Physics 9, Fall 2018, Final Exam. **Name:** BILL

This two-hour, closed-book exam has 25% weight in your course grade. You can use one sheet of your own handwritten notes and a calculator.

Work alone. Keep in mind that here at Penn, every member of the University community is responsible for upholding the highest standards of honesty at all times: offering or accepting help with this exam would be a serious violation of Penn's Code of Academic Integrity.

Please show your work on these sheets. Continue your work on the reverse side if needed. The last page of the exam contains a list of equations and numbers that you might find helpful. Feel free to approximate $g = 10\,\mathrm{m/s^2}$ if you wish.

1. (10%) A violin string has length $L = 0.328$ meters and mass $m = 0.452 \times 10^{-3}$ kg (that's 0.452 grams). I think I've used realistic values for a violin string.

(a) If the string is kept under tension $T = 51.15\,\mathrm{N}$, what is the fundamental (smallest harmonic) frequency at which it vibrates? (As a check, this will turn out to be the **D** above middle **C**.)

$$v_{wave} = \sqrt{\frac{T}{m/L}} \qquad v_{wave} = \lambda f \qquad \lambda_{fundamental} = 2L$$

$$f_{fundamental} = \frac{1}{2L}\sqrt{\frac{T}{m/L}} = \frac{1}{2(0.328\,\mathrm{m})}\sqrt{\frac{(51.15\,\mathrm{N})(0.328\,\mathrm{m})}{0.452\times10^{-3}\,\mathrm{kg}}}$$

$$= \boxed{293.7\ \mathrm{Hz}}$$

check: this is $1.12\times$ ($\sqrt[6]{2}$) middle C $\approx 262\,\mathrm{Hz}$

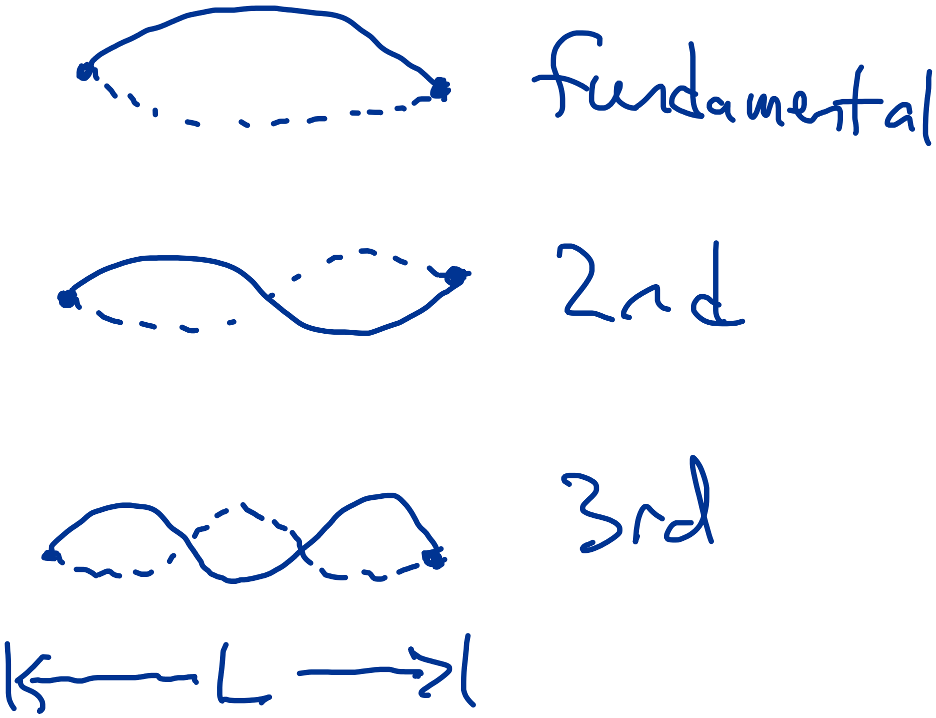
(b) At what other frequencies (higher harmonics) could the string vibrate at this fixed length?

2x, 3x, 4x, ...

587 Hz, 881 Hz, 1175 Hz, ...

(Problem continues on next page.)

(c) Sketch the motion of the string for the lowest harmonic and the next two higher harmonics (i.e. for the three lowest possible frequencies).



Now suppose that the vibration of the violin string at its fundamental frequency causes sound to propagate through air from the violin to your ear. The speed of sound in room-temperature air is $v_s = 343\,\mathrm{m/s}$.

(d) What is the frequency in air of these sound waves?

Same frequency: $f = \boxed{294\ \mathrm{Hz}.}$

(e) What is the wavelength in air of these sound waves?

$$V_{sound} = \lambda f$$

$$\lambda = \frac{V_{sound}}{f} = \frac{343\ \mathrm{m/s}}{294\ \mathrm{Hz}} = \boxed{1.17\ \mathrm{m}}$$

2. (10%) Standing at a distance $r = 2.8$ meters from a loudspeaker at an outdoor rock concert, you measure the intensity of the sound waves to be an astonishing $I = 0.316\,\mathrm{W/m^2}$.

(a) At this distance, what is the corresponding **intensity level,** measured in decibels?

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right) = 10 \log_{10}\left(\frac{0.316\,\frac{W}{m^2}}{10^{-12}\,\frac{W}{m^2}}\right)$$

$$= 115\,\mathrm{dB} = \mathrm{I.L.}$$

(b) Assuming that the "music" from the loudspeaker forms spherical waves that spread out equally in all directions, what is the total acoustical power emitted by the loudspeaker?

$$I = \frac{\text{Power}}{4\pi r^2} \Rightarrow \text{power} = 4\pi r^2 I$$

$$\text{power} = (4\pi)(2.8\,m)^2\left(0.316\,\frac{W}{m^2}\right) = \boxed{31.1\,W}$$

(Problem continues on next page.)

(c) Suppose that your uncle is wisely sitting quite a bit farther back, a distance $r_u = 28\,\mathrm{m}$ from the loudspeaker. Assuming that the acoustical power spreads out equally in all directions and is not absorbed or reflected by any obstacles, what intensity level (in decibels) does your uncle measure for the "music" from the loudspeaker?

$$I @ r_u = \frac{\text{power}}{4\pi r_u^2} = \frac{31.1\,\mathrm{W}}{4\pi (28\,\mathrm{m})^2} = 3.16\times10^{-3}\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

$$\text{I.L.} = \beta = 10\log_{10}\left(\frac{I}{I_0}\right) = \boxed{95\,\mathrm{dB}}$$

check: $I(r_{uncle})/I(r_{me}) = \left(\frac{r_{me}}{r_{uncle}}\right)^2 = \frac{1}{100}$

$10\log_{10}\left(\frac{1}{100}\right) = -20$. $115\,\mathrm{dB} - 20\,\mathrm{dB} = 95\,\mathrm{dB}$ ✓

(d) You sensibly decide to wear ear plugs that permit only a fraction 1.0×10^{-3} (that's 0.1%) of the incident sound intensity (at your original 2.8 m distance) to reach your ears. Now what intensity level (in decibels) do your ears measure?

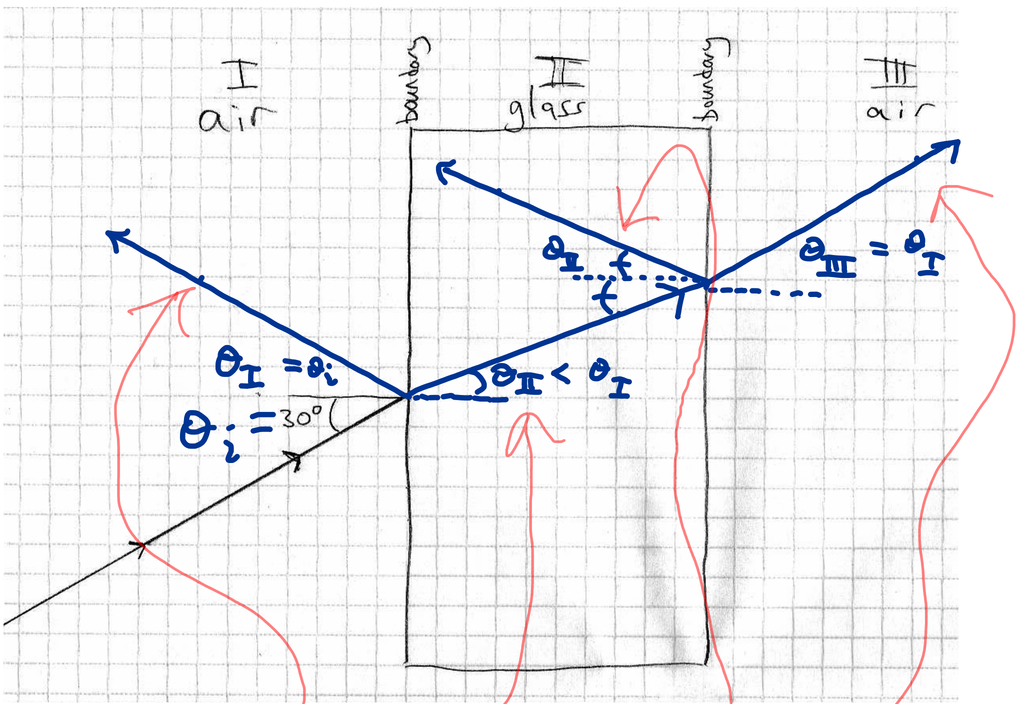
$$115\,\mathrm{dB} - 30\,\mathrm{dB} = \boxed{85\,\mathrm{dB}}$$

(e) What is the "transmission loss" (in decibels) of these ear plugs? (I used a realistic value.)

$$\beta = 10\log_{10}\left(10^{-3}\right) = -30\,\mathrm{dB}$$

T.L. is $\boxed{30\,\mathrm{dB}}$

3. (7%) A beam of laser light starts out in air (region I), passes through a rectangular glass block (region II, n=1.50), then exits to air (region III). At each boundary, most of the light is refracted (bent and transmitted) through to the next region, but some light is also reflected back. (You know this from your own experience using a store window as if it were a mirror.)



(a) Draw the reflected and refracted rays at the boundary between regions I and II (at the left side of the glass block). Make your refracted ray long enough to touch the II–III boundary.

(b) Draw the reflected and refracted rays at the boundary between regions II and III (at the right side of the glass block).

(c) Calculate the angle of each of these four rays with respect to the horizontal and label these four angles in the figure. The incoming ray that I drew makes a 30° angle w.r.t. horizontal. The air/glass boundaries are vertical. (Make sure your calculator is in degrees mode, not radians mode.)

$n_I \sin\theta_I = n_{II} \sin\theta_{II} \Rightarrow 1.00 \sin 30° = 1.50 \sin\theta_{II} \Rightarrow \theta_{II} = 19.5°$

I/II : reflected = incident = 30°, refracted = 19.5°

II/III : reflected = incident = 19.5°, refracted = 30°

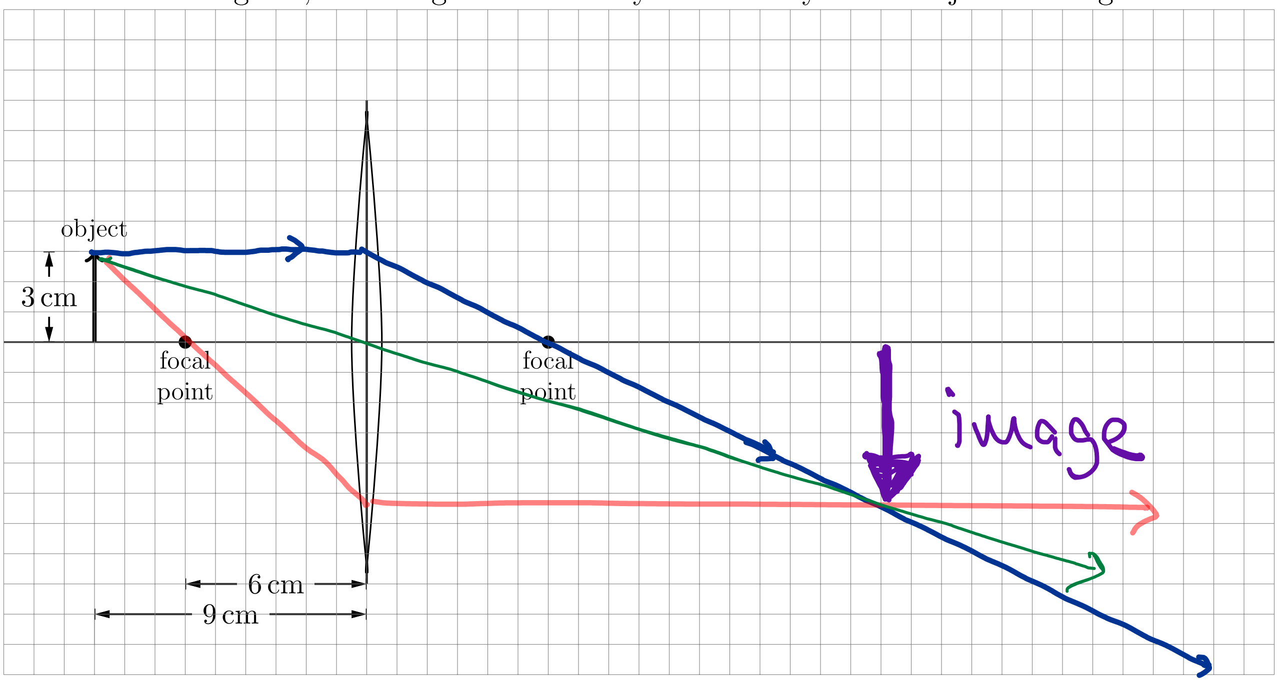
✓ (d) Does light travel faster in air or in glass? Do your refracted rays confirm that at a boundary the light "bends toward the slower medium" as Richard Muller describes?

refracted @ I/II bends forward toward (slow) glass.

refracted @ II/III bends backward toward (slow) glass.

4. (8%) An object that is 3.0 cm tall is placed 9.0 cm to the left of a converging lens whose focal length is 6.0 cm, as shown in the figure.

(a) Draw a scale diagram, including the three easy-to-draw rays from object to image.



(b) Is the image real or virtual?

(c) Is the image inverted or non-inverted?

(d) Is the image enlarged or reduced?

(e) Use the thin-lens equation to check your answers from your drawing, i.e. use equations to calculate the **image position**, the **magnification**, and the **image height**.

$$d_i = \frac{f\,d_o}{d_o - f} = \frac{(6.0\,\text{cm})(9.0\,\text{cm})}{9.0\,\text{cm} - 6.0\,\text{cm}} = \boxed{18\,\text{cm} = d_i}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{18\,\text{cm}}{9\,\text{cm}} = \boxed{-2 = m}$$

$$h_i = m\,h_o = (-2)(3\,\text{cm}) = \boxed{-6\,\text{cm} = h_i}$$

This agrees pretty well with sketch.

5. **(4%)** In order to maintain a reasonable water pressure for nearby homes, your town maintains a water tower whose water level is 40.0 meters above ground level. The pressure at the top of the tower is 1 atm = 101325 Pa = 101325 N/m^2.

(a) When everybody's faucet is turned off (so the water in the pipes is not moving) what is the water pressure in the pipes at your house (at ground level)? Express your answer in pascals.

$$P_{house} = P_{tower} + \rho g h_{tower}$$

$$= 101325 \frac{N}{m^2} + \left(1000 \frac{kg}{m^3}\right)\left(9.8 \frac{N}{kg}\right)(40m)$$

$$= \boxed{493325 \text{ Pa}}$$

(b) Re-express your answer in atmospheres.

$$493325 \text{ Pa} \times \frac{1 \text{ atm}}{101325 \text{ Pa}}$$

$$= \boxed{4.87 \text{ atm}}$$

6. (6%) A hydraulic jack consists of an enclosed volume of oil, into which two cylindrical pistons are inserted. Assume that the two pistons contact the oil at the same height. The first piston has radius 3.5 mm, while the second piston has radius 35.0 mm. The larger piston supports the weight of a car whose mass is 1020 kg. (I like this problem because it shows how one lifts very heavy objects, by using a larger mechanical advantage than one usually gets with levers, pulleys, a block & tackle, etc.)

(a) What is the force in newtons corresponding to the weight of a 1020 kg car?

$$F = mg = (1020\ \text{kg})(9.8\ \tfrac{N}{kg}) = \boxed{10\ kN}$$

$(9996\ N)$

(b) What force in newtons must I exert on the smaller piston in order to support (or lift at constant speed) the weight of the car? (The little hydraulic jack pictured below lets an ordinary person lift 12 tons!)

$$\frac{F_{wide}}{A_{wide}} = \Delta p_{ressure} = \frac{F_{narrow}}{A_{narrow}}$$

$$F_{narrow} = F_{wide}\,\frac{A_{narrow}}{A_{wide}} = F_{wide}\left(\frac{r_{narrow}}{r_{wide}}\right)^2$$

$$= (9996\,N)\left(\frac{3.5\times10^{-3}\,m}{35.0\times10^{-3}\,m}\right)^2 = 99.96\,N$$

$$\approx \boxed{100\,N}$$

7. **(8%)** A horizontal water pipe narrows in inner radius from 25.4 mm to 12.7 mm. The water flows at 1.64 m/s in the wide part of the pipe.

(a) How fast is the water flowing in the narrow part of the pipe? (Hint: water is not created or destroyed anywhere in the pipe.)

$$A_{wide} v_{wide} = A_{narrow} v_{narrow} \Rightarrow v_{narrow} = \frac{A_{wide}}{A_{narrow}} v_{wide}$$

$$v_{narrow} = \left(\frac{r_{wide}}{r_{narrow}}\right)^2 v_{wide} = 4 v_{wide} = \boxed{6.56 \text{ m/s}}$$

(b) Ignoring viscosity, what is the pressure difference (in N/m^2) between the wide and narrow parts?

$$P_n + \frac{1}{2}\rho v_n^2 = P_w + \frac{1}{2}\rho v_w^2 \Rightarrow \frac{1}{2}\rho(v_n^2 - v_w^2) = P_w - P_n$$

$$P_w - P_n = \frac{1}{2}\left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left[\left(6.56 \frac{\text{m}}{\text{s}}\right)^2 - \left(1.64 \frac{\text{m}}{\text{s}}\right)^2\right] = \boxed{20172 \text{ Pa}}$$

$$P_w - P_n \simeq 20.2 \text{ kPa}$$

(c) Express your answer to part (b) in atmospheres (or fraction of an atmosphere).

$$\frac{20172 \text{ Pa}}{101325 \text{ Pa/atm}} = \boxed{0.20 \text{ atm}}$$

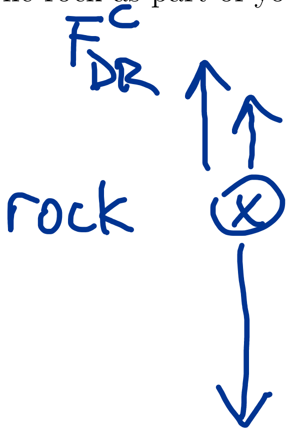
(d) Which part (wide or narrow) is under higher pressure? (To check your answer, ask yourself how the flow speed compares, and think of the homework problem involving a flat, horizontal rooftop on a very windy day.)

$P_w > P_n$: wide part is under higher pressure

familiar : fast flow $\rightarrow$ low pressure

8. (7%) A Lake Michigan scuba diver finds that a rock whose mass is 9.28 kg appears to "weigh" only 6.18 kg when fully submerged in (fresh) water. In other words, whereas the force required to lift the rock in air is 90.9 N, the force required to lift the rock under water is only 60.6 N.

(a) Why does the rock appear to be less heavy when it is under water? Draw a **free-body diagram** for the rock as part of your explanation.



$F^{buoyant} = \rho_{water}\, g\, V_{rock}$ (upward buoyant force)

$F^G_{Er} = mg$ (downward gravitational force by Earth on rock)

Upward buoyant force partially offsets "weight" = downward gravitational force. So necessary upward contact force by diver on rock is smaller than rock's full weight.

(b) What is the density of the rock?

$$(6.18\,\text{kg})g + \rho_{water} V_{rock}\, g = M_{rock}\, g$$

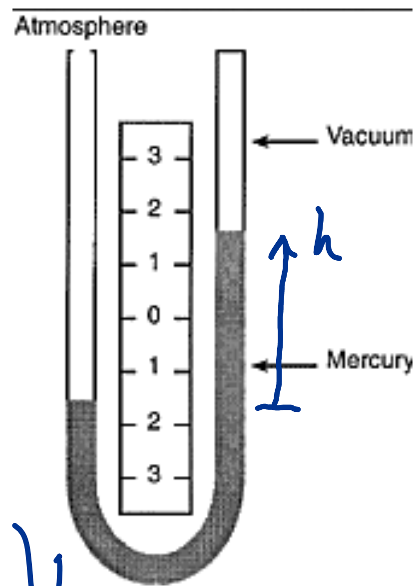
$$\rho_{water} V_{rock} = M_{rock} - 6.18\,\text{kg}$$

$$V_{rock} = \frac{M_{rock} - 6.18\,\text{kg}}{\rho_{water}} = \frac{9.28\,\text{kg} - 6.18\,\text{kg}}{1000\,\text{kg/m}^3} = 0.0031\,\text{m}^3$$

$$\rho_{rock} = \frac{M_{rock}}{V_{rock}} = \boxed{2994\,\text{kg/m}^3}$$

(about 3x the density of water)

9. (4%) Imagine a U-shaped tube filled with liquid mercury ($\rho = 13534\ \text{kg/m}^3$, which is about $13.5\times$ the density of water). The left end of the tube is open to atmospheric pressure ($P_{\text{atm}} = 101325\ \text{N/m}^2$), while the right end is evacuated ($P \approx 0$).



(a) How much higher is the mercury level on the right side than on the left side?

$$P_{left} = P_{right} + \rho g h$$

$$101325\,\frac{N}{m^2} = 0 + \left(13534\,\frac{kg}{m^3}\right)\left(9.8\,\frac{N}{kg}\right)h$$

$$h = \boxed{0.764\,m}$$

(b) A low-pressure system passes through, bringing with it a period of stormy weather, and decreasing the outside air pressure from 1.000 atm to 0.970 atm. Now how much higher is the mercury level on the right side than on the left side?

$$(0.970)\,101325\,\frac{N}{m^2} = \left(13534\,\frac{kg}{m^3}\right)\left(9.8\,\frac{N}{kg}\right)h$$

$$h = \boxed{0.741\,m}$$

10. (6%) Suppose that the insulating properties of the 176 m^2 roof of a house come mainly from an "R-50" layer of insulation. (R-50 means 50 $\frac{\text{foot}^2\cdot\text{hour}\cdot{}^\circ\text{F}}{\text{Btu}}$, which in metric units is 8.8 $\frac{\text{m}^2\cdot{}^\circ\text{C}}{\text{W}}$.)

(a) If the inside-outside temperature difference is 20°C, how much heat per unit time (in watts) is lost through the roof?

$$\frac{dQ}{dt} = \frac{A\,\Delta T}{R} = \frac{(176\,m^2)(20°C)}{8.8\,m^2\,°C/W} = \boxed{400\,W}$$

(b) How thick a layer of fiberglass would achieve this R-value? (Remember to use the metric R-value!) The thermal conductivity of fiberglass is $k = 0.048\,\text{J}/(\text{s}\cdot\text{m}\cdot{}^\circ\text{C})$.

$$R = \frac{\ell}{k} \Rightarrow \ell = Rk = \left(\frac{8.8\,m^2\,°C}{W}\right)\left(\frac{0.048\,W}{m\,°C}\right)$$

$$\ell = \boxed{0.422\,m} \quad (42\,cm \text{ of fiberglass})$$

(c) What is the R-value (in metric units) of a 2.0 cm (that's 0.020 m) thick layer of wood? The thermal conductivity of wood is $k = 0.10\,\text{J}/(\text{s}\cdot\text{m}\cdot{}^\circ\text{C})$.

$$R = \frac{\ell}{k} = \frac{0.020\,m}{0.10\,J/(s\,m\,°C)} = \boxed{0.2\,\frac{m^2\,°C}{W}}$$

(That would be ≈ "R-1" in US units, which is quite poor insulation, unless there is a pocket of trapped air.)

11. (10%) An ideal Carnot heat pump must deliver thermal energy to your home's under-floor heating system at a rate of 10.0 kW to keep the system's circulating hot water at 40°C. This pump uses the outside air at −10°C as its low-temperature reservoir and is powered by electricity that costs 15.0 cents per kilowatt-hour.

(a) What is the COP of an ideal Carnot heat pump operating with $T_{\text{cold}} = -10°\text{C}$ and $T_{\text{hot}} = +40°\text{C}$? (An ideal Carnot heat pump doesn't exist, but it sets an upper bound on what is possible.)

$$\text{ideal COP}_{\text{heating}} = \frac{T_{hot}}{T_{hot} - T_{cold}} = \frac{(273+40)\,K}{((273+40)-(273-10))\,K}$$

$$= \frac{313\,K}{50\,K} = \boxed{6.26}$$

(b) How much electrical power does the heat pump consume to supply the necessary 10.0 kW of thermal power?

$$\frac{dW}{dt} = \frac{dQ/dt}{COP} = \frac{10\,kW}{6.26} = \boxed{1597\,W}$$

(Problem continues on next page.)

(c) How much does the electrical power cost to heat your house for 24 hours?

$$(1597\,W)\left(\frac{\$0.15}{kW\,h}\right)\left(\frac{kW}{1000\,W}\right)(24\,h) = \boxed{\$5.75}$$

(d) What would be your answers for parts (b) and (c) if instead of the (unrealistic) Carnot ideal, we used a (more realistic) COP = 3.6 ?

$$dW/dt = \frac{dQ/dt}{COP} = \frac{10000\,W}{3.6} = \boxed{2778\,W}$$

$$(2.778\,kW)\left(\frac{\$0.15}{kW\,h}\right)(24\,h) = \boxed{\$10.00}$$

In the U.S., one often quotes the Energy Efficiency Ratio (EER) instead of the Coefficient of Performance (COP). Whereas a COP measures both Q and W in joules (or power in watts, equivalently), an EER measures Q in British Thermal Units (BTU) and W in watt-hours. Thus, a COP of 1.0 equals an EER of 3.412.

(e) What are the EER values corresponding to the COP values you used in parts (a) and (d)?

$COP = 6.26 \rightarrow EER = \boxed{21.4}$ (ideal)

$COP = 3.6 \rightarrow EER = \boxed{12.3}$ (realistic)

12. (10%) Most wire used for electric light fixtures in U.S. homes is "14 gauge" copper wire, having radius $r = 0.814\,\mathrm{mm}$ (that's $0.814 \times 10^{-3}\,\mathrm{m}$), with maximum allowed current set by a 15-amp circuit breaker in the basement.

(a) What is the resistance (using correct units: ohms) of a 25-meter length of "14 gauge" copper wire? The electrical conductivity of copper is $\sigma = 6.0 \times 10^{7}\ \mathrm{A/(V \cdot m)}$.

$$R = \frac{\ell}{\sigma A} = \frac{\ell}{\sigma \pi r^2} = \frac{(25\,\mathrm{m})}{(6\times10^7\,\mathrm{A/(Vm)})\pi(0.814\times10^{-3}\,\mathrm{m})^2}$$

$$= \boxed{0.20\,\Omega} \qquad (\Omega = \mathrm{V/A})$$

(b) What is the voltage drop (sometimes called, quite fittingly, the "IR drop") across the resistance you calculated in part (a), assuming that the electric current is 15 amps?

$$V = IR = (15\,\mathrm{A})(0.20\,\Omega) = \boxed{3.0\,\mathrm{V}}$$

(c) When a current of 15 amperes flows through the resistance you calculated in part (a), how much power is dissipated in this resistance? (This tells you the power that would be dissipated in a 25-meter-long run of 14-gauge wire at the maximum allowed current of 15 A. This power heats the wall and ceiling cavities, creating a fire hazard if the power dissipated in the wires is too large.)

$$\text{power} = I^2R = (15\,\mathrm{A})^2(0.20\,\Omega) = \boxed{45\,\mathrm{W}}$$

(d) Most wire used for household electrical outlets in U.S. homes is "12 gauge" copper wire, having radius $r = 1.026\,\mathrm{mm}$ (that's $1.026 \times 10^{-3}\,\mathrm{m}$), with maximum allowed current set by a 20-amp circuit breaker in the basement. What is the resistance of a 25-meter length of "12 gauge" copper wire?

$$R = \frac{\ell}{\sigma\pi r^2} = \frac{25\,\mathrm{m}}{(6\times10^7/(\Omega\mathrm{m}))\pi(1.026\times10^{-3}\,\mathrm{m})^2}$$

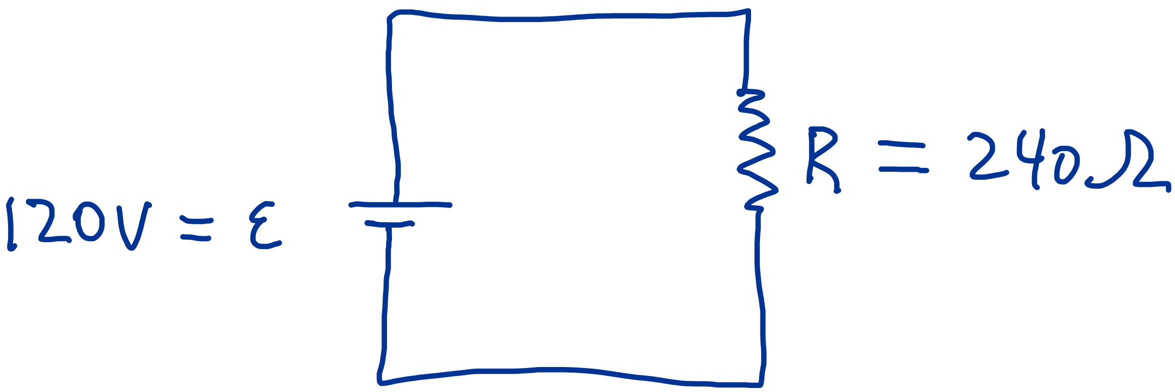
$$= \boxed{0.126\,\Omega}$$

(e) When a current of 20 amperes flows through the resistance you calculated in part (d), how much power is dissipated in this resistance?

$$\text{power} = I^2R = (20\,\mathrm{A})^2(0.126\,\Omega) = \boxed{50.4\,\mathrm{W}}$$

13. (10%) In U.S. household electrical wiring, the electric potential supplied by an ordinary electrical outlet or light fixture is 120 volts.[1]

(a) Draw a schematic diagram showing a 120 volt battery wired to a 240 Ω light bulb. (For the light bulb, you can just draw a 240 Ω resistor if you find that easier.)



(b) What current flows through the 240 Ω light bulb? (Use correct units!)

$$V = IR \Rightarrow I = \frac{V}{R} = \boxed{0.5\,A}$$

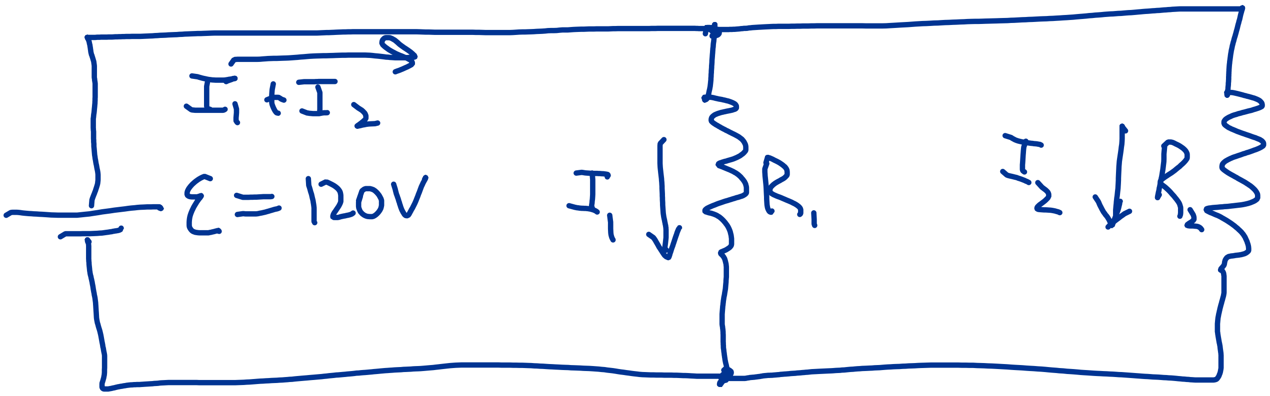
(c) How much power is dissipated by the 240 Ω light bulb? (This should be a familiar number for an old-fashioned medium-bright light bulb. Use correct units!)

$$P = I^2 R = (0.5\,A)^2 (240\,\Omega) = \boxed{60\,W}$$

(Problem continues on next page.)

[1] Household electricity uses "alternating current" (AC), which differs from the "direct current" (DC) of a battery. So "120 volts AC" actually refers to the root-mean-square value of a 60 Hz sine wave whose amplitude is $\sqrt{2} \times 120$ volts. But for most calculations, you get the right answer if you just pretend that you are working with a 120 volt battery.

(d) Draw a schematic diagram showing a single 120 volt battery wired **in parallel** to two 240 Ω light bulbs (or resistors if you prefer).



$R_1 = R_2 = 240\,\Omega$

(e) How much current flows through each (individual) light bulb? (One way to work this out would be to use the junction rule and the loop rule, but you can use whatever method you prefer.)

Voltage across each bulb is 120V.

$I = \frac{V}{R} = \boxed{0.5\text{A}}$ through each bulb.

(f) How much power is dissipated by each (individual) light bulb? (Since household electrical outlets are all wired in parallel with one another, what you find should be what you would normally expect when you plug several identical light bulbs into separate electrical outlets.)

power $= I^2 R = (0.5\text{A})^2 (240\,\Omega) = \boxed{60\text{W}}$

dissipated by each bulb

(g) ~~How much~~ current flows through the battery?

$\boxed{1.0\text{A}}$ through battery is the sum of the currents through the 2 bulbs

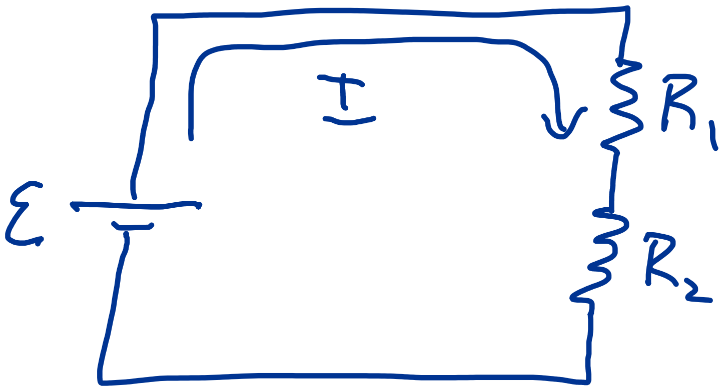
(h) How much power is supplied by the battery?

power $= (I_{\text{battery}})(\mathcal{E}_{\text{battery}}) = \boxed{120\text{W}}$

check: Same as total power dissipated by both light bulbs

(Problem continues on next page.)

(i) Draw a schematic diagram showing a single 120 volt battery wired **in series** to two 240 Ω light bulbs (or resistors). (This is **not** how household lights are wired, except in unusual cases such as strings of holiday lights.)



$R_1 = R_2 = 240\,\Omega$

$\mathcal{E} = 120\,V$

(j) How much current flows through each light bulb? (One way to work this out would be to use the loop rule, but you can use whatever method you prefer.)

$\mathcal{E} - IR_1 - IR_2 = 0 \Rightarrow I = \frac{\mathcal{E}}{2R} = \frac{120\,V}{2 \times 240\,\Omega}$

$I = \boxed{0.25\,A}$

(k) How much power is dissipated by each (individual) light bulb?

$\text{power} = I^2 R = (0.25\,A)^2 (240\,\Omega) = \boxed{15\,W}$

(l) What is the voltage drop across each (individual) light bulb?

$IR_1 = IR_2 = (0.25\,A)(240\,\Omega) = \boxed{60\,V}$

check: $IR_1 + IR_2 = \mathcal{E}$ ✓

(m) How much current flows through the battery?

Only one branch: $I = \boxed{0.25\,A}$

(n) How much power is supplied by the battery?

$\text{power} = I_{batt}\,\mathcal{E}_{batt} = (0.25\,A)(120\,V) = \boxed{30\,W}$

check: this equals total power dissipated by 2 bulbs ✓

possibly useful numbers: $9.8\ \mathrm{m/s^2}$ $343\ \mathrm{m/s^2}$ $6.022\times 10^{23}/\mathrm{mol}$ $1.602\times 10^{-19}\ \mathrm{C}$ $101325\ \mathrm{N/m^2}$

For heating,

$$\frac{Q_{\rm out}}{W} = \frac{Q_{\rm out}}{Q_{\rm out} - Q_{\rm in}} \le \frac{T_H}{T_H - T_L}$$

For cooling,

$$\frac{Q_{\rm in}}{W} = \frac{Q_{\rm in}}{Q_{\rm out} - Q_{\rm in}} \le \frac{T_L}{T_H - T_L}$$

For a heat engine,

$$\frac{W}{Q_{\rm in}} = \frac{Q_{\rm in} - Q_{\rm out}}{Q_{\rm in}} \le \frac{T_H - T_L}{T_H}$$

$$c = \lambda f$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$R = \frac{L}{\sigma A}$$

$$v_{\rm wave} = \sqrt{\frac{T}{m/L}}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad \Rightarrow \qquad d_i = \frac{f\,d_o}{d_o - f}$$

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right) \qquad\qquad I_0 = 10^{-12}\ \mathrm{W/m^2}$$

surface area of sphere $= 4\pi r^2$ area of circle $= \pi r^2$

$PV = nRT$, where $R = 8.31\ \frac{\mathrm{J}}{\mathrm{mol\cdot K}} = 0.0821\ \frac{\mathrm{L\cdot atm}}{\mathrm{mol\cdot K}}$

$1\ \mathrm{g/cm^3} = 1\ \mathrm{kg/L} = 1000\ \mathrm{kg/m^3}$ $273.15\ \mathrm{K} = 0^\circ\mathrm{C}$

In[354]:=
```
Off[Reduce::ratnz];
```

Problem 1

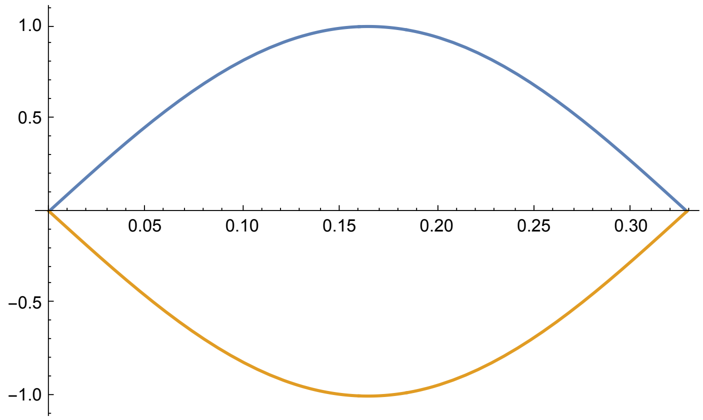
(a) The fundamental frequency is f_0 **= 293.7 Hz**. (As a check, one whole-step (6th root of 2) above middle C (262 Hz) is 294 Hz.)

(b) Higher harmonics are **587 Hz, 881 Hz, 1175 Hz, ...**

(c) Lowest harmonic and next two higher harmonics:

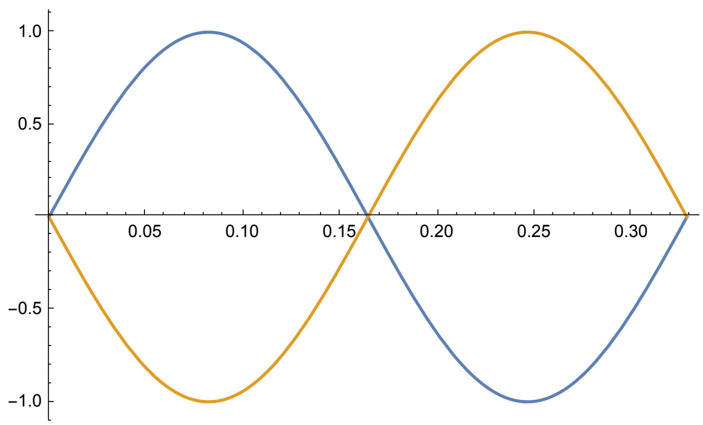
In[372]:=
```
Plot[{Sin[π x / 0.328], -Sin[π x / 0.328]}, {x, 0, 0.328}]
```

Out[372]=



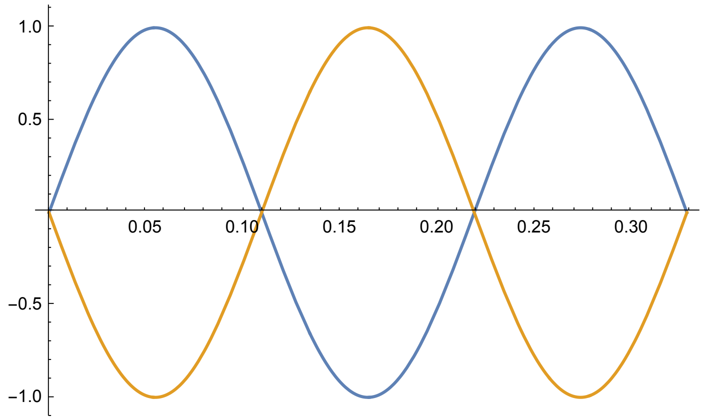
In[374]:=
```
Plot[{Sin[2 π x / 0.328], -Sin[2 π x / 0.328]}, {x, 0, 0.328}]
```

Out[374]=



In[375]:=
```
Plot[{Sin[3 π x / 0.328], -Sin[3 π x / 0.328]}, {x, 0, 0.328}]
```

Out[375]=



(d) Frequency in air is the same **294 Hz** as on the violin string.
(e) Wavelength in air is **λ = 1.17 meter.**

In[361]:=
```
ClearAll["Global`*"];
meter = Quantity["meter"];
second = Quantity["second"];
newton = Quantity["newton"];
kilogram = Quantity["kilogram"];
soln = ToRules[Reduce[{
    v_wave == Sqrt[tension / mass_per_unit_length],
    length == 0.328 meter,
    mass == 0.452 × 10^-3 kilogram,
    tension == 51.15 newton,
    mass_per_unit_length == mass / length,
    λ0 == 2 length,
    v_wave == λ0 f0
  }]]
```

Out[366]= {λ0 → 0.656 m , v_wave → 192.66 m/s ,
tension → 51.15 N , mass_per_unit_length → 0.00137805 kg/m ,
mass → 0.000452 kg , length → 0.328 m , f0 → 293.688 per second}

In[369]:=
```
wholeStep = 2^(1/6);
262.0 × wholeStep
```

Out[370]= 294.085

In[371]:=
```
{2 f0, 3 f0, 4 f0, 5 f0} /. soln
```

Out[371]= {587.377 per second , 881.065 per second , 1174.75 per second , 1468.44 per second}

In[376]:=
```
Reduce[{
  vsound == 343 meter / second,
```

```
  vsound == λ f,
  f == 293.7 / second
 }]
```

Out[376]= $\lambda == 1.16786\,\mathrm{m}$ && $vsound == 343.\,\mathrm{m/s}$ && $f == 293.7$ per second

Problem 2

(a) Intensity level is **115 dB**.
(b) Total acoustical power emitted is **31.1 watts**.
(c) Intensity level at uncle is **95 dB**. (Check: 10× as far away should be 1/100 the intensity, which should be 20dB lower than where I am sitting.)
(d) Wth ear plugs, I hear intensity level **85dB**.
(e) Ear plug transmission loss is **30dB**. Check: 115dB-30dB = 85dB.

In[394]:=
```
ClearAll["Global`*"];
meter = Quantity["meter"];
watt = Quantity["watt"];
soln = ToRules[Reduce[{
    intensityNear == 0.316 watt / meter^2,
    referenceIntensity == 10^-12 watt / meter^2,
    decibelsNear == 10.0 Log10[intensityNear / referenceIntensity],
    intensityNear == speakerPower / (4 π radiusNear^2),
    radiusNear == 2.8 meter,
    radiusFar == 28.0 meter,
    decibelsFar == 10.0 Log10[intensityFar / referenceIntensity],
    intensityFar == speakerPower / (4 π radiusFar^2),
    intensityWithEarPlugs == 10^-3 intensityNear,
    decibelsWithEarPlugs ==
     10.0 Log10[intensityWithEarPlugs / referenceIntensity]
   }]]
```

Out[397]= {speakerPower → $31.1324\,\mathrm{W}$, referenceIntensity → $1.\times10^{-12}\,\mathrm{kg/s^3}$,
radiusNear → $2.8\,\mathrm{m}$, radiusFar → $28.\,\mathrm{m}$, intensityWithEarPlugs → $0.000316\,\mathrm{kg/s^3}$,
intensityNear → $0.316\,\mathrm{kg/s^3}$, intensityFar → $0.00316\,\mathrm{kg/s^3}$,
decibelsWithEarPlugs → 84.9969, decibelsNear → 114.997, decibelsFar → 94.9969}

Problem 3

(a) Draw rays on exam PDF.
(b) Draw rays on exam PDF.
(c) At region I/II boundary, reflected ray has same angle (**30°**) as incident ray, and refracted ray has angle **19.5°** w.r.t. surface normal. At region II/III boundary, reflected ray has same angle (19.5°) as incident ray, and refracted ray has angle 30° w.r.t. surface normal.

(d) Light travels faster in air than in glass. So glass is the slower medium. So it makes sense that at the I/II boundary the refracted ray bends "toward the glass" (the slower medium), i.e. it bends away from the interface, and at the II/III boundary the refracted ray again bends "toward the glass" (the slower medium), i.e. it bends back toward the interface.

In[398]:=
```
ClearAll["Global`*"];
soln = ToRules[Reduce[{
    nair Sin[θair Degree] == nglass Sin[θglass Degree],
    nair == 1.0,
    nglass == 1.50,
    θair == 30.0,
    0 < θglass < 90
   }, {θglass}]]
```

Out[399]=
```
{nair → 1., nglass → 1.5, θair → 30., θglass → 19.4712}
```

Problem 4

(a) Draw rays on exam PDF.
(b) The image is **real**, as di>0.
(c) The image is **inverted**, as hi<0.
(d) The image is **reduced**, as Abs[hi/ho] < 1.
(e) Image position is **+9.0 cm** (to the right of the lens), magnification is **-1/2**, and image height is **-2.0cm**.

In[400]:=
```
ClearAll["Global`*"];
soln = Reduce[{
   1 / f == 1 / do + 1 / di,
   hi / ho == -di / do,
   f == +6.0,
   do == +18.0,
   ho == +4.0
  }]
```

Out[401]=
```
ho == 4. && hi == -2. && f == 6. && do == 18. && di == 9.
```

Problem 5

(a) At house, pressure is **493 kPa**.
(b) This is **4.87 atm**.

In[407]:=
```
ClearAll["Global`*"];
meter = Quantity["meter"];
kilogram = Quantity["kilogram"];
newton = Quantity["newton"];
soln = ToRules[Reduce[{
    towerHeight == 40.0 meter,
    towerPressure == 1.0 atm,
    atm == 101325 newton / meter^2,
```

```
    towerPressure + ρ g towerHeight == housePressure,
    ρ == 1000.0 kilogram / meter^3,
    g == 9.8 newton / kilogram,
    houseAtm == housePressure / atm
   }]]
```

Out[411]= {ρ → 1000. kg/m^3, towerPressure → 101 325. Pa, towerHeight → 40. m, housePressure → 493 325. Pa, g → 9.8 m/s^2, atm → 101 325. Pa, houseAtm → 4.86874}

Problem 6

(a) Weight of car is **10kN.**
(b) Force I must exert on smaller piston is **100N**.

```
In[412]:= ClearAll["Global`*"];
kilogram = Quantity[1.0, "kilogram"];
newton = Quantity[1.0, "newton"];
meter = Quantity[1.0, "meter"];
soln = ToRules[Reduce[{
    g == 9.8 newton / kilogram,
    mass == 1020.0 kilogram,
    fLargePiston == mass * g,
    fLargePiston / aLargePiston == fSmallPiston / aSmallPiston,
    aLargePiston == π (35.0 × 10^-3 meter)^2,
    aSmallPiston == π (3.5 × 10^-3 meter)^2
   }]]
```

Out[416]= {mass → 1020. kg, g → 9.8 m/s^2, fSmallPiston → 99.96 N, fLargePiston → 9996. N, aSmallPiston → 0.0000384845 m^2, aLargePiston → 0.00384845 m^2}

Problem 7

(a) Using continuity equation, flow speed in narrow part is **6.56 m/s.**
(b) Pressure difference between wide and narrow parts is **20172 Pascals**.
(c) This difference equals **0.20 atmospheres**.
(d) The **wide** part is under higher pressure.

```
In[433]:= ClearAll["Global`*"];
pascal = Quantity[1.0, "pascal"];
atm = 101 325 pascal;
meter = Quantity[1.0, "meter"];
second = Quantity[1.0, "second"];
kilogram = Quantity[1.0, "kilogram"];
```

```
mm = 10^-3 meter;
soln = ToRules[Reduce[{
    awide == π rwide^2,
    anarrow == π rnarrow^2,
    rwide == 25.4 mm,
    rnarrow == 12.7 mm,
    vwide == 1.64 meter / second,
    awide × vwide == anarrow × vnarrow,
    ρwater == 1000 kilogram / meter^3,
    pwide + (1 / 2) ρwater vwide ^ 2 == pnarrow + (1 / 2) ρwater vnarrow ^ 2,
    pdiff == pwide - pnarrow,
    pdiffatm == pdiff / atm
   }]]
```

Out[440]= {ρwater → 1000. kg/m^3 , vwide → 1.64 m/s , vnarrow → 6.56 m/s , rwide → 0.0254 m , rnarrow → 0.0127 m , pnarrow → pwide + -20 172. Pa , pdiff → 20 172. Pa , awide → 0.00202683 m^2 , anarrow → 0.000506707 m^2 , pdiffatm → 0.199082}

Problem 8

(a) Draw FBD on exam PDF. Buoyant force points upward. Rock's weight points downward. Magnitude of upward force exerted by diver is magnitude of rock's weight **minus** magnitude of buoyant force.
(b) The density of the rock is **2994 kg / m^3**, or about 3.0× the density of fresh water.

In[451]:=
```
ClearAll["Global`*"];
newton = Quantity[1.0, "newton"];
meter = Quantity[1.0, "meter"];
kilogram = Quantity[1.0, "kilogram"];
soln = ToRules[Reduce[{
    fbuoyant == waterDensity × rockVolume × g,
    g == 9.8 newton / kilogram,
    waterDensity == 1000.0 kilogram / meter^3,
    rockMass == rockDensity × rockVolume,
    rockMass == 9.28 kilogram,
    submergedApparentMass == 6.18 kilogram,
    submergedApparentMass × g == rockMass × g - fbuoyant
   }]]
```

Out[455]= {waterDensity → 1000. kg/m^3 , submergedApparentMass → 6.18 kg , rockVolume → 0.0031 m^3 , rockMass → 9.28 kg , rockDensity → 2993.55 kg/m^3 , g → 9.8 m/s^2 , fbuoyant → 30.38 kg m/s^2 }

Problem 9

(a) The mercury level on the right side is **0.764 meter** higher than on the left side.
(b) As the storm passes through, mercury level is **0.741 meter** (741 mm).

In[468]:=
```
ClearAll["Global`*"];
newton = Quantity[1.0, "newton"];
meter = Quantity[1.0, "meter"];
atm = 101325 newton / meter^2;
kilogram = Quantity[1.0, "kilogram"];
soln = ToRules[Reduce[{
    pRight + ρ g h == pLeft,
    pRight == 0,
    pLeft == 1.0 atm,
    ρ == 13534 kilogram / meter^3,
    g == 9.8 newton / kilogram,
    pRight + ρ g hstorm == 0.970 atm
   }]]
```

Out[473]=
```
{ρ → 13534. kg/m^3 , pRight → 0 Pa , pLeft → 101325. Pa ,
 hstorm → 0.741031 m , h → 0.763949 m , g → 9.8 m/s^2 }
```

Problem 10

(a) Heat per unit time (thermal power) lost through roof is **400 watts**.
(b) Necessary thickness of fiberglass is **0.422 meter** (42 cm).
(c) Metric R-value of 2.0cm thick wood is **0.2 m^2°C/W** which turns out to be R 1.1 in US customary units, so a single layer of wood (with no trapped layer of air) is a very poor insulator.

In[495]:=
```
ClearAll["Global`*"];
second = Quantity[1.0, "second"];
meter = Quantity[1.0, "meter"];
cm = 0.01 meter;
celsius = Quantity[1.0, "celsius"];
watt = Quantity[1.0, "watt"];
soln = ToRules[Reduce[{
    thermalPower == area × ΔT / rvalue,
    area == 176 meter^2,
    rvalue == 8.8 meter^2 × celsius / watt,
    rvalue == thickness / thermalConductivity,
    thermalConductivity == 0.048 watt / (meter × celsius),
    ΔT == 20 celsius,
    woodThermalConductivity == 0.10 watt / (meter × celsius),
```

```
    woodThickness == 2.0 cm,
    woodRvalue == woodThickness / woodThermalConductivity
   }]]
```

Out[501]= {woodThickness → 0.02 m , woodThermalConductivity → 0.1 kg m/(s³ °C) ,
woodRvalue → 0.2 s³ °C/kg , ΔT → 20. °C , thickness → 0.4224 m , thermalPower → 400. W ,
thermalConductivity → 0.048 kg m/(s³ °C) , rvalue → 8.8 s³ °C/kg , area → 176. m² }

Problem 11

(a) Carnot heat pump ideal COP is **6.26.**
(b) Needed electrical power is **1597 watts**.
(c) Cost to heat house for 24 hours is **$5.75**.
(d) Using COP=3.6, needed electric power is **2778 watts**, and 24-hour cost is **$10.00**.
(e) The EER values are **21.4 (ideal) and 12.3 (realistic)**.

```
In[524]:= ClearAll["Global`*"];
watt = Quantity[1.0, "watt"];
second = Quantity[1.0, "second"];
kelvin = Quantity[1.0, "kelvin"];
dollar = Quantity[1.0, "dollar"];
hour = 3600 second;
cent = 0.01 dollar;
kWh = 1000 watt × hour;
soln = ToRules[Reduce[{
    tHot == (+40 + 273) kelvin,
    tCold == (-10 + 273) kelvin,
    copIdealHeating == tHot / (tHot - tCold),
    thermalPower == 10 000 watt,
    electricalPower == thermalPower / copIdealHeating,
    unitCost == 15 cent / kWh,
    electricCost == electricalPower × (24 hour) × unitCost,
    copRealistic == 3.6,
    elecPowerRealistic == thermalPower / copRealistic,
    elecCostRealistic == elecPowerRealistic × (24 hour) × unitCost
   }]]
```

Out[532]= {unitCost → $4.16667 × 10⁻⁸ s²/(kg m²) , tHot → 313. K ,
thermalPower → 10 000. W , tCold → 263. K , electricCost → $5.75 ,
electricalPower → 1597.44 W , elecPowerRealistic → 2777.78 W ,
elecCostRealistic → $10.00 , copRealistic → 3.6, copIdealHeating → 6.26}

In[533]:=
```
copIdealHeating × 3.412 /. soln
```
Out[533]=
```
21.3591
```

In[534]:=
```
copRealistic × 3.412 /. soln
```
Out[534]=
```
12.2832
```

Problem 12

(a) A 25m length of 14-gauge copper wire has resistance **0.20 ohm**.
(b) The voltage drop after 15 amps running through this wire is **3.0 volt**.
(c) Dissipated power in walls is **45.0 watts**.
(d) Resistance of 25m length of 12-gauge copper wire is **0.126 ohm**.
(e) The power dissipated in the 25m length of 12-gauge wire at 20 amps is 5**0.4 watts.**

In[607]:=
```
ClearAll["Global`*"];
amp = Quantity[1.0, "ampere"];
volt = Quantity[1.0, "volt"];
meter = Quantity[1.0, "meter"];
mm = 0.001 meter;
soln = ToRules[Reduce[{
    σcopper == 6.0 × 10^7 amp / (volt × meter),
    length == 25.0 meter,
    R14gauge == length / (σcopper × area14gauge),
    area14gauge == π radius14gauge^2,
    radius14gauge == 0.814 mm,
    irDrop14gauge == (15 amp) × R14gauge,
    power14gauge == (15 amp)^2 × R14gauge,
    R12gauge == length / (σcopper × area12gauge),
    area12gauge == π radius12gauge^2,
    radius12gauge == 1.026 mm,
    irDrop12gauge == (20 amp) × R12gauge,
    power12gauge == (20 amp)^2 × R12gauge
   }]]
```

Out[612]=
$\{\sigma\text{copper} \to 6. \times 10^7 \text{ per meter per ohm}, \text{radius14gauge} \to 0.000814\text{ m}, \text{radius12gauge} \to 0.001026\text{ m}, \text{R14gauge} \to 0.200166\text{ W/A}^2, \text{R12gauge} \to 0.125992\text{ W/A}^2, \text{power14gauge} \to 45.0373\text{ W}, \text{power12gauge} \to 50.3969\text{ W}, \text{length} \to 25.\text{ m}, \text{irDrop14gauge} \to 3.00249\text{ V}, \text{irDrop12gauge} \to 2.51985\text{ V}, \text{area14gauge} \to 2.08161 \times 10^{-6}\text{ m}^2, \text{area12gauge} \to 3.30708 \times 10^{-6}\text{ m}^2\}$

Problem 13

(a) Draw schematic diagram on exam PDF.
(b) 0.5 amp
(c) 60 watts
(d) Draw schematic diagram on exam PDF.
(e) 0.5 amp
(f) 60 watts
(g) 1.0 amp
(h) 120 watts
(i) Draw schematic diagram on exam PDF.
(j) 0.25 amp
(k) 15 watts
(l) 60 volts
(m) 0.25 amp
(n) 30 watts

In[620]:=
```
ClearAll["Global`*"];
volt = Quantity[1.0, "volt"];
amp = Quantity[1.0, "ampere"];
ohm = Quantity[1.0, "ohm"];
v = 120 volt;
R = 240 ohm;
i = v / R
```

Out[626]= 0.5 V/Ω

In[627]:=
```
power = i^2 R
```

Out[627]= 60. V^2/Ω

In[628]:=
```
iseries = v / (R + R)
```

Out[628]= 0.25 V/Ω

In[629]:=
```
powerOneBulbSeries = iseries^2 R
```

Out[629]= 15. V^2/Ω