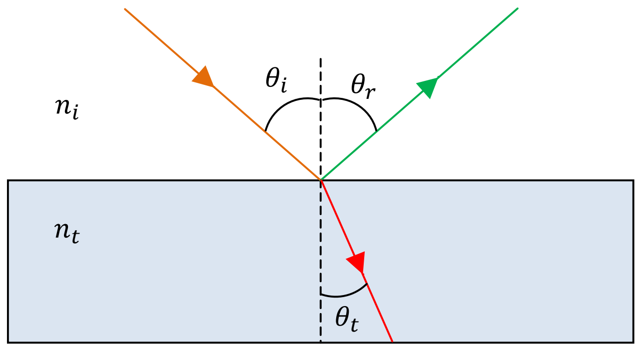
# Thin Lenses

## Reflection and Refraction

When light passes from one medium to another, part of the light is reflected and the rest is transmitted. Light rays that are transmitted undergo refraction (bending) at the interface of the two media. The directions of light rays at the interface of two media are defined with respect to a line normal (perpendicular) to the interface as shown below.




The angles that the incident ($i$) and reflected ($r$) rays make with the normal are the same. That is, $\theta_i = \theta_r$. The angles that the incident ($i$) and transmitted ($t$) rays make are related by Snell's Law:

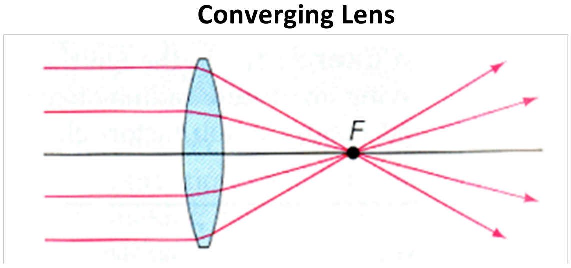
$$n_i \sin\theta_i = n_t \sin\theta_t$$

Here, $n_i$ and $n_t$ are the index of refraction for the medium that hosts the incident and transmitted ray, respectively.

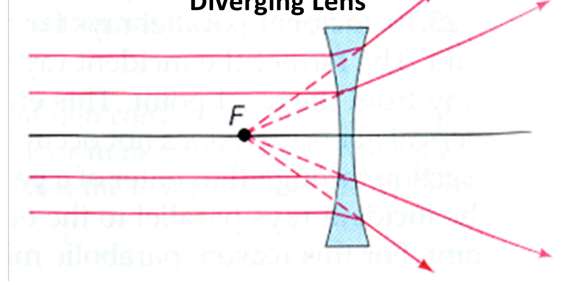
## Lenses

Refraction and Snell's Law is the basis for how a lens works. A lens is an optical device which transmits and refracts light. Lenses can be categorized by how they transmit light rays. A converging lens transmits parallel light rays to a single point called the focal point $F$. On the other hand, parallel light rays diverge after passing through a diverging lens. The diverging light rays are transmitted such that they appear to emerge from a focal point $F$ located behind the lens.

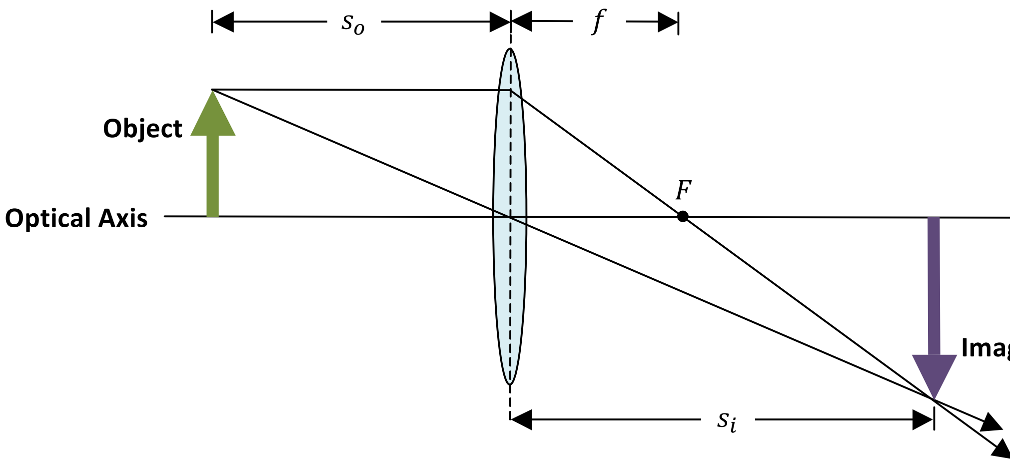
**Converging Lens**




**Diverging Lens**



A lens can form an image of an extended object as shown below for a converging lens. The focal point $F$ is located at a distance $f$, known as the focal length, from the center of the lens. The object is located at a distance $s_o$, known as the object distance, from the lens. A ray emitted parallel to the optical axis of the lens will pass through the focal point $F$. A ray that passes through the center of the lens will pass through the lens undeflected. The point at which these two rays cross determines the position of the image, which is located at a distance $s_i$, known as the image distance, from the lens.




The relationship between all these distances is given by the Thin Lens Formula:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

The magnification $m$ of the lens is given by

$$m = -\frac{s_i}{s_o}$$

The negative sign tells us that when $s_i$ and $s_o$ are positive, the image will be inverted.

**Prelab Exercise**

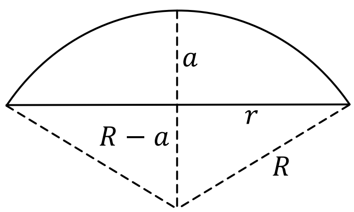
You have a converging lens that has a focal length of $f = 30\text{cm}$. Calculate the image distance $s_i$, the magnification $m$ and indicate whether the image is upright or inverted for each of the following object distances.

1. $s_o = 1f$
2. $s_o = 3f$
3. $s_o = 10f$
4. $s_o = 100f$
5. $s_o = \infty$
6. What is the percent difference in $s_i$ for an object located at $s_o = 10f$ and $s_o = \infty$? What about $s_o = 100f$ and $s_o = \infty$? When is it justified to consider at object to be located at "optical infinity"?

**Lensmaker's Formula**

The Lensmaker's Formula provides a recipe for engineering a lens to have a particular focal length. This formula relates the focal length $f$ to the index of refraction of the lens $n$ and to the radii of curvature $R_1$ and $R_2$ for each side of the lens. The Lensmaker's Formula is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

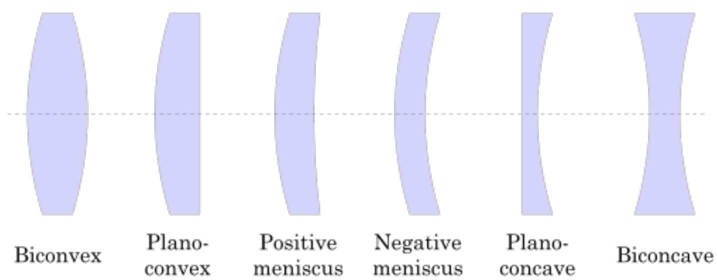


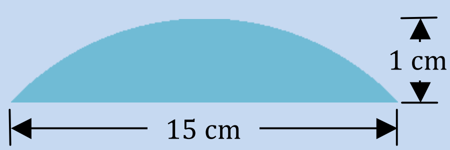

The radius of curvature $R$ can be found using the figure to the left. Using the Pythagorean Theorem $R^2 = (R-a)^2 + r^2$. Solving for $R$ gives

$$R = \frac{r^2 + a^2}{2a}$$

**Types of Lenses**

Lenses are named according to the radii of curvature of each side.



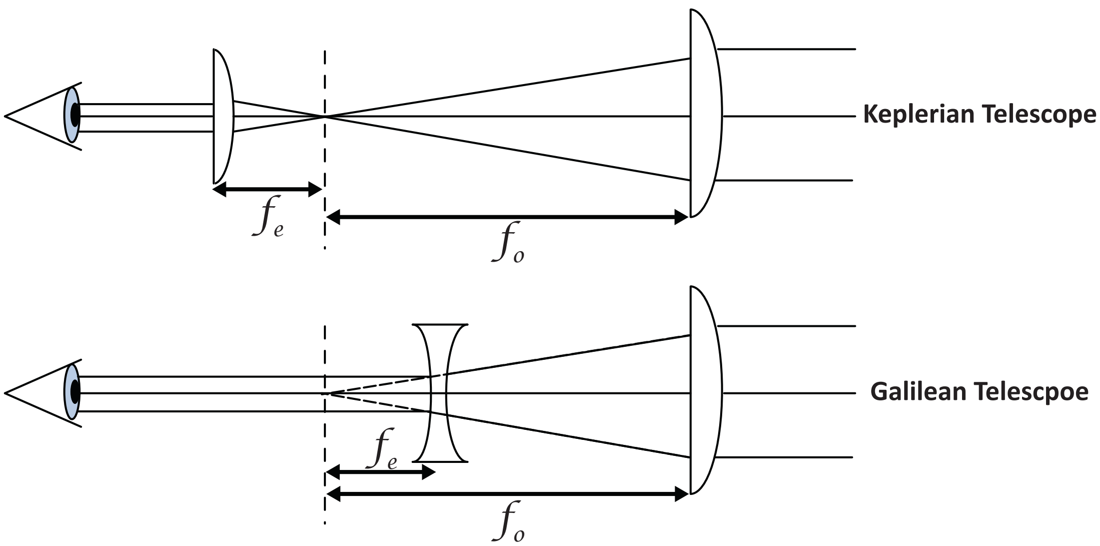




**Prelab Exercise**

7. You have a plano-convex lens 1 cm high with a diameter of 15 cm as shown to the left. Its index of refraction $n = 1.5$. What is the radius of curvature of the lens? What is the focal length?

**Telescopes**

A simple telescope consists of only two lenses: an objective lens, which focuses the image, and the eyepiece, which magnifies it. Below are two types of simple telescopes. The Galilean telescope uses a convergent lens for the objective and a divergent lens for the eyepiece. The lenses are located a distance away from each other equal to the difference in their focal lengths. The Keplerian telescope uses a convergent lens for both the objective and eyepiece. The lenses are located a distance away from each other equal to the sum of their focal lengths. The advantage of the Keplerian telescope is that it provides a wider field of view compared to the Galilean telescope. However, unlike the Galilean telescope the image produced by a Keplerian telescope is inverted.




**Lens Aberrations**

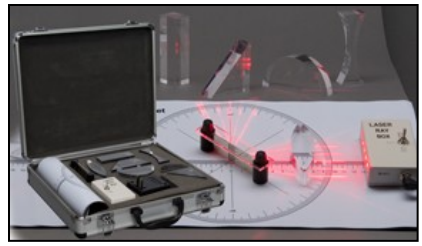
Lens aberrations are deformations of the image formed when light travels through a lens. Spherical aberration is a common lens aberration that occurs with spherical lens surfaces. The above formulas are valid for rays that make small angles with the lens surfaces. These rays are referred to as paraxial rays. The behavior of light rays that make larger angles with the lens surface do not follow the above formulas. As a result, non-paraxial rays will focus at a different point and result in a blurred image.

## Goals of this Lab

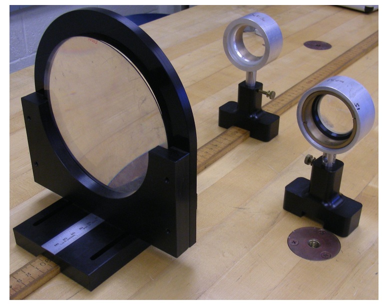
1. **Observe and understand how lenses refract light**
2. **Measure the focal length of a lens using several techniques**
3. **Measure the index of refraction of a lens**
4. **Construct a telescope**

## Lab Materials

**Laser Ray Box and Lenses**
A laser ray box can produce 1, 3 or 5 laser beams that can be refracted by different lenses.



**Telescope Kit**
A large converging objective lens and two eyepieces, converging and diverging, in mounts on a meter stick.



## Lab Procedure

### Index of Refraction

You will measure the index of refraction of a plano-convex lens.

1. Unroll the scale and place it on the table.
2. Turn the laser ray box on and set it for a single ray.
3. Position the large, rectangular plano-convex lens such that the planar side faces the ray box.
4. Orient the lens so that you can observe the incident and transmitted (refracted) ray.
5. Record the angles of the incident and transmitted ray and calculate the index of refraction for the lens material using Snell's law.

**Questions**

1. What is the index of refraction for your lens? The lens is made of acrylic. Look up the index of refraction for acrylic and compare this with your measured value.

**Ray Tracing**

You will observe how light rays are bent by lenses.

1. Use the laser ray box to shine 3-5 light rays through the plano-convex, biconcave and biconvex lens separately. Observe how each ray is bent by the lens.
2. Measure the focal length of each lens using ray tracing.
3. Construct the Galilean and Keplerian telescopes using the appropriate lenses and observe the path of the rays as they shine through each.

**Prelab Exercise**

8. An extended object emits light rays in all directions. What is the object distance $s_o$ if all light rays are parallel to each other when they arrive at a lens? How can these parallel light rays be used to determine the focal length of a lens?

**Questions**

2. When shining 5 light rays, are the outermost 2 rays focused to the same point as the others? Why?
3. What is the focal length of each lens?

**Lensmaker's Formula**

1. Trace an outline of the large plano-convex lens on a piece of paper.
2. Use the Lensmaker's Formula and any measurements necessary to calculate its index of refraction.

**Questions**

4. How does your measured index of refraction compare to the value you obtained in the previous step? What are possible sources of error?

**Telescopes**

1. Go out into the hallway and arrange the two converging lenses so that the objective lens, (the big one) is in front of the other and they share the same optical axis (similar to the diagram for the Keplerian telescope on p. 3).

2. Position an index card between the lenses. Move the lenses along their optical axis until light from the window at either end of the hallway is focused on each side of the index card. When this occurs, you can then be sure that the lenses are separated by the sum of their focal lengths.

3. Remove the card and use the telescope to observe a distant object.

4. Now construct a Galilean telescope by using a divergent lens for the eyepiece. You already know the focal length of your objective lens. You can't directly measure the focal length of the divergent lens. However, you can move the eyepiece along the common optical axis of the telescope until you obtain a focused image.

**Questions**

5. What is the focal length of your objective lens and both eyepieces?
6. Which lens was the most effective eyepiece? Why?
7. Why does the Galilean telescope have such a narrow field of view?

**Feedback**

Do you have any comments, complaints, or suggestions about this lab? Feel free to email our Lab Manager directly at Pharnish@physics.upenn.edu .