$\Sigma M_{DF} = 0 = CE(1) - (1\text{kN})(2) - (2\text{kN})(3)$

$\Rightarrow \boxed{CE = 8\text{kN}}$

$DF = CE + CF\cos 45° = 8\text{kN} + 3\text{kN}$

$\boxed{DF = 11\text{kN}}$

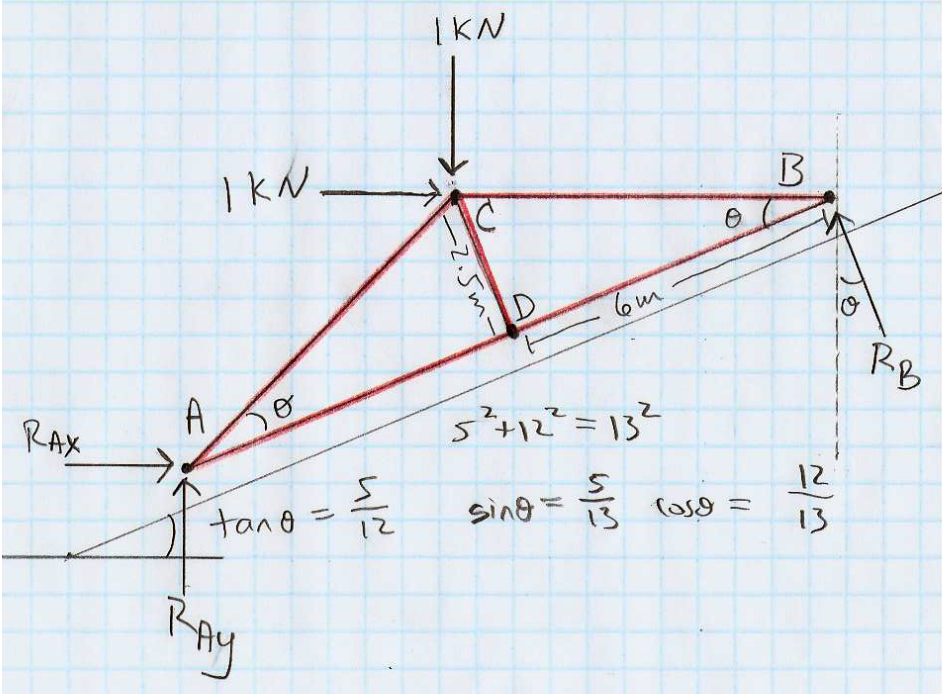
2.38 An inclined king-post truss supports a vertical and horizontal force at *C*. Determine the support reactions developed at *A* and *B*.



This is not really a "truss problem," since we're not asked to solve for the internal forces in the truss, but it is an example of a pretty tricky equilibrium problem.

Let's try working through this together in class. (I think it's deviously tricky!)

Notice, from the given dimensions: the angle of the incline is the same as the interior angle at joints A and B of the truss. Also notice pin/hinge support at A and roller support at B.


1KN
1KN
B
C
θ
2.5m
D
6m
θ
R_B
A
θ
R_AX
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
R_Ay

① $0 = \underset{\text{on truss}}{\sum F_x} = R_{Ax} + 1\,kN - R_B \sin\theta$

$$\Rightarrow R_{Ax} = R_B \sin\theta - 1\,kN$$

② $0 = \underset{\text{on truss}}{\sum F_y} = R_{Ay} - 1\,kN + R_B \cos\theta$

$$\Rightarrow R_{Ay} = 1\,kN - R_B \cos\theta$$

③ $0 = \underset{\circlearrowleft}{\sum M_A} = R_B(12m) - 1\,kN(12m\sin\theta) - 1\,kN(6.5m\cos 2\theta)$

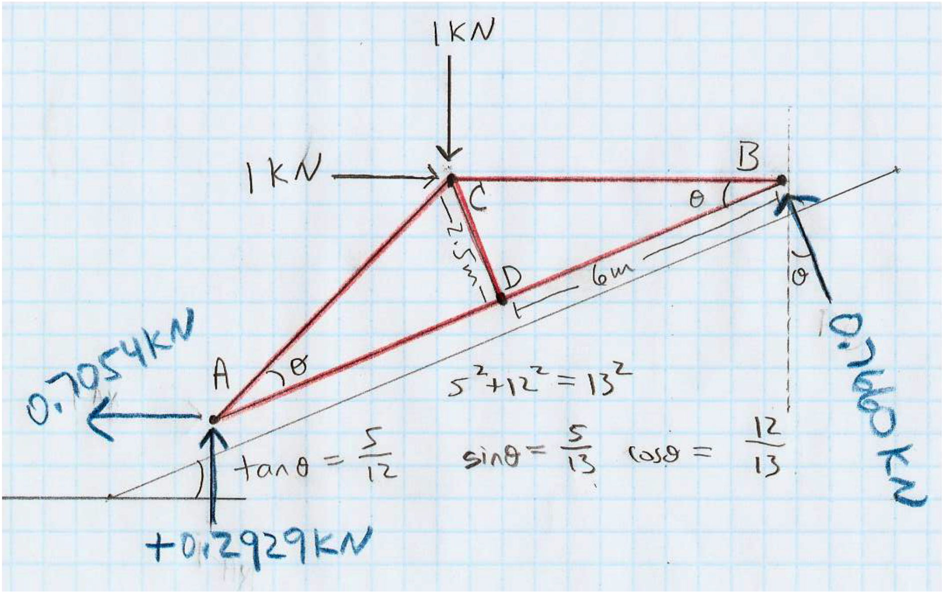
$$\Rightarrow R_B = 1\,kN(\sin\theta) + \frac{6.5}{12}\,kN(\cos 2\theta) = 0.7660\,kN$$

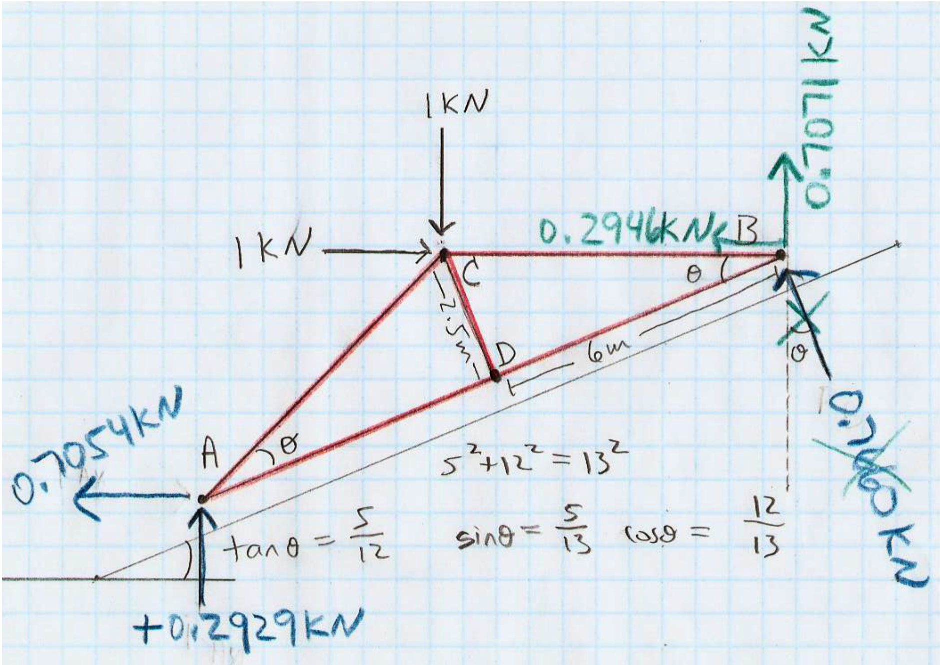
$$(R_B)_x = -R_B \sin\theta = -0.2946\,kN$$

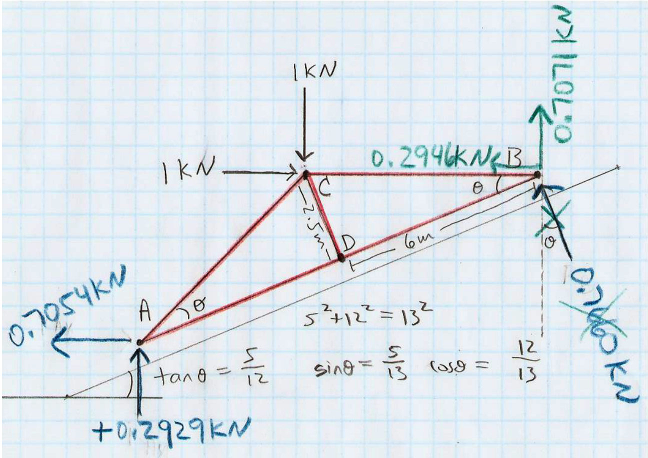
$$(R_B)_y = R_B \cos\theta = 0.7071\,kN$$

$$R_{Ax} = (0.2946 - 1)\,kN = -0.7054\,kN$$

$$R_{Ay} = (1 - 0.7071)\,kN = 0.2929\,kN$$


1KN
1KN
B
C
θ
θ
2.5m
D
6m
θ
0.7054KN
A
θ
0.7660 KN
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
+0.2929KN


1KN
1KN
0.2946KN
0.7071KN
A
B
C
D
θ
2.5m
6m
0.7054KN
+0.2929KN
0.7660 KN
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$



Here's what puzzled me last time I looked at this statics problem: You can evaluate the moment (about A) due to the horizontal 1 kN force in two ways — with the same result, of course:

- how we did it: $-1\,\mathrm{kN} \times 6.5\,\mathrm{m}\,\sin(2\theta) = 4.615\,\mathrm{kN}\cdot\mathrm{m}$
- clever, tricker way: $-1\,\mathrm{kN} \times 12\,\mathrm{m}\,\sin\theta = 4.615\,\mathrm{kN}\cdot\mathrm{m}$

We discovered a geometrical proof that $\sin(2\theta) = 2\cos\theta\sin\theta$.