Physics 8/9, 2017/2018, equation sheet

work in progress

(Chapter 1: foundations)

1 year $\approx 3.156 \times 10^7$ s

circumference of Earth $\approx 40 \times 10^6$ m (radius = 6378 km)

mass of Earth $\approx 6.0 \times 10^{24}$ kg

speed of light $c = 2.9979 \times 10^8$ m/s

mass of proton or neutron $\approx$ 1 amu (“atomic mass unit”) $= 1\ \frac{\text{g}}{\text{mol}} = \frac{0.001\ \text{kg}}{6.022\times 10^{23}} = 1.66 \times 10^{-27}$ kg

Some exact definitions: 1 inch = 0.0254 meter. 1 foot = 12 inches. 1 mile = 5280 feet.

Weight of 1 kg = 2.205 pounds.

Other unit conversions: try typing e.g. “1 mile in centimeters” or “1 gallon in liters” into google!

(Chapter 2: motion in one dimension)

x component of displacement: $\Delta x = x_f - x_i$

If object goes from x_i to x_f, changing direction at intermediate points x_a and x_b, then distance traveled is $d = |x_a - x_i| + |x_b - x_a| + |x_f - x_b|$

velocity: $v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$

average velocity $= \frac{\text{displacement}}{\text{time}}$ $\qquad v_{x,\text{av}} = \frac{x_f - x_i}{t_f - t_i}$

average speed $= \frac{\text{distance traveled}}{\text{time}}$

If $ax^2 + bx + c = 0$, then $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

(Chapter 3: acceleration)

$g = 9.8$ m/s^2. If x axis points *upward*, then $a_x = -g$ for free fall near earth's surface.

If x axis points *downward* along inclined plane, then $a_x = g \sin\theta$ for object sliding down inclined plane (inclined at angle θ w.r.t. horizontal).

For constant acceleration:

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$v_{x,f}^2 = v_{x,i}^2 + 2 a_x \, (x_f - x_i)$$

(Chapter 4: momentum)

Momentum $\vec{p} = m\vec{v}$. Constant for *isolated* system: no external pushes or pulls (later we'll say "forces"). Conservation of momentum in isolated two-body collision implies

$$m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

which then implies (for isolated system, two-body collision)

$$\frac{\Delta v_{1x}}{\Delta v_{2x}} = -\frac{m_2}{m_1}$$

If system is not isolated, then we *cannot* write $\vec{p}_f - \vec{p}_i = 0$. Instead, we give the momentum imbalance caused by the external influence a name ("impulse") and a label ($\vec{J}$). Then we can write $\vec{p}_f - \vec{p}_i = \vec{J}$.

(Chapter 5: energy)

In chemistry, a calorie is 1 cal = 4.18 J. In nutrition, a "food Calorie" is 1 Cal = 4180 J.

The energy of motion is *kinetic* energy:

$$K = \frac{1}{2} m v^2$$

For an *elastic* collision, kinetic energy K is constant. For a two-body elastic collision, the *relative speed* is unchanged by the collision, though obviously the relative velocity changes sign. Thus, for a for a two-body elastic collision along the x axis (Eqn. 5.4),

$$(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i})$$

.

For a *totally inelastic collision*, the two objects stick together after collision: $\vec{v}_{1f} = \vec{v}_{2f}$. This case is easy to solve, since one variable is eliminated.

In the real world (but not in physics classes), most collisions are *inelastic* but are not totally inelastic. K is not constant, but $v_{12,f} \neq 0$. So you can define a *coefficient of restitution*, e, such that $e = 1$ for elastic collisions, $e = 0$ for totally inelastic collisions, and $0 < e < 1$ for inelastic collisions. Then you can write (though it is seldom useful to do so)

$$(v_{1x,f} - v_{2x,f}) = -e\ (v_{1x,i} - v_{2x,i})$$

If you write down the momentum-conservation equation (assuming that system is isolated, so that momentum is constant) for a two-body collision along the x axis,

$$m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

and the equation that kinetic energy is also constant in an elastic collision,

$$\frac{1}{2} m_1 v_{1x,i}^2 + \frac{1}{2} m_2 v_{2x,i}^2 = \frac{1}{2} m_1 v_{1x,f}^2 + \frac{1}{2} m_2 v_{2x,f}^2$$

you can (with some effort) solve these two equations in two unknowns. The quadratic equation for energy conservation gives *two* solutions, which are equivalent to

$$(v_{1x,f} - v_{2x,f}) = \pm(v_{1x,i} - v_{2x,i})$$

In the "+" case, the two objects miss each other, as if they were two trains passing on parallel tracks. The "−" case is the desired solution. In physics, the "other" solution usually means *something*, even if it is not the solution you were looking for.

(Chapter 6: relative motion)

Center of mass:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Center of mass velocity (equals velocity of ZM frame):

$$v_{ZM,x} = \frac{m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

(The following chapter 6 results are less important, but I list them here anyway.)

Convertible kinetic energy: $K_{\text{conv}} = K - \frac{1}{2} m v_{CM}^2$

Elastic collision analyzed in ZM ("$*$") frame:

$$v_{1i,x}^* = v_{1i,x} - v_{ZM,x}, \quad v_{2i,x}^* = v_{2i,x} - v_{ZM,x}$$
$$v_{1f,x}^* = -v_{1i,x}^*, \quad v_{2f,x}^* = -v_{2i,x}^*$$
$$v_{1f,x} = v_{1f,x}^* + v_{ZM,x}, \quad v_{2f,x} = v_{2f,x}^* + v_{ZM,x}$$

Inelastic collision analyzed in ZM frame (restitution coefficient e):

$$v_{1f,x}^* = -e v_{1i,x}^*, \quad v_{2f,x}^* = -e v_{2i,x}^*$$

(Chapter 7: interactions)

For two objects that form an isolated system (i.e. interacting only with one another), the ratio of accelerations is

$$\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

When an object near Earth's surface moves a distance Δx in the direction away from Earth's center (i.e. upward), the change in gravitational potential energy of the Earth+object system is

$$\Delta U = mg\Delta x$$

(Chapter 8: force)

The rate of change of the momentum of object A is the vector sum of forces exerted **on** object A.

Force (newtons: 1 N = 1 kg $\cdot$ m/s^2) is rate of change of momentum:

$$\sum \vec{F}_{\text{on A}} = \frac{\mathrm{d}\vec{p_A}}{\mathrm{d}t}$$

Impulse (change in momentum due to external force):

$$\vec{J} = \Delta\vec{p} = \int \vec{F}_{\text{ext}}\ \mathrm{d}t$$

Equation of motion for a single object A:

$$\sum \vec{F}_{\text{on A}} = m_A\ \vec{a}_A$$

Equation of motion for CoM of several objects (depends only on forces exerted by objects external to the system on objects inside the system, i.e. the vector sum of external forces):

$$\sum \vec{F}_{\text{ext}} = m_{\text{total}}\ \vec{a}_{CM}$$

Gravitational potential energy near earth's surface (h = height):

$$U_{\text{gravity}} = mgh$$

Force of gravity near earth's surface (force is $-\frac{\mathrm{d}U_{\text{gravity}}}{\mathrm{d}x}$):

$$F_x = -mg$$

Potential energy of a spring:

$$U_{\text{spring}} = \frac{1}{2}k(x - x_0)^2$$

where x_0 is the "relaxed length" of the spring, and k is the "spring constant" (units for k are newtons per meter).

Hooke's Law (force is $-\frac{\mathrm{d}U_{\text{spring}}}{\mathrm{d}x}$):

$$F_{\text{by spring ON load}} = -k(x - x_0)$$

(Chapter 9: work)

Work done on a system by an external, nondissipative force in one dimension:

$$W = \int_{x_i}^{x_f} F_x(x) dx$$

which for a constant force reduces to

$$W = F_x \Delta x$$

Power is rate of change of energy (measured in watts: 1 W = 1 J/s):

$$P = \frac{\mathrm{d}E}{\mathrm{d}t}$$

In one dimension, power delivered by constant external force is

$$P = F_{\mathrm{ext},x} v_x$$

(Chapter 10: motion in a plane)

Various ways to write a vector:

$$\vec{A} = (A_x, A_y) = A_x\ (1,0) + A_y\ (0,1) = A_x\ \hat{\imath} + A_y\ \hat{\jmath}$$

Can separate into two $\perp$ vectors that add up to original, e.g.

$$\vec{A}_x = A_x\ \hat{\imath}, \quad \vec{A}_y = A_y\ \hat{\jmath}$$

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Scalar product ("dot product") is a kind of multiplication that accounts for how well the two vectors are aligned with each other:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = |\vec{A}|\ |\vec{B}|\ \cos(\theta_{AB})$$

In one dimension, we learned

$$W = F_x\ \Delta x \quad \rightarrow \quad \int F_x(x)\ \mathrm{d}x$$

Sometimes the force is not parallel to the displacement: for instance, work done by gravity if you slide down a hill. In two dimensions,

$$W = \vec{F} \cdot \vec{D} = F_x\ \Delta x + F_y\ \Delta y$$

which in the limit of many infinitessimal steps becomes

$$W = \int \vec{F}(\vec{r}) \cdot \mathrm{d}\vec{r} = \int (F_x(x,y)\ \mathrm{d}x + F_y(x,y)\ \mathrm{d}y)$$

Similarly, in two dimensions, power must account for the possibility that force and velocity are not perfectly aligned:

$$P = \vec{F} \cdot \vec{v}$$

Static friction and kinetic (sometimes called "sliding") friction:

$$F^{\text{Static}} \le \mu_S \; F^{\text{Normal}}$$

$$F^{\text{Kinetic}} = \mu_K \; F^{\text{Normal}}$$

"normal" & "tangential" components are $\perp$ to and $\parallel$ to surface.

For an inclined plane making an angle θ w.r.t. the horizontal, the normal component of gravity is $F^N = mg\cos\theta$ and the (downhill) tangential component is $mg\sin\theta$. The frictional force on a block sliding down the surface then has magnitude $\mu_K \; mg\cos\theta$ and points uphill if the block is sliding downhill. You have to think about whether things are moving and if so which way they are moving in order to decide which direction friction points and whether the friction is static or kinetic.

(Chapter 11: motion in a circle)

For motion in a circle, acceleration has a *centripetal* component that is perpendicular to velocity and points toward the center of rotation. If we put the center of rotation at the origin $(0, 0)$ then

$$x = R\cos\theta \qquad y = R\sin\theta$$

$$\vec{r} = (x, y) = (R\cos\theta, R\sin\theta) = R\;(\cos\theta, \sin\theta)$$

The "angular velocity" ω is the rate of change of the angle θ

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$$

The units for ω are just s^{-1} (which is the same as radians/second, since radians are dimensionless). Revolutions per second are $\omega/(2\pi)$, and the period (how long it takes to go around the circle) is $2\pi/\omega$. The velocity is

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \omega R\;(-\sin\theta, \cos\theta), \qquad |\vec{v}| = \omega R$$

The magnitude of the centripetal acceleration (the required rate of change of the velocity vector, to keep the object on a circular path) is

$$a_c = \omega^2 R = \frac{v^2}{R}$$

and the centripetal force (directed toward center of rotation) is

$$|\vec{F}_c| = ma_c = m\omega^2 R = \frac{mv^2}{R}$$

Moving in a circle at constant speed (velocity changes but speed does not!) is called *uniform circular motion*. For UCM, $\vec{a} \perp \vec{v}$, and $\omega =$ constant. Then

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\omega^2 R\ (\cos\theta, \sin\theta) = -\frac{v^2}{R}\ (\cos\theta, \sin\theta)$$

(For non-UCM case where speed is not constant, $\vec{a}$ has an additional component that is parallel to $\vec{v}$.)

We can also consider circular motion with non-constant speed, just as we considered linear motion with non-constant speed. Then we introduce the *angular acceleration*

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

and we can derive results that look familiar but with substitutions

$$x \to \theta, \quad v \to \omega, \quad a \to \alpha, \quad m \to I, \quad p \to L$$

if α is constant (which is a common case for constant torque), then:

$$\theta_f = \theta_i + \omega t + \frac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\ (\theta_f - \theta_i)$$

Rotational inertia ("moment of inertia") (see table below):

$$I = \sum m r^2 \quad \to \quad \int r^2 \mathrm{d}m$$

Kinetic energy has both translational and rotational parts:

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

Angular momentum:

$$L = I\omega = m\ v_\perp\ r_\perp$$

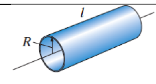
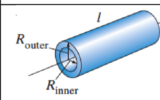
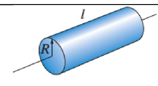
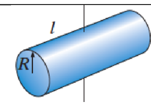
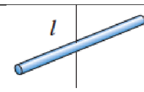
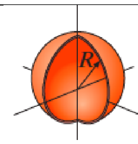
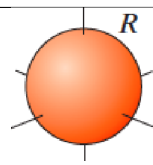
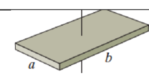
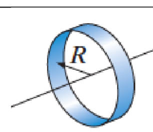
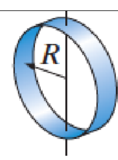
(where $\perp$ means the component that does not point toward the "reference" axis — which usually is the rotation axis)

If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance ℓ from c.o.m.), the rotational inertia is larger, because the object's c.o.m. revolves around a circle of radius ℓ and in addition the object rotates about its own center of mass. This larger rotational inertia is given by the *parallel axis theorem:*

$$I = I_{\mathrm{cm}} + M\ell^2$$

where I_{cm} is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's c.o.m.

(Chapter 12: torque)

configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder ⊥ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod ⊥ to axis		$(1/12)m\ell^2$
hollow sphere		$(2/3)mR^2$
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2 + b^2)$
thin hoop about its axis		mR^2
thin hoop ⊥ to axis		$(1/2)mR^2$

Torque:

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$$

$$\tau = I\alpha$$

Work and power:

$$W = \tau\ (\theta_f - \theta_i)$$

$$P = \tau\omega$$

Equilibrium:

$$\sum \vec{F} = 0, \quad \sum \vec{\tau} = 0$$

(Chapter G9: static equilibrium, etc.)

Static equilibrium (all forces/torques acting ON the object sum to zero):

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum \tau = 0$$

Young's modulus: $\frac{\Delta L}{L_0} = \frac{1}{E} \left(\frac{\text{force}}{\text{area}}\right)$

(Onouye/Kane material)

Add relevant Onouye/Kane equations here!

(Chapter 13: gravity) (Chapter 13 is optional/XC this year)

Gravity:

$$F = \frac{G m_1 m_2}{r^2}$$

where $\vec{F}$ points along the axis connecting m_1 to m_2.

$$G = 6.67 \times 10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}$$

is a *universal* constant — the same on Earth, on Mars, in distant galaxies, etc.

$$g = 9.8\ \text{m/s}^2 = \frac{G M_e}{R_e^2}$$

shows that an apple falling onto Newton's head results from the same force that governs the motion of the Moon around Earth, Earth around the Sun, etc.

For an orbit, gravity provides the centripetal force, so

$$\frac{m v^2}{R} = \frac{G M m}{R^2}$$

Gravitational potential energy for objects 1 and 2 is

$$U = -\frac{G m_1 m_2}{r} \quad \text{(note the sign)}$$

which $\to 0$ as $r \to \infty$. The objects are *bound* if $K + U < 0$.

If $K + U \geq 0$, they escape each other. They just barely escape if $K + U = 0$

$$\frac{1}{2} m v_{\text{escape}}^2 = \frac{GmM}{R}$$

in which case $K \to 0$ when $R \to \infty$.

G.P.E. of e.g. a spacecraft of mass m in the field of two large objects (e.g. earth and moon) of mass M_1 and M_2:

$$U = -\left(\frac{GM_1 m}{R_{M1,m}} + \frac{GM_2 m}{R_{M2,m}} \right)$$

For a central force that goes like $F \propto 1/R^2$, the forces from a uniform spherical shell add (if you're outside the shell) up to one force directed from the center of the shell. So a rigid sphere attracts you as if it were a point mass.

If you're inside the shell, the sum of the forces adds up to zero.

(Chapter 15: periodic motion)

Oscillations (mostly illustrate using mass and spring). Combining $F = ma$ with $F = -kx$, we get $m\ddot{x} = -kx$. (A dot is shorthand for derivative with respect to time.) Using $\omega = \sqrt{k/m}$, you can rewrite as $\ddot{x} = -\omega^2 x$, which has solution

$$x = A\cos(\omega t + \phi), \quad v_x = \dot{x} = -\omega A \sin(\omega t + \phi)$$

You can also write it in terms of frequency f, using $\omega = 2\pi f$:

$$x = A\cos(2\pi f \ t + \phi), \quad v_x = -2\pi f A \sin(2\pi f t + \phi)$$

where f is frequency (cycles per second) and ω is "angular frequency" (radians per second). When a frequency is given in Hz (hertz), it always means f, not ω. The A above middle C on a piano has frequency $f = 440$ Hz, and the buzzing you hear from electrical appliances is 60 Hz (or a small-integer multiple, e.g. 120 Hz).

So frequency is $f = \frac{\omega}{2\pi}$ (how many times the thing vibrates per second), period is $T = \frac{1}{f} = \frac{2\pi}{\omega}$ (how many seconds elapse per vibration). The maximum displacement is *amplitude* A, measured in meters. The maximum speed is ωA (units are meters/second). The initial phase, ϕ, tells you where you are in the oscillation at $t = 0$. If at $t = 0$ you have $x > 0$ but $v_x = 0$, then $\phi = 0$. If at $t = 0$ you have $v_x > 0$ but $x = 0$, then $\phi = \pi/2$ (90°). The energy is $K + U = \frac{1}{2} m\omega^2 A^2$.

For a pendulum, you get $\theta = A\cos(\omega t + \phi)$, with $\omega = \sqrt{g/\ell}$. This requires two approximations: first, that θ is small enough that $\sin\theta \approx \theta$; second, that the mass is concentrated at a point at the end of the string, i.e. that the shape of the mass does not contribute to the rotational inertia of the pendulum.

If the second approximation does not hold (e.g. the rod is about as heavy as the mass on the end), then you have a "physical pendulum" with $\omega = \sqrt{mg\ell_{cm}/I}$, where ℓ_{cm} is the distance from the pivot to the CoM, and I is the rotational inertia **about the pivot** (not about the CoM).

For damped oscillations, the *energy* decays away with a factor $e^{-t/\tau}$, where the symbol τ in this case means "decay time constant," (not torque!). The quality factor $Q = 2\pi f\tau$ tells you how many oscillation periods it takes for the oscillator to lose a substantial fraction ($1 - e^{-2\pi} \approx 99.8\%$) of its stored energy.

For two springs connected in parallel (side-by-side), $k = k_1 + k_2$. For two springs connected in series (end-to-end), $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$, or equivalently $k = \frac{k_1 k_2}{k_1 + k_2}$.

Waves (not until spring semester)

Wavelength λ, frequency f, and speed c of wave propagation are related by

$$c = \lambda f$$

For transverse waves on a taut string of mass per unit length m/L, speed c of wave propgation is

$$c = \sqrt{\frac{T}{m/L}}$$

Sound

For waves that spread out in three dimensions without reflection or absorption, intensity I at distance r is given in terms of source power P by

$$I = \frac{P}{4\pi r^2}$$

Intensity level, β, of sound (in decibels) is given by

$$\beta = 10\ \text{dB}\ \log_{10}\left(\frac{I}{I_0}\right)$$

where $I_0 = 10^{-12}$ W/m^2 is the threshold of human hearing and $\log_{10}$ means taking the base-ten logarithm.

Speed of sound in air is $c_{\text{sound}} = \sqrt{B/\rho}$, or 343 m/s at 20°C and 331 m/s at 0°C, where B is the bulk modulus and ρ is the density (mass/volume).

For observer moving away from (toward) stationary source, Doppler-shifted frequency is (use upper sign for "away from" and lower sign for "toward")

$$f_{\text{observed}} = f_{\text{emitted}} \left(1 \mp \frac{v_{\text{observer}}}{c_{\text{sound}}}\right)$$

For source moving away from (toward) stationary observer,

$$f_{\text{observed}} = \frac{f_{\text{emitted}}}{1 \pm v_{\text{source}}/c_{\text{sound}}}$$

Angle of shock wave for sonic boom is given by $\sin\theta = c/v$.

Light

Angle of incidence (w.r.t. surface normal) equals angle of reflection. Incident ray, reflected ray, refracted ray, and surface normal all lie in a plane.

Speed of light in vacuum: $c = 299792458$ m/s $\approx 3.00 \times 10^8$ m/s. Speed of light is c/n in medium with index of refraction n.

For geometrical (ray) optics, light obeys *principle of least time,* which is also known as Fermat's principle.

Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Lens & mirror summary (light always enters from LHS):

converging lens	$f > 0$	$d_i > 0$ is RHS	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
diverging lens	$f < 0$	$d_i > 0$ is RHS	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
converging mirror	$f > 0$	$d_i > 0$ is LHS	$f = R/2$
diverging mirror	$f < 0$	$d_i > 0$ is LHS	$f = -R/2$

Horizontal locations of object, image (beware of sign conventions!):

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad \Rightarrow \qquad d_i = \frac{d_o f}{d_o - f}$$

Magnification (image height / object height):

$$m \ = \ \frac{h_i}{h_o} \ = \ -\frac{d_i}{d_o} \ = \ \frac{f}{f - d_o}$$

Lenses: $R_{1,2} > 0$ for "outie" (convex), < 0 for "innie" (concave).

Real image: $d_i > 0$. Virtual image: $d_i < 0$. Real image means light really goes there. Virtual: rays converge where light doesn't go.

Lens maker's equation (usually $n_0 = 1$ for air; flat surface $R = \infty$):

$$\frac{1}{f} = \left(\frac{n}{n_0} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Focusing "power" (in diopters: $1\ \mathrm{D} = (1\ \mathrm{m})^{-1}$) for a lens is $1/f$.

Brewster's angle (reflected light is polarized if $\theta_i > \theta_B$): $\theta_B = \arctan(n_2/n_1)$

For a thin film surrounded by air (e.g. soap bubble), a film of thickness $\lambda/4n$ will have maximum reflection for normal incidence, where n is the film's index of refraction.

Rayleigh criterion: a lens (or other aperture) of diameter D can resolve angles no smaller than

$$\theta_{\mathrm{min}} = \frac{1.22\ \lambda}{D}$$

Visible light: $\lambda_{\mathrm{red}} \approx 630$ nm, $\lambda_{\mathrm{green}} \approx 540$ nm, $\lambda_{\mathrm{blue}} \approx 450$ nm, $\lambda_{\mathrm{violet}} \approx 380$ nm.

For a two-lens telescope, where the objective lens and the eyepiece have focal lengths f_o and f_e, respectively, the magnification is $M = -f_o/f_e$.

When monochromatic light passes through two narrow slits that are separated by distance δ, and is viewed on a screen at a large distance L, the separation Δx between adjacent maxima in the interference pattern is $\Delta x = \lambda L/\delta$. Equvalently, the angle θ between adjacent maxima is given by $\sin\theta = \lambda/\delta$. Notice that $\Delta x/L \approx \sin\theta$.

Fluids

pressure: $P = F/A$. $1\ \mathrm{Pa} = 1\ \mathrm{N/m^2}$. $1\ \mathrm{atm} = 101325\ \mathrm{Pa} = 760$ mm-Hg.

Pascal's principle: if an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

Archimedes's principle: the buoyant force on an object immersed (or partially immersed) in a fluid equals the weight of the fluid displaced by that object.

Equation of continuity: $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Bernoulli's equation (neglects viscosity, assumes constant density):

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Viscosity (symbol η, unit = Pa·s), where F is the frictional force between two parallel plates of area A, separated by distance d, moving at relative velocity v, is defined by

$$F = \frac{\eta A v}{d}$$

Reynolds number Re indicates presence of turbulence. For Re < 2300, flow is *laminar*. For Re > 4000, flow is *turbulent*. For $2300 <$ Re < 4000, turbulent flow is possible ("onset of turbulence"). For average fluid speed $\overline{v}$ in a cylinder of radius r,

$$\mathrm{Re} = \frac{2r\rho\overline{v}}{\eta}$$

Poiseuille's equation for volume rate of flow Q (unit = m^3/s) for a viscous fluid undergoing laminar flow in a cylindrical tube of radius R, length L, end-to-end pressure difference $P_1 - P_2$, is

$$Q = \frac{\pi R^4 \, (P_1 - P_2)}{8\eta L}$$

Surface tension $\gamma = F/L$ is force per unit length, tending to pull the surface closed. You can also regard γ as the energy cost per unit increase in surface area.

Useful tables: densities (Giancoli Table 10-1, page 276); viscosities (Mazur Table 18-1; or Giancoli Table 10-3, page 295); surface tensions (Mazur Table 18-2; or Giancoli Table 10-4, page 297).

Kinetic theory, heat, thermodynamics

Atomic mass unit: 1 u $= 1.66 \times 10^{-27}$ kg. Proton mass: $m_p = 1.6726 \times 10^{-27}$ kg. Neutron mass: $m_n = 1.6749 \times 10^{-27}$ kg. As you saw if you did the extra-credit reading on Special Relativity (Einstein's $E = mc^2$, etc.), the mass of a nucleus is slightly smaller than the sum of the masses of its protons and neutrons, because of the negative binding energy that holds the nucleus together. So you can argue that mass is really just one more form of energy! The atomic mass unit u is defined to be $\frac{1}{12}$ of the mass of a ^{12}C nucleus, which is a bound state of 6 protons and 6 neutrons.

Avogadro's number: $N_A = 6.022 \times 10^{23}$. Just as 12 of something is called a dozen, 6.022×10^{23} of something is called a mole. The mass of a mole of protons is 1.007 grams, i.e. almost exactly a gram. A mole of atomic mass units is $N_A \times 1\ \text{u} = 1.0000\ \text{g} = 1.0000 \times 10^{-3}$ kg.

A Fahrenheit degree is $\frac{5}{9}$ of a Celsius degree, and 0°C is 32°F. According to the Wikipedia, the Fahrenheit scale is considered obsolete everywhere except the United States, the Cayman Islands, and Belize.

The Kelvin scale measures absolute temperature. A change of one Kelvin is the same as a change of 1°C, but with an offset such that 0°C = 273.15 K.

Thermal expansion: $\Delta L = \alpha L_0 \Delta T$, $\Delta V = \beta V_0 \Delta V$. Typically $\beta = 3\alpha$. (Here α is the linear coefficient of thermal expansion, and β is the volume coefficient of thermal expansion.)

Thermal stress (if ends are not allowed to move when object is heated or cooled): $F/A = E\alpha\Delta T$

Ideal gas law (works where density is low enough that the gas molecules interact primarily with the walls of the container, and not so much with one another):

$$PV = nRT$$

where T is in Kelvin and n is in moles. If you measure P in Pa (same as N/m^2) and V in m^3, then $R = 8.315\ \frac{\text{J}}{\text{mol}\cdot\text{K}}$. If you measure P in atm and V in liters, then $R = 0.0821\ \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$. A mol of ideal gas at STP (1 atm, 0°C) has a volume of 22.4 L.

The volume per mole of ideal gas at temperature T at 1 atm is

$$\frac{V}{n} = \frac{RT}{P} = \frac{(0.0821\ \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}})T}{1\ \text{atm}} = (22.4\ \text{L})\left(\frac{T}{273\ \text{K}}\right) = (0.0224\ \text{m}^3)\left(\frac{T}{273\ \text{K}}\right)$$

If you measure N in molecules (not moles), P in N/m^2, V in m^3, and T in Kelvin, then $\boxed{PV = Nk_BT}$, where $k_B = 1.38 \times 10^{-23}$ J/K is **Boltzmann**'s constant. The root-mean-squared speed of a gas molecule, v_{rms} is given by $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_BT$, with T in Kelvin. So the average K.E. of a gas molecule is proportional to absolute temperature.

Because of the random motions of molecules, a concentrated blob of ink, spray of perfume, etc., will **diffuse** from a region of high concentration to a region of low concentration. The number per unit time of molecules diffusing through area A is $\frac{\mathrm{d}N}{\mathrm{d}t} = AD\frac{\mathrm{d}C}{\mathrm{d}x}$, where C is the concentration of molecules per unit volume, and D is called the diffusion constant (unit is m^2/s).

Increasing the temperature of a given mass of a given substance requires heat $Q = mc\Delta T$, where c is the **specific heat capacity** of the substance. (Mind the sign: you get heat back out if you decrease the temperature.)

Melting or evaporating a mass m of a substance requires heat $Q = mL$, where L is the **latent heat** (of fusion for melting, of vaporization for boiling). (Mind the sign: you get heat back out for condensation or for freezing.)

Remember that **energy is conserved** (always, now that we know how to account for thermal energy). Work W represents the transfer of mechanical energy into ($W > 0$) or out of ($W < 0$) a system. Heat Q represents the transfer of thermal energy into ($Q > 0$) or out of ($Q < 0$) a system. If we call the internal energy (including thermal energy) of the system U, then

$$\Delta U = W_{\mathrm{in}} + Q_{\mathrm{in}} - W_{\mathrm{out}} - Q_{\mathrm{out}}$$

is just the statement that energy is conserved.

Heat (symbol Q, standard (S.I.) unit = joules) is the transfer of thermal energy from a warmer object to a cooler object. Heat can be transferred via conduction, convection, and radiation.

Conduction is the incoherent movement (similar to diffusion) of thermal energy through a substance, from high T to low T. The heat conducted per unit time is

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{k\ A\ \Delta T}{\ell} = \frac{A\ \Delta T}{R}$$

where k is thermal conductivity, A is cross-sectional area (perpendicular to direction of heat flow), ℓ is the thickness (parallel to direction of heat flow), and ΔT is the temperature difference across thickness ℓ. We can also define **R-value**, $\mathbf{R} = \ell/\mathbf{k}$, and then use the second form written above. Be careful: if an R-value is given in imperial units ($\mathrm{foot}^2 \cdot \mathrm{hour} \cdot {}^\circ\mathrm{F}/\mathrm{Btu}$), you must multiply it by 0.176 to get S.I. units ($\mathrm{m}^2 \cdot \mathrm{K}/\mathrm{W}$).

Convection means e.g. I heat some water in a furnace, then a pump mechanically moves the hot water to a radiator: it is the transfer of thermal energy via the coherent movement of molecules. Convection also occurs if I heat some air, which then becomes less dense and rises (because of buoyancy, which is caused by gravity), moving the thermal energy upward. "Heat rises" because increasing temperature usually makes things less dense, hence more buoyant.

Radiation is the transfer of heat via electromagnetic waves (visible light, infrared, ultraviolet, etc.), which can propagate through empty space. For a body of emissivity e ($0 \le e \le 1$, $0 =$ shiny, $1 =$ black) at temperature T (kelvin), with surface area A, the heat radiated per unit time is

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = e\sigma A T^4$$

where $\sigma = 5.67 \times 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot K}$ is the Stefan-Boltzmann constant.

Useful tables: expansion coefficients (Giancoli Table 13-1, page 388); saturated vapor pressure of water (Giancoli Table 13-4, page 406); specific heat capacities (Giancoli Table 14-1, page 421); latent heats (Giancoli Table 14-3, page 425); thermal conductivities (Giancoli Table 14-4, page 429).

Useful tables for elasticity, etc.: elastic modulus (Giancoli Table 9-1, page 254); ultimate strength (Giancoli Table 9-2, page 258).

Work done BY a gas is $W_{\mathrm{out}} = \int P\ \mathrm{d}V$. Work done ON a gas is $W_{\mathrm{in}} = -\int P\ \mathrm{d}V$. Mazur's convention is $W = -\int P\ \mathrm{d}V$, i.e. if you don't specify "in" or "out" then W means $W_{\mathrm{in}} - W_{\mathrm{out}}$.

The state (P, V, T, S, energy) of a steady device is the same at the end of each cycle. Since the energy is unchanged after going around one complete cycle, all of the changes in energy must add up to zero, so then

$$W_{\mathrm{in}} + Q_{\mathrm{in}} = W_{\mathrm{out}} + Q_{\mathrm{out}}$$

If a system transfers thermal energy Q_{out} to its environment at constant temperature T_{out}, the change in the entropy of the environment is $\Delta S_{\mathrm{env}} = Q_{\mathrm{out}}/(k_B T_{\mathrm{out}})$. (This is using Mazur's definition, $S = \ln\Omega$, for entropy, which is usually called the "statistical entropy." Most other books instead use $S = k_B \ln\Omega$, which is called the "thermodynamic entropy." Books that use $S = k_B \ln\Omega$ will instead write $\Delta S_{\mathrm{env}} = Q_{\mathrm{out}}/T_{\mathrm{out}}$.)

If a system absorbs thermal energy Q_{in} from its environment at constant temperature T_{in}, the change in the entropy of the environment is $\Delta S_{\mathrm{env}} = -Q_{\mathrm{in}}/(k_B T_{\mathrm{in}})$.

If thermal energy is transferred at non-constant temperature, you can use calculus to figure out $\Delta S = \int \frac{1}{T}\,\mathrm{d}Q$. If thermal energy Q flows from system A at temperature T_A to system B at temperature T_B (and without any mechanical work done on or by either system), then $\Delta S_A = -Q/(k_B T_A)$, and $\Delta S_B = +Q/(k_B T_B)$.

If N molecules of ideal gas go from an equilibrium state with temperature T_i and volume V_i to a new equilibrium state with temperature T_f and volume V_f, the change in entropy of the gas is (where C_V is heat capacity per particle at constant volume)

$$\Delta S_{\text{gas}} = S_f - S_i = \frac{N C_V}{k_B} \ln(T_f/T_i) + N \ln(V_f/V_i)$$

The efficiency of a heat engine is

$$\eta \ = \ \frac{W_{\text{out}} - W_{\text{in}}}{Q_{\text{in}}} \ = \ \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} \ \le \ \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}}$$

In the special case of an ideal (or Carnot, or "reversible") heat engine, the "$\le$" becomes "=" (which you can prove by using $\Delta S_{\text{env}} = \frac{Q_{\text{out}}}{k_B T_{\text{out}}} - \frac{Q_{\text{in}}}{k_B T_{\text{in}}} = 0$).

A heat pump moves thermal energy from a "low" temperature T_L to a "high" temperature T_H. The coefficient of performance (COP) for heating is (note that "out" means the output of the heat pump, not the outdoors)

$$\text{COP}_{\text{heating}} \ = \ \frac{Q_{\text{out}}}{W_{\text{in}} - W_{\text{out}}} \ = \ \frac{Q_{\text{out}}}{Q_{\text{out}} - Q_{\text{in}}} \ \le \ \frac{T_{\text{out}}}{T_{\text{out}} - T_{\text{in}}} \ = \ \frac{T_H}{T_H - T_L}$$

where the "$\le$" is "=" for an ideal heat pump. For COP for cooling is

$$\text{COP}_{\text{cooling}} \ = \ \frac{Q_{\text{in}}}{W_{\text{in}} - W_{\text{out}}} \ = \ \frac{Q_{\text{in}}}{Q_{\text{out}} - Q_{\text{in}}} \ \le \ \frac{T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}} \ = \ \frac{T_L}{T_H - T_L}$$

where once again "$\le$" becomes "=" if $\Delta S_{\text{env}} = 0$ for a complete cycle.

Electricity

The electric charge on a proton is $+e = 1.602 \times 10^{-19}$ C (C = coulomb). The electric charge on an electron is $-e$.

Coulomb's law: charged particle B exerts electrostatic force ON charged particle A:

$$\vec{F}^{\text{elec}}_{\text{B on A}} \ = \ -\frac{1}{4\pi\epsilon_0}\,\frac{q_A q_B}{r_{AB}^2}\,\hat{r}_{A\to B} \ = \ \frac{q_A q_B}{4\pi\epsilon_0}\,\frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|^3}$$

where I used $\hat{r}_{A\to B} = -\frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|}$. The constant is $\epsilon_0 = 8.854\times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2}$, or equivalently

$$k = \frac{1}{4\pi\epsilon_0} = 8.988\times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C}^2}$$

The force is repulsive (force on A points away from B) if the two charges have the same sign, and is attractive (force on A points toward B) if the two charges have opposite signs.

The electrostatic force exerted by N other particles ON particle A is

$$\vec{F}_{\text{on A}} = \sum_{i=1}^{N} \frac{q_A q_i}{4\pi\epsilon_0}\ \frac{\vec{r}_A - \vec{r}_i}{|\vec{r}_A - \vec{r}_i|^3}$$

Writing out the components of the force (on A) for clarity:

$$F_x = \sum_i \frac{q_A q_i}{4\pi\epsilon_0}\ \frac{x_A - x_i}{((x_A - x_i)^2 + (y_A - y_i)^2 + (z_A - z_i)^2)^{3/2}}$$

$$F_y = \sum_i \frac{q_A q_i}{4\pi\epsilon_0}\ \frac{y_A - y_i}{((x_A - x_i)^2 + (y_A - y_i)^2 + (z_A - z_i)^2)^{3/2}}$$

$$F_z = \sum_i \frac{q_A q_i}{4\pi\epsilon_0}\ \frac{z_A - z_i}{((x_A - x_i)^2 + (y_A - y_i)^2 + (z_A - z_i)^2)^{3/2}}$$

The electric field $\vec{E}$ at a point $\vec{r} = (x, y, z)$ is the electrostatic force-per-unit-charge ($\vec{E} = \vec{F}/q$) that a test charge q would experience if placed at position $\vec{r}$. The electric field created by N particles is

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0}\ \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3}$$

or writing out the components,

$$E_x(x,y,z) = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0}\ \frac{x - x_i}{((x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2)^{3/2}}$$

$$E_y(x,y,z) = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0}\ \frac{y - y_i}{((x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2)^{3/2}}$$

$$E_z(x,y,z) = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0}\ \frac{z - z_i}{((x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2)^{3/2}}$$

The electric field $\vec{E}(\vec{r})$ due to a charge q placed at the origin $(0, 0, 0)$ is

$$\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0\ r^2}\ \hat{r}$$

where $\hat{r}$ is the unit vector pointing radially away from the origin.

Note that for a point that is outside of a uniform spherical shell of charge ("shell" = the thin surface of a sphere), $\vec{E}$ is the same as if all of the charge were at the center of the sphere. A uniform spherical shell of radius R contributes nothing (zero) to the electric field for $r < R$. (You may remember an analogous result for gravity: `http://en.wikipedia.org/wiki/Shell_theorem`) Using Gauss's law is the easiest way to show that this is true.

Gauss's law states that the electric flux (the "flow" of $\vec{E}$ field lines) through the closed surface of an arbitrary volume is

$$\Phi_E \equiv \int_{\text{surface}} E_\perp\ \mathrm{d}A = Q_{\text{enclosed}}/\epsilon_0$$

To calculate Φ_E, you sum up the area of the enclosing surface, weighting each area by the normal component (i.e. $\perp$ to the surface) of $\vec{E}$. Gauss's law is most useful if you can choose each face of your enclosing surface so that either (a) $\vec{E}$ is $\parallel$ to the face (in which case the flux through that face is zero), or (b) $\vec{E}$ is constant and $\perp$ to the face, in which case you just multiply the area of the face by E (or $-E$) for outward (or inward) pointing $\vec{E}$.

Below are one figure and one "procedure box" from Eric Mazur's chapter 24, describing how to use Gauss's law.

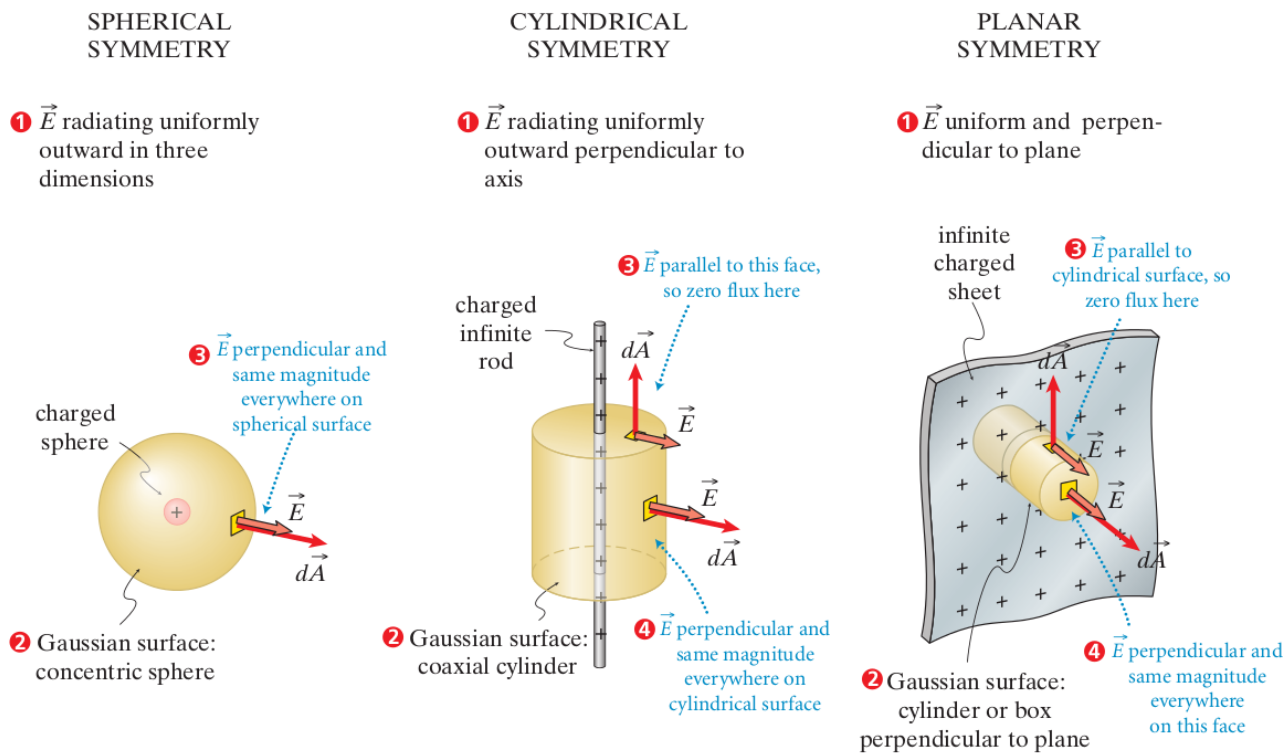


Figure 24.27 Applying Gauss's law to determine the electric fields of charge distributions exhibiting spherical, cylindrical, or planar symmetry.

PROCEDURE: Calculating the electric field using Gauss's Law

Gauss's Law allows you to calculate the electric field for charge distributions that exhibit spherical, cylindrical, or planar symmetry without having to carry out any integrations.

1. Identify the symmetry of the charge distribution. This symmetry determines the general pattern of the electric field and the type of Gaussian surface you should use (Figure 24.27)

2. Sketch the charge distribution and the electric field by drawing a number of field lines, remembering that the field lines start on positively charged objects and end on negatively charged ones. A two-dimensional drawing should suffice.

3. Draw a Gaussian surface such that the electric field is either parallel or perpendicular (and constant) to each face of the surface. If the charge distribution divides space into distinct regions, draw a Gaussian surface in each region where you wish to calculate the electric field.

4. For each Gaussian surface determine the charge q_{enc} enclosed by the surface.

5. For each Gaussian surface calculate the electric flux Φ_E through the surface. Express the electric flux in terms of the unknown electric field E.

6. Use Gauss's Law (Eq. 24.8) to relate q_{enc} and Φ_E and solve for E.

You can use the same general approach to determine the charge carried by a charge distribution given the electric field of a charge distribution exhibiting one of the three symmetries in Figure 24.27. Follow the same procedure, but in Steps 4–6, express q_{enc} in terms of the unknown charge q and solve for q.

The electric field lines from a point charge spread out in three dimensions. You can use Gauss's law to show that at a distance r from a single point charge q, the electric field has magnitude $E(r) = \frac{q/\epsilon_0}{4\pi r^2}$, which is the result you knew already.

The electric field lines from an infinitely long line of charge spread out in only two dimensions. You can use Gauss's law to show that at a distance r from a line charge whose charge-per-unit-length is q/L, the electric field has magnitude $E(r) = \frac{q/\epsilon_0}{2\pi r L}$, i.e. the field falls off only as $1/r$.

The electric field lines from an infinitely wide plane of charge do not spread out at all. (Since there is only one dimension available for getting away from the + charge, they must remain parallel.) You can use Gauss's law to show that at a distance r from a plane charge whose charge-per-unit-area is q/A, the electric field has magnitude $E(r) = \frac{q/\epsilon_0}{2A}$, i.e. the field has constant magnitude $\frac{q}{2A\epsilon_0}$. (If the plane of charge sits on only one surface of a conductor, as in a parallel-plate capacitor, then there is no factor of two, because the field is nonzero only between the two plates: then $E = \frac{q}{\epsilon_0 A}$.)

Electric potential (a.k.a. voltage) is analogous to the elevation on a topo map. Electric potential is *potential energy per unit charge* for a small test charge q

$$V = U/q$$

The work that you need to do to move a charge q across a potential difference ΔV (like pushing a ball up a hill, but mind the sign of q) is

$$W = q\Delta V$$

The electric field always points in the "downhill" direction: the direction in which V decreases most rapidly.

$$E_x = -\frac{\mathrm{d}V}{\mathrm{d}x}, \quad E_y = -\frac{\mathrm{d}V}{\mathrm{d}y}, \quad E_z = -\frac{\mathrm{d}V}{\mathrm{d}z}$$

So I can measure the strength of an electric field equivalently as $\frac{\text{newtons}}{\text{coulomb}}$ or $\frac{\text{volts}}{\text{meter}}$. (These two units are equal.) The electric field $\vec{E}$ measures

- force per unit charge
- how rapidly potential varies with position
- direction in which potential decreases most rapidly

If the point (x, y, z) is a distance d_i away from each of N point charges q_i, then (taking $V = 0$ at $r = \infty$, and integrating $-E(r)$ from $r = \infty$ to $r = d_i$)

$$V(x, y, z) = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0 d_i}$$

where $d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$.

Energy units useful for connecting physics with chemistry: since $e = 1.602\times10^{-19}$ C, an electron volt is 1 eV $= 1.602 \times 10^{-19}$ J. And since a mole is 6.022×10^{23} of something, 1 kJ/mol $= (1000\text{ J})/(6.022 \times 10^{23}) = 1.66 \times 10^{-21}$ J.

When you accumulate electric charge Q on a single conductor (taking $V = 0$ at $r = \infty$), the potential V of the conductor is proportional to the accumulated charge Q. When you accumulate charge $+Q$ on one conductor and charge $-Q$ on a nearby conductor, the potential difference ΔV between the two conductors is proportional to Q. In either case, the constant of proportionality is a geometrical factor called the **capacitance**:

$$Q = CV$$

Usually you figure out C by drawing a picture of the conductor(s) with charge Q in place, then using Gauss's law to figure out $\vec{E}$, then using $E_x = -\mathrm{d}V/\mathrm{d}x$, etc., and integrating to find V. Then you divide to get $C = Q/V$.

For a single conducting **sphere of radius** R, you find $\mathbf{C = 4\pi\epsilon_0 R}$. For two **parallel plates** of area A separated by distance d, you find $\mathbf{C = \epsilon_0 A/d}$. If you understand Gauss's law, it can be fun to derive that for two long coaxial cylinders of length L and radii r_1 and r_2, the capacitance is $C = 2\pi\epsilon_0 L/\ln(r_2/r_1)$.

The potential energy stored in a capacitor is $U = \frac{1}{2}CV^2 = \frac{1}{2}Q^2/C$

If you stick an electrical insulator (called a *dielectric*) between the plates of a capacitor, the factor ϵ_0 in the capacitance is replaced by $\epsilon = K\epsilon_0$, where K is known as the **dielectric constant**. The way this comes about is that the + and − charges inside the dielectric material separate just a tiny bit, responding in proportion to the external electric field. They are only able to move maybe a few angstroms in response to the external field, so their movement only partially cancels out $\vec{E}$. This partial cancellation replaces $\vec{E}$ inside the dielectric with $\vec{E}/K$. So then when you compute ΔV for a given Q, you get a number that is smaller by a factor of K. Smaller ΔV for a given K means that $C \to KC$.

Ohm's law for a resistor: $\Delta V = IR$, where ΔV is the voltage drop across the resistor (the potential difference between the two terminals of the resistor), and I is the current through the resistor.

For a resistor of length L and constant cross-sectional area A, made from material of **conductivity** σ, the resistance is $R = L/(\sigma A)$. Or in terms of **resistivity** $\rho = 1/\sigma$, $R = \rho L/A$.

Suppose that a direct current I flows through a circuit element, from terminal a to terminal b, and that the volage *drop* across that circuit element is $\Delta V = V_b - V_a > 0$. Then the **power dissipated** by (or perhaps stored in or otherwise consumed by) that circuit element is $I\Delta V$. (In the case of alternating current, you can often just replace I and ΔV by their rms values and use $I_{\rm rms}\Delta V_{\rm rms}$ for power, but in general you need to account for the possibility that I and ΔV are out of phase with one another by some angle ϕ, in which case the power is $I_{\rm rms}\Delta V_{\rm rms}\cos(\phi)$. In the unlikely event that you want the details behind this, see Mazur's chapter 32—which we won't have time to cover, but you're welcome to read it for extra credit.)

Combining this last result with Ohm's law, the **power dissipated in a resistor** is $P = I^2R = V^2/R$.

Junction rule for circuits in steady state (charge conservation): $\sum I_{\rm in} = \sum I_{\rm out}$. This is like saying that per unit time, the number of cars entering an intersection equals the number of cars leaving the intersection—which is true for a steady flow of traffic.

Loop rule for circuits (energy conservation): $\sum \Delta V = 0$. As you go around the loop, add up the voltage *gains*: $\Delta V = -IR$ for each resistor, and $\Delta V = +\mathcal{E}$ for each battery, if your loop exits the battery at the (+) terminal and re-enters the battery at the (−) terminal. (If your loop goes through the battery in the opposite direction, then $\Delta V = -\mathcal{E}$.) This is like saying that if I go backpacking for a few days in Yosemite, taking a "loop" route that begins and ends at the same trail head, the sum of my uphill ascents equals the sum of my downhill descents, because I end at the same elevation at which I began.

A consequence of the above two rules (you can draw series and parallel circuits, find the current drawn from a battery of given $\mathcal{E}$, and find $R_{\rm combined} = \mathcal{E}/I_{\rm battery}$) is that the combined resistance R for two resistors R_1 and R_2 **in series** is $R = R_1 + R_2$. Another consequence is that **in parallel**, $1/R = 1/R_1 + 1/R_2$.

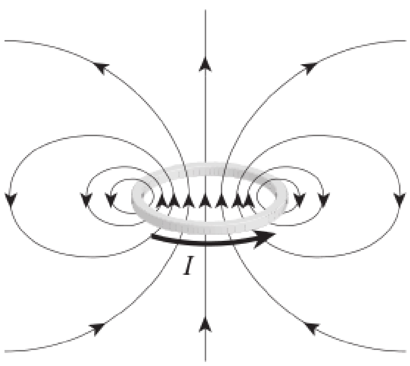
Using the results from the previous paragraph, you can show that putting n copies of the same resistor R_1 in series gives you $R = nR_1$. Putting n copies of the same resistor R_1 in parallel gives you $R = R_1/n$.

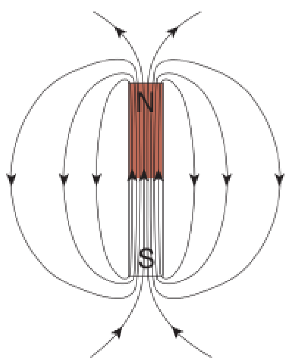
Magnetism

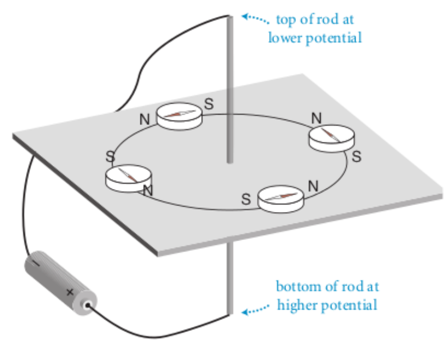
Opposite poles of magnets attract; like poles repel. So N attracts S; N repels N.

Magnetic field lines flow out of the north pole of a magnet and back into the south pole. (Inside the magnet, the field lines continue from S to N, completing the loop. Because magnetic charges ("magnetic monopoles") don't exist, magnetic field lines always form closed loops.) So the magnetic field outside of a bar magnet points away from the north pole and toward the south pole.

The arrow of a compass is the north pole of a magnet. It actually points toward the **south** magnetic pole of the earth, which is located near the geographic north pole, confusingly enough. (Because opposite poles attract, there's no way to avoid this, alas.) If you put a compass into a magnetic field, the arrow (with the "N" marking) will line up with the $\vec{B}$ field direction (or for a flat compass, the projection of the $\vec{B}$ field direction into the plane of the compass).







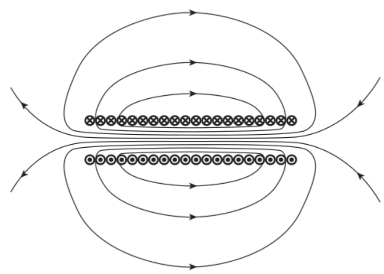
$\vec{B}$ field lines encircle an electric current (or a charged particle in motion). If you orient your right thumb in the direction in which positive current flows (or positively charged particles move), the fingers of your right hand will curl around in the direction of $\vec{B}$.

The right-hand rule is usually all you need to figure out the direction of $\vec{B}$. For those rare cases (not in this course) in which you need to figure out the magnitude, use Ampere's law: going around a closed loop, sum up the distance traveled times the component of $\vec{B}$ in the direction of travel. This sum (or integral) equals the constant $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T/(A \cdot m)}$ times the net current enclosed by your Amperian loop.

$$\oint_{\substack{\text{closed}\\\text{loop}}} \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0\ I_{\text{enclosed}}$$

Using Ampere's law, you can derive the magnitude of $\vec{B}$ at distance r around a long, straight wire carrying current I to be $B = (\mu_0 I)/(2\pi r)$.

More importantly, Ampere's law also tells you that the magnitude of $\vec{B}$ **inside** a long solenoid (a cylinder with wire wrapped around it, the most common shape for an electromagnet) is $B = (N/L)\mu_0 I$, where N/L is the number of turns of wire per unit length and I is the current flowing in the wire.



The force on a moving charged particle due to magnetic field $\vec{B}$ is $\vec{F} = q\vec{v} \times \vec{B}$, where q is the particle's electric charge and v is the particle's velocity. The magnitude of $\vec{F}$ is $|\vec{F}| = qvB\sin\theta$, where θ is the angle between $\vec{v}$ and $\vec{B}$. The force is largest when $\vec{v}$ and $\vec{B}$ are perpendicular and is zero when $\vec{v}$ and $\vec{B}$ are parallel or antiparallel. You can figure out the direction of $\vec{F}$ for positive q by first pointing the fingers of your right hand in the $\vec{v}$ direction, then curling them in the $\vec{B}$ direction. Your right thumb will then point in the $\vec{F}$ direction. (For negative q, your right thumb winds up pointing in the $-\vec{F}$ direction.) The magnetic force is always perpendicular both to $\vec{v}$ and to $\vec{B}$.

An equivalent version of the above force law is more useful for calculating the force on a current-carrying wire in a magnetic field: $\vec{F} = I\vec{\ell} \times \vec{B}$. You use the same right-hand rule as in the previous paragraph, with the direction of positive current instead of the velocity. If the current flow is perpendicular to $\vec{B}$, then the magnitude of the force is $F = I\ell B$. Or if the current flow makes an angle θ with $\vec{B}$, then $F = I\ell B\sin\theta$.

The *magnetic flux* through a surface is of area A is $\Phi_B = BA\cos\theta$, where θ is the angle between the $\vec{B}$ field lines and the surface normal. (The "surface normal," which you may encounter in computer graphics or CAD software, is a vector that is locally perpendicular to the surface; for example, the surface normal of a spherical surface always points radially.) So the flux is maximum when $\vec{B}$ is normal (perpendicular) to the surface, and is zero when $\vec{B}$ is parallel to the surface. If the surface is not flat or $\vec{B}$ is not constant, you divide the surface up into many small pieces and add them. So more formally $\Phi_B = \int \vec{B} \cdot \mathrm{d}\vec{A}$, where $\vec{A}$ points along the surface normal. Intuitively, you can think of Φ_B as proportional to the net number of $\vec{B}$ field lines that pierce the surface in the "outward" direction (you have to call one side of the surface the outside and one the inside, and count up outgoing minus ingoing field lines).

A closed loop of wire defines a surface. (Think of the surface of the soap bubble that would form with the loop as a frame.) When the magnetic flux through that surface changes, an emf (i.e., a voltage) is induced around the loop: $\mathcal{E} = -\mathrm{d}\Phi_B/\mathrm{d}t$. You can change Φ_B by changing the magnitude of $\vec{B}$, by changing the direction of $\vec{B}$, or by changing the orientation of the loop. The minus sign reminds us that the induced current flows in the direction that creates a $\vec{B}$ field that opposes the change in magnetic flux. So if the loop is in the xy plane, and $\vec{B}$ points along the z axis and is increasing, then $\mathcal{E}$ induces a clockwise current (as seen from $+z$), because a clockwise current creates a magnetic field pointing along in the $-z$ direction, which is opposite $\mathrm{d}\vec{B}/\mathrm{d}t$. (See "Faraday's law" and "Lenz's law" in the book.)

If the wire is coiled around so that the $\vec{B}$ field lines pass through the coil N times, then Φ_B is N times as large.

In a transformer, the primary coil is connected to a source of AC voltage V_p, and the secondary coil is connected to a load to which you want to supply voltage V_s. Then $V_s/V_p = N_s/N_p$, where N_s is the number of times the secondary coil is wound around the iron core, and N_p is the number of times the primary coil is would around the iron core. The purpose of the iron is to ensure that all of the magnetic field lines passing through one coil also pass through the other coil. $V_s > V_p$ is called "step up," and $V_s < V_p$ is called a "step down" transformer. For an ideal transformer (most real transformers nowadays are not far from ideal), 100% of the power supplied by the primary circuit is delivered to the secondary circuit, i.e. $V_p I_p = V_s I_s$.

What I hope you remember about electricity and magnetism from this course is (a) mainly how circuits work, and (b) this synopsis:

- Electric charge creates an electric field.
- Moving electric charge (i.e. electric current) creates a magnetic field.
- Force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ on charged particles.
  - electromagnet, speaker, doorbell, mass spectrometer, electric motor
- Changing magnetic flux induces "emf" (voltage) $\mathcal{E}$ in loop
  - electric generator, AC transformer
- Equivalently, changing magnetic field creates an electric field.
- Changing electric field creates a magnetic field.

- Last two together allow EM waves to propagate
  - wireless telegraph, radio, cell phone, and even light

Quantum mechanics / atoms

Light is emitted and absorbed in discrete "quanta" called photons. The energy E_γ of a photon is given by

$$E_\gamma = hf = \frac{hc}{\lambda}$$

where $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$ is Planck's constant, $c = 2.9979 \times 10^8\ \mathrm{m/s}$ is the speed of light, f is frequency (in cycles/second, or Hz), and λ is wavelength (in meters).

The spectrum of thermal (black-body) radiation is very broad, but the *peak* (most probable) wavelength is inversely proportional to temperature: hotter objects tend to emit higher-energy (thus lower wavelength) photons:

$$\lambda_{\text{peak}} = \frac{2.90 \times 10^{-3}\ \mathrm{m \cdot K}}{T}$$

with λ_{peak} in meters and T in kelvins. See `http://en.wikipedia.org/wiki/Wien%27s_displacement_law` . You can see qualitatively where this comes from by assuming that the photon energy corresponding to λ_{peak} is $E_\gamma = \alpha k_B T$, where k_B is Boltzmann's constant from thermal physics, and α is some numerical constant. It turns out that $\alpha = 4.96$ gives you the experimentally correct result: $\lambda_{\text{peak}} = (hc)/(4.96 k_B T)$.