Physics 9, fall 2020, reading due before 2020-09-09

(Whenever you start reading this document, you might want to double-check that there is not a more recent version online. I put the document's update date on the bottom edge of each page. I am hoping to add a link to a short lightboard video illustrating how I like to draw free-body diagrams, which are a useful bookkeeping mechanism for forces.) `http://positron.hep.upenn.edu/p9/files/reading_20200909.pdf`

Physics 9 assumes that you have previously seen (either in Physics 8 or in a good high-school physics course) the basics of how to use Newton's laws to analyze motion. But most of the Physics 9 material **does not** make very extensive use of your Physics 8 (or similar) knowledge, so **please don't worry** if your previous physics knowledge feels quite rusty. Together, we will fill in the gaps in our knowledge as we work cooperatively on problem solving during the synchronous parts of Physics 9.

Your formal reading assignment for 2020-09-09 (online response web-form due before noon) is to read Giancoli's chapter 4 ("motion and force"). But in today's notes (which you should read carefully), I will do a broader review that I hope you will find helpful. While Giancoli's chapter 4 discusses and illustrates the decomposition of force vectors into their components (a big topic in Physics 8), that sort of vector manipulation is unimportant for Physics 9. Our brief review of forces will mostly involve one-dimensional problems.

This "review" at the start of the semester will be the most math-heavy part of the course. The subsequent material will generally be less difficult and less math-intensive. Again, don't worry: working together with me and with a group of your classmates will (I believe) make refreshing your earlier physics memory an enjoyable and stress-free experience.

Some useful numbers and dimensions.

You might find many of these numbers useful throughout the course, though they can also be readily found in a web search or with Wolfram Alpha.

1 year $\approx 3.156 \times 10^7$ s

circumference of Earth $\approx 40 \times 10^6$ m (radius = 6378 km)

mass of Earth $\approx 6.0 \times 10^{24}$ kg

speed of light $c = 2.9979 \times 10^8$ m/s mass of proton or neutron $\approx$ 1 amu ("atomic mass unit") $= 1\ \frac{\text{g}}{\text{mol}} = \frac{0.001\ \text{kg}}{6.022\times10^{23}} = 1.66\times10^{-27}$ kg

Some exact definitions: 1 inch = 0.0254 meter. 1 foot = 12 inches. 1 mile = 5280 feet.

Weight of 1 kg = 2.205 pounds.

Other unit conversions: try typing e.g. "1 mile in centimeters" or "1 gallon in liters" into google or into Wolfram Alpha.

Unit conversions: The trick is to make use of the fact that a ratio of two equal values, like $\frac{1\,\text{inch}}{2.54\,\text{cm}}$, equals 1. So you "multiply by 1" and then cancel the unwanted units until you are left with the desired units. For example, to convert 1 mile into meters, we can write [notice that each ratio in parentheses equals 1]

$$1\,\text{mile}\times\left(\frac{5280\,\text{foot}}{\text{mile}}\right)\times\left(\frac{12\,\text{inch}}{\text{foot}}\right)\times\left(\frac{0.0254\,\text{m}}{\text{inch}}\right)=1609.3\,\text{m}\ .$$

Significant digits: we will generally not worry about significant digits in this course, but you should be aware that if I were to write that the mass of my dog Alfie is 37.7352863 kg, I would be implying a level of precision that is unrealistic, since a meal or a visit to the back yard can easily change Alfie's mass by about 0.1 kg or so. A good rule of thumb for Physics 9 is to use 4, 5, or 6 total digits for intermediate calculations (to avoid rounding errors), but to write down 3 or 4 total digits in a final answer: for example, 37.7 kg or 37.74 kg.

(describing motion in one dimension)

We usually represent an object's motion in one dimension by a function $x(t)$, where t is the time (usually measured in seconds) and x is the position along the x-axis (usually measured in meters). Often using a graph of x vs t is more intuitive than writing a mathematical function. It also helps to draw a diagram indicating which direction in space corresponds to increasing x and what point along the x-axis corresponds to $x = 0$. For instance, to describe a walk from my office to the campus swimming pool, I might make the x-axis point west along Walnut Street and choose $x = 0$ to be the corner of 33rd and Walnut streets, where David Rittenhouse Lab is located.

If I start out at "(i)nitial" time t_i and initial position x_i and I walk (along the x-axis) to "(f)inal" position x_f, arriving at final time t_f, the x component of my **displacement** is $\Delta x = x_f - x_i$.

If an object goes from x_i to x_f, changing direction at intermediate points x_a and x_b, then **distance traveled** (in one dimension) is $d = |x_a - x_i| + |x_b - x_a| + |x_f - x_b|$.

Velocity is the "rate of change of position with respect to time." The x component of (instantaneous) velocity is $v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$.

Speed (a scalar) is the magnitude of velocity (a vector). In one dimension, $v = |v_x|$.

average velocity $= \frac{\text{displacement}}{\text{time interval}}$. The x component of average velocity is $v_{x,\mathrm{av}} = \frac{x_f - x_i}{t_f - t_i}$.

average speed $= \frac{\text{distance traveled}}{\text{time interval}}$.

Solving quadratic equations: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(more than one dimension)

We live in three-dimensional space. It is pretty common (but not in this course) to define a Cartesian coordinate system using some reference point on Earth's surface as the origin (maybe the corner of 33rd and Walnut streets) and then let the x-axis point east, the y-axis point north, and the z-axis point up from there.

Since we are so fond of drawing on two-dimensional sheets of paper, we will often just use x and y, with the x-axis usually pointing in some relevant horizontal direction and with the y-axis usually pointing upward. (This is a common choice, but sometimes a different choice of axes simplifies a given problem.)

In two dimensions, (instantaneous) velocity has x component $\mathrm{d}x/\mathrm{d}t$ and y component $\mathrm{d}y/\mathrm{d}t$. We can use ordered-pair notation to write 2D position as a vector $\vec{r} = (x, y)$. Then velocity is the vector $\vec{v} = (v_x, v_y) = (\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t})$. The magnitude of the velocity vector is speed, $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$. We use vectors often in Physics 8, but they arise very rarely in Physics 9, so I will say very little here about vectors. For our purposes, a vector is a physical quantity that has both a magnitude (such as "34.5 kilometers per hour") and a direction in space (such as "northeast").

(acceleration)

Acceleration (a vector) is the rate of change of velocity (which is also a vector). The x component of acceleration is $a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$. The y component of acceleration is $a_y = \frac{\mathrm{d}v_y}{\mathrm{d}t} = \frac{\mathrm{d}^2y}{\mathrm{d}t^2}$. Acceleration is the derivative of velocity with respect to time, and velocity is the derivative of position w.r.t. time; therefore, acceleration is the second derivative of position w.r.t. time, which we write using the confusing (but conventional) notation $a_x = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$.

The reason why physics books talk so often about acceleration is that the acceleration of an object directly relates to the forces acting on that object, as we will see below.

When discussing gravity near Earth's surface, we introduce a constant $g = 9.8\ \mathrm{m/s^2}$, which is "the acceleration due to Earth's gravity." If the y axis points *upward*, then $a_y = -g$ for free fall near Earth's surface. So for this scenario we use a constant negative value for a_y. "Free fall" occurs when the only force exerted on an object is the graviational force exerted by Earth on that object (as we'll see below). If we neglect air resistance (as we usually do), then "free fall" describes the motion of a projectile (such as a baseball) from the instant after the projectile is released until the instant before the projectile lands. A projectile in free fall near Earth's surface has constant vertical acceleration ($a_y = -g$) and constant horizontal velocity ($v_x = \text{constant}$). The only time we will see projectile motion in Physics 9 is when we describe the parabolic arc of water sprayed from a fire hose (or similar example).

Because an object's motion, for some interval of time, is in many useful cases the result of a constant force, it is often useful to describe motion in which an object's acceleration is constant. For **constant acceleration** (written here for the case where acceleration in the y direction, a_y, is constant):

$$v_{y,f} = v_{y,i} + a_y t$$

$$y_f = y_i + v_{y,i} t + \frac{1}{2} a_y t^2$$

$$v_{y,f}^2 = v_{y,i}^2 + 2a_y\ (y_f - y_i)$$

The third equation comes from combining the first two equations and eliminating t. Here we take $t_i = 0$ and $t_f = t$, where "i" stands for "initial" and "f" stands for "final."

(momentum)

An object's momentum (also a vector) is $\vec{p} = m\vec{v}$, where m is the object's mass. (We usually measure mass in kilograms. An object's mass is also known as the object's inertia). If no forces act on an object (or if the forces acting on that object sum (as vectors) to zero), then that object's momentum is constant. If no *external* forces act on a system of several objects, then the total momentum of that system of objects is constant. The total momentum (vector) of a system of objects equals the sum of the constituent objects' momentum vectors. In Physics 8 we often use momentum to understand what happens when two objects collide; we will seldom use momentum in Physics 9.

(energy)

In chemistry, a **calorie** is 1 cal = 4.18 J. In nutrition, a "**food Calorie**" is 1 Cal = 4180 J. The standard unit of energy is a **joule** (J).

The energy of motion is **kinetic energy**: $K = \frac{1}{2}mv^2$, where m is the object's mass and v is the object's speed.

Technically, $\frac{1}{2}mv^2$ is an object's *translational* kinetic energy. In Physics 8 we also discuss *rotational* kinetic energy, which we will not need in Physics 9.

When an object near Earth's surface is displaced by Δy in the direction away from Earth's center (i.e. with the y-axis pointing upward), the change in gravitational potential energy of the Earth+object system is $\Delta U_{\text{grav}} = mg\Delta y$. Speaking less precisely, it is convenient to say that an object whose height abuove Earth's surface is y has **gravitational potential energy** $U_{\text{grav}} = mgy$.

(Technically, G.P.E. is a property of the Earth+object system, not of the object itself. Also, the expression mgy is only correct if the object is close to Earth's surface, i.e. if the distance above Earth's surface is much much smaller than Earth's radius. Also, you are free to add a constant offset to potential energy, as a constant U offset has no effect on the physics, so sometimes it is convenient to choose a different altitude y at which U_{grav} is defined to be zero.)

(force)

Newton's first law (the least useful one): When the motion of an object is described with respect to an inertial reference frame, the velocity of that object will be constant if the vector sum of forces acting on that object is zero. More commonly, one says, "an object at rest remains at rest, and an object in motion remains in motion at constant speed in a straight line, unless acted upon by a nonzero net force."

The first law is just a special case of the second law (below). So the modern view is to say that the second law only holds when we measure objects' velocities w.r.t. an **inertial reference frame**. That means we use a set of coordinate axes that are neither acclerating nor rotating w.r.t. "the fixed stars." In practice, "the fixed stars" are moving away from us as the universe expands, but in effect we mean w.r.t. Earth's solar system. And in practice, if we neglect Earth's rotation and Earth's motion around the Sun, we can use a set of coordinate axes fixed w.r.t. a point on Earth's surface. (This is fine for describing a baseball or a bicycle. One needs to account for Earth's own rotation or acceleration when describing the motion of a tropical storm, a Foucault pendulum, an artillery shell, a naval gun, or ocean tides.)

Newton's second law (the most useful one): Suppose we are describing the motion of an object named "A." The acceleration of object A equals the vector sum of forces *acting on A* divided by the mass of A.

$$\vec{a}_A = \frac{\sum \vec{F}_{\text{on A}}}{m_A}$$

Always remember that the acceleration of object A depends only on forces exerted *by other objects on A*, not on forces exerted on other objects by A. Below, we will use free-body diagrams to help us keep track of the forces acting *on* a given object.

In Physics 8 (but not so much in Physics 9), we also write Newton's second law in terms of momentum: The rate of change of the momentum of object A is the vector sum of forces exerted **on** object A. Force (newtons: $1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$) is rate of change of momentum:

$$\sum \vec{F}_{\text{on A}} = \frac{\text{d}\vec{p_A}}{\text{d}t}$$

A result that is useful (in Physics 8) when describing a system of several objects (for example, two toy cars connected by a spring) is that the acceleration of the *center of mass* of several objects depends only on forces exerted by objects external to the system on objects inside the system, i.e. the vector sum of **external forces**:

$$\sum \vec{F}_{\text{ext}} = m_{\text{total}}\ \vec{a}_{CM}$$

where m_{total} is the sum of the masses of the several objects in the system.

Newton's third law (the most misunderstood one): A force is always exerted between two objects – by each object on the other. When a force is exerted between two objects A and B, the force $\vec{F}_{\text{by A on B}}$ exerted by A on B and the force $\vec{F}_{\text{by B on A}}$ exerted by B on A are equal in magnitude and opposite in direction:

$$\vec{F}_{\text{by A on B}} = -\vec{F}_{\text{by B on A}}$$

Because a force is always exerted *between two objects,* the forces $\vec{F}_{\text{by A on B}}$ and $\vec{F}_{\text{by B on A}}$ are really two facets of the same force. If I throw a 6 kg medicine ball, I can't help recoiling from the throw. When I push on the medicine ball, the medicine ball pushes back on me. The ball is not pushing back as a response to my push; rather, the force between me and the ball affects both me and the ball. The push between me and the ball has two parts: the part that affects my motion is $\vec{F}_{\text{ball on me}}$, while the part that affects the ball's motion is $\vec{F}_{\text{me on ball}}$. Newton's third law says that $\vec{F}_{\text{ball on me}} = -\vec{F}_{\text{me on ball}}$.

Newton's third law is equivalent to the statement that if objects A and B push or pull on one another in outer space (where they interact only with one another, not with any other objects), the resulting change in A's momentum will be equal and opposite the resulting change in B's momentum: $\Delta\vec{p}_A = -\Delta\vec{p}_B$.

To use Newton's second law correctly, you need to add up the correct list of forces, and you need to get the signs (or directions) right. It is largely a bookkeeping problem. A **free-body diagram** is a useful visual accounting device that helps you to keep track of all forces acting *on* a given object. The FBD for object A includes only those forces acting *on* object A. (The force exerted by A on B belongs on the FBD for B, not on the FBD for A; but the force exerted by B on A belongs on the FBD for A.) We draw many, many FBDs in Physics 8. Since free-body diagrams are a visual topic, I will not elaborate on them here. Instead, I will discuss them in an upcoming lightboard lecture, and we will practice drawing them in an upcoming synchronous class meeting (and therefore also on our first problem set).

Link to lightboard video on free-body diagrams: **(insert here)**

An important special case of Newton's second law is when $\vec{a} = 0$, called **equilibrium**. For example, when you build an architectural structure, you want all of its components to stay in place. That means you want them not to move. In other words, every component's velocity should be zero and should remain zero. Thus, a necessary condition is that every component must have zero acceleration. For component A to have zero acceleration, all forces acting on component A must sum to zero. (There is also a rotational constraint: all torques must sum to zero. We discuss that at great length in Physics 8 but ignore it here.) The free-body diagram for an object in equilibrium has the appealing visual quality that the total length of all up-pointing force arrows equals the total length of all down-pointing force arrows, and so on. We will work out some one-dimensional examples together, such as analyzing the forces (called "load tracing" in architectural structures) for three blocks stacked one top of one another.

As we noted above, **gravitational potential energy** near Earth's surface (where y = height) is $U_{\rm grav} = mgy$. The force of gravity near Earth's surface is given by its y component: $F_y^{\rm grav} = -mg$, where the y-axis points upward. Notice that $F_y = -\frac{{\rm d}U}{{\rm d}y}$, which may appeal to you if you took Math 114.

The **potential energy stored in a spring** is $U_{\rm spring} = \frac{1}{2}k(x - x_0)^2$ where x_0 is the "relaxed length" of the spring, and k is the "spring constant" (units for k are newtons per meter). The force exerted by a spring on its load is given by Hooke's Law:

$$F_{x,(\text{by spring ON load})} = -k\ (x - x_0)$$

If you increase x (the position of the end of the spring) beyond its relaxed value, then the spring will exert on you a force in the $-x$ direction; if you decrease x below its relaxed value, then the spring will exert on you a force in the $+x$ direction. That's why the minus sign is there in the force equation. If you love math, you may enjoy noticing that $F_x = -\mathrm{d}U/\mathrm{d}x$.

We will revisit the Physics 8 concept of **work** when we see how a heat engine converts thermal energy (e.g. from burning fuel) into mechanical energy. Formally, the work done on a system by an external force is $W = \int F_x(x)\,\mathrm{d}x$. Work is the integral of force w.r.t. displacement. We usually consider only a constant force, where $W = F_x \Delta x$, so we can just multiply instead of using calculus. When an external force does work on a system, the energy of the system increases. Work is a transfer of energy into (or out of) a system. Work, like energy, is measured in joules (J).

Power (measured in watts: 1 W = 1 J/s) is the rate of change of the energy of a system: $P = \mathrm{d}E/\mathrm{d}t$. (Sometimes, to be more precise, one says that power is the rate at which an external force does work on a system: work per unit time.)

In one dimension, the power delivered by a constant external force is $P = F_{\mathrm{ext},x} v_x$.

(motion in a circle)

You do not need to follow the details of these derivations. (The math in this course will be nowhere near as tricky as the math that I will do in this section.) At the end of this section, we will use the main results to talk about satellite orbits. The derivations are here in case you want to see where the results come from.

For motion in a circle (such as a satellite in orbit around Earth), the acceleration has a *centripetal* component that is perpendicular to velocity and points toward the center of rotation. If we put the center of rotation at the origin $(0, 0)$ then

$$x = R\cos\theta \qquad y = R\sin\theta$$

$$\vec{r} = (x, y) = (R\cos\theta, R\sin\theta) = R\,(\cos\theta, \sin\theta)$$

We measure the angle θ in radians. The "angular velocity" ω is the rate of change of the angle θ

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$$

The units for ω are just s^{-1} (which is the same as radians/second, since radians are dimensionless). Revolutions per second are $\omega/(2\pi)$, and the period (how long it takes to go around the circle) is $2\pi/\omega$. The velocity is

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \left(\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t}\right) = R\left(-\sin\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}, \cos\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\right) = \omega R\,(-\sin\theta, \cos\theta), \qquad |\vec{v}| = \omega R$$

The magnitude of the centripetal acceleration (the required rate of change of the velocity vector, to keep the object on a circular path) is

$$a_c = \omega^2 R = \frac{v^2}{R}$$

and the centripetal force (directed toward center of rotation) is

$$|\vec{F}_c| = ma_c = m\omega^2 R = \frac{mv^2}{R}$$

Moving in a circle at constant speed (velocity changes but speed does not!) is called *uniform circular motion.* For UCM, the acceleration $\vec{a}$ is perpendicular to the velocity $\vec{v}$, and the angular velocity ω is constant. Then

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\omega^2 R\ (\cos\theta, \sin\theta) = -\frac{v^2}{R}\ (\cos\theta, \sin\theta)$$

For the non-UCM case where speed is not constant, the acceleration $\vec{a}$ has an additional component that is parallel to the velocity $\vec{v}$. That case is not needed for Physics 9.

Whew! What can we do after all that effort? (Reminder: the math in this course will be nowhere near as tricky as what I did in this section.)

First, the variable ω measured in radians/second will reappear when you read Giancoli's chapter 11 ("vibrations and waves") next week. There is a neat mathematical connection that if you project out only the x component (or only the y component) of uniform circular motion, the result is precisely the sinusoidal motion that you will see in chapter 11 for a swinging pendulum or a mass bobbing up and down on the end of a string.

Second, we can talk about a satellite in a circular orbit around Earth, as you will read in Muller's chapter 3 ("gravity, force, and space"). We will do that as soon as we mention gravity, below.

Third, toward the end of the course, we will discuss the closely analogous scenario of an electron in orbit around an atomic nucleus, where the attraction (the centripetal force) is an electrical force rather than a gravitational force.

(gravity)

Next weekend, when you read Richard Muller's chapter 3 ("gravity, force, and space"), you will learn about satellite orbits. We don't want to get too bogged down in equations, but we can use a little bit more math than Muller's book does.

Every two objects in the universe attract one another via a gravitational force. In ordinary terrestrial life, most gravitational forces are much too small to perceive: we only notice the gravitational force exerted by Earth on us and by Earth on other terrestrial objects.

Newton's universal law of gravitation states that object A (mass m_A) and object B (mass m_B) exert on one another a gravitational force of magnitude

$$F^{\mathrm{grav}} = \frac{G m_A m_B}{r^2}$$

where $\vec{F}^{\mathrm{grav}}$ points along the axis connecting object A to object B, and where r is the distance between objects A and B (between A's center of mass and B's center of mass). The force is "attractive," meaning the force exerted by A on B points toward A, while the force exerted by B on A points toward B.

$$G = 6.67 \times 10^{-11} \, \frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}$$

is a *universal* constant — the same on Earth, on Mars, in distant galaxies, etc. Plugging in Earth's mass M_e for m_A and Earth's radius R_e for r and then dividing out m_B, we can write the terrestrial "little g" in terms of the universal "big G" as:

$$g = 9.8 \ \mathrm{m/s^2} = \frac{G M_e}{R_e^2}$$

which shows that an apple falling onto Newton's head results from the same force that governs the motion of the Moon around Earth, Earth around the Sun, etc.

For an orbit, gravity provides the centripetal force, so

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

The gravitational potential energy for objects A and B is

$$U_{\mathrm{grav}} = -\frac{G m_A m_B}{r} \quad (\text{note the sign}, \mathrm{U_{grav}} < 0)$$

which $\to 0$ as $r \to \infty$. The objects are *bound* (will not escape one another) if $K + U < 0$, i.e. if the sum of kinetic and potential energies is less than zero.

If $K + U \geq 0$, they escape each other. They just barely escape if $K + U = 0$, which lets us define the **escape velocity** for a satellite of mass m from a planet or star of mass M:

$$\frac{1}{2} m v_{\mathrm{escape}}^2 = \frac{GmM}{r}$$

in which case $K \to 0$ when $r \to \infty$.

We can evaluate the gravitational potential energy $U_{\text{grav}} = -Gm_A m_B/r$ for a small altitude y above Earth's surface, $r = R_e + y$, using a Taylor series (Math 104) to express the fact that y/R_e is a small number:

$$U_{\text{grav}} = -\frac{GmM_e}{R_e + y} = -\frac{GmM_e}{R_e(1+\frac{y}{R_e})} \approx -\frac{GmM_e}{R_e}\left(1 - \frac{y}{R_e}\right)$$

$$U_{\text{grav}} \approx -\frac{GmM_e}{R_e} + m\left(\frac{GM_e}{R_e^2}\right)y = -\frac{GmM_e}{R_e} + mgy$$

The first term is just a constant that does not depend on the altitude y of the object of mass m. So we are back to our more familiar terrestrial expression for gravitational potential energy: $U_{\text{grav}} = mgy$, which is a good approximation as long as y is very small compared with Earth's radius.

Again, you won't have to do math that is anywhere near this complicated. But for those of you who enjoy math, I wanted to show you where some of the key results come from.